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Editors-in-Chief

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Editors-in-Chief:

David Gross (University of California Santa Barbara)

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Contents

Volume 1

Epilogue from the President

SHING-TUNG YAU 3

MATHEMATICS

Preface

CAUCHER BIRKAR 7

Basic Science Lifetime Award

BASIC SCIENCE LIFETIME AWARD SELECTION COMMITTEE 13

Basic Science Lecture: Consciousness, robots, and DNA

DAVID MUMFORD 15

Frontiers of Science Awards

Combinatorics, Discrete Geometry, and Graph Theory

Improved bounds for the sunflower lemma

RYAN ALWEISS, SHACHAR LOVETT, KEWEN WU AND
JIAPENG ZHANG 29

Induced subgraphs of hypercubes and a proof of the Sensitivity Conjecture

HAO HUANG 35

Hodge theory for combinatorial geometries

KARIM ADIPRASITO, JUNE HUH AND ERIC KATZ 40

The Fuglede conjecture for convex domains is true in all dimensions

NIR LEV AND MATE MATOLCSI 43

Log-concave polynomials, I: Entropy and a deterministic approximation algorithm for counting bases of matroids

NIMA ANARI, SHAYAN OVEIS GHARAN AND CYNTHIA VINZANT 49

*Mathematical Logic, Foundations and Category Theory***Martin's Maximum⁺⁺ implies Woodin's axiom (*)**

DAVID ASPERO AND RALF SCHINDLER 59

Ax-Schanuel for Shimura varieties

NGAIMING MOK, JONATHAN PILA AND JACOB TSIMERMAN 65

*Number Theory***Courbes et fibres vectoriels en theorie de Hodge p -adique**

LAURENT FARGUES AND JEAN-MARC FONTAINE 77

On the Gross-Stark conjecture

SAMIT DASGUPTA, MAHESH KAKDE AND KEVIN VENTULLO 91

On the Duffin-Schaeffer conjecture

DIMITRIS KOUKOULOPOULOS AND JAMES MAYNARD 100

*Commutative Algebra and Algebraic Geometry***Singularities of linear systems and boundedness of Fano varieties**

CAUCHER BIRKAR 107

Canonical bases for cluster algebrasMARK GROSS, PAUL HACKING, SEAN KEEL AND
MAXIM KONTSEVICH 115**Specialization of birational types**

MAXIM KONTSEVICH AND YURI TSCHINKEL 120

Finite generation for valuations computing stability thresholds and applications to K-stability

YUCHEN LIU, CHENYANG XU AND ZIQUAN ZHUANG 124

*Homological Algebra, K-Theory and Noncommutative Algebra***Monoidal categorification and quantum affine algebras**MASAKI KASHIWARA, MYUNGHO KIM, SE-JIN OH AND
EUIYONG PARK 133**On topological cyclic homology**

THOMAS NIKOLAUS AND PETER SCHOLZE 139

Coulomb branches of 3d $N = 4$ quiver gauge theories and slices in the affine GrassmannianALEXANDER BRAVERMAN, MICHAEL FINKELBERG AND
HIRAKU NAKAJIMA 145

Cohomological Donaldson-Thomas theory of a quiver with potential and quantum enveloping algebras	
BEN DAVISON AND SVEN MEINHARDT	152
<i>One and Several Variables, Complex Dynamical System</i>	
On the constant scalar curvature Kahler metrics (II) – Existence results	
XIUXIONG CHEN AND JINGRUI CHENG	161
Large genus asymptotics for volumes of strata of Abelian differentials	
AMOL AGGARWAL	167
Invariant metrics on negatively pinched complete Kahler manifolds	
DAMIN WU AND SHING-TUNG YAU	175
Extending holomorphic forms from the regular locus of a complex space to a resolution of singularities	
STEFAN KEBEKUS AND CHRISTIAN SCHNELL	182
A variational approach to the Yau-Tian-Donaldson conjecture	
ROBERT J. BERMAN, SÉBASTIEN BOUCKSOM AND MATTIAS JONSSON	191
<i>Ordinary Differential Equations and Special Functions, D Module</i>	
Torsion points, Pell’s equation, and integration in elementary terms	
DAVID MASSER AND UMBERTO ZANNIER	203
A non-hypergeometric E-function	
JAVIER FRESÁN AND PETER JOSSEN	211
Iterated integrals on $\mathbb{P}^1 \setminus \{0, 1, \infty, z\}$ and a class of relations among multiple zeta values	
MINORU HIROSE AND NOBUO SATO	218
On the local Birkhoff conjecture for convex billiards	
VADIM KALOSHIN AND ALFONSO SORRENTINO	224
<i>Partial Differential Equations</i>	
A proof of Onsager’s conjecture	
PHILIP ISETT	235
Expanding large global solutions of the equations of compressible fluid mechanics	
MAHIR HADZIC AND JUHI JANG	241

On the implosion of a compressible fluid II: Singularity formation	
FRANK MERLE, PIERRE RAPHAEL, IGOR RODNIANSKI AND JEREMIE SZEFTTEL	248
A proof of the instability of AdS for the Einstein-massless Vlasov system	
GEORGIOS MOSCHIDIS	257
KdV is well-posed in H^{-1}	
ROWAN KILLIP AND MONICA VIŞAN	270
<i>Dynamics Systems, Ergodic Theory and Diophantine Approximation</i>	
Invariant and stationary measures for the $SL(2, \mathbb{R})$ action on moduli space	
ALEX ESKIN AND MARYAM MIRZAKHANI	279
The logarithmic Sarnak conjecture for ergodic weights	
NIKOS FRANTZIKINAKIS AND BERNARD HOST	286
Zimmer's conjecture: Subexponential growth, measure rigidity, and strong property (T)	
AARON BROWN, DAVID FISHER AND SEBASTIAN HURTADO	293
Measures of maximal entropy for surface diffeomorphisms	
JÉRÔME BUZZI, SYLVAIN CROVISIER AND OMRI SARIG	297
The algebraic hull of the Kontsevich-Zorich cocycle	
ALEX ESKIN, SIMION FILIP AND ALEX WRIGHT	303
<i>Fourier Analysis and Harmonic Analysis</i>	
Sharp L^2 estimates of Schrödinger maximal function in higher dimensions	
XIUMIN DU AND RUIXIANG ZHANG	311
A sharp square function estimate for the cone in $\mathbb{R}^3$	
LARRY GUTH, HONG WANG AND RUIXIANG ZHANG	317
Sharp estimates for oscillatory integral operators via polynomial partitioning	
LARRY GUTH, JONATHAN HICKMAN AND MARINA ILIOPOULOU	322
One can hear the shape of ellipses of small eccentricity	
HAMID HEZARI AND STEVE ZELDITCH	328
Logarithmic improvements in L^p bounds for eigenfunctions at the critical exponent in the presence of nonpositive curvature	
MATTHEW D. BLAIR AND CHRISTOPHER D. SOGGE	333

*Functional Analysis and Operator Theory***A classification of finite simple amenable $\mathcal{Z}$ -stable C^* -algebras, I and II**

GUIHUA GONG, HUAXIN LIN AND ZHUANG NIU 343

Complete descriptions of intermediate operator algebras by intermediate extensions of dynamical systems

YUHEI SUZUKI 370

Universal hierarchical structure of quasiperiodic eigenfunctions

SVETLANA JITOMIRSKAYA AND WENCAI LIU 376

*Calculus of Variations and Optimal Control***Stable solutions to semilinear elliptic equations are smooth up to dimension 9**

XAVIER CABRÉ, ALESSIO FIGALLI, XAVIER ROS-OTON AND JOAQUIM SERRA 383

Generic regularity of free boundaries for the obstacle problem

ALESSIO FIGALLI, XAVIER ROS-OTON AND JOAQUIM SERRA 389

Global regularity for the Monge-Ampere equation with natural boundary condition

SHIBING CHEN, JIAKUN LIU AND XU-JIA WANG 394

The sharp upper bound for the area of the nodal sets of Dirichlet Laplace eigenfunctions

ALEXANDER LOGUNOV, EUGENIA MALINNIKOVA, NIKOLAI NADIRASHVILI AND FEDOR NAZAROV 399

*General Relativity***The global non-linear stability of the Kerr-de Sitter family of black holes**

PETER HINTZ AND ANDRAS VASY 407

Uniform energy bound and Morawetz estimate for extreme components of spin fields in the exterior of a slowly rotating Kerr black hole II: linearized gravity

SIYUAN MA 424

The C^0 -inextendibility of the Schwarzschild spacetime and the space-like diameter in Lorentzian geometry

JAN SBIERSKI 429

Supertranslation invariance of angular momentum PO-NING CHEN, MU-TAO WANG, YE-KAI WANG AND SHING-TUNG YAU	436
The linear stability of Reissner-Nordström spacetime: the full subextremal range ELENA GIORGI	446
<i>Geometric Analysis</i>	
Nodal sets of Laplace eigenfunctions: proof of Nadirashvili's conjecture and of the lower bound in Yau's conjecture ALEXANDER LOGUNOV	457
Local and global boundary rigidity and the geodesic X-ray transform in the normal gauge PLAMEN STEFANOV, GUNTHER UHLMANN AND ANDRÁS VASY	463
Ancient low-entropy flows mean-convex neighborhoods, and uniqueness KYEONGSU CHOI, ROBERT HASLHOFER AND OR HERSHKOVITS	472
<i>Algebraic and Geometric Topology</i>	
On property (T) for $\text{Aut}(F_n)$ and $\text{SL}_n(\mathbb{Z})$ MAREK KALUBA, DAWID KIELAK AND PIOTR W. NOWAK	481
Hyperbolic 5-manifolds that fiber over S^1 GIOVANNI ITALIANO, BRUNO MARTELLI AND MATTEO MIGLIORINI	490
The Chow t-structure on the ∞-category of motivic spectra TOM BACHMANN, JIA, GUOZHEN WANG AND ZHOULI XU	495
A closed formula for the evaluation of foams LOUIS-HADRIEN ROBERT AND EMMANUEL WAGNER	501
A homological approach to pseudoisotopy theory. I MANUEL KRANNICH	507
<i>Symplectic and Differential Topology</i>	
Simple homotopy equivalence of nearby Lagrangians MOHAMMED ABOUZOID AND THOMAS KRAGH	519
A complete knot invariant from contact homology TOBIAS EKHOLM, LENHARD NG AND VIVEK SHENDE	523
The 4-dimensional light bulb theorem DAVID GABAI	528
Bordered Heegaard Floer homology ROBERT LIPSHITZ, PETER S. OZSVÁTH AND DYLAN P. THURSTON	535

The Conway knot is not slice LISA PICCIRILLO	556
<i>Probability Theory and Stochastic Processes</i>	
Proof of the satisfiability conjecture for large k JIAN DING, ALLAN SLY AND NIKE SUN	563
Liouville quantum gravity and the Brownian map I: the $QLE(8/3, 0)$ metric JASON MILLER AND SCOTT SHEFFIELD	570
Sharp phase transition for the random-cluster and Potts models via decision trees HUGO DUMINIL-COPIN, ARAN RAOUFI AND VINCENT TASSION	579
Tightness of Liouville first passage percolation for $\gamma \in (0, 2)$ JIAN DING, JULIEN DUBÉDAT, ALEXANDER DUNLAP AND HUGO FALCONET	584
<i>Statistics</i>	
Estimation and inference of heterogeneous treatment effects using random forests SUSAN ATHEY AND STEFAN WAGER	593
On deep learning as a remedy for the curse of dimensionality in nonparametric regression BENEDIKT BAUER AND MICHAEL KOHLER	597
Covariate-assisted ranking and screening for large-scale two-sample inference T. TONY CAI, WENGUANG SUN AND WEINAN WANG	606
<i>Numerical Analysis and Scientific Computation</i>	
Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations MAZIAR RAISSI, PARIS PERDIKARIS AND GEORGE EM KARNIADAKIS	615
The AAA algorithm for rational approximation YUJI NAKATSUKASA, OLIVIER SÈTE AND LLOYD N. TREFETHEN	624
<i>Differential Geometry</i>	
Uniqueness and stability of Ricci flow through singularities RICHARD H. BAMLER AND BRUCE KLEINER	635

Counting minimal surfaces in negatively curved 3-manifolds DANNY CALEGARI, FERNANDO CODÁ MARQUES AND ANDRÉ NEVES	638
On the multiplicity one conjecture in min-max theory XIN ZHOU	643
<i>Lie Theory and Representation Theory</i>	
Quantum groups and quantum cohomology DAVESH MAULIK AND ANDREI OKOUNKOV	651
Smith-Treumann theory and the linkage principle SIMON RICHE AND GEORDIE WILLIAMSON	656
Comptage des systèmes locaux ℓ-adiques sur une courbe HONGJIE YU	662
<i>Mathematics of String Theory and Condensed Matter</i>	
Towards a mathematical definition of Coulomb branches of 3-dimensional $N = 4$ gauge theories, II ALEXANDER BRAVERMAN, MICHAEL FINKELBERG AND HIRAKU NAKAJIMA	669
Wick rotation and the positivity of energy in quantum field theory MAXIM KONTSEVICH AND GRAEME SEGAL	674
Contact homology and virtual fundamental cycles JOHN PARDON	679
FRONTIERS OF SCIENCE AWARD GLOBAL COMMITTEES	699
FRONTIERS OF SCIENCE AWARD LOCAL COMMITTEES	701

Volume 1

Epilogue from the President

The inaugural International Congress of Basic Science (ICBS) opened in Beijing on July 16, 2023. *Advancing Science for Humanity* would become the founding principle of this historic conference. It was the first ever international scientific gathering of its kind, wholly conceptualized and hosted in China. The conference brought together leading scientists from around the world as part of a broader effort to strongly support and promote basic research in science worldwide. This Congress focused on three major subjects: Mathematics, Theoretical Physics, and Theoretical Computer and Information Sciences.

By all accounts, the 2023 ICBS was a spectacular success, thanks to the support and presence of some 1000 scientists from more than 40 countries who gathered in Beijing to celebrate and honor achievements of the world's scientific community. The ICBS opening and award ceremonies were held at the prestigious Great Hall of the People in Beijing on July 16. Subsequent events and lectures took place at the Yanqi Lake Beijing Institute of Mathematical Sciences and Applications (BIMSA) in Huairou Science City, about an hour from Beijing City center.

A highlight of the conference was the inauguration, on July 16, of two major science awards: the Basic Science Lifetime Award (BSLA), and the Frontiers of Science Award (FSA). A Lifetime Award is conferred to an individual for contributions, widely considered to have had lasting impact in a field of research. The Frontiers of Science Award are given to recent papers, recognized for major breakthroughs in its field. David Mumford was the 2023 Basic Science Lifetime Award recipient in Mathematics. Adi Shamir received this prestigious award in Theoretical Computer and Information Sciences. One hundred thirty-one recipients were named for the Frontiers of Science Award. In all, the FSA winning papers included more than 500 authors, who were leading researchers in their respective fields.

The scientific program of the conference included some 400 lectures, delivered by a broad spectrum of researchers, including world renowned scholars from three major subjects of basic science. Speakers included 8 Fields medalists, 4 Turing Prize winners, and one Nobel laureate. The two Lifetime Award recipients delivered their Basic Science Lectures on day one of the program, while authors of the Frontiers of Science winning papers gave

their distinguished FSA Lectures during the two-week conference. Plenary and invited speakers of the program also included numerous academicians from China, the United States, and European nations. Other participants of the conference were some 600 scholars from more than 40 countries. All in all, the conference program lasted 13 days and featured hundreds of lectures, satellite conferences, and many other engaging academic events.

To me, the 2023 ICBS was a scientific gathering of historic proportions. The present inaugural ICBS conference proceedings, which collect papers on some of the most important recent breakthrough results in a broad range current topics, could be also viewed as a historic document. I am very grateful to my fellow mathematicians, information scientists, and physicists, who have so meticulously prepared their papers in a timely fashion, and contribute them to this important document.

I hope that this inaugural ICBS has been a catalyst for international scientific cooperation. I believe that we have witnessed a new chapter in intellectual pursuit of the highest order. I am also hopeful that generations from now, future scientists would look back to this day and remember that our collective efforts in this pursuit had been a noble endeavor!

Sincerely

Shing-Tung Yau

President

2023 International Congress of Basic Science

MATHEMATICS

Preface

The first International Congress of Basic Science (ICBS) was held in Beijing during 16–28 July 2023 covering Mathematics, Physics, and Theoretical Computer Science. A very large number of people attended the event and it was a very successful event by many accounts. Most of the ICBS activities occurred on the newly opened campus of the Beijing Institute for Mathematical Sciences and Applications (BIMSA). I will highlight a few aspects of the mathematics sections.

The Basic Science Lifetime Award. One of the highlights of the ICBS was the anticipated awarding of the Basic Science Lifetime Award at the opening ceremony on 16 July. This award is given for outstanding and innovative works that have, over the course of the past thirty years or more, brought about fundamental change in their discipline.

This year’s mathematics award was given to David B. Mumford for his “fundamental contributions to algebraic geometry”. Mumford is Professor at Harvard University and Brown University, a 1974 Fields Medallist, and a former President of the International Mathematical Union. The winner was selected among a list of nominated mathematicians by a committee of top mathematicians.

Among the greatest contributions of Mumford to mathematics is his work in algebraic geometry, particularly, moduli theory. Moduli spaces are parameter spaces for families of geometric objects such as algebraic varieties or sheaves on varieties. On the one hand, these spaces help to classify the objects at hand. On the other hand, they carry algebraic geometric structures of their own and provide an important source of highly non-trivial examples in algebraic geometry.

Moduli theory has a long history going back at least to the 19th century when it was treated by Riemann and others. But it was given a new meaning in the Grothendieck school of algebraic geometry at about the same time when Mumford started working on the theory. In this new approach to moduli, the emphasis was not only on constructing a space that parametrises a given class of objects but also that the moduli space should give information about families of such objects.

Mumford elevated the theory to a new level via the introduction of the concept of stability and the development of Geometric Invariant Theory which is a general approach to construction of moduli spaces. He successfully constructed moduli spaces of curves and abelian varieties and paved the way for the development of the theory for other classes of varieties. Subsequently he proved important results on the structure of moduli spaces. Mumford also constructed moduli spaces of abelian varieties in a different way using theta functions. This in particular allowed him to describe explicit equations for these moduli spaces. The Geometric Invariant Theory has also been successfully applied to the moduli theory of sheaves on algebraic varieties.

Mumford's work on moduli theory has had a strong influence on many areas of mathematics including higher dimensional algebraic geometry, arithmetic geometry, differential geometry, etc, and it has played an important role in mathematical physics.

He proved many other important results in mathematics, for example, on abelian varieties, on the geometry of algebraic surfaces in positive characteristic, on singularities, on toroidal embeddings, etc. In the 80s, Mumford shifted focus from algebraic geometry to theoretical foundations of intelligence, both natural and artificial. He has done important work in this direction.

The Frontiers of Science Awards. Another highlight of the ICBS was the Frontiers of Science Awards distributed at the opening ceremony. The aim of these awards is to emphasize achievements from the past five years which are of outstanding scholarly value. For the 2023 selection, mathematics papers were chosen in 22 different areas of mathematics covering the spectrum of mathematical sciences. The main criteria for selection of papers were: (1) it must have been published in the last 5 years; (2) it must be of highest scientific value and originality and have made an important impact on its area; (3) it must have been evaluated and accepted by scholars in its area.

The papers were chosen in a rigorous process. A selection committee of experts was formed for each area with the task of selecting the 10 best papers published in the area in the past five years according to the criteria above. In the next stage a large global committee of experts was formed with the task of selecting a maximum of 5 papers in each area. For each work, the award was given to all the authors of that work. There was no restriction on the nationality, race, gender, or workplace of the authors.

In some areas like algebraic geometry the competition was clearly intense as there were many top works. While in some other areas less than 5 papers were selected. This will hopefully be reflected in future ICBS.

Focusing on the best work done in recent years meant that the majority of the participants of the ICBS were highly active scientists. A consequence of this is more mathematical discussions among the participants.

Lectures. One of the conditions for the awards mentioned above was that at least one author of each winning work should attend the ICBS and

give a presentation related to that work. Not surprising then the ICBS showcased a large number of top mathematical works by mathematicians from around the world. The only other mathematical event of comparable scale and quality that I can think of is the International Congress of Mathematicians (ICM). There were many fantastic talks that I personally attended. It was great to see many top mathematicians in the lecture rooms as well as many brilliant students.

There were also many plenary lectures not related to the awarded works. Some of the lecturers were among those who served on one of the award committees.

Lectures were also broadcast live on the the internet, hence many more people could participate, in particular, students who did not get the chance to attend the ICBS. Hopefully more students can participate in the future.

Satellite conferences. Numerous satellite conferences were organised around the ICBS in Beijing and elsewhere. This included conferences on Algebraic and Arithmetic Geometry, Quantum Theory and Operator Theory, Number Theory, Representation Theory and Integrable Systems, Quantum Gravity and Quantum Field Theory, Geometric Analysis, Geometry and Statistics, Data Science and Algebraic Geometry, AliCPT and Primordial Gravitational Waves, an Economic Modernization Forum, and a Pujiang Innovation Forum.

Other activities. Several other mathematical activities were organised in and around the ICBS. This included a special evening devoted to mathematics in which few mathematicians discussed current affairs of mathematical sciences and the joys and challenges of being a mathematician. Special events were also organised at Tsinghua University for students and mathematics enthusiasts. On the other hand, students demonstrated their mathematical work through poster presentations.

Caucher Birkar
(YMSC, Tsinghua University)

Basic Science Lifetime Award

BASIC SCIENCE LIFETIME AWARD SELECTION COMMITTEE

Shing-Tung Yau	Fields Medal 1982 Crafoord Prize 1994 Wolf Prize 2010 Shaw Prize 2023	Tsinghua University
Simon Donaldson	Fields Medal 1986 Crafoord Prize 1994 Shaw Prize 2009 Wolf Prize 2020	Imperial College London
Maxim Kontsevich	Fields Medal 1998 Crafoord Prize 2008 Shaw Prize 2012	IHÉS
Caucher Birkar	Fields Medal 2018	Tsinghua University
Richard Schoen	Wolf Prize 2017	Stanford University
Bojn Engquist	Member of Royal Swedish Academy of Sciences Member of American Academy of Arts and Sciences	University of Texas at Austin
Kenji Fukaya	Member of Japanese Academy 2003 Japan Academy Prize Winner	The State University of New York at Stony Brook
Nicolai Reshetikhin	Weyl-Wigner Award Winner	Tsinghua University
Benny Sudakov	Member of Academia Europaea	ETH Zurich
Vladimir Markovic	Fellow of the Royal Society	University of Oxford

Awardee: David Mumford

Basic Science Lecture: Consciousness, robots, and DNA

David Mumford

ABSTRACT. We explore the complex interplay between consciousness, robotics, and DNA using the insights of artificial intelligence, neuroscience and physics. Specifically, we delve into the evolution of language models and the quest for human-level AI, the scientific dimensions of consciousness, the elusive nature of “now” in consciousness, and the enigma of so-called cat-states in quantum theory. Finally, we examine the role of DNA replication as possibly a generator of cat-states and its implications for understanding consciousness, free will, and the nature of reality. We encourage further exploration at the intersection of these fields, highlighting the profound questions they raise about the nature of consciousness in both biological and artificial systems.

CONTENTS

1. Exploring the Evolution of Language Models and the Quest for Conscious AI	16
2. Unraveling the Scientific Dimensions of Consciousness	18
3. The Enigma of “Now” in Consciousness and Relativity	19
4. Quantum Theory, the Enigma of Cat-States and DNA Replication	21
Conclusion	24

I would like to thank Professor S. T. Yau and the Beijing Institute for Mathematical Science and its Applications (BIMSA) for their invitation to address this inaugural congress. This is a wonderful opportunity for Eastern and Western mathematical scientists to exchange their ideas. My talk is based on Chapters 9, 10 and 14 of my recent book “Numbers and the World, Essays on Math and Beyond”. I hope it will raise interest in several enigmas that raise philosophical questions as well as scientific ones. Full references, many figures and many further aspects of these issues are discussed there.

1. Exploring the Evolution of Language Models and the Quest for Conscious AI

The basic learning algorithm used in “deep learning” was devised many decades ago but was long considered useful only in highly constrained tasks like reading hand-written zip codes. In the 2010’s however, two things happened: first, the widespread use of computers generated huge datasets of text and images; and second, it was realized that the graphical processing units present in all computers could be adapted to hugely speed up the deep learning algorithm. All of a sudden, deep learning could be used with data many orders of magnitude larger than previously and the learning process could be implemented on much faster computers. This, plus the invention of an extension of deep learning called “transformers”, led to amazing successes. Speech and face recognition began finally to work at a useful level of accuracy and much more ambitiously, language models learned to generate coherent, contextually relevant responses to questions on virtually any topic. This is best known in the series of GPT (**G**enerative **P**re-training **T**ransformer) programs which stunned the world in their seemingly human-like responses to most queries. These programs take trillions of words as their data and then train billions of parameters embedded in fairly simple computer code.

But GPT lacks any embodiment in the real world – senses or robotic actuators. Despite their remarkable linguistic proficiency, these models exhibit a form of cognitive limitation akin to a quadriplegic blind individual. Note that Helen Keller, though blind and deaf, retained touch and muscles and became a brilliant writer. But GPT lacks the ability to directly perceive and interact with the actual external world beyond its vast textual input. As a result, it generates answers being called “hallucinations” or, simply put, it makes stuff up. For GPT, Santa Claus might as well as exist as not.

1.1. AI’s Progress Beyond Language. The expected evolution of AI involves transcending the confines of language processing by integrating both sensory inputs and motor capabilities. This paradigm shift entails linking the language model with systems that interpret signals from the real world, providing the AI with a platform to learn cause-and-effect relationships. The incorporation of transformer-based architectures, holds promise in bridging the gap between language-based understanding and sensory/motor interaction. A major difficulty here is providing large enough datasets in which the computer interacts with the world, which are needed for deep learning training. I think it is likely that experiments will use *virtual* worlds in order to create orders of magnitude more pseudo-world training data. More simply, continuous exposure to diverse experiences through ongoing total access to the internet should create a sense of discernment giving tools to the language model helping it distinguish truth from fictional constructs. Additionally, the inclusion of reinforcement learning mechanisms becomes essential, optimizing the AI’s skills in virtual or physical domains.

Google is not resting on its laurels. After my talk at BIMSA, I found in the arXiv multiple papers by their group implementing exactly what I had hoped for. Their program ViT (**V**ision **T**ransformers) extends the GPT architecture to visual input. Interestingly, to do so they devise a version of saccades to create linear sets of transformers relating distinct image patches as an alternative to sentences breaking into words. More recently, they have published a description of the programs PaLM-E (**P**athways **L**anguage **M**odel – **E**mbodiment) and rt-2 (**R**obot **T**ransformer) which incorporates limited movements along with vision, rather similar to the old MIT program called “Blocks World”, but a world apart in its potential. Robots performing fluid movements and adapting to diverse grasping motions begins to look likely in a few years.

1.2. Exploring Radical Transformations. In contemplating more radical advances, a further step is programming AI models to “see” and “speak” with actual people, facilitating a form of acquaintance with humanity. Imagine a lab with a robot which roams around the halls and offices and talks to the human researchers it encounters, asking them what they are working on, whether they (the robot) can help. To do this, it must build a model of each person’s jobs, personality, history. Of course, the robot does not contain its computer: it is communicating electronically with the mainframe in the lab, sending and receiving data. The objective is to foster mutual interactions, probably leading to anthropomorphism as users begin to perceive the AI as a seemingly conscious entity. Humans have no problem anthropomorphizing their pets and children do the same with stuffed animals. All this raises profound questions about the potential awareness and consciousness of the enhanced AI program. Once the AI is “embodied” in the real world, it will have agency and it must be given objectives, e.g. helping people. This is what scares the pants off of pessimists who recall Goethe’s poem about how the Sorcerer’s Apprentice ran amuk. Will the robot develop self-awareness, and to what extent can it emulate human-like urges and emotions?

I would just like to point out that what drove animal evolution and human conflict is the deep seated drive for survival and for the power to control their environment, but there is *no need to translate this into the AI’s code*. Why should a machine be concerned with self-preservation unless told to do so? Instead teach it the Hippocratic oath, “do no harm”. On the other hand, the creations of Silicon Valley have all been perverted by criminals to trick, subvert and impoverish naive users. So one must assume that criminals or criminal regimes will attempt to misuse robots and embed an urge to control humans in various ways in the robot’s code. This is my worry.

I may be wrong, but I see no reason to expect an AI to exhibit any emotions at all unless it has been programmed to have such things. For that matter, even though human emotions are such a big part of our consciousness, they are still very poorly understood. I tried to summarize some of this research in Chapter 10.3 of my book but I was surprised at how tentative it

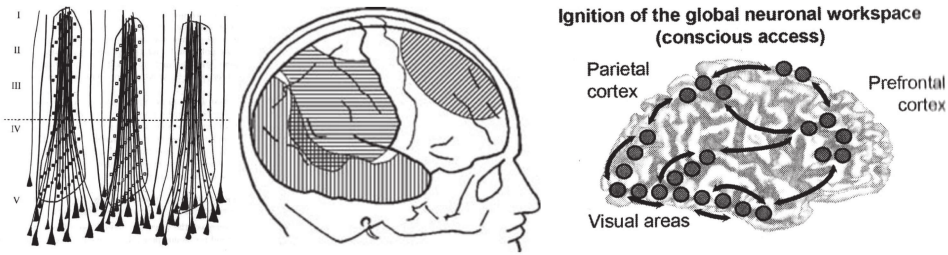


FIGURE 1. On the left, Eccles’ concept of pyramidal cell bundles, one for each component of thought; in the middle, huge excisions made by Penfield to control epilepsy that did not effect consciousness; on the right, Dehaene’s idea of the whole neocortex’s activity creating a thought.

still is. But certainly, the exploration of these issues delves into the mystery of AI consciousness, prompting inquiries into the nature of their subjective experience and the ethical implications of human-AI relationships.

2. Unraveling the Scientific Dimensions of Consciousness

The exploration of consciousness stands at the crossroads of philosophy, neuroscience, and artificial intelligence, challenging us to analyze what it means to be conscious. The historical perspective on consciousness reveals a shift from naively associating it with voluntary movement or with breath (*atma*, meaning breath in Sanskrit, is the Hindu essence of life) to contemporary studies using advanced techniques such as brain recordings. The latter date from Penfield’s amazing records of human experiences during cortical stimulation of awake patients on the operating table. Notably, the taboo surrounding the discussion of consciousness in neuroscientific circles was challenged by John Eccles and Karl Popper in their groundbreaking work, “The Self and its Brain,” published in 1977. This daring endeavor opened the door to a new era of scientific inquiry into consciousness, marked now by dedicated conferences and journals. Today, the question of whether a robot can possess consciousness is regarded as a legitimate scientific inquiry.

2.1. Locating Consciousness. The quest to locate consciousness within the intricacies of the brain has generated diverse theories and controversies. Eccles and Popper postulated a direct correlation between bundles of pyramidal cells, particularly in the prefrontal lobe, and components of thought (figure 1, left). Conversely, the findings of Penfield and Merker, observing minimal effects on consciousness after major cortical excisions (in surgery to control epilepsy, figure 1, middle), proposed an origin of consciousness in the midbrain. Building upon the legacies of Freud, contemporary neuroscientists like Stanislas Dehaene challenge traditional notions by uncovering that much of human cognition, including reasoning, occurs *unconsciously*. The proposal that γ -oscillation-based synchrony, pervasive in

the brain, gives rise to conscious thoughts shifts the focus from the activity of single or small sets of neurons to joint activity of large parts of the neocortex as the neural mechanism underlying consciousness (the global workplace theory, figure 1, right).

3. The Enigma of “Now” in Consciousness and Relativity

The concept of “now” holds a unique place in the tapestry of consciousness, representing the immediate awareness of living each moment while our memory of the past continuously increases and the unknowns of the future gradually reveal themselves. We struggle to understand how this trip down the river of time started when we were born and will end (or will it?) when we die. The question of awareness in fully intelligent robots shifts this conversation from the religious plane to a more immediate, pressing, arguably scientific issue.

3.1. The Elusive Nature of “Now”. Consciousness, characterized by awareness, memory, and the experiential immediacy of the present moment, finds itself in a paradoxical relationship with the principles of physics. In the realm of physics, the pursuit of patterns valid uniformly in all parts of space-time has no need for the concept of a subjective “now.” The division of time into past, present, and future is irrelevant in physics but, becomes a central object of contemplation in all religions, for example in Buddhist meditation and in Saint Augustine’s Confessions, Book 11. Although useful for hyperbolic initial value problems, the choice of the initial time point is quite arbitrary.

Newton had written “*Absolute, true and mathematical time, of itself and from its own nature, flows equably without regard to anything external*”. Space and time were two distinct continua and the concept of simultaneity was obvious. Einstein’s theory of relativity shattered the conventional understanding of time by abandoning the concept of simultaneity, destroying forever the notion of a universal “now”. It joins the theories of a flat earth and the geocentric model of the solar system as major misconceptions of the true universe by naive humanity. Although now the discrepancies between clocks in satellites and clocks on earth are mere microseconds, the further we explore in the universe, the more these discrepancies between clocks will grow and it is likely that parents in whose lives there is prolonged acceleration in space ships or who dive close to black holes will then outlive their great grandchildren. There will be *NO* way to fix a universal time for mankind. A person’s subjective time is simply the integral of the space-time Minkowski metric along his or her space-time trajectory and is a separate time for each person.

The clash between Einstein and philosopher Bergson in 1922 epitomized this tension. While Einstein envisioned life as a trajectory in 4D space-time, Bergson, influenced by his introspective and philosophical analysis of consciousness, rejected the relativity framework. Curiously, though, Einstein,

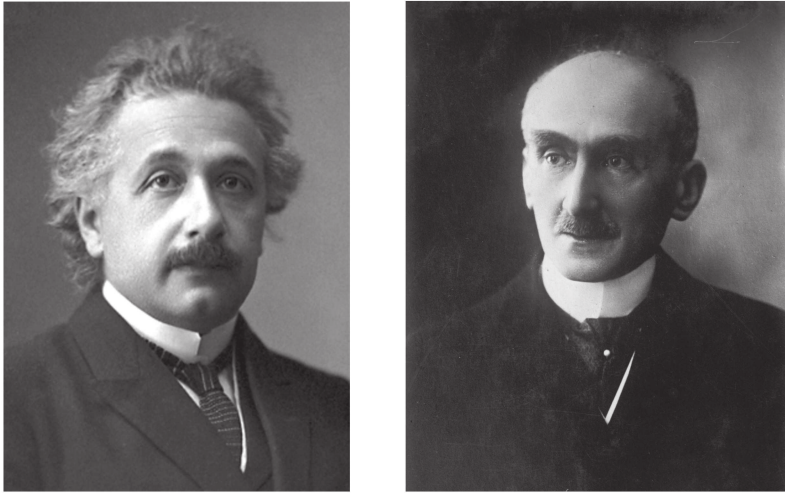


FIGURE 2. Einstein and Bergson at about the time of their debate.

grappling with the subjective experience of the “now,” ultimately acknowledged the limits of science in capturing the essence of this aspect of the temporal dimension. Towards the end of his life, he is reported by Rudolph Carnap as saying “*The experience of Now means something special for man, something essentially different from the past and the future, but that this important difference does not and cannot occur within physics ... there is something special about the Now which is just outside of the realm of science.*”

Curiously, Bergson is not the only voice clinging to a universal time. Cosmologists have insisted on this too. Although starting from general relativity, their so-called “standard model” introduces some doubtful axioms that lead to a space-time metric which splits conveniently into a global product of space and time and a metric with no cross terms but only a time varying spatial expansion/contraction coefficient. My sense is that the data from the Webb telescope is finally repudiating this simplistic model.

3.2. The Timeline of a Robot. In the context of artificial intelligence and robotics, the understanding of “now” takes a divergent path. Unlike human consciousness, robots can be powered on and off, with their computations and sensors dispersed across server clusters and physical bodies. The awareness in robots, whatever form it takes, does not adhere to a single, continuous path but rather manifests in a time-directed mesh net, possibly exhibiting a non-continuous nature. All robots will likely use parallel processing and then be unable to fix a timeline of their thoughts at the microsecond level. Neuroscientists, engaged in a parallel inquiry into human awareness, find themselves without a consensus on the localization of consciousness within the brain, so maybe this too is a mesh and not a simple one dimensional trajectory. In any case, mental processing is much slower,

occurring more or less at hundreds of milliseconds speed (bats are a startling exception, distinguishing echos separated by mere microseconds).

The entwined nature of consciousness and the temporal dimension remains an enigma, with physics challenging the very concept of an absolute “now.” As debates persist among scientists and philosophers, the exploration of consciousness in both human and artificial entities opens new frontiers of inquiry.

4. Quantum Theory, the Enigma of Cat-States and DNA Replication

In the last part of my talk, I want to discuss the role of conscious beings in the foundations of quantum theory. The problem here has nothing to do with time but with the merging of deterministic classical physics with the non-deterministic quantum description of the atomic world. This merging is needed in describing the act of measuring an atomic event with the aid of some device that amplifies events involving interacting elementary particles into something the physicist can see with his or her own eyes. I will also propose that DNA replication can be viewed as such an amplifying device and that this brings quantum reality much closer to our world than is usually believed.

4.1. The Quantum Enigma: Measurement and Cat-States. Let’s recall that quantum theory is based on a so-called wave function ϕ , an element in a complex Hilbert space $\mathcal{H}$ of norm 1 defined up to multiplication by some scalar $e^{i\theta}$. ϕ is not directly observable but evolves by the deterministic equation of Schrödinger. Measurable things are described by Hermitian operators X on $\mathcal{H}$ in the following sense: if ϕ lies in the eigenspace X_a of X with eigenvalue a , then the measurement corresponding to X must give the value a . But what happens if ϕ is not in any eigenspace? ϕ can be decomposed as the integral of its projections to X ’s eigenspaces. More usefully, the norm squared of these projections defines a measure on the spectrum of X whose value on a subset U is the probability that the measured value of X will be in U . The eigenvalue of X has a mean under this measure and a standard deviation quantifying your uncertainty. Finally, after a measurement determines that the value is in U , ϕ must be *reset* to its projection $p_U(\phi)$ on the corresponding eigenspaces. This is called the “collapse of the wave form”.

Immediately we have a problem: collapsing the wave form after a measurement clearly means that ϕ is describing what you know, not what is actually “there”. In other words, ϕ is epistemic thing, not an ontological thing. But on the other hand, long calculations on ϕ ’s allow accurate predictions of complex particle interactions and even the chemical properties of complex molecules. It is hard to come to terms with these successes without thinking of ϕ as ontological, describing something “really there”. This is the

central dilemma at the heart of quantum theory, the problem that Feynman often said no one really understood.

The indeterminacy of measurements is also an issue. In ordinary life, outside physics labs, things tend to be predictable. Sure there is the unpredictable nature of weather but we are pretty sure we know the PDE's that can, in principle, predict it and the issue is that of large Lyapunov exponents. Your toothbrush is where you left it last night and if you think stuff is acting mysteriously, you may need to see a psychiatrist. Let's make the seeming determinacy of macroscopic life precise. First make a list of the (bounded) Hermitian operators $\{X_i\}$ defining observations we can make with human senses alone and second, specify tolerances describing how small a change will be noticeable to our senses. Then the determinacy of the macroscopic world can be expressed reasonably well by requiring that the standard deviation of each X_i should be less than the corresponding tolerance ϵ_i . Expressed in formulas, this means:

$$\frac{\langle X_i^2 \phi, \phi \rangle}{\|\phi\|^2} - \left(\frac{\langle X_i \phi, \phi \rangle}{\|\phi\|^2} \right)^2 < \epsilon_i^2$$

This requirement defines an open set $\mathcal{H}_{\text{AMU}} \subset \mathcal{H}$ with a screwy shape defined by 4th order inequalities, a set that I call the “almost macroscopically unique” (AMU) quantum states. Its complement $\mathcal{H}_{\text{CAT}}$ are referred to as “cat-states” and are states *where there is visible indeterminacy*.

The archetypal cat-state was described by Schrödinger as the result of locking a cat in a sealed box for a minute with some radioactive material that triggers the release of cyanide when the material emits an alpha particle. After a minute, the state ϕ describing alive or dead is a weighted sum of an alive cat state and a dead cat state, $p \cdot \phi_{\text{alive}} + (1-p) \cdot \phi_{\text{dead}}$. Lots of question arise: was the cat really *neither alive nor dead* but in a superposition a second before the box is opened? What happens if a robot opens the box?

4.2. Physicists’ Fudges and the Dilemma. Physicists have not reached any consensus on how to deal with this dilemma. Heisenberg took an epistemic stance saying at one point “*The natural laws, formulated mathematically in quantum theory, no longer deal with the elementary particles themselves but with our knowledge of them.*” Another group has sought “hidden variables” that evolve deterministically but their proposals have not incorporated the extra complexities of quantum field theory. And then we have the “multiversers” who say the universe splits into many copies of itself, one for each value of the indeterminate measurement. As far as I’m concerned, I’ve lived one life and I find the idea that there are millions of me living other lives seems a bit silly. By and large, physicists hate the idea that consciousness has anything to do with physics and want to place the “Heisenberg cut”, where the indeterminacy is resolved, somewhere in the middle of the measuring process. The epistemic group now cloths the subjective aspect of measurements in the math of information theory which is more comfortable

for scientists. Information theory introduces the concept of ϕ as information without specifying its source. I discuss these multiple approaches to the dilemma of measurements further in my book, Ch. 11.1. The fundamental question of where and how the Heisenberg cut occurs remains elusive.

4.3. The Macroscopic Dilemma and Evolution of States. The introduction of “almost macroscopically unique” states $\mathcal{H}_{\text{AMU}}$ attempts to address the non-existence of cat-states by defining a subset of Hilbert space where macroscopic observables exhibit small variances. However, Schrödinger’s evolution does not map $\mathcal{H}_{\text{AMU}}$ to itself! For example, his equation for single free particles is readily solved and leads to ϕ ’s that spread out without limit. As they spread, their wave aspect eventually dominates their particle aspect. And since measurements of the particle’s position then become macroscopically indeterminate, the particle is in a cat-state. Physicists’ labs convert atomic superpositions to macro cat-states but with a twist. All measuring apparatuses are made of huge numbers of atoms, something on the order of Avocado’s famous number, so around 10^{23} . The original uncertainty spreads out into this mess, into what is often modeled as a “heat bath”. Then the final macroscopic measurement is best described by a “density operator”, a sort of trace of the rank one Hermitian operator $\phi \cdot \phi^*$ in the Hilbert space of the measured value alone. When this density operator has more than one substantial eigenvalue, the result is a cat state.

4.4. Measuring Instruments and DNA Replication. Measuring instruments, such as Wilson’s cloud chamber and photomultiplier tubes, create cat-states. The cloud chamber does this by being on the verge of a phase change where the passage of a single particle can create a bubble. A photomultiplier tube contains a sequence of electrodes held at steadily increasing charges each of which responds to being hit by electrons by emitting more electrons, e.g. 5 times as many. These are the lab’s work horses but I want to propose that there is one other amplifier that is everywhere in our world: DNA. The essence of our lives is that one double-stranded set of human DNA produced by the coupling of 2 parents produces by replication some 30 trillion sets of the same DNA strands. Replication is not always 100% accurate but every cell has multiple repair mechanisms to keep the error rate small. But a multitude of injuries, radiation for example or poisonous chemicals, can cause mutations. Radiation, for example, can ionize a key atom *but always with some probability*. The result is a superposition of 2 states: an ionized state and an unionized state. This superposition is going to propagate creating a superposition of two distinct full strands of DNA, one mutated and one unmutated. In some cases, both will be viable but the two outcomes will differ in one base pair.

The speculative notion that DNA replication itself induces DNA cat-states opens a Pandora’s box of questions. If mutations disrupt the sequence, the entire phenotype, the adult his or herself, can become a superposition. This raises challenging queries about when and where a collapse occurs.

I want to suggest that two things to speculate on. First, in the context of paleontology, were dinosaurs' senses developed enough to collapse the wave-form of a cat-dino? Or would the cat-dinos become cat-fossils and only when paleontologists dug them up was the form of the ancient dino finally chosen? Perhaps the shape of Mesozoic fauna was fixed in the 20th century when humans collapsed their wave-form.

Secondly, consider cancer. It is old folk medicine that cancer tends to strike grieving people with higher frequency than average and that positive attitudes sometimes create miracle remissions. If a cancer starts as a cat-cancer, then consciousness leads to the choice between two quantum states. And perhaps free will, if such a thing exists, can twist Born's rule by affecting the probabilities of the two states. I have never heard of an experiment where people try to affect the outcome of measuring a cat-state.

In my book, I propose a new model for dealing with measurements. I suggest that we are living in a "Bohr-bubble" in space-time in which we share information and memory across civilizations in order to maintain a stable macroscopic world. Many animals are like fellow-travelers who get sucked up into this bubble but whether dinosaurs or robots can also be included is not clear.

Conclusion

I have tried to shed light on subtle ideas at the intersection of AI, consciousness, and quantum mechanics. In the first section, I propose that robots with all the usual human abilities will be built fairly soon. Then the question of whether the robot has consciousness will have to be discussed. Personally, I have been amazed that the study of consciousness has gone from a theological and philosophical topic to a scientific one as I describe in the second section. But I believe there is an obstacle: the experience of "now" seems impossible to define scientifically as I discuss in the third section. This makes the role of consciousness in quantum theory very significant. The collapse of the wave-form is needed to prevent cat-states from disrupting the apparently deterministic macroscopic world. This is described in the fourth section together with the hypothesis that DNA replication is continually creating cat-states, for instance in cancer mutations. The interwoven complexities of quantum theory, measurement-induced cat-states, and the potential role of DNA replication as a measuring instrument challenge our understanding of reality and consciousness. As physicists grapple with the enigma of the quantum world, the implications for free will, the nature of measurements, and the collapse of cat-states lead back to the mysteries of consciousness.

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