Graphon-based Synthetic Power System Model and its Application in System Risk Analysis

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***Abstract*—The high confidentiality level of power system's data has motivated the ongoing research on the generation of synthetic power grids that mimic actual systems. The existing synthetic models rely on specific geographical and parametric assumptions, which leads to non-generalizable models that overfit the observed data. To fill up this research gap, this paper proposes the use of graphon, a non-parametric graph processing method, to generate graph samples of different sizes with similar topological and electrical characteristics as actual power systems. We first estimate the graphon based on realistic parameters of the observed actual power system. Then as an example of a use case, we sample multiple graphs from the graphon in order to provide a general assessment of the power system vulnerabilities.**

***Index Terms*—Graphon, generative model, power systems, risk analysis, and vulnerability.**

I. Introduction and Motivation

The reliable and efficient operations of smart cities highly depend on improved monitoring and management of energy usage, increased efficiency, and enhanced power grid resilience. When smart cities' critical infrastructures are optimized for safety and security, they can improve the quality of life for its citizens through enhanced public services and access to a sustainable environment. However, a fundamental limitation in research on power systems is the restricted access to the confidential data associated with actual power grids. For instance, in the U.S., power system's data related to the production, generation, transmission, and distribution of energy fall under the Critical Energy/Electricity Infrastructure Information (CEII), and therefore are not made available even for research purposes [1]. When partial power system data is made accessible, it often does so under a strong non-disclosure agreement. Therefore, any study done on actual power systems cannot be made public. Along this direction, some efforts were made to create synthetic power systems that mimic the characteristic features of actual power grids such as [2]–[4]. However, these developed models rely on several assumptions such as the geographical area, the region structure, the topological and electrical statistics, etc., with a limited number of test cases provided. Moreover, it is hard to assess how these synthetic models scale to massive power systems. Another limitation is that research is conducted on specific synthetic test cases, and therefore, the final results become dependent on the systems used in the study. To fill this research gap, this paper introduces graphons [5], a non-parametric graph processing method, to model and predict how power systems massively expand by taking the graphs to the limit, i.e., to an infinite number of vertices and edges. This graph theory concept allows to generate random power graphs that are consistent with the observed actual power graph. Since graphons treat power systems as graph objects, we will present a method to statistically equip graphs with electrical parameters. Therefore, statistically consistent power graphs of different sizes with topological and electrical information can be generated to assess power systems vulnerabilities.

Graphons, which are short for "graph functions", are limiting objects of a sequence of graphs with a large number of nodes, in which members of the same family share similar structures even if their corresponding number of nodes is different [6]. They are considered to be non-parametric models since the number of parameters that describe the network structure and characteristics does not need to be fixed or even finite, which in turn avoids overfitting the model representation [7]. The main motivation behind graphons is that instead of analyzing an individual graph, we can analyze and work on the mathematical equation that governs the graph. This means that instead of studying an individual power system, one can study the power graph function that characterises all the properties of such a system and analyze its multiple aspects. Thus, we can work on all graph samples generated from the same graphon as if they were the same object. Graphons can also be used to model power systems that continuously evolve in space and time due to topological modifications, seasonal reconfigurations, integration of renewable energy sources, new buses and electrical lines addition, etc.

In this paper, we use parameters of actual power systems to estimate their graphons. Then, we provide a general assessment of power vulnerabilities using the graphons. This reveals the most critical components that maximize the damage. This analysis is visualized by constructing a risk graph that averages over all the graphs sampled from the same graphon.

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A. Related Work

Since the availability of power system's data is restricted due to confidentiality, several papers proposed to generate synthetic power systems that mimic actual ones in terms of topological structures and electrical features. In [2], the authors generated synthetic power systems using publicly available information, the locations of generation substations, and the structural statistics of real power systems. The generated test cases were augmented to build larger systems. In [4], the authors developed a stochastic spatio-temporal evolving model using parameters extracted from the Western U.S. and ERCOT power grids. In [3], the authors proposed a generative spatio-temporal expanding stochastic power grid model that is mapped to a circular geographical region, where power elements were distributed using iterated Poisson tessellations. In [8], the authors suggested a bus type entropy-based approach for assigning generating, load, and connection buses as well as for producing electrical characteristics for synthetic power systems. Other works such as [9], [10] generated synthetic power systems based on real-world data.

The main limitations of the related works are that they rely on certain assumptions that are specific to the region to which the power system is mapped to. In addition, the aforementioned works do not describe how power systems largely scale and massively grow in terms of electrical elements and power lines. Since they are constructed based on multiple structural and electrical parameters of actual power grids, they tend to overfit the learned model. This paper bridges these gaps by using a non-parametric graphon, from which a sequence of sample graphs of any size can be statistically and consistently generated. The proposed synthetic power graphs are generated by estimating the graphon model from the graph of an actual power system with no additional parameters assumptions. Hence, no assumptions of topological or electrical parameters are considered as in the case of the aforementioned related works. Once the graph is sampled from the graphon, it can be mapped to a geographical region with physical boundaries. Moreover, instead of working with individual test cases, we only need to deal with a measurable function that describes such systems. To the best of our knowledge, we were able to identify a single paper [11] that used graphons mainly for modeling power dynamics to analyze the stability of synchronized nodes to a common frequency. Instead of using a generalized graphon model for all power systems based on small-world networks characteristics as in [11], we specifically estimate the graphon from an actual power system.

B. Contributions and Organization

The main contributions are summarized next. We start by predicting the graphon model from the graph of an actual power grid using different graphon estimators proposed in literature. We compare these estimators with the small-world graphon model, and we select the estimator with the lowest mean squared error (MSE). The obtained estimated graphon is then used to sample power graphs to which we statistically assign electrical parameters. Then, as one application of the graphon model, we conduct a general risk assessment study to reveal the main vulnerable components. In this regard, we first compute a weighted feature score for each node in terms of topological and electrical features using the analytical hierarchical process (AHP). Then, we conduct the vulnerability analysis on the graphs to identify the target nodes with the greatest damage impact. The damage values of the target nodes and their feature scores are finally used to build the risk graph.

The remainder of this paper is organized as follows. Section II describes the process of generating synthetic power systems based on graphons. Section III presents an application use case for risk assessment in power systems. Section IV discusses the obtained results. Section V concludes the paper.

II. Graphon-Based Synthetic Power System Models

In this section, we describe how to generate synthetic power systems based on non-parametric graphon models.

A graphon can be represented as a symmetric measurable function mapping the unit square to the unit interval, $W : [0,1]^2 \to [0,1]$. It may be compared to the weight matrix W of an infinitely large graph, whose node variables are $(x, y) \in [0,1]$, and the weights of the edges are represented by $W(x, y) = W(y, x)$ [6]. In specific, For a specific power system, we can estimate its graphon model using one of these consistent estimators proposed in literature:

- Stochastic blockmodel approximation (SBA) [12], where nodes are first clustered into blocks based on the distance estimate between graphon slices. Then, an empirical histogram is applied to estimate the graphon.
- Universal Singular Value Thresholding (USVT) [13] uses singular value decomposition of observations between pairs of nodes followed by thresholding over these values.
- Sort and Smooth (SAS) [14], where the graph empirical degree is first sorted and then the observed data is smoothed using total variation minimization.
- Largest Gap [15], where network blocks are estimated using the largest gap criterion on consecutive normalized degrees, followed by a histogram on the estimated blocks.
- Matrix completion [16] applies a completion scheme on the observed adjacency graph by treating the non-existing edges as missing entries.

Power grids exhibit small-world characteristics [17], [18]. The authors in [4] approximated power distribution systems accurately by having their nodal degree distribution follow the shifted sum of exponential distribution, $f(d)$, where d denotes the node degree such that

$$f(d) = \sum_i \frac{\eta_i}{\mu_K} e^{-\frac{d-k_i}{\mu_K}\mathbb{1}(d \geq k_i)}, \qquad (1)$$

where μ_K is the average number of edges formed by node i, η_i are the probabilities of node i taking different degree values k_i, and $\mathbb{1}(.)$ is the indicator function. The parameters k_i and η_i are obtained by matching with real power grids [4]. Then, based on the r-nearest-neighbour graphs and the rewiring of the short-range connections among nodes, we use the small-world network characteristics as a general model for all power systems to model the graphon as [11]

$$\mathcal{L}_{p,r}(x,y) = pG_{1/2}(x-y) + (1-2p)G_r(x-y), \quad (2)$$

where $(x,y) \in [0,1]$ are the node variables that are mapped to latent node labels, p is the probability sampled from $f(d)$ representing the edge probability between a pair of nodes, r is a parameter between 0 and 0.5, and

$$G_r(x-y) = \begin{cases} 1 & \text{if dist}(x-y) < r \\ 0 & \text{else,} \end{cases}$$

where $\text{dist}(x-y) = \min(|x-y|, 1-|x-y|)$. Therefore, the small-world graphon can be considered as a general model for all power systems, while the estimated graphons from actual systems are considered specific to the actual observed systems.

Once we infer the graphon model, we can sample multiple graphs from it by drawing n samples uniformly from $[0,1]$ representing node variables and mapping them to latent node labels. Then, edges between each pair of nodes are randomly added according to the edge probability $W(x,y)$ obtained from the estimated graphon. In order to assign electrical parameters to power nodes and edges, we generate a random set of active/reactive power values using the exponential distribution [19]. At this point, we obtain the normalized nodal degree and normalized active/reactive power values of each node for both the actual and synthetic systems. The probability values are matched between both systems, and the load values are re-ordered to the appropriate nodes by comparing the probability mass function (PMF) of the two normalized variables in the synthetic power system with the PMF of the normalized variables in the real power system. Then, buses are assigned the real unnormalized active/reactive power values depending on their degrees [19]. Additionally, we use empirical data from IEEE bus systems and the NYISO system in [20, Table V], which models line impedance using various distributions depending on the system, to give line impedance values. The obtained values are then probabilistically matched with the real values of the actual power system. Finally, we verify if the system load exceeds the steady-state loading capacity by running a continuation power flow that gradually increases the loading/generation. To get a convergent power flow solution, we scale down all load values when the load exceeds the steady-state loading limit.

III. Application in System Risk Analysis

In this section, we describe how to use the graphon-based synthetic power system models in Section II to provide a general system risk assessment that reveals the power components that maximize the system damage.

We start by assigning each node in the graph a weighted feature score based on topological and electrical metrics. These scores are used to build the average risk power graph.

We use the following topological metrics: i) the average distance between any two nodes is measured by the average path length; ii) the clustering coefficients show how much a node tends to form a cluster; iii) the betweenness centrality measures how many times a particular node appears in the shortest path between two nodes; iv) the closeness centrality shows how close a node is to all other nodes; v) the degree centrality, which is the number of nodes and edges that directly influence the node status; vi) the efficiency, which indicates the effectiveness of sending data between any two nodes; vii) the eccentricity centrality, where a low eccentricity of a given node indicates that all other nodes are nearby; and viii) the centroid centrality, which denotes the node's central position in a region of high node density.

Regarding the electrical metrics, we define: i) the electrical degree centrality, which shows how many power flows directly affect a node's status; ii) the electrical betweenness centrality, which shows how far a node is from other pairs of nodes when power is assumed to travel via the shortest pathways between them; and iii) the effective graph resistance, which measures the expense of transferring a power flow between two nodes.

With the different metrics presented above, we use AHP to identify the weight impact of each one of them on the overall system vulnerability in order to accurately calculate the node feature scores [21]. The AHP weights are obtained by constructing the table of pairwise comparisons of the normalized metrics, where diagonal entries are equal to 1s, since each metric is as important as itself. The remaining entries of the table are filled by comparing the normalized score of each metric to the normalized score of another metric. Then, the obtained comparison table is normalized by dividing each column entry by its column sum. Then, each row is summed and its average is obtained, which corresponds to the obtained weight. Finally, using the obtained AHP weights, we calculate the weighted feature score for each node.

Next, we describe how we perform vulnerability analysis on a graph. We use a method similar to the one outlined in [22] to obtain the best combination of target nodes with the greatest system damage. The damage impact is evaluated as a weighted measure of the following metrics: i) the percentage of drop in net-ability, which gauges how effectively the power system can deliver power [22], ii) the connectivity impact, which shows how many nodes are still linked following a node loss [23], and iii) the topological damage which, according to [24], is calculated as the normalized efficiency loss following a failure.

The vulnerability strategy for obtaining the combination of nodes that maximize the system damage is described next. We begin with a collection of nodes whose elements are $m = 1$ target node and work our way up to sets of m-node combinations. There are $\binom{N_P}{m}$ different single node combinations available. We want to select the nodes that are the most vulnerable. In order to do this, we retrieve the first set of nodes with the highest damage values, where V can be adjusted to a tiny value between 4 and 8 in order to condense the search space [22]. If $V = 8$, the top 8 nodes with the highest damage values would make up the first set, where each node in set 1 is a single target node.

The initial step in creating all the $m = 2$-node combinations is to combine each node from the set of nodes with the top V

largest feature score values with the nodes from the preceding set, or set 1. Then, we look through all of these potential combinations to find the ones that damage the system the most. To create all the possible m-node combinations, we combine the node combinations from the previous $(m-1)^{\text{th}}$ set with each node from the set of nodes with the top V largest feature score values. This generalizes on generating the subsequent m^{th} set of m-node combinations. The top $(m+1)V/2$ combinations with the highest system damage values constitute the m^{th} set.

Then, for each sampled graph, we calculate the feature scores for all the nodes, and we obtain the combination sets of nodes with the greatest damage impact along with their corresponding damage values as previously described. To construct the risk graph, we average the feature scores and the damage values for all the combination sets obtained over all the sampled graphs. Then, we calculate the vertex occurrence frequency (VOF), which specifies the number of times a certain node appears in all of the target combination sets. Similarly, we calculate the edge occurrence frequency (EOF), which specifies the number of times a pair of nodes appear in all of the target combination sets [25]. The VOF and EOF weights are then input to a graph visualisation tool to obtain the risk graph. Such a risk graph can be considered as a generalized risk assessment for all power graphs sampled from the graphon, rather than being specific to certain systems.

IV. Results

We conduct simulations using MATLAB on the 39-bus New England system, which is a power system in the New England area of the United States. Fig. 1 shows a comparison of the different graphon estimators presented in Section II for the observed power system. Using Eq. (2) and parameters $k_i = [1, 2, 3, 4, 5]$ and $\eta_i = [0.2308, 0.3077, 0.3590, 0.0769, 0.0256]$ obtained from the 39-bus New England system, we plot in the upper left figure the small-world graphon, which is a general model for power systems. The horizontal axis of Fig. 1 represents a random variable x and the vertical axis represents a random variable y, both of which are uniformly distributed between 0 and 1. A latent mapping maps the specific values of x and y with the node labels. The color in the graphon represents the intensity of the edge weight, $W(x, y)$, indicated by the vertical color bar. A more yellowish color indicates that a pair of nodes are strongly connected. Fig. 1 shows a strong connection between nodes along the upper edge of the graphon with weakly connected nodes across. Such a characteristic is of the small-world graphs, where most nodes are not neighbors of each other and the neighbors of a specific node are likely to be neighbors of each other, indicating a highly clustered graph. Nodes are reachable from each other with a small number of hops, indicating a low average characteristic path length. These small-world characteristics make power systems more stable and robust to cascading outages. In specific, the graphon estimators show that they retain the small-world features with MSE ($\pm$ standard deviation) averaged over 100 independent trials of 0.137 ± 0.0018, 0.128 ± 0.0021, 0.127 ± 0.0017, 0.126 ± 0.0017, and 0.172 ± 0.002 for the SAS, SBA, Matrix Completion, Largest Gap, and USVT estimators, respectively. The USVT estimator returned the least resemblance to the small-world graphon, mainly due to the high rank of the small-world adjacency matrix. The largest gap estimator had the closest resemblance to the small-world graphon, since this method seeks low-rank structures of the adjacency matrix [14]. To compare the topological similarities with the 39-bus New England system, we sample

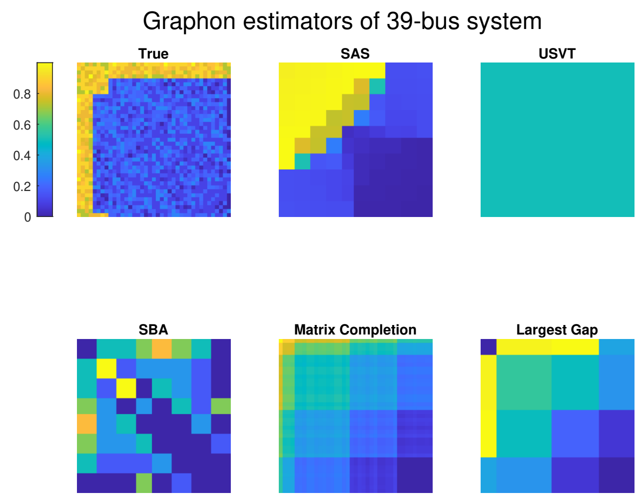

Fig. 1. Graphon estimators for the 39-bus New England.

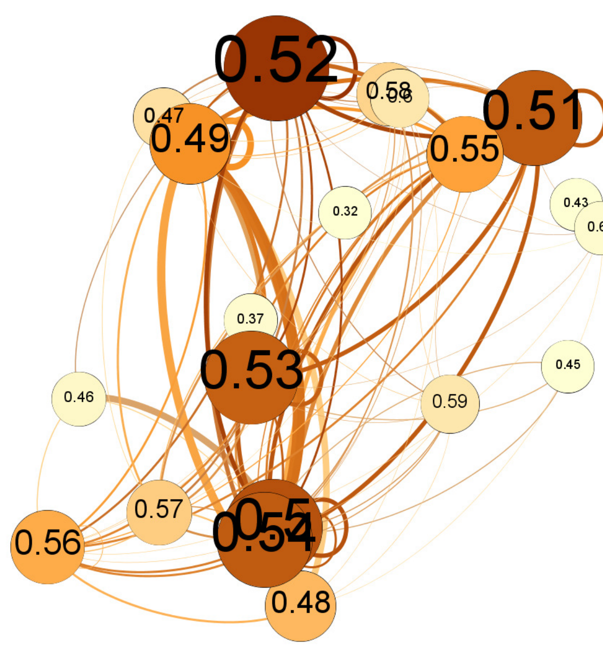

Fig. 2. Average risk graph representation based on 100 graphs sampled from the induced graphon using Gephi tool.

100 graph samples from the 'Largest Gap' graphon estimator and we compute the cumulative distribution function (CDF) of the eigenvalues spread, the graph diameter, the betweenness centrality, the closeness centrality, the nodal degree, and the clustering coefficients. Then, we use the similarity index $S_{f,g} = \langle f \cdot g \rangle / (\langle f \cdot f \rangle + \langle g \cdot g \rangle - \langle f \cdot g \rangle)$, where $\langle f \cdot g \rangle$ is the inner product of the functions f and g corresponding to the CDF of the topological characteristics of 39-bus New England system and the graph sample, respectively. Finally, we average the similarity scores over 100 graph samples. We found an average similarity of 87.94% with the 39-bus system.

Next, we construct the average representation of the power risk graph using the Largest Gap estimator. For this purpose, for each sampled graph, we obtain the combination sets of nodes and their corresponding damage values using $V = 8$ and $m = \{1, \cdots, 6\}$. We also compute the feature scores of each node, and we average them in addition to the obtained damage values over all the 100 graph samples. Fig. 2 shows the main vulnerable components based on the feature scores. The bigger and darker the vertices and the links are, the more critical they are in terms of contributing to the overall damage. In specific, those nodes with scores ranging from 0.49 to 0.56 are the most critical in terms of maximizing the damage. Finally, we show the risk graph of a sampled graph (Fig. 3) from the graphon in Fig. 4. We can see that nodes $\{8, 13, 17, 20, 28, 32, 33, 38\}$ are the most critical ones. These nodes are consistent with the nodes in Table 1, which have feature scores in the range of 0.57 and 0.48. Table 2 shows the combination sets of target nodes with the greatest damage impact obtained by comparing the nodes feature scores of the sampled graph with the average feature scores of the 100 sampled graphs. The damage values correspond to the average damage values obtained from the 100 sampled graphs based on the nodes feature scores.

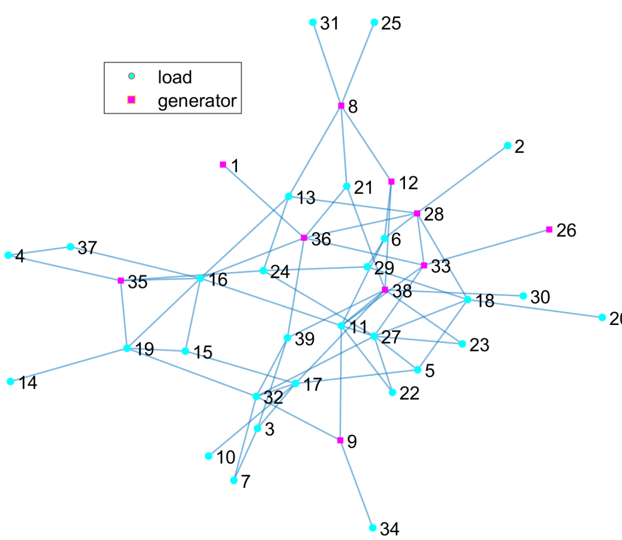

Fig. 3. A graph example sampled from the estimated graphon.

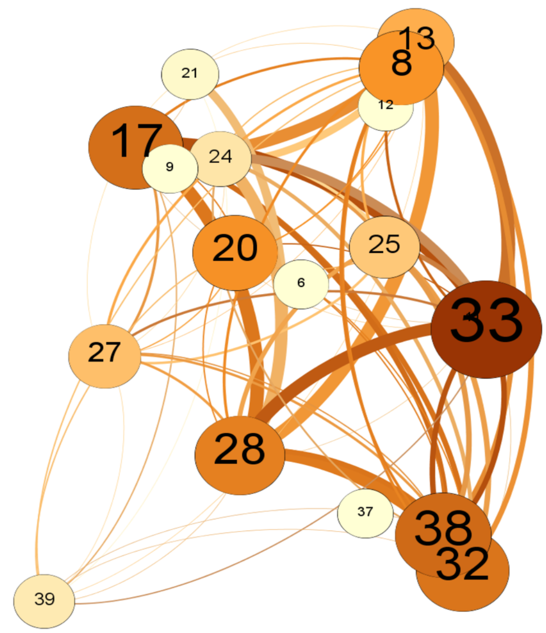

Fig. 4. Average risk graph representation of a graph sampled from the induced graphon using Gephi tool.

V. Conclusions

In this paper, we have estimated the graphon model from an actual power system graph to generate synthetic power systems without relying on additional geographical, topological and/or electrical assumptions. Given any graph sampled from the graphon, we presented a method that allows system operators to protect the most vulnerable power components based on the power nodes feature scores solely, without the need to run extensive cascading failure simulations. These results find usefulness in the energy sector of smart cities that collect and analyze massive volumes of energy data from a variety of sources, including sensors, meters, and voltage detectors. This information is utilized to make sound decisions and improve city operations by providing reliable public services in an efficient and secure manner.

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TABLE I

NODES FROM THE SAMPLED GRAPH WITH FEATURE SCORES RANGING FROM 0.57 TO 0.48.

Node #	38	32	33	17	28	13
Feature score	0.57	0.53	0.51	0.5	0.48	0.48

TABLE II

THE COMBINATION SETS OF TARGET NODES FOR THE SAMPLED GRAPH OF FIG. 3

Set 1	**% Dam**	**Set 2**	**% Dam**	**Set 3**	**% Dam**	**Set 4**	**% Dam**	**Set 5**	**% Dam**	**Set 6**	**% Dam**
24	53.77	28,17	57.4	32,17,33	60.56	38,17,25,8	63.63	38,33,17,8,20	66.52	27,17,25,38,20,32	69.08
6	52.78	17,33	56.52	33,38,17	59.79	38,17,32,8	62.77	38,17,32,8,33	65.65	38,13,28,17,33,27	68.18
9	52.24	28,13	56.11	32,28,33	59.37	38,17,32,33	62.41	38,28,33,8,20	65.25	38,13,32,8,17,28	67.75
23	52.01	17,13	55.88	32,28,17	59.22	38,28,17,32	62.19	38,33,25,32,20	64.96	38,17,25,32,20,33	67.29
35	51.65	28,24	55.62	38,28,33	58.95	32,33,25,8	61.94	38,33,25,8,20	64.54	38,28,25,33,20,32	66.89
12	51.4	24,17	55.4	33,13,17	58.74	32,28,33,8	61.84	27,33,25,32,20	64.31	38,32,17,8,20,33	66.43
37	51.17	13,24	55.2	17,33,25	58.59	38,28,33,32	61.56	38,28,32,33,20	64.04	27,21,13,28,17,39	66.14
22	50.83	33,24	55.02	33,28,17	58.37	32,17,33,8	61.38	38,33,25,17,20	63.88	27,13,33,38,17,32	65.98
		13,33	54.85	17,13,28	58.15	33,28,25,38	61.23	38,28,17,33,20	63.73	27,13,33,28,20,38	65.77
		21,28	54.63	32,33,25	58.1	33,32,25,38	61.07	27,33,17,8,20	63.58	27,28,33,8,20,38	65.66
		33,28	54.44	17,13,32	57.91	38,33,25,8	60.91	27,28,33,8,20	63.43	39,28,33,8,20,38	65.42
		32,28	54.34	28,13,32	57.83	38,13,17,33	60.76	38,13,33,8,20	63.28	27,28,33,32,20,38	65.22
				33,38,25	57.67	27,28,33,32	60.68	38,13,17,32,20	63.18	27,13,28,8,33,39	65.08
				28,24,25	57.59	27,33,25,8	60.54	38,13,32,28,33	63.07	38,13,33,8,20,32	64.93
				38,13,17	57.51	27,28,17,32	60.43	32,33,17,8,20	62.95	38,33,17,8,32,27	64.83
				17,24,32	57.42	32,13,28,33	60.35	38,33,32,8,20	62.8	38,24,25,17,33,27	64.68
						38,24,28,33	60.26	38,17,25,8,20	62.67	38,25,8,20,32,27	64.44
						32,24,33,8	60.2	38,17,13,33,20	62.61	38,32,28,8,33,17	64.22
						32,28,17,8	60.07	27,38,28,33,20	62.48	27,28,33,8,20,17	64.11
						38,33,17,8	60.04	39,17,33,8,20	62.36	39,28,17,33,20,38	63.95
								32,13,33,8,20	62.3	27,13,17,33,20,32	63.79
								38,13,32,8,33	62.2	39,13,33,8,20,32	63.61
								27,28,33,32,20	62.01	27,17,33,8,20,38	63.44
								38,13,33,32,20	61.74	39,17,32,8,33,38	63.29
										27,21,13,8,28,39	63.08
										38,28,32,33,20,17	62.86
										11,32,28,33,20,17	62.4
										38,13,28,8,20,17	61.94

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