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Bulk black hole dark matter

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ABSTRACT

The dark dimension provides a mechanism to resolve the cosmological hierarchy problem and assembles a colosseum for dark matter contenders. In a series of recent publications we investigated whether primordial black holes (PBHs) perceiving the dark dimension could constitute all of the dark matter in the universe. A key assumption of these investigations is that PBHs remain confined to the brane during the entire evaporation process. As a consequence, the abundance of PBHs evaporating at the present epoch is severely constrained by observations of both the extragalactic and Galactic γ-ray backgrounds. As a natural outgrowth of these investigations, herein we relax the assumption of brane localized PBHs and reexamine the evaporation process of PBHs which are allowed to escape into the dark dimension. We show that the escape of PBHs from the brane is almost instantaneous. Armed with this pivotal finding we reassess the allowed mass range of PBHs to assemble all cosmological dark matter, which is estimated to be $10^{11} \lesssim M_{\rm BH}/{\rm g} \lesssim 10^{21}$.

1. Introduction

The set of effective field theories (EFTs) that look consistent according to all available low-energy criteria, but do not arise from an ultraviolet complete theory that includes quantum gravity has become known as the swampland [1]. There are several conjectures for fencing off the swampland [2–5]. For example, the distance conjecture (DC) asserts that in any consistent theory of quantum gravity, when venturing to large distances in four-dimensional (4D) Planck units within scalar field space, a tower of particles will become light at a rate that is exponential in the field space distance [6]. Concatenated to the DC is the anti-de Sitter (AdS) distance conjecture, which hitches the dark energy density to the mass scale m characterizing the infinite tower of states, $m \sim |\Lambda|^\alpha$, as the negative AdS vacuum energy $\Lambda \to 0$, with α a positive constant of $\mathcal{O}(1)$ [7]. Furthermore, if this scaling behavior stays around valid in de Sitter (or quasi de Sitter) space, an unbounded number of massless modes also would pop up in the limit $\Lambda \to 0$.

The AdS-DC in dS space provides an expressway to elucidate the cosmological hierarchy problem, $\Lambda/M_p^4 \sim 10^{-122}$, because it connects the size of the compact space $R_\perp$ to the dark energy scale $\Lambda^{-1/4}$ via

$$R_\perp \sim \lambda \, \Lambda^{-1/4} \,, \qquad (1)$$

where the proportionality factor is estimated to be within the range $10^{-1} < \lambda < 10^{-4}$ and where M_p is the Planck mass [8]. As a matter of fact, (1) stems from constraints of both theory and experiment. Firstly, the accompanying Kaluza–Klein (KK) tower contains massive spin-2 bosons, and thus the Higuchi bound [9] sets an absolute upper bound to α. Secondly, explicit string calculations of the vacuum energy (see e.g. [10–13]) lead to a lower limit on α. More precisely, these two stringy constraints imply $1/4 \leq \alpha \leq 1/2$. Now, when these theoretical arguments are combined with experimental observations (e.g. constraints on deviations from Newton's gravitational inverse-square law [14] and neutron star heating [15]) we arrive at the conclusion encapsulated in (1): *The cosmological hierarchy problem can be addressed if there exists one extra dimension of radius $R_\perp$ in the micron range, and the lower bound for $\alpha = 1/4$ is basically saturated [8].* A theoretical amendment on the connection between the cosmological and KK mass scales confirms $\alpha = 1/4$ [16]. A second conclusion encapsulated in (1) is that the KK tower of the new (dark) dimension lays bare at the mass scale $m_{\rm KK} \sim 1/R_\perp$.

The 5-dimensional Planck scale (or species scale where gravity becomes strong [17–33]) provides an upper bound for the ultraviolet

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cutoff of EFTs of gravity coupled to a number of light particle species and within the dark dimension scenario is found to be

$$M_* \sim m_{\rm KK}^{1/3} \, M_p^{2/3} \,. \tag{2}$$

The dark dimension heralds a new era of particle phenomenology and cosmology [34–56]. For example, primordial black holes (PBHs) with a Schwarzschild radius smaller than a micron could be potential dark matter candidates [35,37,50,53]. If such PBHs were evaporating at the present epoch, then their abundance would be strongly constrained by observations of both the extragalactic and Galactic γ-ray backgrounds [57,58]. The null results from searches in γ-ray experiments, imply that only for the black hole mass range

$$10^{15} \lesssim M_{\rm BH}/{\rm g} \lesssim 10^{21} \,, \tag{3}$$

PBHs could make all cosmological dark matter [53]. This bound was derived within the probe-brane approximation, which ensures that the black hole is bind to the brane during the evaporation process. However, a thorough study of the evolution of the PBH-brane system, once the black hole is given an initial velocity (that mimics the recoil effect due to graviton emission in the bulk), suggests that the brane would bend around the PBH forcing it to eventually escape into the dark dimension once the two portions of the brane come in contact and reconnect [59–62]. Motivated by this pivotal finding, in this paper we explore the possibility of enriching the dark dimension with bulk black hole dark matter.

The layout is as follows. We begin in Section 2 with a synopsis of PBH formation in standard 4D inflation and after that we lay out the hypotheses for PBH formation within the dark dimension scenario. In Section 3 we reexamine the Hawking evaporation process considering particle emission onto the brane and in the bulk. In Section 4 we estimate the time scale for PBHs escaping the brane in the mass range of interest. Armed with our finding, we reassess limits on the abundance of PBH dark matter by focusing on black holes which could escape from the brane into the bulk, and therefore would avoid constraints from γ-ray experiments. The paper wraps up in Section 5 with some conclusions.

Before proceeding, we pause to note that the dark dimension can be understood as a line interval with end-of-the-world 9-branes in M-theory attached at each end, which is equivalent to a semicircular 11th dimension endowed with $S^1/\mathbb{Z}_2$ symmetry [63]. It is worthy of mention that if the dark dimension is warped then the brane could behave as if the tension were infinite, resulting in the impossibility for PBHs to leave the brane [64].

2. Primordial black hole production

Arguably, the most well-motivated model for PBH production makes use of quantum fluctuations generated during inflation [65–67]. *Planck* measurements of the CMB tightly constrain temperature fluctuations on large scales, because the power spectrum of the comoving curvature perturbation $\mathcal{R}$ is almost scale invariant: $P_{\mathcal{R}}(k) = \mathcal{A}(k/k_0)^{n_s-1}$, with amplitude $\ln(10^{10}\mathcal{A}) = 3.044 \pm 0.014$, spectral index $n_s = 0.965 \pm 0.004$, and pivot scale $k_0 = 0.05~{\rm Mpc}^{-1}$ [68]. However, observational constraints on smaller scales (which cannot be probed by CMB data) are loose enough to allow for large fluctuations, with amplitude of power spectrum $\mathcal{O}(0.01)$ [69]. In this section we review the necessary conditions for obtaining large curvature fluctuations that could ignite PBH formation with appreciable abundance today.

To lay out the foundation of the discussion we first note that the order of magnitude estimate of the black hole mass $M_{\rm BH}$ can be guessed by equating the scaling of the cosmological energy density with time t in the radiation dominated epoch,

$$\rho \sim M_p^2/t^2 \,, \tag{4}$$

to the required density in a region of mass $M_{\rm BH}$ which is able to collapse within its Schwarzschild radius

$$\rho \sim M_p^6/M_{\rm BH}^2 \,. \tag{5}$$

Drawing a connection between (4) and (5) we can see that at production PBHs would have roughly the cosmological horizon mass [57]

$$M_{\rm BH} \sim t M_p^2 \sim 10^{15} \left(\frac{t}{10^{-23}~{\rm s}} \right)~{\rm g} \,. \tag{6}$$

Substituting in (6) representative cosmological time scales we can roughly estimate that black holes produced at the Planck time (10^{-43} s) would have the Planck mass ($M_p \sim 10^{-5}$ g), black holes produced at the QCD epoch (10^{-5} s) would have a solar mass ($1~M_\odot$), and black holes produced at $t \sim 1$ s would have a mass of $10^5 M_\odot$, which is comparable to the mass of the black holes thought to reside in galactic nuclei. The back-of-a-napkin calculation delivering (6) suggests that PBHs could span an enormous mass range. Even though the spectrum of masses of these PBHs remains unspecified, on cosmological scales PBHs could behave like a typical cold dark matter particle.

Taking into account the cosmological expansion, the present day PBH density parameter in units of the critical density $\Omega_{\rm PBH}$ can be related to the initial collapse fraction at redshift z by

$$\beta = \frac{\Omega_{\rm PBH}}{\Omega_r} \, \frac{1}{(1+z)} \sim 10^{-6}~\Omega_{\rm PBH} \, \left(\frac{t}{\rm s} \right)^{1/2} \sim 10^{-18}~\Omega_{\rm PBH} \, \left(\frac{M_{\rm BH}}{10^{15}~{\rm g}} \right)^{1/2} , \tag{7}$$

where $\Omega_r \sim 10^{-4}$ is the present-day density parameter of radiation and in the last step we have used (6) [70]. Note that the $(1+z)$ factor derives from the fact that the radiation density scales as $(1+z)^4$, while the PBH density scales as $(1+z)^3$. This implies that $\beta(M_{\rm BH})$ must be tiny even if PBHs provide all of the dark matter [71].

The criterion for PBH formation is most easily specified in terms of the density contrast $\delta \equiv (\rho - \bar\rho)/\bar\rho$, where ρ is the density of the region and $\bar\rho$ is the average background density. A PBH will form if the density contrast exceeds a critical value, δ_c. The PBH abundance at formation epoch can then be interpreted as the fraction of the universe with regions dense enough to produce PBHs,

$$\beta \equiv \int_{\delta_c}^\infty P(\delta)\, d\delta, \tag{8}$$

where $P(\delta)$ is the probability distribution function, which describes the likelihood that a given fluctuation have an over-density δ, and we assume that a perturbation will collapse to form PBH if its amplitude is larger than a critical value δ_c [72]. It is generally assumed that the density fluctuations have a Gaussian distribution

$$P_G(\delta) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(\delta-\mu)^2/2\sigma^2}, \tag{9}$$

and are spherically symmetric, where μ is the mean and σ^2 is the variance of the distribution. Equating (7) to (8), we can estimate the required mass variance σ^2 of (9) that can give rise to large population of PBH today. Setting $\mu = 0$ and integrating (8) it follows that

$$\beta = \int_{\delta_c}^\infty \frac{d\delta}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\delta^2}{2\sigma^2}\right) = \frac{1}{2}\,{\rm Erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma}\right) \simeq \frac{\sigma}{\sqrt{2\pi}\delta_c} \, \exp\left(-\frac{\delta_c^2}{2\sigma^2}\right), \tag{10}$$

where ${\rm Erfc}(x) = 1 - {\rm Erf}(x)$ is the complementary error function and in the last equality we have assumed $\delta_c \gg \sigma$, which is a good approximation for all practical purposes. Indeed, following [73] we set $\delta_c \sim 0.4$, and so for an all-dark-matter interpretation with $M_{\rm BH} \sim 10^{15}$ g we arrive at $\sigma \sim 0.045$, where we have taken the present-day dark-matter density from Planck's measurements $\Omega_{\rm DM}h^2 \sim 0.120 \pm 0.001$ [68], with a dimensionless Hubble constant $h \sim 0.73$ [74,75].

Now, we can estimate the primordial power spectrum of the curvature perturbation at the time of PBH formation, which is of order the square of the mass variance,

$$\mathcal{P}_{\mathcal{R}}(k_{\rm PBH}) \sim \sigma^2 \,, \tag{11}$$

and so for Gaussian fluctuations the amplitude of the scalar power spectrum needed at scales $k_{\rm PBH}$ relevant for PBH formation is found to be $\mathcal{P}_\mathcal{R}(k_{\rm PBH}) \sim 10^{-3}$ [76,77]. We can then infer that the assumption $\Omega_{\rm PBH} = \Omega_{\rm CDM}$ requires a very large amplification of the curvature spectrum between large CMB and small PBH-formation scales; namely,

$$\Delta\mathcal{P}_\mathcal{R} \equiv \frac{\mathcal{P}_\mathcal{R}(k_{\rm PBH})}{\mathcal{P}_\mathcal{R}(k_{\rm CMB})} \sim 10^6\,. \tag{12}$$

Noticeably, one- and two-field slow-roll inflationary dynamics could actually generate a peak in the curvature power spectrum [78–81]. This is possible if the effective dynamics of the inflation field presents a near-inflection point which slows down the field right before the end of inflation. This simple and quite generic feature of the potential makes the inflation enter into an ultra-slow-rolling stage during a short range of e-folds that gives rise to a prominent spike in the fluctuation power spectrum at scales much smaller than those probed by CMB experiments. Concerns were raised regarding one-loop corrections to the large power spectrum in single-field inflation [82,83]. However, these corrections have been found to be negligible when the transition from the ultra-slow-roll to the slow-roll phase is smooth [84–87].[1]

An alternative proposal for PBH production within slow-roll inflation, advocates for nucleation of bubbles of metastable vacuum during a time interval that is short compared to the inflationary Hubble time [95]. These bubbles would then expand exponentially during inflation to super-horizon size, and later collapse into black holes when the expansion of the universe is decelerating. As a consequence, even though $M_{\rm BH}$ is exponentially sensitive to the moment bubbles form during inflation, the resulting PBH mass spectrum can be nearly monochromatic. If bubble nucleation occurs near the middle of inflation, $M_{\rm BH}$ can fall in the range given in (3), in which PBHs could make all cosmological dark matter.

The preceding discussion has been framed within the context of standard 4D inflation. This implies that the size of the dark dimension during the inflationary epoch should have been much smaller than a micron (most likely at M_*^{-1}); otherwise, as we observed in [39], the dark dimension scenario would be inconsistent with the Higuchi bound [9]. On the other hand, to have 5D black hole formation and evaporation, one should also assume that the size of the dark dimension became much bigger by some 4D potential that was created immediately after the end of inflation. Alternatively, it is always possible that the PBHs are produced in the bulk to start with. This situation will be more appealing within the proposal introduced elsewhere [39,47,56], in which we postulated that the dark dimension may have undergone a uniform rapid expansion, together with the three-dimensional non-compact space, by regular exponential inflation driven by an (approximate) higher dimensional cosmological constant.[2] If this were the case, then primordial fluctuations during inflation of the compact space could lead to the production of black holes in the bulk. An investigation along these lines is obviously important to be done, but it is beyond the scope of this paper. Herein, we proceed under the hypothesis that 5D PBHs could be formed within the dark dimension scenario. Shortly after the PBHs are formed they experience Hawking evaporation. It is this that we now turn to study.

3. Hawking evaporation

In the mid-70's Hawking pointed out that a black hole emits thermal radiation as if it were a black body, with a temperature inversely proportional to its mass [96,97]. However, in the neighborhood of the horizon the black hole produces an effective potential barrier that backscatters part of the emitted radiation, modifying the thermal spectrum. The so-called "greybody factor" $\sigma^{(s)}(\omega)$, which controls the black hole absorption cross section, depends upon the spin of the emitted particles s, their energy ω, and $M_{\rm BH}$ [98–100]. In this section we carry out a numerical analysis of the evaporation of black holes perceiving the dark dimension to determine the emission rate onto the brane and in the bulk.

We assume that the black hole is spherically symmetric and can be treated as a flat $(4+n)$ dimensional object. This assumption is valid for extra dimensions that are larger than the $(4+n)$-dimensional Schwarzschild radius

$$r_s = \frac{1}{M_*}\left[\frac{M}{M_*}\right]^{1/(n+1)}\left[\frac{8\,\Gamma\left(\frac{n+3}{2}\right)}{(n+2)\,\pi^{(n+1)/2}}\right]^{1/(n+1)}, \tag{13}$$

where $\Gamma(x)$ is the Gamma function [101–103]. The higher-dimensional Schwarzschild black hole behaves like a thermodynamic system [104], with temperature $T_H \sim (n+1)/(4\pi r_s)$ and entropy $S = (4\pi M_{\rm BH} r_s)/(n+2)$ [105].

The emission rate of particles of spin s is found to be

$$\frac{dN^{(s)}}{dt} = \sigma^{(s)}(\omega)\,\left[\exp\left(\frac{\omega}{T_H}\right) - (-1)^{2s}\right]^{-1}\,\frac{d^{d-1}k}{(2\pi)^{d-1}}\,, \tag{14}$$

where k is the $(d-1)$-momenta of the particle living in d dimensions, and

$$\begin{aligned}\sigma^{(s)}(\omega) &= \sum_l \frac{2^{n+1}\,\pi^{(n+1)/2}\,\Gamma\left(\frac{n+3}{2}\right)}{\omega^{n+2}\,n!}\,\frac{(2l+n+1)\,(l+n)!}{l!}\,|\mathcal{A}_l^{(s)}(\omega)|^2\\ &= \frac{2^n}{\pi}\,\frac{\left\{\Gamma\left(\frac{n+3}{2}\right)\right\}^2}{(\omega r_s)^{n+2}}\,A_{\rm hor}\,\mathcal{N}_l\,|\mathcal{A}_l^{(s)}(\omega)|^2\,,\end{aligned} \tag{15}$$

and where

$$\mathcal{N}_l = \frac{(2l+n+1)\,(l+n)!}{l!\,(n+1)!}\,, \tag{16}$$

is the multiplicity of states corresponding to the same partial wave l,

$$\begin{aligned}A_{\rm hor} &= r_s^{n+2}\int_0^{2\pi} d\varphi \prod_{k=1}^{n+1}\int_0^{\pi} \sin^k\theta_{k+1}\,d\theta_{k+1} = 2\pi\,r_s^{n+2}\prod_{k=1}^{n+1}\sqrt{\pi}\,\frac{\Gamma(\frac{k+1}{2})}{\Gamma(\frac{k+2}{2})}\\ &= 2\pi\,r_s^{n+2}\,\frac{\pi^{(n+1)/2}}{\Gamma\left(\frac{n+3}{2}\right)}\end{aligned} \tag{17}$$

is the horizon area of the $(4+n)$-dimensional black hole, and $\mathcal{A}_l^{(s)}(\omega)$ the absorption coefficient [106]. Note that the number of dimensions d in which a particular field lives should not be confused with dimensionality of spacetime $(4+n)$. For massless particles, $|k| = \omega$, and the phase-space integral reduces to an integral over the energy of the emitted particle ω. For massive particles, $|k|^2 = \omega^2 - m^2$, which implies that for a black hole to emit a particle of mass m, its temperature $T_H \geq m$.

Bearing this in mind, the evaporation rate of a given particle species of spin s in the frequency window $(\omega, \omega + d\omega)$ is given by [107]

$$\frac{d^2N^{(s)}}{dt\,d\omega} = \frac{1}{2\pi}\sum_l \mathcal{N}_l\,|\mathcal{A}_l^{(s)}(\omega)|^2\,\left[\exp\left(\frac{\omega}{T_H}\right) - (-1)^{2s}\right]^{-1}. \tag{18}$$

The absorption coefficients for different particle species have been calculated in [108,109] and for emission of particles on the brane, in the low frequency limit $\omega r_s \ll 1$, are given by

$$|\mathcal{A}_l^{(0)}(\omega)|^2 = \frac{16\pi}{(n+1)^2}\left(\frac{\omega r_s}{2}\right)^{2l+2}\frac{\left\{\Gamma(\frac{l+1}{n+1})\right\}^2\,\left\{\Gamma(1+\frac{l}{n+1})\right\}^2}{\left\{\Gamma(\frac{1}{2}+l)\right\}^2\,\left\{\Gamma(1+\frac{2l+1}{n+1})\right\}^2}\,, \tag{19}$$

$$|\mathcal{A}_l^{(\frac{1}{2})}(\omega)|^2 = \frac{2\pi\,(\omega r_s)^{2l+1}\,2^{-(4l+2)/(n+1)}}{2^{2l}\,\{\Gamma(l+1)\}^2}\,, \tag{20}$$

[1] For additional input on the one-loop correction from the short modes onto the large-scale power spectrum, see [88–94].

[2] Note that the amplitude of the power spectrum observed in the branes should be suppressed by the bulk volume in order to convert M_* to 4D M_p.

and

$$|A_l^{(1)}(\omega)|^2 = \frac{(2\omega r_s)^{2+2l}}{(n+1)^2} \frac{\left\{\Gamma\left(\frac{l}{n+1}\right)\right\}^2 \left\{\Gamma\left(\frac{l+1}{n+1}\right)\right\}^2 \{\Gamma(l+2)\}^2}{\left\{\Gamma\left(\frac{2l+1}{n+1}\right)\right\}^2 \{\Gamma(2l+2)\}^2}\,. \quad (21)$$

The absorption coefficient for graviton emission in the bulk is estimated to be

$$|\mathcal{A}_l^{(s)}(\omega)|^2 = 4\pi \left(\frac{\omega r_H}{2}\right)^{2l+n+2} \frac{\left\{\Gamma\left(1+\frac{l}{n+1}-G\right)\right\}^2 \left\{\Gamma\left(1+\frac{l}{n+1}+G\right)\right\}^2}{\left\{\Gamma\left(l+\frac{n+3}{2}\right)\right\}^2 \left\{\Gamma\left(1+\frac{2l}{n+1}\right)\right\}^2}\,, \quad (22)$$

where

$$G = \frac{1}{2(n+1)} \sqrt{(n+2)^2 - \frac{q+\tilde{p}+\tilde{w}}{[2\,m+(n+2)(n+3)]^2}} \quad (23)$$

for $s = 0$, and

$$G = \frac{(1+k)(n+2)}{4(n+1)} \quad (24)$$

for $s = 1$ and $s = 2$, with $m \equiv l(l+n+1) - n - 2$, $q \equiv (n+2)^4(n+3)^2$, $z \equiv 16m^3 + 4m^2(n+2)(n+4)$, $p \equiv (n+2)(n+3)\left[4m(2n^2+5n+6) + n(n+2)(n+3)(n-2)\right]$, $w \equiv -12m(n+2)\,[m(n-2) + n(n+2)(n+3)]$, $\tilde{p} = p - z(n+2)^2(n+3)^2/(4m^2)$, and $\tilde{w} = w - z(n+2)(n+3)/m$ [110].

In Fig. 1 we show the evaporation rate for particles of spin s from PBHs perceiving the dark dimension, considering benchmark black hole masses. One can check by inspection that for large values of ω, the differences in emission rate for all brane localized species become smaller, as one would expect because in the high frequency limit, the greybody factors for all species approach the geometric optics limit. The curves have been truncated at the frequency in which the low frequency approximation, adopted to derive the greybody factors, breaks down. Note that the radiation of particles with higher spin is suppressed at low frequencies because of the larger barrier such particles have to surmount. Note also that $d^2N^{(s)}/dt\,d\omega$ depends only weakly on $M_{\rm BH}$ and M_*. For a comparison with previous estimates of the particle emission rate, see Appendix.

4. Filling the dark matter void with bulk black holes

In this section, in line with our stated plan, we turn to revise previous estimates of the fraction of dark matter that could be composed of PBHs. Higher-dimensional black holes radiate Standard Model fields and gravitons on the brane as well as gravitons into the bulk. The total power emitted in both channels scales as

$$-\frac{dM_{\rm BH}}{dt} \sim \kappa_n\, T_H^{n+4}\, r_s^{n+2} \sim \kappa_n\, T_H^2\,, \quad (25)$$

where κ_n encodes the effect of greybody factors [111]. As can be seen in Fig. 1, κ_n depends only weakly on T_H.

A point worth noting at this juncture is that during the process of black hole evaporation $M_{\rm BH}$ decreases and T_H rises. In what follows, we adopt a quasi-stationary approach to the evaporation process, which implies that the black hole has time to come into equilibrium at each new temperature before the next particle is emitted. We also rely on the semi-classical assumption of self-similarity; namely, we assume that in the course of evaporation, a black hole gradually shrinks in size while maintaining the standard semi-classical relations between its parameters, such as its mass, the Schwarzschild radius, and the Hawking temperature. Note that this is a very strong assumption, because we will estimate the evaporation rate exclusively for a black hole of a fixed radius. With this assumption in mind, (25) can be solved for the lifetime

$$\tau_{\rm BH} \sim \frac{1}{\kappa_n\, M_*} \left(\frac{M_{\rm BH}}{M_*}\right)^{(n+3)/(n+1)}\,. \quad (26)$$

Now, the recoil effect due to graviton emission imparts the black hole a relative kick velocity v with respect to the brane, allowing the PBH to escape into the bulk. The time scale of the escape of a black hole is estimated to be [61]

$$\tau_{\rm esc} \sim \frac{1}{\kappa_n} \frac{r_s}{v}\,. \quad (27)$$

If the origin of the recoil is the inhomogeneous emission of particles during black hole evaporation, a rough estimate of the kick velocity gives

$$v \sim \frac{f_{M_{\rm BH}}}{\sqrt{N_{\rm emitted}}}\,, \quad (28)$$

where $f_{M_{\rm BH}}$ is the fraction of the evaporated mass and $N_{\rm emitted} \sim f_{M_{\rm BH}} M_{\rm BH}/T_H$ is the number of emitted particles [112]. Substituting (13) and (28) into (27) it follows that

$$\tau_{\rm esc} \sim \frac{1}{\kappa_n\, \sqrt{f_{M_{\rm BH}}}\, M_*} \left(\frac{M_{\rm BH}}{M_*}\right)^{(n+4)/(2n+2)}\,. \quad (29)$$

Now, $f_{M_{\rm BH}}$ at $\tau_{\rm esc}$ is estimated to be $f_{M_{\rm BH}} \sim \tau_{\rm esc}/\tau_{\rm BH}$, and so we arrive at

$$\tau_{\rm esc} \sim \frac{1}{\kappa_n\, M_*} \left(\frac{M_{\rm BH}}{M_*}\right)^{(2n+7)/(3n+3)} \quad (30)$$

and

$$f_{M_{\rm BH}} \sim \left(\frac{M_*}{M_{\rm BH}}\right)^{(n+2)/(3n+3)}\,. \quad (31)$$

Taking $n = 1$ and $M_* \sim 10^{10}$ GeV, we can conclude that PBH of $M_{\rm BH} \sim 10^{15}$ g would almost immediately escape the brane.

To a very good approximation we can then assume the black holes will only emit gravitons in the bulk, and so the semi-analytic studies carried out in [113,114] yield $\kappa_1 \sim 0.06$.[3] Now, setting $n = 1$, $M_* \sim 10^{10}$ GeV, and considering the age of the universe of 13.8 Gyr [68] we can parametrize (26) as

$$\tau_{\rm BH} \sim 13.8 \left(\frac{\kappa_1}{0.06}\right)^{-1} \left(\frac{M_{\rm BH}}{10^{11.37}\ {\rm g}}\right)^2 \ {\rm Gyr}\,. \quad (32)$$

Because PBHs evaporate almost entirely into gravitons in the bulk, the constraints from gamma-ray observations [57,58] can be safely neglected. Taking into account the systematic error associated with the order of magnitude uncertainty in the species scale $10^9 \lesssim M_*/{\rm GeV} \lesssim 10^{10}$, we conclude that the mass range for PBHs to make all the cosmological dark matter is then

$$10^{11} \lesssim M_{\rm BH}/{\rm g} \lesssim 10^{21}\,. \quad (33)$$

Note that the minimum $M_{\rm BH}$ required for bulk PBHs to make all cosmological dark matter is in agreement with the estimate derived in [53] using scaling arguments.

In closing, we stress that if there were 5D primordial near-extremal black holes in nature, then it would be possible to lower the minimum mass allowing a PBH all-dark-matter interpretation, because near-extremal black holes are colder and longer-lived [50]. Note, however, that near-extremal black holes cannot escape from the brane into the bulk since they carry electromagnetic charge. It is also important to stress that if PBHs experience the memory burden effect [117,118], then the minimum $M_{\rm BH}$ allowing a 5D PBH all-dark-matter interpretation can also be relaxed [119–122]. Note that the limits from γ-ray observations adopted in [53] to constrain this possibility would be evaded if PBHs escape from the brane.

[3] This semi-analytic estimate is in agreement with those derived in [115,116].

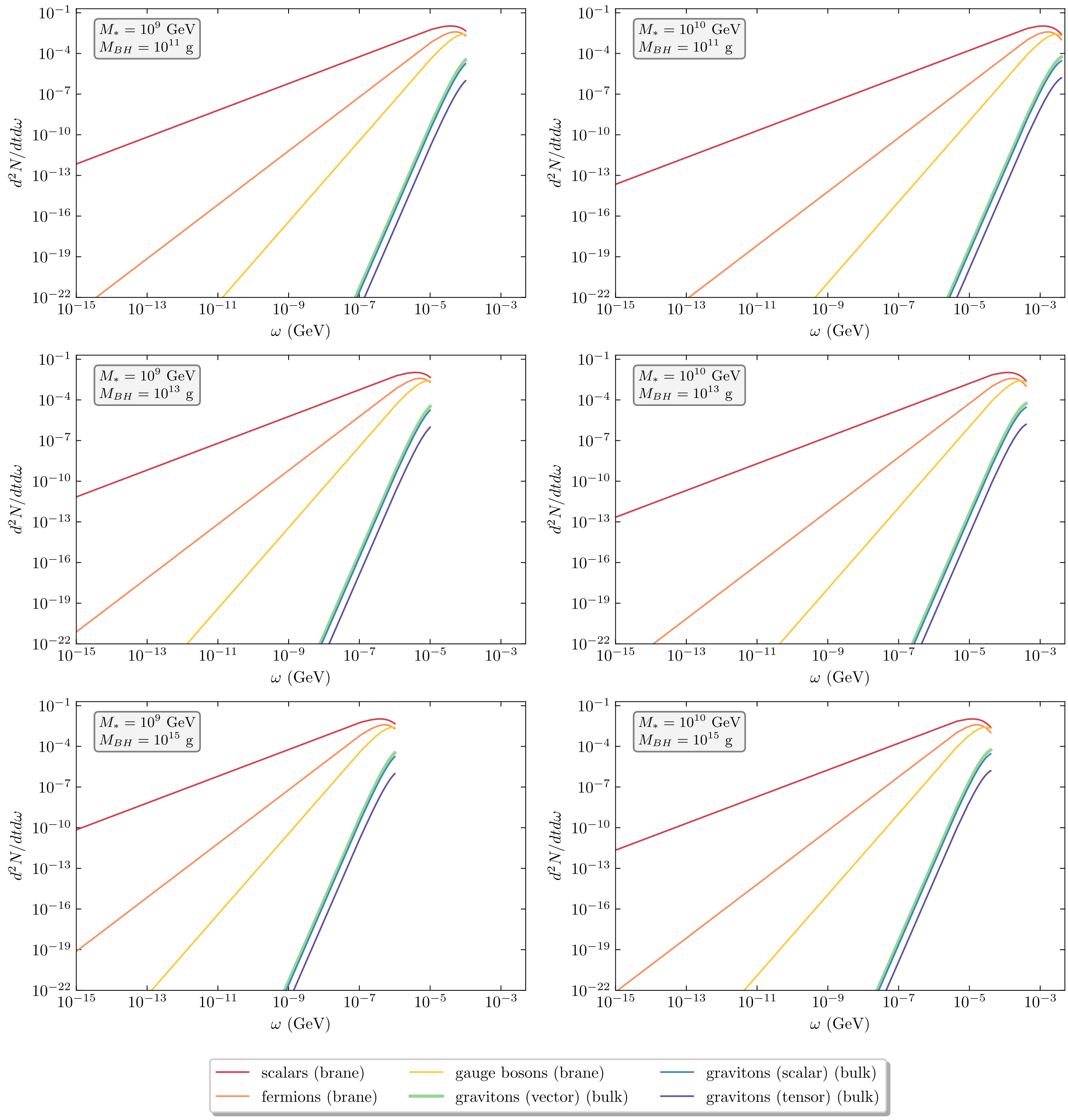

Fig. 1. Evaporation rate for particles of spin s. We have taken $n = 1$, a species scale $M_* = 10^9$ GeV (left) and $M_* = 10^{10}$ GeV (right), considering from top to bottom $M_{\rm BH}/{\rm g} = 10^{11}, 10^{13}, 10^{15}$.

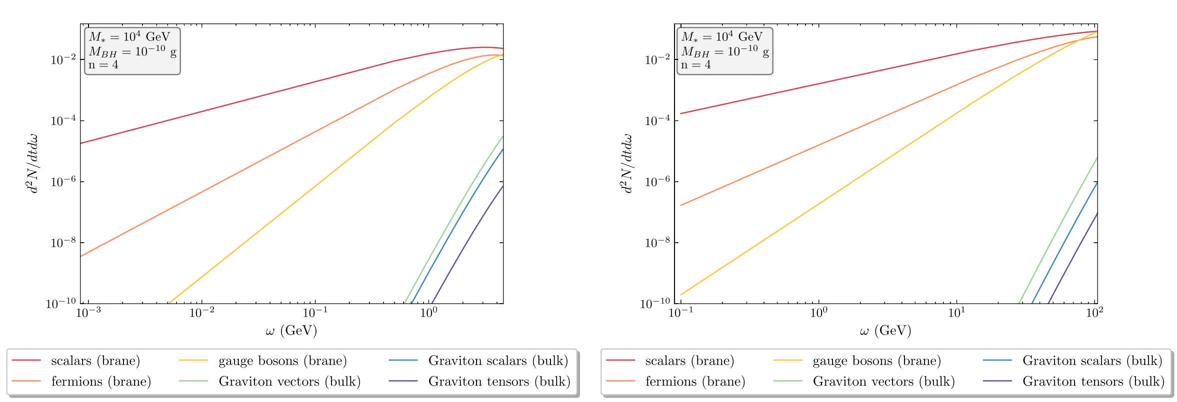

Fig. 2. Particle emission rate of Schwarzschild, near extremal, and quantum black holes; for $d = 4$ (left) and $d = 5$ (right).

5. Conclusions

We have reexamined the Hawking evaporation process of PBH perceiving the dark dimension. We have shown that PBHs are expected to escape from the brane almost instantaneously. Motivated by this essential finding, we have revised the allowed mass range of PBHs that could assemble all cosmological dark matter. We found that a PBH all-dark-matter interpretation would be possible in the mass range $10^{11} \lesssim M_{\rm BH}/{\rm g} \lesssim 10^{21}$.

We end with an observation. Complementary to the PBHs, it was observed in [38] that the universal coupling of Standard Model fields to the massive spin-2 KK excitations of the graviton in the dark dimension provides an alternative dark matter contender. The cosmic evolution of the dark graviton gas is primarily dominated by "dark-to-dark" decays, yielding a specific realization of the dynamical dark matter framework [123]. A remarkable close relation between PBHs and the dark gravitons has been pointed out in [37]. We note that the ideas discussed in this paper make more noticeable the PBH $\leftrightharpoons$ dark graviton gas connection, as in both scenarios the cosmological evolution of the dark matter sector would take place mostly in the bulk. Actually, PBHs originally formed on the brane could act as a source of KK gravitons. Note that for $M_{\rm BH}$ in the range of interest the Hawking temperature,

$$T_H \sim \frac{1}{r_s} \sim \left(\frac{M_{\rm BH}}{10^{12}\ {\rm g}}\right)^{-1/2}\ {\rm MeV}\,, \tag{34}$$

is consistent with experimental constraints on the natural scale in which the KK gravitons on the dark dimension are generated with enough abundance [38,46,48].

CRediT authorship contribution statement

Luis A. Anchordoqui: Conceptualization, Investigation, Writing – original draft. **Ignatios Antoniadis:** Conceptualization, Investigation, Writing – original draft. **Dieter Lüst:** Conceptualization, Investigation, Writing – original draft. **Karem Peñaló Castillo:** Conceptualization, Investigation, Writing – original draft.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Luis Anchordoqui reports financial support was provided by National Science Foundation. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

In this Appendix we compare our estimates of $d^2N^{(s)}/dtd\omega$ with those obtained in [111] for $n = 2$ and $n = 4$. In Fig. 2 we show the instantaneous emission of species on the brane and in the bulk for two and four extra dimensions, with $M_{\rm BH} = 10^{-10}$ g and $M_* = 10$ TeV. One can check by inspection of Fig. 2 and Fig. 1 in [111] that there is good agreement between the two estimates.

Data availability

No data was used for the research described in the article.

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