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Airfoil Oscillator System: Data-Driven System Identification

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With the computational resources becoming available, data-driven methods have emerged as powerful means for equation discovery and model construction. Sparse regression methods such as SINDy (Sparse Identification for Nonlinear Dynamical Systems) can be used for developing reduced-order models of nonlinear systems. In this study, the authors examine how SINDy can be used for developing low-dimensional models for airfoil systems, which experience unsteady aerodynamic loads and flutter instabilities. For a system of multiple closely spaced airfoil oscillators, analytical models are not readily available to determine flutter instabilities and one has to take recourse to experimental and numerical means. In this work, as a starting point, data collected through simulations of unsteady aerodynamics of a single airfoil oscillator system are considered and a reduced-order model is constructed based on this data.

I. Nomenclature

$\mathbf{x}$	:	vector of state variables
$\dot{\mathbf{x}}$	:	time derivative of state variables
$\boldsymbol{\Theta}$	:	data matrix for a library of nonlinear functions
$\boldsymbol{\xi}$	:	vector of weights for the user-defined nonlinear functions
λ	:	sparsity promoting parameter
U_∞	:	freestream speed
h, θ	:	plunge and pitch degrees of freedom
b, L	:	semi chord length and span length of the wing
m, m_f, I_p	:	mass per unit length of wing, fixture mass. and moment of inertia about the mounting point
x_θ	:	distance between the center of mass and the mounting location normalized by the length b
k_h, k_θ	:	plunge stiffness and pitch stiffness coefficients
d_h, d_θ	:	plunge damping and pitch damping coefficients
c_L, L	:	aerodynamic lift coefficient and aerodynamic lift
c_M, M	:	pitching moment coefficient and pitching moment coefficient
ρ_∞	:	air density

II. Introduction

Flutter oscillations produced in elastic bodies with airfoil cross-sections can be used for wind energy harvesting. This harvested energy can be used for running low power electronic equipment in remote and inhospitable locations. Analytical models of a single airfoil are reasonably well established to carry out nonlinear instability analysis. However, analytical models for multiple closely spaced airfoil oscillators, which can be used for nonlinear instability analysis are not readily available. In the case of such systems, numerical simulations have shown that limit cycle oscillations can be produced at freestream speeds lower than the critical flutter speed observed in the case of a system with a single airfoil oscillator [1]. For applications such as energy harvesting, this phenomenon can be leveraged for substantial benefits. Multiple airfoil oscillators can not only be used to improve the overall power output of the system but also extend the range of freestream speeds over which the systems can be operated. Reduced-order models can provide insights into the nature of aerodynamic coupling between the considered airfoil oscillators and also help explain why a drop in the critical flutter speed occurs for closely spaced airfoil oscillators. Indeed, by using a potential flow framework, one can show that the aerodynamic interactions between bound vortex sheets vary with respect to the inter-oscillator spacing in an inverse squared manner. Hence, nonlinear variations are expected to play a role. In this work, the authors use the

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simulation data to examine the system behavior by developing reduced-order models using sparse nonlinear system identification tools. The used methodology is discussed in the following sections.

III. Nonlinear System Identification

In recent years, data-driven methods have emerged as a powerful tool for model construction. By taking advantage of the available high performance computing capabilities, large quantitites of scientific data can be used to develop an understanding of the dynamics of a physical system, in particular, for those systems whose dynamics has not been previously well understood. To address this, system identification of nonlinear aerodynamic systems is undertaken. In particular, the Sparse Identification of Nonlinear Dynamical Systems (SINDy) paradigm is used [2]. This paradigm may be described by using the following equations:

$$\dot{\mathbf{x}} = f(\mathbf{x}) \tag{1}$$

$$f(\mathbf{x}) \approx \sum_{i=1}^{N} \Theta_i(\mathbf{x})\xi_i \tag{2}$$

$$\boldsymbol{\xi} = \underset{\bar{\xi}}{\operatorname{argmin}} \frac{1}{2}||\dot{\mathbf{x}} - \boldsymbol{\Theta}\,\bar{\boldsymbol{\xi}}||_2^2 + \lambda||\bar{\boldsymbol{\xi}}||_1 \tag{3}$$

For a dynamical system described by Eq. (1), having obtained data for the time derivatives of the system states, $\dot{\mathbf{x}}$, an approximate expression for $f(\mathbf{x})$, is sought. Initially, a linear approximation for $f(\mathbf{x})$ as prescribed in Eq. (2) can be considered. Here, bold letters refer to vectors and bold capitalized letters refer to matrices. The various nonlinear functions, $\Theta_i(\mathbf{x})$, are stacked in columns of the matrix, $\boldsymbol{\Theta}$, and these functions are weighted by the unknown coefficients, ξ_i. The choice of candidate functions is flexible and can be determined by the user. After using the above-mentioned approximation for the dynamical system, a sparse solution is sought for the coefficients $\bar{\boldsymbol{\xi}}$. To this end, a least squares minimization problem with an ℓ_1-norm regularized cost function is set up as shown in Eq. (3).

$$\begin{aligned} \dot{\mathbf{x}} &= f(\mathbf{x}; U_\infty) \\ \dot{U}_\infty &= 0 \end{aligned} \tag{4}$$

Aeroelastic flutter is an example of a physical manifestation of the Hopf bifurcation, wherein a fixed point solution of the dynamical system under consideration experiences a loss of stability as a control parameter such as the flow speed U_∞ is varied and a limit cycle motion follows [3]. Hence, the desired reduced-order dynamical models would need to include not just the effect of changes in the state variables but also account for the dependence on parameters such as the freestream speed as well. To incorporate this effect in the system identification, the extended dynamical system can be considered as shown in Eq. (4). Here, the SINDy framework has been used to obtain reduced-order models of dynamical systems, whose solutions may undergo Hopf bifurcations.

For systems having implicit dynamics and rational function nonlinearities, the above sparse regression problem can be reformulated into the form [4]

$$\boldsymbol{\Theta}(\mathbf{x}, \dot{\mathbf{x}})\,\boldsymbol{\xi} = 0 \tag{5}$$

where it is of interest to find the sparsest vector $\boldsymbol{\xi}$ for any given dataset for a dynamical system. The implicit formulation, also known as the SINDy-PI algorithm, is needed for modeling unsteady aerodynamic loads experienced by airfoil shaped bodies.

IV. Results

In order to leverage data-driven methods, there is an imperative need to collect high quality data. To generate the raw data for this process, numerical simulations are carried out using an Unsteady Vortex Lattice Method (UVLM) based formulation. The stable fixed point solutions obtained at freestream speeds lower than the critical freestream speed and limit cycles obtained at freestream speeds higher than the critical flutter speed are recorded. The pitch and plunge response states along with the lift and moment coefficients are recorded. An important advantage of using numerical simulation data versus experimental data is that accurate observations of the time derivatives of the state variables, $\mathbf{x}$

can be obtained and the introduction of noise in the data can be limited. At freestream speeds approximately equal to the critical flutter speed, the pitch oscillation amplitudes are expected to be low for supercritical flutter. This would possibly preclude any possibilities for flow separation, and hence, the UVLM scheme is expected to suffice. A two degree-of-freedom airfoil oscillator as shown in Figure 1 is considered. The structural parameters of the airfoil oscillator system considered for the above system are listed in Table 1. The parameters are the same as the airfoil oscillators used in the study by Roccia *et al.* [1] with the electromechanical terms ignored. Fluid-structure interaction (FSI) simulations were carried out by quasi-statically varying the freestream speeds in the range of 8.81-9.30 m/s in steps of 0.01 m/s. For the considered airfoil oscillator system, the flutter speed was found to be 8.94 m/s. The data from the FSI simulations constituted the training data used for predicting the expressions for the aerodynamic lift and moment co-efficients. With the fifty samples of inflow speed that were considered, 40 cases were used to train the model and 10 cases were used for evaluating the errors made in the predictions from the data-driven model.

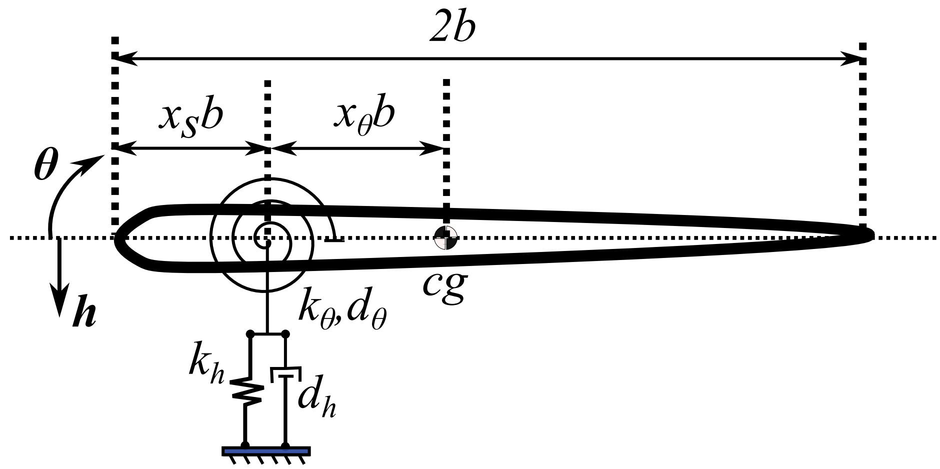


Fig. 1 Model of a two degree-of-freedom airfoil oscillator that can undergo pitch and plunge motions.

Table 1 Table of design parameters

Parameter	Values
m	1.7799 kg/m
m_f	2.8425 kg/m
I_p	7.0645×10^{-3} kgm
b	0.1250 m
x_θ	0.2600
L	0.5000 m
k_h	4.6808×10^{3} N/m^2
k_θ	1.67540 N/rad
d_h	9.6110×10^{-1} Ns/m^2
d_θ	1.32504×10^{-2} Ns

A. Library selection

The primary objective of the study at this stage is to use system identification tools for deriving closed-form expressions for aerodynamic loads by using dynamic response data derived from numerical simulations. For this purpose a library of test functions is required. To that end, based on established expressions for aerodynamic loads at the flutter boundary, the aerodynamic lift and moment coefficients, c_L and c_M, are assumed to be a function of the pitch and plunge degrees of freedom and the freestream speed, as shown in the equations that follow.

$$c_L = f(h, \dot{h}, \ddot{h}, \theta, \dot{\theta}, \ddot{\theta}, U_\infty) \tag{6}$$

$$c_M = f(h, \dot{h}, \ddot{h}, \theta, \dot{\theta}, \ddot{\theta}, U_\infty) \tag{7}$$

A library of test functions, consisting of linear pitch-plunge deformation, velocity and acceleration terms, $(.)_{h,\theta}$, and nonlinear terms consisting of products of the aforementioned linear terms and the freestream speed terms U_∞ and U_∞^2 is established as shown in the equation provided next.

$$\mathbf{\Theta} = [(.)_{h,\theta},\ (.)_{h,\theta}U_\infty,\ (.)_{h,\theta}U_\infty^2] \tag{8}$$

B. Data-driven system identification

The system identification is carried out by adjusting the sparsity knob parameter λ in Eq. (3). This parameter is used to control the system sparsity. A low value of the parameter λ leads to the inclusion of a high number of nonlinear terms in the resulting dynamical system identified by using SINDy. For each value of the sparsity parameter, the accuracy of the identified system is examined by comparing the model prediction with test data. The variation of the error with respect to the sparsity parameter is shown in Figure 2. It is seen that an increase in the sparsity resulted in reduced accuracy of the identified system. This possibly implies that a high number of terms in the library of functions are required to capture the dynamics of the system and that the library of functions that was used was well adapted for the considered data.

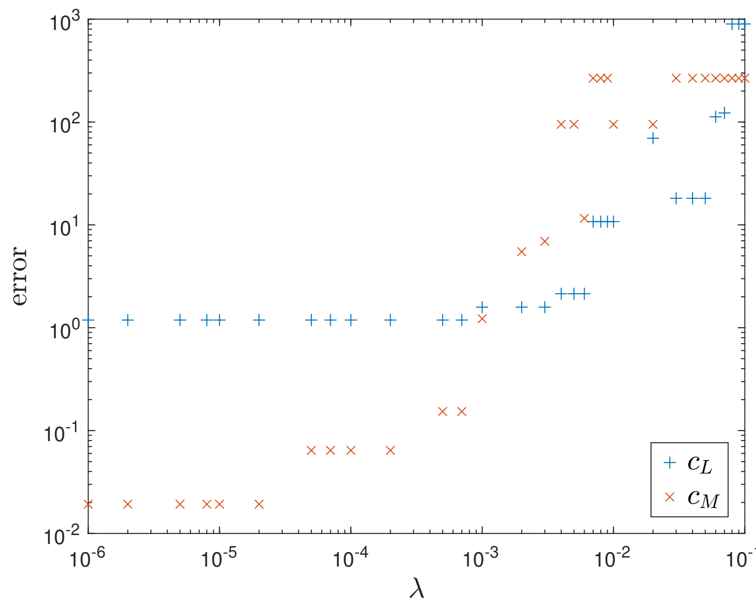


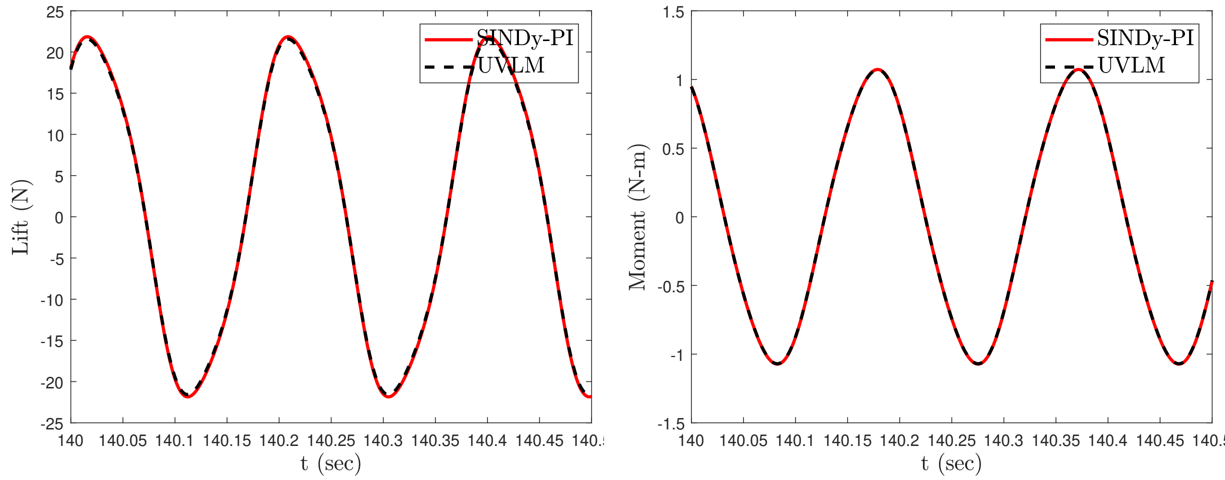
Fig. 2 Error variations observed with respect to hyperparameter λ for expressions obtained using SINDy-PI [4].

The expressions for the lift and moment coefficients, which had lowest errors associated with them, were found to be the following:

$$c_L = 0.213\ddot{h} + 0.13\ddot{\theta} + 5.160 \times 10^{-2}U_\infty\ddot{h} - 2.046 \times 10^{-2}U_\infty\ddot{\theta} + 1.993U_\infty\dot{h} + 0.305U_\infty\dot{\theta} - 3.905 \times 10^{-3}U_\infty^2\ddot{h} + 7.389 \times 10^{-4}U_\infty^2\ddot{\theta} - 0.216U_\infty^2\dot{h} - 3.317 \times 10^{-2}U_\infty^2\dot{\theta} + 4.303U_\infty^2 h + 4.375 \times 10^{-3}U_\infty^2\theta - 1.731 \times 10^{-6} \tag{9}$$

$$c_M = 0.107\ddot{h} + 9.145\times10^{-3}\ddot{\theta} - 1.523\times10^{-2}U_\infty\ddot{h} - 1.156\times10^{-3}U_\infty\ddot{\theta} - 5.108\times10^{-2}U_\infty\dot{h} - 2.495\times10^{-3}U_\infty\dot{\theta} + 5.991\times10^{-4}U_\infty^2\ddot{h} + 4.351 \times 10^{-5}U_\infty^2\ddot{\theta} + 5.594 \times 10^{-3}U_\infty^2\dot{h} + 3.212 \times 10^{-4}U_\infty^2\dot{\theta} + 1.742 \times 10^{-3}U_\infty^2 h - 6.431 \times 10^{-3}U_\infty^2\theta \tag{10}$$

In Figure 3, the aerodynamic lift and moment coefficients obtained by using the abovementioned expressions are compared with the aerodynamic lift and moment coefficients obtained from the raw data at a freestream speed of $U_\infty = 9.2\ m/s$. The test data is seen to compare well with the results from the expressions identified by using the data-driven approach. The raw test data used along with the results from the abovementioned expressions for lift and moment are used to obtain the plots shown in red in Figure 3. Here, aerodynamic lift, (L), and pitching moment, (M), are defined as $L = c_L b \rho_\infty U_\infty^2$ and $M = 2c_M b^2 \rho_\infty U_\infty^2$.

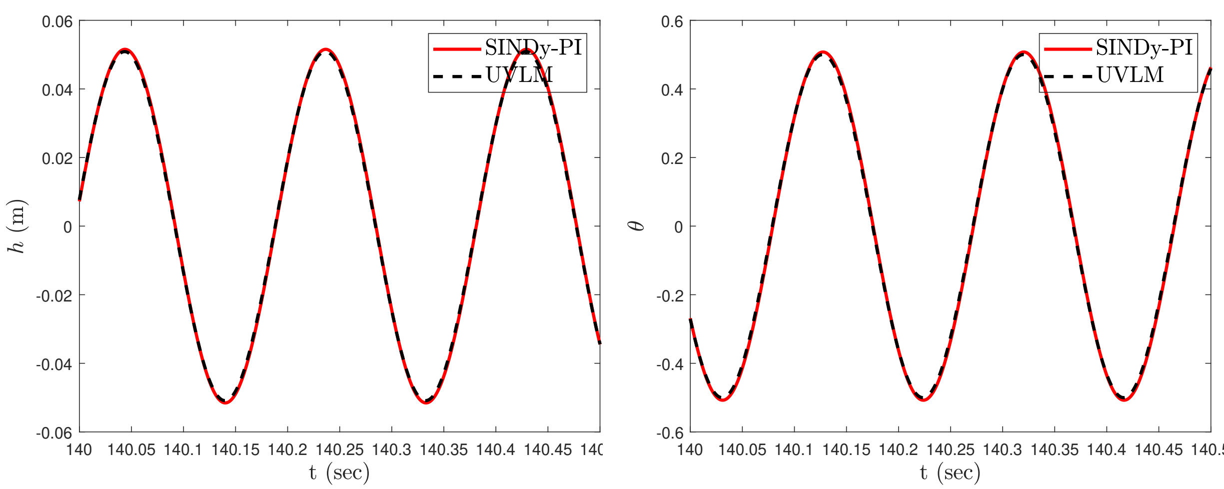


((a)) ((b))

Fig. 3 (a) Aerodynamic lift variation and (b) pitching moment variation obtained by using data-driven system identification and FSI simulations.

Moving forward, it is of interest to find out whether the expressions found earlier can be used as a generalized expression for the aerodynamic lift and pitching moments. For this purpose, the airfoil oscillator systems are numerically integrated, with the assumption that the structural properties of the airfoil oscillator system, as prescribed in Eq. (11), are known.

$$
\begin{aligned}
(m + m_f)\ddot{h} + m x_\theta b \ddot{\theta} + d_h \dot{h} + k_h h = L \\
m x_\theta b \ddot{h} + I_p \ddot{\theta} + d_\theta \dot{\theta} + k_\theta \theta = M
\end{aligned} \tag{11}
$$



((a)) ((b))

Fig. 4 (a) Plunge response and (b) pitch response obtained using data driven system identification and using FSI simulations.

For the purpose of integration, the implicit numerical integration scheme, *ode15i* in MATLAB, was used. When the analytical expressions for the aerodynamic lift and moment presented in Eqs. (9) and (10), were used for numerical integration, numerical instabilities were encountered. However, if the system is integrated by using the aerodynamic loads plotted in Figure 3, the response was seen to closely match the response obtained using the numerical simulations, as shown in Figure 4. This possibly is a result of an over-fitted system, wherein small deviations in the evaluated aerodynamic loads from the actual aerodynamic loads obtained by using the numerical simulations result in the progressive growth of errors in the system response.

V. Closure

The applicability of the SINDy-PI framework for the purpose of developing reduced-order models for unsteady aerodynamic loads acting on an airfoil oscillator has been explored in this work. The sparse regression paradigm was used to to obtain nonlinear analytical expressions for the aerodynamic loads. Given a set of raw dynamical response data, the identified model was shown to accurately predict the aerodynamic loads. However, while accurate for the data sets used to build the model, the predictions from this model may be limited due to overfitting. Nevertheless, the analytical expressions obtained earlier can be used as a starting point towards further refinement and development of reduced-order models.

Acknowledgments

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