Chapter 11

Orienting to Student Sense-Making:

Using Simulations to Support the Development of Equitable Mathematics Teaching

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ABSTRACT

This chapter reports on work from a decade-long project to develop and study the use of teaching simulations focused on the teaching practices of eliciting and interpreting student thinking to support preservice teachers' (PSTs') learning. The chapter describes how teaching simulations focused on these practices allow teacher educators to support PSTs in orienting to student sense-making that is at the heart of equitable mathematics instruction. The teaching simulation approach is described. Examples illustrate how the approach is designed and facilitated in ways that make visible PSTs' engagement with three teaching performance areas (eliciting the student's process, using mathematical knowledge and skill, and conveying respect for the student as a mathematical thinker and learner) that are crucial for more equitable mathematics instruction. Connections between each of the performance areas and more equitable eliciting and interpreting of student thinking are described alongside the ways in which teacher educators can provide feedback that supports PSTs' development.

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INTRODUCTION

For over a decade, our research project has developed and studied the use of teaching simulations focused on the teaching practices of eliciting and interpreting student thinking to support preservice teachers' (PSTs) learning. In teaching, "teachers pose questions or tasks that provoke or allow students to share their thinking about specific academic content in order to evaluate student understanding, guide instructional decisions, and surface ideas that will benefit other students" (TeachingWorks, 2024). In other words, eliciting and interpreting student thinking is the work teachers do to learn about and make sense of student thinking. These practices are central to everyday classroom teaching (Association of Mathematics Teacher Education [AMTE], 2017; National Council of Teachers of Mathematics [NCTM], 2014).

In this chapter, we describe how teaching simulations focused on these practices allow teacher educators to work carefully on developing more equitable mathematics teaching situated in practice. By teaching simulation, we mean an approximation of practice (Grossman et al., 2009) that places a PST in a situation that requires authentic engagement in the work of teaching while at the same time standardizing or controlling the context of that teaching in ways that support focus on a particular practice, element of content, and/or student interaction. In the following introductory sections, we first define eliciting and interpreting in the context of advancing more equitable instruction. Second, we articulate why teaching simulations can help support the development of eliciting and interpreting student thinking. Third, we position our teaching simulations as a formative assessment in teacher education. Fourth, we articulate in more detail the focus of the paper.

Eliciting and Interpreting Student Thinking to Advance More Equitable Instruction

At the foundation of our work, we view teaching as involving the interactions between and among teachers, students, and the content situated inside of the school environment (Cohen et al., 2003; Lampert, 2001). In teaching, some common structures and practices privilege particular groups of students (Louie, 2017, 2018) and affect how students see themselves as mathematical thinkers and doers (Boaler & Staples, 2008). Significant disparities in learning opportunities exist based on race, class, language, gender, and ability status (e.g., Banilower et al., 2008; Berry et al., 2013; Flores, 2007; Gutierréz, 2018; Herbel-Eisenmann & Shah, 2019; McAfee, 2014; Sadker et al., 2009). Achieving more equitable mathematics teaching requires broadening the purposes of learning mathematics, creating equitable structures in mathematics, and developing deep mathematical understanding (NCTM, 2020). In a broad sense, we define more equitable mathematics teaching as instruction that disrupts the status quo of who is seen as good in mathematics and elevates the voices and ideas of students from marginalized communities and students whose mathematical ideas differ from those often privileged in mathematics classrooms. To support this goal, we have designed and used teaching simulations to support PSTs in valuing the mathematical thinking of students whose mathematical ideas differ from those often privileged in mathematics classrooms. We do this to increase the likelihood that such students have opportunities to develop and share their mathematical sense-making and justification, experience their thinking being valued, make sense of others' reasoning, and see themselves as mathematical thinkers and doers.

Next, we turn to our focus to eliciting and interpreting students' mathematical thinking. We define the practice and articulate its role in supporting our vision of more equitable mathematics teaching. Drawing upon our prior work (Boerst et al., 2020; Shaughnessy & Boerst, 2018a, 2018b), we unpack eliciting and interpreting student thinking to include eight related teaching performance areas: (1) eliciting a student's process, (2) interpreting the student's process, (3) probing a student's understanding, (4) interpreting a student's understanding, (5) attending to the student's thinking, (6) applying Mathematical Knowledge for Teaching (Ball et al., 2008), (7) using mathematical knowledge and skills, and (8) conveying respect for the student as a mathematical thinker and learner. In this paper, we focus on eliciting a student's process, using mathematical knowledge and skills, and conveying respect for the student as a mathematical thinker and learner. These areas will be unpacked later in this paper. More generally, Jacobs and Philipp (2004) have referred to eliciting student thinking as exploring the student's thinking. We see eliciting and interpreting student thinking as a precursor to responding to or extending a student's thinking instructionally.

Equitable instruction requires that teachers ask questions that invite children to share their ideas, listen to what children are saying, and ask follow-up questions that are responsive and purposefully connect students' ideas with mathematical processes, concepts, and practices. Respect for students' ways of making sense of mathematics and building on students' mathematical resources is at the core of our conception of equitable instruction. These practices require curiosity about the thinking of others, the disposition to listen and relate across differences (e.g., racial, cultural, gender), and eagerness to consider diverse ways of thinking about and making sense of disciplinary content (Aguirre et al., 2013). Akin to impactful practices such as formative assessment (Black & Wiliam, 1998), when done well, these practices facilitate teachers' connections with and attention to the children they teach (Carpenter, 1996; Jacobs et al., 2010; Jacobs et al., 2011) and yield information about student thinking that is core to formulating instructional next steps that actively build on students' current understandings. By surfacing the resources that students bring, eliciting student thinking provides a mechanism to disrupt patterns of oppression and inequity (Aguirre et al., 2013; Ball, 1988; Lampert et al., 2013), whereas less thorough eliciting can reinforce deficit-oriented interpretations and lead teachers to pursue less rigorous instructional goals (Battey & Franke, 2015). Furthermore, eliciting and interpreting student thinking supports the development of students' mathematical identities, including broadening their senses of what it means to be mathematically competent, their learning of mathematics, and their flourishing in school (NCTM, 2014), and provides students with the opportunity to engage with mathematics in ways that impact their learning, including opportunities to articulate and explain their thinking (e.g., Webb et al., 2008, 2009) and to be pressed to provide detail and justification of their ideas (Webb et al., 2014).

Teaching Simulations

In our practice as teacher educators (TEs), we have found that a significant challenge for teachers is learning about, understanding, valuing, and making sense of student thinking that differs from their own (Boerst et al., 2020; Shaughnessy & Boerst, 2018b; Shaughnessy et al., 2021). While representations of practice (Grossman et al., 2009), such as videos, are often used to support learning about eliciting and interpreting student thinking, simulations are a mechanism for engaging in the doing of teaching. Simulations can be constructed to embody teaching situations that might not be uniformly available across field sites (Shaughnessy et al., 2019). Simulations position TEs close to the action in terms of time and distance (Shaughnessy et al., 2019), making them conducive to providing feedback and gathering useful information for designing subsequent learning experiences.

At their core, our simulations are designed strategically to support PSTs' growth by simulating encounters with particular mathematical identities that would not otherwise be assured through the contexts in which they learn to teach (e.g., field sites). Our teaching simulations focus on uncovering PSTs' skills in positioning children as sense-makers, as evidenced by PSTs' use of questions and tasks to learn about a child's thinking through representations of children's mathematical identities involving alternative algorithms or partial mathematical conceptions. To provide a concrete example, the "standard" approach that is taught for solving subtraction problems in the United States involves what is typically called "borrowing" (National Research Council, 2001). This approach differs from strategies used in other communities and cultures, where the strategy focuses on reasoning about numbers relationally and adjusting the original problem en route to the same answer (Ron, 1998). PSTs often experience challenges connecting their knowledge of "standard" algorithms with different (but equally valid) approaches (Son, 2016). When students use approaches that differ from the teacher's own preference or goals, as is often the case when teacher and student identities differ, there is a risk that the students' approaches and understanding will be dismissed.

Simulations can push teachers to make sense of students' thinking and strategies and map students' mathematical understanding onto their own. Further, to learn about students' methods and understandings, teachers must navigate the ways that children communicate their thinking, which is shaped by linguistic resources, interactional norms, and non-verbal cues. Engaging in a simulation focused on student mathematical thinking reveals PSTs' own identities as mathematical thinkers and communicators. This provides teacher educators with a window into PSTs' development and opportunities for formative feedback that can promote growth in eliciting and interpreting student thinking that differs from PSTs' own thinking or communication patterns. We recognize that this approach does not portray students' full identities. Still, we see work with mathematical identities as a key step in building PSTs' skills with being able to see the strengths and potential of students with identities different from their own. Furthermore, we assert that engaging with these differing views of mathematics through a simulated student interaction is more likely to produce a change in actual practice than simply reading about the ways children from different communities think and communicate about mathematics.

Teacher educators can use simulations to create opportunities to observe PSTs' engagement in equitable teaching (DeFino, 2025; Shaughnessy et al., 2018; Wilhelm et al., 2025) and to provide feedback to those PSTs. We hypothesize that such feedback can support PSTs in (1) improving their deployment of practices central to equitable teaching and (2) raising their consciousness about why those practices advance equity in the classroom. When simulations are strategically designed in terms of the featured mathematical approach, the accuracy of how the approach is deployed, and the nature and degree of student understanding demonstrated (Shaughnessy & Boerst, 2018a), simulations have the potential to promote growth in eliciting and interpreting student thinking that differs from PSTs' own thinking. These situations can also present genuine challenges to PSTs' developing commitments to equitable instruction. How PSTs react in the moment to the student's approach and reasoning and later characterize the student's approach and reasoning provides the TE a window into the PSTs' growing ability to treat children as sense-makers and respect the mathematical resources they bring to the situation (Shaughnessy & Boerst, 2018b).

Teaching Simulations as Formative Assessments

Simulations can provide early, frequent, and substantive formative assessment opportunities that are embedded in the doing of teaching (Boerst et al.., 2020). Formative assessment has been shown to impact learning significantly (Black & Wiliam, 1998), and it has been identified as a crucial component in teacher preparation (AMTE, 2017; Darling-Hammond et al., 2005). Importantly, engagement in teaching simulations alone is insufficient for making such experiences educative. Studies of the development of expertise have found that practice opportunities alone do not sufficiently support novices to improve and that practice opportunities need to be coupled with structured, directive coaching (Ericsson & Pool, 2016). We refer to structured, directive coaching as "formative feedback." Teaching simulations are a rich site for simultaneously providing practice opportunities and formative feedback. It has been shown that PSTs who received feedback on their performances in teaching simulations focused on establishing a productive learning environment had significant improvements in skills relative to PSTs who only reflected on their simulation performance (Cohen et al., 2020). Thus, we frame our teaching simulations as formative assessment opportunities for teaching. While beyond the scope of this chapter, we see our teaching simulations as fitting into a broader set of activities that are used to support PSTs' learning of how to elicit and interpret student thinking in ways that advance more equitable instruction.

Focus of the Chapter

Our chapter is intended to be conceptual. We aim to illustrate how our teaching simulation structure provides a mechanism to reveal PSTs' current practices for eliciting and interpreting student thinking so that they can be observed by teacher educators and made available for a formative feedback conversation. We do not make claims about an individual PST in a broad sense, nor do we make claims about a group of PSTs. Instead, we aim to illustrate how our teaching simulation structure provides a means to support teacher educators (TEs) in noticing aspects of a PST's eliciting of student thinking and providing feedback in ways that advance engagement in more equitable mathematics teaching. While we could have focused on any of the performance areas, because of space constraints, we focus on three teaching performance areas of eliciting and interpreting student thinking requiring distinct sets of skills: eliciting a student's process, using mathematical knowledge and skills, and conveying respect for the student as a mathematical thinker and learner.

OUR TEACHING SIMULATION APPROACH

Authentic engagement within a simulation requires positioning the PST to engage in work they can recognize as teaching within a common instructional situation. Eliciting and interpreting a student's thinking in the context of looking at a student's response to a math problem is an example of such authenticity. In the simulations we design, a PST interacts with a "standardized student" (a TE taking on the role of a student using a well-defined set of decision rules for responding) around a specific piece of written work (see Shaughnessy & Boerst, 2018a for a more detailed description of how we design the simulation).

Our live actor teaching simulations involve two TEs. One TE takes on the role of the standardized student (referred to as the student or standardized student). The second TE (referred to as the TE) facilitates by appraising the performance and providing formative feedback. As shown in Figure 1, our teaching simulations have four parts from the perspective of a PST. First, a PST examines a piece of student work and plans questions to ask the student to learn about their process and their understanding of the process and underlying mathematical ideas. In the second part, the PST has five minutes to interact with the standardized student to elicit the student's thinking. In the data we feature in this chapter, the student has used a procedure, sometimes known as the "column addition method," to solve the problem shown in Figure 2. The student adds the digits in each column, starting with the tens. The student interprets the 623 in the written work as 6 "tens" and 23 "ones." The student knows that 23 ones can also be thought of as 2 tens and 3 ones. Then, the student combines the 6 tens and the 2 tens (from the 23 ones). This yields the final answer of 83. The student can explain why the 6 and the 2 must be combined. We train the "standardized student" to take on a particular student's way of reasoning about and understanding the mathematics task. Third, the TE interviews the PST to learn about their interpretations of the student's thinking (e.g., the student's process and understanding) and their understanding of the mathematics underlying the student's process (e.g., generalizability) (see Shaughnessy & Boerst, 2018a for more detail on the interview protocol). Fourth, the TE engages the PST in a feedback conversation about the performance in which they aim to support the PST in learning to engage in more equitable mathematics teaching, with attention to considering when, how, why, and whether they might use particular eliciting moves.

Figure 1. Activity Structure From the PST Perspective

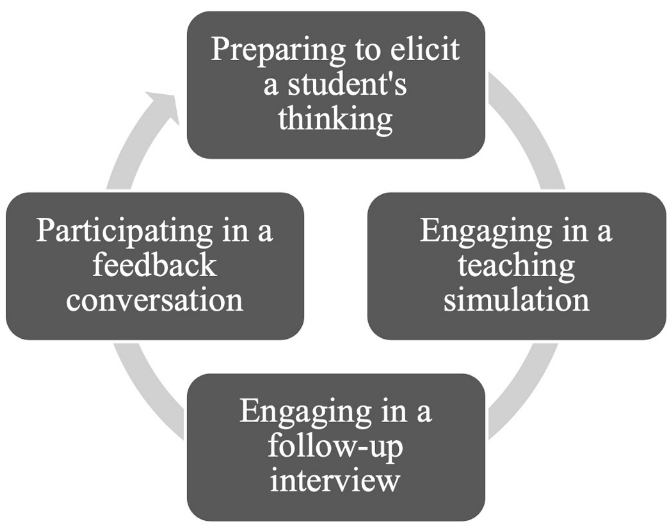
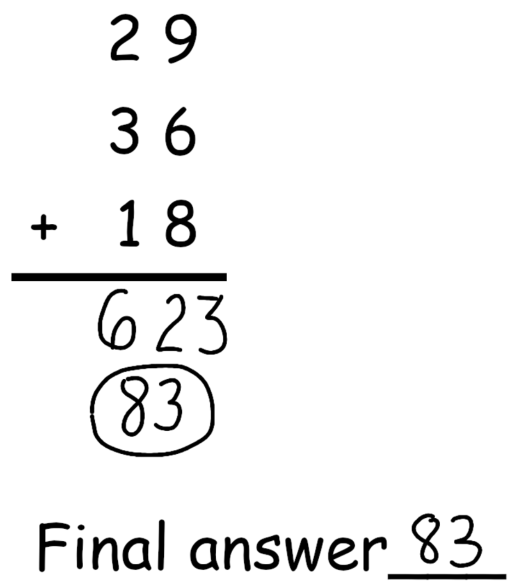


Figure 2. A Student's Work on a Multi-Digit Addition Problem



Our approach allows us to gather evidence of a PST's knowledge and skills in eight performance areas, three of which are featured in this chapter: eliciting a student's process; using mathematical knowledge and skills; and conveying respect for the student as a mathematical thinker and learner. In Table 1, we elaborate a definition of each of these performance areas with a rationale and information on where the performance can be seen. Table 2 provides an overview of simulation components that support opportunities to see each of these performance areas.

Table 1. Teaching Performance Areas, Their Rationales, and Where Enactment Occurs

Teaching Performance Area	Rationale	Where Enactment Occurs
Eliciting the student's process for solving the math problem	Even though some of the process is evident in the written work, it is important to elicit more information about the process because a teacher might not be able to see everything in the written work OR might make assumptions that do not correspond with what the student did.	Within the simulation The PST has the student talk about or otherwise show the process being used.
Using mathematical knowledge and skills	Students can share processes for solving problems their teachers have yet to see. Teachers need to be able to use their own mathematical skills to analyze whether a strategy is mathematically sound. Being able to generalize about the extent to which a student's strategy "works" is key for helping students see themselves as doers of mathematics.	Within the simulation and the follow-up interview PSTs demonstrate this by using mathematical knowledge to solve problems, using mathematical language and representations with integrity, and generalizing the extent to which the student's process would work.
Conveying respect for the student as a mathematical thinker and learner	Redirecting a student toward a strategy they did not use can communicate to the student that their strategy is wrong or undesirable and that doing mathematics means following a specific process, often the algorithms traditionally taught in US schools. Learning and doing math involves invention, hypothesizing, and other leaps of reasoning or stretching of processes that can involve errors. Errors are a necessary and natural part of learning mathematics, and errors should not be used to question or doubt the student's ability to learn mathematics or characterize a student's status as a learner.	Within the simulation and the follow-up interview PSTs demonstrate respecting the student as a mathematical thinker and learner by focusing on the student's strategy and interpreting that strategy through an asset-based frame.

Table 2. Simulation Components that Support Opportunities to See Areas of Practice

Simulation Component	Description
Student work sample	The student work used as the basis for the simulation brings together an algorithm (one of many possibilities), a way of representing the problem in writing (that conveys or masks certain elements), and a computational result (accurate/inaccurate) in ways that shape what can be asked and interpreted.
Simulation protocol	The protocol defines what the "standardized student" understands (specifically and more generally), the sequenced steps in the student's process, the responses to commonly asked questions, and the characteristics of how the student interacts with the PST. Thus, the protocol creates a predictable context in which PSTs engage and a guide for what the teacher educator can be listening and watching for.
Standardized student training	Every "standardized student" participates in training to observe (through descriptions, video, and modeling) and practice (engaging in scenarios) how to use elements of the simulation protocol in interaction with a PST.
Simulation facilitator professional development (facilitation focus)	Every TE participates in professional development to observe (through descriptions, video, and modeling) and practice (engaging in scenarios) how to administer the simulation, interview, and use a checklist to record their observations.
Observation checklist	Each performance area has criteria defined in terms of what to look for in a particular simulation scenario and keyed to information in the simulation protocol. Information from the observation checklist feeds into feedback supports and suggestions that the teacher educator can use.

To support the TE in appraising the performance, they are provided with an observational checklist for the simulation and interview. Information from the observational checklist is aligned to each performance area and tied to feedback suggestions that the TE can use. We are developing and piloting a formative feedback support system with components (see Table 3) to support TEs in identifying foci for their feedback and sharing that feedback with PSTs.

Table 3. Formative Feedback Support System Components

Component	Description
Performance snapshots	Text generated in response to the observational checklist that connects proficiency levels with the PST's performance in each of the teaching performance areas.
Performance summaries	Text generated in response to the observational checklist that succinctly summarizes the PST's performance in each of the teaching performance areas.
Suggestions for "what to do next"	Text generated in response to the observational checklist that suggests possible routes of action to use in subsequent teaching opportunities. These routes of action are connected with teaching performance areas.
Simulation facilitator professional development (feedback focus)	Every TE participates in professional development to observe feedback (through descriptions and video) and generate feedback (engaging in scenarios) they might give in response to performances with different characteristics.
Suite of performance information displays	Displays of information that organize and aggregate information from the use of the observation checklist (and notes written by the TE during the administration). These dynamic displays are intended to flexibly scaffold the feedback that the TE determines is most relevant and useful.
Facilitator math notes	Text scaffolds designed to support the TE in talking about key aspects of the mathematics within a particular simulation scenario.

CONTEXT FOR THE DATA SHARED

The data shared in this chapter come from ongoing work with TEs and PSTs at three universities across the United States. As part of this broader project, we are supporting TEs in using the teaching simulations with their PSTs and learning how PSTs respond to feedback.

SUPPORTING LEARNING TO ENGAGE IN EQUITABLE MATHEMATICS TEACHING

We share examples of three teaching performance areas within eliciting and interpreting student thinking. These examples illustrate how interaction with the standardized student and follow-up interview routinely provide opportunities to observe and provide feedback on facets of teaching tied to learning to teach mathematics equitably. For each teaching performance area, we define the area and provide an overview of the ways in which PSTs demonstrate strong beginning practice through teaching in the simulation and/or interpretations of what they learned about the student's mathematical thinking during the simulation. Then, we situate and share an example of a PST's teaching from the interaction with the standardized student and/or follow-up interview that offers robust opportunities for growth-oriented feedback. After describing how the example illuminates important facets of learning to teach mathematics equitably, we share how we provide feedback on those facets in ways that practically and conceptually support PSTs in becoming more aware, knowledgeable, and skillful practitioners of equitable teaching. It is important to note that there were many examples of PSTs' strengths with regard to eliciting and interpreting student thinking. Each of the PSTs in the examples shared in this chapter received feedback on the strengths they displayed in other teaching performance areas. However, because this chapter focuses on how the simulations, paired with feedback, can foster more equitable mathematics teaching, we have necessarily chosen examples that illustrate how feedback can support PSTs in areas with room for improvement.

Performance Area: Eliciting the Student's Process

In this section, we begin by defining the teaching performance area of eliciting the student's process and provide a characterization of strong beginning practice that is grounded in over a decade of work using these simulations with PSTs. Then, we share an example of a PST's teaching from the interaction with the standardized student that provides a robust opportunity for growth-oriented feedback. Finally, we unpack how we provide feedback in ways that practically and conceptually support PSTs in becoming more aware, knowledgeable, and skillful practitioners of equitable teaching.

Defining and Illustrating Strong Beginning Eliciting of the Student's Process

Eliciting the student's process involves teaching moves that encourage and support the student in sharing the process they used to solve a math problem. While some of the components of the process used are discernable from the student's written work (e.g., the 2, 3, and 1 were added to create the 6), other components of the process are less obvious (e.g., the sequence in which the numbers were added or the process of getting from 623 to the final answer of 83). Equitable mathematics teaching requires teachers to engage with and work from the ideas and resources that the student brings to solving the problem. When teachers ask questions to learn more about the less apparent components and confirm those more evident components, these questions can yield a fuller picture of the student's process and gather the evidence needed to support any interpretations of what the student is doing (Boerst et al., 2020).

We have found in past work (Shaughnessy & Boerst, 2018b) that when we observe PSTs engage in the eliciting of the student's process in the context of the Column Addition simulation, a combination of generic questions (e.g., What did you do first? What did you do next?...) and specific questions (e.g., What did you do with the 623 to arrive at 83?) are often used to gain a fuller picture of the sequenced steps of solving the problem. Many PSTs ask a series of questions that elicit all of the steps of the process from the student, including questions that confirm steps that might be discernable from the written work. Some PSTs also ask the student to solve the problem again or solve another problem that requires combining (e.g., 21 + 45 + 16). Some PSTs ask the student to narrate their process as they re-solve the problem or solve a new problem. These instructional choices allow the PST to see the student use their process and to ask more detailed questions that position the student as the mathematical authority and as doing mathematics that the teacher is interested in learning more about. Feedback on such performances involves noting the completeness of the information gathered about the process and reinforcing asking questions that confirm what might otherwise have been assumed. Using confirming questions is also a means to guard against assumptions that can undermine the foundation for later instruction.

An Opportunity for Growth in Eliciting the Student's Process

While less common, instead of asking the student questions, some PSTs confirm the process in a way that generates less information about the student's thinking and positions the student differently. One such approach, which we refer to as "filling" (short for "filling in the student's thinking"), involves the PST stating the steps they believe are a part of the student's process and seeing if the student agrees or disagrees with these hypotheses. The example below shows how a PST (Anna) fills in student thinking as she aims to learn about the student's process.

> **Anna:** Oh. So correct me if I'm right. You— This is your carried number [pointing to the 2 in 23] and you just brought it down with the six?
> **Student:** I don't know what you mean by carried, but when I added this all up, I got 23, so I just wrote it all right here.
> **Anna:** Oh, I see. Okay. So then, this is what you did? You did six plus two and you got eight?
> **Student:** Yep.
> **Anna:** And then you brought down the three and you got 83?
> **Student:** Yep, 83.

The "yep" in this transcript is the signal the standardized student gives to indicate that the PST is filling. Fills indicate when a PST is not eliciting and building on ideas the student shares.

Advancing Equitable Mathematics Teaching Through Formative Feedback on Eliciting the Student's Process

Given the power dynamic between teachers and students, when a student responds "yep" after a PST fills their thinking, the "yep" does not necessarily confirm a step in the student's actual process. The "yep" could also show deference to the teacher's ideas. The simulation allows TEs to see when filling happens, notice what parts of the process are filled, and observe the impact of filling on learning about other aspects of the process.

When TEs notice that a PST is filling in student thinking, they can provide feedback aimed at supporting improvement in teaching. This feedback could include a discussion of the way that filling prevents learning directly from the student about their thinking. A TE might choose to name what was filled and whether the fill undermined knowing about the sequence of the student's process (e.g., a fill that indicates starting with the ones place means that the PST is less likely to learn that the student starts with the tens place on the left). The feedback can include alternatives to filling that will recenter the questioning on learning from the student about their process.

Focusing on "filling" as a part of feedback to PSTs is a lever for moving toward more equitable mathematics teaching. The PST has opportunities to learn to notice filling, which they may have previously perceived to be a strong tool for eliciting information from students. PSTs can learn how filling can decenter students and their thinking, position the teacher as the mathematical authority, and potentially lead to faulty interpretations of the student's process. This reinforces the hierarchical status quo between teachers and students. The aim is to support PSTs in moving from filling in student thinking to asking questions that reveal student thinking, position students as sense-makers, and, in turn, support more equitable instruction.

Performance Area: Using Mathematical Knowledge and Skill

First, we define the teaching performance area of using mathematical knowledge and skill and provide a characterization of strong beginning practice. Then, we share an example of a PST's teaching that provides a robust opportunity for growth-oriented feedback. We close by unpacking how we view the feedback as supporting PSTs engagement in equitable teaching.

Defining and Illustrating Strong Beginning Using Mathematical Knowledge and Skill

Using mathematical knowledge and skill to elicit and interpret a student's thinking involves a combination of being able to solve for oneself the problem at the center of the simulation, reasoning about the mathematical ideas that are in play, and generalizing about the extent to which the student's process works. In the Column Addition simulation context, using mathematical knowledge and skill means understanding place value, regrouping values to ensure combining digits with the same place value, and equating the results of left-to-right and right-to-left approaches to the problem. Equitable mathematics teaching in this context requires teachers to strive to make sense of the mathematics that the student is using, to understand how 6 tens and 23 ones can be combined to make 83, to realize that 83 is the correct answer, and that the approach that the student is using could yield correct answers for any addition problem.

PSTs engaging in the Column Addition simulation often recognize that the student's answer is correct by using their own approaches to confirm the student's answer. PSTs may connect their own ideas about place value and addition algorithms with the student's process. PSTs talk about 623 as being six tens and 23 ones. They internally connect the student's process to their preferred approach and come to notice that there is just a minimal difference between the way that the student is handling the 2 tens and the way that they themselves would handle it (e.g., "They are just leaving the 2 that I would carry at the bottom, but they still add it to the 6 just like I would"). In terms of equitable mathematics teaching, PSTs actively puzzle about what the student is doing, asking questions that show that they are foregrounding the student's ideas and using the mathematical language that the student uses while preserving the integrity of the mathematics content. PSTs are either convinced, or nearly so, that the student's process, while potentially cumbersome when the magnitude of the numbers increase, will result in correct sums. These are the kinds of things that are central to the feedback discussion between the TE and PST. This discussion advances equitable mathematics teaching by highlighting how the PST's own mathematical knowledge and skills can be used in ways that seek to understand and privilege what the student brings to the computational situation.

An Opportunity for Growth in Using Mathematical Knowledge and Skill

The interaction with the standardized student and the follow-up interview also provides opportunities to support improvement in using mathematical knowledge and skills. Such opportunities arise when PSTs engage in ways that do not sufficiently leverage their mathematical knowledge and skills or when PSTs' knowledge and skills are roadblocks to listening to the student. Engagement in ways that fail to leverage the PST's mathematical knowledge and skill in service of supporting more equitable instruction can often be observed at the outset of the interview that follows the simulation. For example, a PST may fail to leverage their mathematical knowledge and skill by not doing the computation, either using the student's process or their own, to determine if the answer is accurate. To illustrate this, we consider the follow-up interview with a PST, Ariel.

> **TE:** So now I'm going to ask you some questions about what you learned about the student's thinking from that interaction. Did he arrive at the correct answer?
> **Ariel:** No.
> **TE:** And so what would the correct answer be? And you are welcome to use paper and pencil.

[Ariel works out the problem using the US standard addition algorithm]

> **Ariel:** So 83. Oh wait, he did get it [surprised, smiling]. Oh! See, I didn't do the work. But he didn't get to it correctly.

Not only did the PST (Ariel) initially believe that the answer was wrong, but she admitted she did not apply her mathematical knowledge or skills to make that determination. The TE surfaced this with a simple follow up: "What would the answer be?" Ariel's assumption that the answer was wrong raises a serious challenge to equitable mathematics teaching. Why was the answer assumed to be wrong? Was it simply because the process was unfamiliar? Feedback guided by this question can support reflection on the problematic nature of that assumption and the importance of a generous or curious initial interpretation that the answer is correct until otherwise disproven.

Later in the interview, the same PST (Ariel) had a chance to apply the student's process to a new problem (27 + 48). This provided another space to observe Ariel using her mathematical knowledge and skills, and, as the example below shows, to see again how the student's process can yield the correct answer.

> **Ariel:** So he would do— he would do 15 and then six. And then it seems like he added these two, so he would add these two, and it would be 67. So that's how he would solve it.
> **TE:** Which two numbers did you add here?
> **Ariel:** 5 and 1 because those two make a two-digit. So he added the ones that made the two digits like two— Oh, I see. Yes. So he would add 6 and 1, so that would be 75. Oh my God, he got it right again.
> **TE:** He got it right again?
> **Ariel:** Well, yeah. His process.

The transcript shows Ariel's surprise that the process worked. The first example of the process working did not shift Ariel's perspective on its utility. After this second instance, later in the interview, Ariel was asked if the student's process would work in all such problem situations.

> **TE:** So that makes sense. If we bounded it to numbers where that didn't happen, where it was— where the tens places didn't end up in a two-digit sum, would it work?
> **Ariel:** Yes. I feel like, so far, I'm seeing the trend.
> **TE:** So far, you are seeing the trend. Do you have any ideas about why it works?
> **Ariel:** I don't because I've never seen this strategy, so I just— It's new to me, so no. I'm not sure.
> **TE:** So your gut is that it would work because—
> **Ariel:** It seems to be working.

Here, through the questioning, the PST was convinced that the process will generalize.

Advancing Equitable Mathematics Teaching Through Formative Feedback on Using Mathematical Knowledge and Skill

The information provided through the follow-up interview positioned the facilitator to provide feedback on how using mathematical knowledge and skills can advance more equitable mathematics teaching. The facilitator can talk with the PST about the need to (and how to) move beyond the accumulation of confirming cases to seek a deeper understanding of how the process works that transcends cases. This encourages the PST to apply their knowledge and skills and develop new insights. Crucially, it also signals to the PST that the student's approach is worthy of serious mathematical consideration. As such, the feedback materially contributes to developing a mathematical orientation that advances equitable mathematics teaching.

Performance Area: Conveying Respect for the Student

First, we define conveying respect for the student as a mathematical thinker and learner and provide a characterization and example of strong beginning practice. Then, we share an example of a PST's teaching that provides a robust opportunity for growth-oriented feedback. To close, we unpack how we view the feedback as supporting PSTs' engagement in equitable teaching. In this section we illustrate how both the interaction with the standardized student and the later interview provide opportunities for learning to engage in equitable teaching.

Defining and Illustrating Strong Beginning Conveying Respect for the Student

We recognize that "respect" is a loaded term to which individuals bring their own interpretation and meaning. When we reference conveying respect for the student while eliciting and interpreting a student's thinking, we are referring to the actions the teacher takes to acknowledge and honor the student as a mathematical thinker and learner. This relies, among other things, on the content of teacher talk, and what they choose to take up/not take up from what students say and do. While the general notion of respect is contested and socially constructed, we narrowly define respect in this context as an anti-deficit framing of the student's mathematical ideas, understandings, and processes. In the interaction with the standardized student, the PST formulates *talk about the student's mathematics while interacting with the student*. In the follow-up interview, the PST *talks about the student's mathematics and, in some ways, characterizes the student* in conversation with the facilitator outside of the student's presence. In both the simulation and follow-up interview context, observable characteristics of such talk provide evidence of an asset-based framing of the student and the student's ideas. That respect is at the core of equitable mathematics teaching. In this simulation, the student is using an algorithm that is likely different from the one that the PST routinely uses. Does the PST foreground the student's or their own approach during the simulation? Does the PST dig into the student's method, or do they undermine the method by asking about its provenance, asking the student to use another process, or asking the student primarily about the drawbacks of the student's process? During the follow-up interview, does the PST focus on deficits by characterizing the student's process as inferior or problem-prone, characterizing the student as confused or needing reteaching that does not build on their current understanding? These observable signs of disrespect present barriers to eliciting and interpreting student thinking and equitable mathematics teaching. Opportunities to observe them allow for feedback that pinpoints areas for improvement, articulation of why changes are needed, and dialogue focused on alternatives.

Of course, the interactions with standardized students and follow-up interviews also provide opportunities to notice engagement in teaching and communicating interpretations that embody respectful interaction and can be built on to advance engagement in equitable mathematics teaching. For example, one PST, Andrea, listened to how the student named and assigned value to the 623.

Andrea: I do have a question about your 23 and your six hundred and twenty-three.
Student: Oh, it's not six hundred and twenty-three.
Andrea: It's not six hundred and twenty-three?
Student: No, because I said this was six tens.

Andrea appeared to realize that the student did not see the 623 as six hundred and twenty-three. Andrea explicitly named the dissonance and positioned the student as the one who explains why the number cannot be six hundred twenty-three.

Andrea continued to use the student's way of talking about the numbers as the conversation moved along.

Andrea: So how did you get from your 23 and your 6 tens to 83?
Student: Well, I saw right here that I needed to do some combining before I got to the answer.
Andrea: Can you tell me a little bit more about what you mean by combining?
Student: Sure. Well, like I kinda had to add the six and the two together because they're both tens.
Andrea: Okay, so you added your six and your two together because they're both tens?
Student: Mm-hm.

This example also shows the PST, Andrea, picked up on the student's use of "combining." Andrea asked the student to explain the meaning of combining. As the simulation continued, Andrea revoiced what the student said about why the student could combine digits in the problem.

In the follow-up interview, when commenting on their interpretation of the student's understanding of the need to add 6 tens to the 2 tens to arrive at the answer of 83, the PST, Andrea, appeared determined to continue using the student's formulation of combining, consciously choosing not to use a term that they more typically would use.

Andrea: Well, I feel like he emphasized a lot, like that it was six tens and that there were 23 ones and that he— I forget what the word was, it wasn't regrouping. It was maybe like combining, right?

Advancing Equitable Mathematics Teaching Through Formative Feedback on Respecting the Student

These are affirming examples of respecting the student—the kind of respect at the heart of equitable mathematics teaching. This PST, Andrea, shifted away from their way of framing ideas to take up the student's language, ideas, and sense-making. Andrea appeared curious about the student's ways of thinking and positioned the student as one who explains and whose ideas are revoiced. Feedback to

PSTs who interact with the student in these ways focuses on connecting examples from their eliciting and interpreting to respecting the student and their thinking and reinforcing the stance that asset-based frames are needed when in the presence of the student and when talking with others about the student.

An Opportunity for Growth in Respecting the Student When Interacting With the Student

Just as the interaction with the standardized student can surface evidence of respect for the student's ways of thinking, talking, and doing mathematics, it also provides opportunities to notice and provide feedback on cases where the interaction is not as respectful. This is not evidenced simply by the absence of moves that convey respect, but by teaching moves that demonstrate a deficit orientation towards the student and their mathematics. In the simulation, a PST sometimes quickly moves from questions to learn about the student's process and understanding to questions that require the student to defend the approach or to try an alternative. We turn to the case of Kendall, a PST, to illustrate this scenario.

Kendall: Where did you begin?
Student: I added the tens. So I added the 2, 3, and the 1, and I got six.
Kendall: So tell me more about that.
Student: What more do you want to know?
Kendall: Why did you begin to add in the tens place?
Student: Well, you could really add starting on either side, and you would get the same answer, but I usually just add from the left.

Kendall wanted the student to justify why they started on the left and persisted later with a rephrasing of asking why the student started with the tens. The addition algorithm traditionally taught in US schools begins the computation by adding the ones and the questioning here is focused on why the student is not adding the ones first.

As the interaction continued, the PST, Kendall, misremembered where the student started the addition process. This required the student to actively contradict, to which Kendall responded that in the problem at hand, the student "would have to start with the ones."

Kendall: I love how you started in the ones.
Student: I started in the tens.
Kendall: Well, here you would have to start in the ones.

When corrected, Kendall stated that the student needed to start by adding in the ones. Still later, Kendall questioned why the student was not "carrying."

Kendall: So why didn't you think about carrying instead of putting 23 here, why didn't you think about carrying the two?

Eventually, the simulation ended with Kendall directly encouraging the student to do the problem differently.

Kendall: What would happen if you did carry the 2? Would you understand what— what you could do there?
Student: Well, is my way— My way won't work?
Kendall: Not that it won't— I mean, it's not the proper way to add. If it were me, I— [trails off]

At its conclusion, the student was told that their process for solving the problem is "not the proper way to add."

Advancing Equitable Mathematics Teaching Through Formative Feedback on Respecting the Student When Interacting With the Student

Such questions and comments disrespect what the student brings to the interaction and strongly contrast with equitable mathematics teaching that values and builds from what students bring to the doing of mathematics. Feedback in this situation could focus on encouraging the PST to prioritize learning about the student's approach without directly pressing the student to use another process. The feedback could promote reflection about what makes any process "the proper" one and consider the impact of making a student work so hard just for their process to be heard, let alone having their process characterized as undesirable.

An Opportunity for Growth in Respecting the Student During the Interview

The follow-up interview also makes examples of disrespecting the student's mathematical thinking observable and available for feedback. The example below is part of a follow-up interview. In the simulation that preceded it, the PST elicited the student's process and understanding of that process, including the student's understanding that the 6 and 2 represented tens that needed to be combined, without evidence of deficit thinking toward the student. However, in the follow-up interview, the PST, Kamila, appeared to dismiss that the combining demonstrated understanding.

Kamila: Well, he understood he had to add all the tens first and then all the ones, but he didn't understand that he had to regroup to the tens.

When Kamila was later asked to generalize whether the student's process would work for all two-digit addition problems, Kamila indicated that the student "kind of regrouped, but did not regroup on top," only to follow later saying that the student "didn't understand that he had to regroup to the tens." The student's understanding, which was elicited during the simulation, was not considered an understanding simply because of where it is recorded on the paper in the process that the student is using. This demonstrates a lack of respect for the student's understanding.

During the same interview, disrespect was shown in other instances. When asked to construct a problem that could be used to confirm the student's process, Kamila created a problem with numbers chosen purposefully so that the student would start in the ones place.

Kamila: They could start with just the ones place . . . to know where the ones and tens are at . . . so he could know to start in the ones place, instead of the tens place.

Advancing Equitable Mathematics Teaching Through Formative Feedback on Respecting the Student When Talking About the Student as a Mathematical Thinker and Learner

Feedback to the PST can focus on helping them to appreciate the understanding that is shared and not be distracted by surface-level features that do not impact the underlying understanding. Similar to the earlier example from interacting with the student, the follow-up interview can also surface when the purpose of a problem is to direct the student to start addition with the digits in the ones place rather than to learn more about the student's current process. In this instance, feedback to the PST could include contrasting the purpose of the eliciting with the function of questioning that directs the student away from their own thinking.

CONCLUSION

Teacher educators wrestle with how best to support PSTs with the enactment of teaching practice. Using data collected in the context of work with PSTs, we suggest that a way to address this challenge is to design and use teaching simulations to support the PSTs in learning to engage in eliciting and interpreting student thinking in ways that support more equitable mathematics teaching. In the chapter, we described our approach to using simulations in these ways. We highlighted how the approach can support TEs in seeing the instructional moves of PSTs and their linkages to more equitable mathematics teaching, and in giving feedback to support enhanced instruction. While we included examples from three performance areas related to eliciting and interpreting student thinking, such examples exist for all eight performance areas. We view the ability to make facets of equitable mathematics teaching concretely visible, doable, and improvable in the work of teaching as a central driver of the design and implementation of the simulation approximation of practice, as well as the focus and composition of feedback.

The set of examples shared in this paper shows how the interaction with the standardized student and follow-up interview provide opportunities for PSTs to demonstrate their skill with a teaching practice that is critical for more equitable mathematics teaching, and how the subsequent feedback session makes space for TEs to support PSTs in moving toward more equitable instruction. Additionally, these simulation sessions provide opportunities to connect specific teaching moves to issues of equity in close proximity to the enactment of practice. Finally, the simulation design features ensure that PSTs are engaged in situations that often result in inequitable teaching moves (e.g., unfamiliar algorithms, students that do not readily share information), ensuring opportunities for TEs to address these critical issues.

We have found that the design of the teaching simulation matters for supporting work on more equitable mathematics teaching. As illustrated in Table 3, the design includes the specificity of the student work, how the student shares their mathematical thinking, and the design of the questions that the TE poses in the follow-up interview. The teaching simulation we shared in this paper was purposefully designed to have the student use and understand a process that is likely to differ from the one used by PSTs. The student's ability to articulate their understanding can be a crucial resource for the PST's learning. Without it, the PST may dismiss the unfamiliar process the student uses. With it, the PST has the opportunity to access mathematical resources within the teacher-student interaction to support their own learning and advance their appreciation of students as sense-makers. As shown in this chapter, this context provides opportunities to see how PSTs elicit student thinking and respect the student as a mathematical thinker and learner. It also provides a rich context to explore how PSTs use their mathematical knowledge in service of learning about and making sense of student thinking. All of these are crucial areas for supporting teaching that ensures that students have opportunities to share their mathematical sense-making and justification, experience their thinking being valued, make sense of others' reasoning, and see themselves as mathematical thinkers and doers.

While we have illustrated the utility of a particular set of design features, we have found other scenarios to be rich as well. For instance, we have found it fruitful to have a student use a more "standard" algorithm with a typical error to see whether PSTs are able to learn about the entirety of the student's process and understanding rather than fixating on the mistake that was made or assuming that it must be a minor error since the process is both familiar and likely known by the PST to generalize. Using the "standard" algorithm design feature alongside variable student understanding (e.g. the student does not understand the reasoning behind a component of the algorithm) allows TEs to see whether the PST will make assumptions about student understanding since the PST knows it works and why it works. These assumptions can be explicitly seen through the interaction with the standardized student and the follow-up interview. The assumptions are then available for targeted and explicit feedback to enhance future equitable mathematics teaching engagement.

At the same time, we acknowledge that some aspects of equitable teaching practice focused on eliciting and interpreting student thinking are not addressed in the teaching simulation. In particular, while we recognize the identity of the person in the role of the standardized student is likely taken up as part of the simulation, we have deliberately designed the teaching simulations such that they are focused on aspects of the student's mathematical identity (e.g., their mathematical processes, understandings, ways of communicating about mathematics, etc.) and not particular aspects of student identity beyond mathematics (e.g., race/ethnicity, gender, multilingual learner status). This design decision is important because taking on social identities that are different from one's own is inherently problematic due to the likelihood of essentializing and failing to represent lived experiences. Yet, as evidenced in this chapter, there is much to focus on and learn related to more equitable mathematics teaching when the simulation focuses on mathematical identity.

In our work, these teaching simulations are part of a broader set of teacher education activities designed to support more equitable mathematics teaching focused on eliciting and interpreting student thinking. For instance, in our courses, we show videos with actual students with varied identities and have PSTs practice listening to and interpreting the mathematical thinking of students with mathematical and cultural identities different from their own. Our PSTs engage in eliciting student thinking in classroom settings with actual children as well. Thus, teaching simulations are an important complement to (not a replacement for) other teacher education activities.

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