Revisiting C and CP violation in $\eta \to \pi^+\pi^-\pi^0$ decay

Jun Shi,[1,2] Jian Liang,[1,2,*] and Susan Gardner[3]

[1]Key Laboratory of Atomic and Subatomic Structure and Quantum Control (MOE), Guangdong Basic Research Center of Excellence for Structure and Fundamental Interactions of Matter, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China

[2]Guangdong-Hong Kong Joint Laboratory of Quantum Matter, Guangdong Provincial Key Laboratory of Nuclear Science, Southern Nuclear Science Computing Center, South China Normal University, Guangzhou 510006, China

[3]Department of Physics and Astronomy, University of Kentucky, Lexington, Kentucky 40506-0055, USA

(Received 20 July 2024; accepted 29 August 2024; published 27 September 2024)

The decay $\eta \to \pi^+\pi^-\pi^0$ is an ideal process in which to study flavor-conserving C and CP violation beyond the Standard Model. We deduce the C- and CP-odd quark operators that contribute to $\eta \to \pi^+\pi^-\pi^0$ originating from the mass-dimension-six Standard Model effective field theory. The corresponding hadron-level operators that generate a nonvanishing $I = 0$ amplitude at order p^6 in the chiral effective theory are presented for the first time, to the best of our knowledge, in addition to the leading-order operators ascribed to the $I = 2$ final state. By fitting the KLOE-2 and the most recent BESIII experimental data, we determine the coefficients of the lowest-order $I = 0$ and $I = 2$ amplitudes and estimate the potential new physics energy scale. We also perform an impact study of the future $\eta \to \pi^+\pi^-\pi^0$ experiments.

DOI: 10.1103/PhysRevD.110.055039

I. INTRODUCTION

C and CP violation have long been regarded as essential conditions in baryogenesis [1], which generates the observed matter-antimatter asymmetry in the universe [2]. However, this baryon asymmetry of the universe cannot be explained within the Standard Model (SM), because its electroweak phase transition cannot be of first order [3,4] and its mechanism of CP violation, through a single phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, is far from sufficient [5–9] to explain its size, though exceptions may exist [9,10]. This motivates the continuing search for new CP-violating sources beyond the SM (BSM). Previous BSM CP-violation studies have focused mainly on flavor-changing processes, such as in B, D, and K meson decays, e.g., [11–22], and on electric dipole moments, e.g., [23–30], which are flavor-conserving P- and T (CP)-violating observables. But studies of flavor-conserving C and CP violation are scarce. Considering the fact that the SM CP-violating mechanism lies in the flavor-changing weak coupling, the flavor-conserving C and CP violation might be more sensitive to physics BSM. An explicit example would be a charge asymmetry in $\eta \to \pi^+\pi^-\pi^0$ decay.

The decay $\eta \to \pi^+\pi^-\pi^0$ is flavor conserving, and its parity is conserved due to angular momentum conservation. This decay can occur via either C-conserving but isospin-breaking or C-violating processes [31]. The latter comes from the interference of the C-conserving and the C-violating amplitudes, which is proportional to the BSM C- and CP-violating coefficients [32], rather than the coefficients squared as in the branching ratio of a pure CP-violating process. Thus, new physics may be more appreciable in $\eta \to \pi^+\pi^-\pi^0$ decay, making this channel an ideal arena to study flavor-conserving C and CP violation at low energy.

The possibility of C and CP violation in $\eta \to \pi^+\pi^-\pi^0$ first received attention after the discovery of CP violation through the $K_L \to \pi^+\pi^-$ decay in 1964 [33], to test the validity of the proposed new interaction [34,35] inducing the CP-violating K_L decay. However, at that time the C-conserving amplitude was incorrectly believed to go through a virtual electromagnetic interaction [31,36,37], which was later proved to be negligibly small [38–42], and the process is dominated by strong interaction [43,44]. For over five decades since then, C and CP violation in $\eta \to \pi^+\pi^-\pi^0$ decay was not further investigated by theorists, with theoretical studies focusing on better descriptions of the final-state interactions within the SM to extract the light-quark mass ratio precisely [45–55]. Very recently, theoretical studies of C and CP violation in $\eta \to \pi^+\pi^-\pi^0$

*Contact author: jianliang@scnu.edu.cn

decay have been made by Gardner and Shi [32] and Akdag *et al.* [56], using different phenomenological frameworks, in which similar patterns of the C-violating amplitudes with $I = 0$ and $I = 2$ final states are obtained. In searching for the origin of the CP-violating mechanism, Refs. [57,58] derive all the C- and CP-odd quark-level operators from the dimension-six SM effective fheory (SMEFT) [59], while Ref. [60] lists similar operators based on low energy effective theory (LEFT). The latter work also matches the operators to hadron level in chiral perturbation theory (ChPT), and by combining with the results of Ref. [56], a naive dimensional analysis (NDA) of the new physics scale has been carried out. Here we improve upon this first analysis through the use of SMEFT, as we shall explain.

Experimentally, the charge asymmetry in $\eta \to \pi^+\pi^-\pi^0$ decay can be identified by observing the population asymmetry A_{LR} in the Dalitz plot distribution with respect to the mirror line $t = u$ with $t \equiv (p_{\pi^-} + p_{\pi^0})^2$ and $u \equiv (p_{\pi^+} + p_{\pi^0})^2$ [61]. Other C-asymmetry observables include the quadrant asymmetry A_Q and sextant asymmetry A_S, which probe the final $I = 2$ and $I = 0$ states [31,36], respectively. Except for three early experiments [62–64] that reported C-asymmetry signals, with a significance of less than 3σ, no C violation is observed in other experiments [61,65–70], including the recent high statistics measurements from KLOE-2 Collaboration [71] and BESIII [72]. Future η related experiments with much higher statistics from the JLab Eta Factory (JEF) experiment [73–76], REDTOP [77–79] Collaboration, and the eta factory of the High-Intensity heavy-ion Accelerator Facility (HIAF) in China [80] are on the way.

In this work, we begin with the BSM C and CP-odd quark-level operators pertinent to $\eta \to \pi^+\pi^-\pi^0$ decays from the dimension-six SMEFT [59], in which the coefficients are suppressed by the new physics scale as $1/\Lambda^2$. This is different from the $1/\Lambda^4$ scaling behavior in the LEFT work [60], because LEFT cannot resolve the difference between Λ and the mass of the SM weak gauge bosons. Next, we develop the hadron-level operators at order p^2 in ChPT accounting for the $I = 2$ final state, as well as the lowest-order operators generating a nonvanishing $I = 0$ amplitude at order p^6. Although examples of the former are presented in Ref. [60], the latter are presented here for the first time. With the $I = 0$ and $I = 2$ ChPT amplitudes, we fit the KLOE-2 data [71] together with the most recent BESIII data [72] directly to obtain the coefficients of the order p^2 and p^6 amplitudes, which are used to estimate the potential new physics scale. Finally, an impact study of future $\eta \to \pi^+\pi^-\pi^0$ experiments is carried out, which provides some guidance to the upcoming η factory experiments [73–80]. According to our study, it should be possible to observe C and CP violation in $\eta \to \pi^+\pi^-\pi^0$ decay in the near future.

This paper is organized as follows. In Sec. II, we introduce the quark-level operators and the matching to ChPT operators contributing to C and CP violation of $\eta \to \pi^+\pi^-\pi^0$. In Sec. III, we explain the fitting procedure to determine the coefficients of C-violating amplitudes with specific final-state isospin. Then we carry out NDA of the new physics scale from these coefficients and also an impact study for future experiments in Sec. IV. A brief summary is provided in Sec. V.

II. C- AND CP-ODD OPERATORS

In this section, we first briefly recall the derivation of the C- and CP-odd flavor-conserving quark-level operators from SMEFT, and then show their matching to ChPT operators, from which the lowest-order $I = 2$ and $I = 0$ C-violating amplitudes of $\eta \to \pi^+\pi^-\pi^0$ are obtained.

The SMEFT Lagrangians read

$$\mathcal{L} = \mathcal{L}_{\rm SM}^{(4)} + \sum_i \frac{C_i}{\Lambda^{D-4}} \mathcal{O}^D, \tag{1}$$

where the operators $\mathcal{O}^D$ have mass-dimension $D > 4$ with a suppression by the new physics scale Λ and C_i are the corresponding dimensionless Wilson coefficients. The SMEFT shares the same gauge symmetries and building blocks as the SM, which are the $SU(2)_L$ left-handed doublets $q_{Lp} = (u_{Lp}, d_{Lp})^T$ and $l_{Lp} = (\nu_{Lp}, e_{Lp})^T$; the right-handed singlets u_{Rp}, d_{Rp}, and e_{Rp} with p denoting the generation; the Higgs field φ; the $SU(3)_C$ gauge field G_μ; and the $SU(2)_L \times U(1)_Y$ gauge fields W_μ^I and B_μ. In the particular case of $\eta \to \pi^+\pi^-\pi^0$, the dimension-five operator [81] does not appear since it breaks the lepton number and contains only Higgs and lepton fields. Thus, we need to start from dimension-six operators. In Refs. [57,58] we have already investigated all the dimension-six SMEFT operators obtained in Ref. [59] and derived the P- and CP-odd as well as the C- and CP-odd quark-level operators at the scale just below the weak gauge boson mass. Here we present the essential procedure for the readers' convenience and list the lowest-dimensional C- and CP-odd operators that contribute to $\eta \to \pi^+\pi^-\pi^0$.

The dimension-six SMEFT operators [59] most pertinent to flavor-conserving C and CP violation [57,58] can be expressed as the T-odd combination

$$\begin{aligned}\Omega = \frac{i}{\Lambda^2}\{&\bar{q}_{Lp}\sigma^{\mu\nu}[\mathrm{Im}(C_{quB\varphi}^{pr})B_{\mu\nu} + \mathrm{Im}(C_{quW\varphi}^{pr})\tau^I W_{\mu\nu}^I]\tilde{\varphi}u_{Rr}\\ &+ \bar{q}_{Lp}\sigma^{\mu\nu}[\mathrm{Im}(C_{qdB\varphi}^{pr})B_{\mu\nu} + \mathrm{Im}(C_{qdW\varphi}^{pr})\tau_I W_{\mu\nu}^I]\varphi d_{Rr}\\ &- \mathrm{H.c.}\},\end{aligned} \tag{2}$$

where τ_I are Pauli matrices; $\tilde{\varphi} = i\tau_2\varphi^*$; and $C_{quB\varphi}^{pr}$, $C_{qdB\varphi}^{pr}$, $C_{quW\varphi}^{pr}$, and $C_{qdW\varphi}^{pr}$ are the related Wilson coefficients with subscripts indicating the operator constituents and superscripts indicating the generations of the quark fields.

We have omitted terms with two weak gauge fields since they will prove to be of higher mass-dimension. After the electroweak symmetry is spontaneously broken, so that the Higgs field acquires its vacuum expectation value, we rotate B_μ and W^I_μ to the physical fields $W^\pm_\mu$, Z_μ, and A_μ and obtain

$$\Omega \sim \frac{\sqrt{2}vi}{\Lambda^2}[\mathrm{Im}(C^{pp}_{quZ\varphi})\bar{u}_{Lp}\sigma^{\mu\nu}u_{Rp}\partial_\mu Z_\nu + \mathrm{Im}(C^{pp}_{qdZ\varphi})\bar{d}_{Lp}\sigma^{\mu\nu}d_{Rp}\partial_\mu Z_\nu + \sqrt{2}\mathrm{Im}(C^{pr}_{quW\varphi})\bar{d}_{Lp}\sigma^{\mu\nu}u_{Rr}\partial_\mu W^-_\nu + \sqrt{2}\mathrm{Im}(C^{pr}_{qdW\varphi})\bar{u}_{Lp}\sigma^{\mu\nu}d_{Rr}\partial_\mu W^+_\nu] + \mathrm{H.c.}, \quad (3)$$

where $C^{pr}_{quZ\varphi} = [c_w C^{pr}_{quW\varphi} - s_w C^{pr}_{quB\varphi}]$, $C^{pr}_{qdZ\varphi} = -[c_w C^{pr}_{qdW\varphi} + s_w C^{pr}_{qdB\varphi}]$, $s_w \equiv \sin\theta_W$, $c_w \equiv \cos\theta_W$, and θ_W is the Weinberg angle. We also omit all the terms with A_μ because we suppose quark operators should dominate BSM effects in $\eta \to \pi^+\pi^-\pi^0$. The operators of Eq. (3) are CP-odd, but they do not have definite P or C transformation properties since the weak gauge bosons couple to both vector and axial-vector quark bilinears. We then integrate out $W^\pm$ and Z when the energy is just below their mass and make the following replacements:

$$W^+_\mu \to \frac{g}{\sqrt{2}M_W^2}\bar{d}_{Lx}\gamma_\mu V^*_{px}u_{Lp}, \quad (4)$$

$$W^-_\mu \to \frac{g}{\sqrt{2}M_W^2}\bar{u}_{Lp}\gamma_\mu V_{px}d_{Lx}, \quad (5)$$

$$Z_\mu \to \frac{g_Z}{M_Z^2}\left[\bar{u}_{Lp}\gamma_\mu\left(\frac{1}{2}-\frac{2}{3}s_w^2\right)u_{Lp} + \bar{d}_{Lp}\gamma_\mu\left(-\frac{1}{2}+\frac{1}{3}s_w^2\right)d_{Lp} + \bar{u}_{Rp}\gamma_\mu\left(-\frac{2}{3}s_w^2\right)u_{Rp} + \bar{d}_{Rp}\gamma_\mu\left(\frac{1}{3}s_w^2\right)d_{Rp}\right], \quad (6)$$

where $g_Z = g/\cos\theta_W$, V_{px} are the CKM matrix elements, and we sum over the generation indices, omitting the t quark. Finally, we pick out the lowest mass-dimensional flavor-conserving C- and CP-odd quark-level operators as

$$\Omega^{\not{C}P} \sim \frac{i\sqrt{2}v}{\Lambda^2}\left\{\frac{g_Z}{4M_z^2}(\mathrm{Im}(C^{pp}_{quZ\varphi})\bar{u}_p\sigma^{\mu\nu}\gamma_5 u_p + \mathrm{Im}(C^{pp}_{qdZ\varphi})\bar{d}_p\sigma^{\mu\nu}\gamma_5 d_p)\partial_\mu(\bar{d}_r\gamma_\nu\gamma_5 d_r - \bar{u}_r\gamma_\nu\gamma_5 u_r) + \frac{g}{4M_W^2}\mathrm{Im}(C^{pr}_{quW\varphi} - C^{rp}_{qdW\varphi})[\bar{d}_p\sigma^{\mu\nu}u_r\partial_\mu(\bar{u}_r\gamma_\mu V_{rp}d_p) - \bar{u}_r\sigma^{\mu\nu}d_p\partial_\mu(\bar{d}_p\gamma_\mu V^*_{rp}u_r)] - \frac{g}{4M_W^2}\mathrm{Im}(C^{pr}_{quW\varphi} + C^{rp}_{qdW\varphi})[\bar{d}_p\sigma^{\mu\nu}\gamma_5 u_r\partial_\mu(\bar{u}_r\gamma_\mu\gamma_5 V_{rp}d_p) + \bar{u}_r\sigma^{\mu\nu}\gamma_5 d_p\partial_\mu(\bar{d}_p\gamma_\mu\gamma_5 V^*_{rp}u_r)]\right\}. \quad (7)$$

Given our interest in $\eta \to \pi^+\pi^-\pi^0$, we let $p = r = 1$ and then, according to their flavor structure, we write down the operators in three kinds:

$$i\bar{\psi}_i\sigma^{\mu\nu}\gamma_5\psi_i\partial_\mu(\bar{\psi}_j\gamma_\nu\gamma_5\psi_j), \quad (8)$$

$$i\bar{\psi}_i\sigma^{\mu\nu}\gamma_5\psi_j\partial_\mu(\bar{\psi}_j\gamma_\nu\gamma_5\psi_i) + i\bar{\psi}_j\sigma^{\mu\nu}\gamma_5\psi_i\partial_\mu(\bar{\psi}_i\gamma_\nu\gamma_5\psi_j) \quad (i \neq j), \quad (9)$$

$$i\bar{\psi}_i\sigma^{\mu\nu}\psi_j\partial_\mu(\bar{\psi}_j\gamma_\nu\psi_i) - i\bar{\psi}_j\sigma^{\mu\nu}\psi_i\partial_\mu(\bar{\psi}_i\gamma_\nu\psi_j) \quad (i \neq j), \quad (10)$$

where ψ_i denotes the quark field with flavor i and $V_{ud} \approx 1$. Using the equations of motion, integration by parts, and Fierz identities [82], it can be proved that these three operators can be converted into one single form $\bar{\psi}_i\overleftrightarrow{\partial}_\mu\gamma_5\psi_i\bar{\psi}_j\gamma^\mu\gamma_5\psi_j$, where we employ the Hermitian derivative $i\bar{\psi}\overleftrightarrow{\partial}_\mu\psi \equiv \bar{\psi}i(\partial_\mu\psi) - i(\partial_\mu\bar{\psi})\psi$. Thus, we rewrite Eq. (7) for use in first-generation processes as

$$\Omega^{\not{C}P} \sim \frac{vg}{M_W^2}\frac{1}{\Lambda^2}c_{ij}\bar{\psi}_i\overleftrightarrow{\partial}_\mu\gamma_5\psi_i\bar{\psi}_j\gamma^\mu\gamma_5\psi_j, \quad (11)$$

where the Wilson coefficients c_{ij} are real with flavor indices and we use the coefficient $vg/(M_W^2\Lambda^2)c_{ij}$ since $g_Z \approx g$ and $M_W \approx M_Z$. The structure of this quark-level operator, which is dimension-seven in itself, has also been reported in the early literature [83–87] and is adopted in Ref. [60], but only with SMEFT can the new physics scale dependence be properly handled. Please note that in Ref. [60], they start from LEFT and trace their operators back to dimension-eight SMEFT, which has a different $(1/\Lambda^4)$ dependence on the BSM scale.

Now we show the matching of the quark-level operators in Eq. (11) to ChPT ones; we neglect all QCD evolution effects in evolving the quark-level operators to the chiral matching scale Λ_χ. Following Refs. [60,88–91], the C- and CP-odd ChPT operators are constructed by rewriting the quark-level operators in chiral irreducible representations and replacing the quark field¡s with chiral building blocks coupled to the spurions $\lambda_i^{(\dagger)}$ and $\lambda_{L/R,j}$ with flavor indices $i, j \in u, d, s$. After carefully examining all the possible forms, we find the following three operators with a nonzero contribution to $\eta \to \pi^+\pi^-\pi^0$ at order p^2 and at leading order in the number of colors N_c:

$$\mathcal{L}_{p^2}^{\not{C}P} = \frac{ivg}{M_W^2}\frac{1}{\Lambda^2}c_{ij}[g_1\langle(\lambda_i D^2\bar{U}^\dagger\bar{U}\lambda_{L,j}\bar{U}^\dagger + \lambda_i^\dagger D^2\bar{U}\bar{U}^\dagger\lambda_{R,j}\bar{U}) - \text{H.c.}\rangle$$
$$+ g_2\langle(\lambda_i D_\mu\bar{U}^\dagger D^\mu\bar{U}\lambda_{L,j}\bar{U}^\dagger + \lambda_i^\dagger D_\mu\bar{U}D^\mu\bar{U}^\dagger\lambda_{R,j}\bar{U}) - \text{H.c.}\rangle$$
$$+ g_3\langle(\lambda_i\bar{U}^\dagger D^2\bar{U}\lambda_{L,j}\bar{U}^\dagger + \lambda_i^\dagger\bar{U}D^2\bar{U}^\dagger\lambda_{R,j}\bar{U}) - \text{H.c.}\rangle]. \quad (12)$$

Here the large N_c extension [92–99] has been applied to include η' and $\bar{U} = \exp(\bar{\Phi}/F_0)$ with

$$\bar{\Phi} = \begin{pmatrix} \frac{1}{\sqrt{3}}\eta' + \sqrt{\frac{2}{3}}\eta + \pi^0 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & \frac{1}{\sqrt{3}}\eta' + \sqrt{\frac{2}{3}}\eta - \pi^0 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & \frac{2}{\sqrt{3}}\eta' - \sqrt{\frac{2}{3}}\eta \end{pmatrix}, \quad (13)$$

which is useful for future use. The effective coefficients g_i are of mass-dimension-five. We note a similar result can be found in Ref. [60], though we differ in the contribution of a g_3 term. With Eq. (12) in hand, the spurions can be set to their physical values, $\lambda_u^{(\dagger)}, \lambda_{L/R,u} = \text{diag}(1,0,0)$, $\lambda_d^{(\dagger)}, \lambda_{L/R,d} = \text{diag}(0,1,0)$, and $\lambda_s^{(\dagger)}, \lambda_{L/R,s} = \text{diag}(0,0,1)$. Expanding $\bar{U}^{(\dagger)}$ to $\bar{\Phi}^4$, these three operators together produce the following C- and CP-odd $\eta\to\pi^+\pi^-\pi^0$ interaction

$$\frac{ivg}{M_W^2}\frac{1}{\Lambda^2 F_0^4}2\mathcal{N}_{p^2}\partial^\mu\pi^0(\pi^+\partial_\mu\pi^- - \pi^-\partial_\mu\pi^+)\eta, \quad (14)$$

where

$$\mathcal{N}_{p^2} = 4\sqrt{\frac{2}{3}}(c_{uu} - c_{ud} - c_{du} + c_{dd})(-g_1 + g_2 - g_3). \quad (15)$$

The corresponding amplitude is

$$\mathcal{M}_{p^2}^{\not{C}}(s,t,u) = i\frac{vg}{M_W^2}\frac{1}{\Lambda^2 F_0^4}\mathcal{N}_{p^2}(t-u) \equiv i\alpha(t-u), \quad (16)$$

where $s = (p_{\pi^+} + p_{\pi^-})^2$ and α is of mass-dimension -2. Generally the ChPT amplitude does not map to a certain final state with specific isospin, but we can decompose it into an $I=0$ amplitude and $I=2$ amplitude according to their isospin structures [60,100] as

$$\mathcal{M}_{I=0}^{\not{C}} = \frac{1}{\sqrt{6}}[-\mathcal{M}^{\not{C}}(t,s,u) + \mathcal{M}^{\not{C}}(s,t,u) - \mathcal{M}^{\not{C}}(u,t,s)], \quad (17)$$

$$\mathcal{M}_{I=2}^{\not{C}} = -\frac{1}{2\sqrt{3}}[\mathcal{M}^{\not{C}}(t,s,u) + 2\mathcal{M}^{\not{C}}(s,t,u) + \mathcal{M}^{\not{C}}(u,t,s)]. \quad (18)$$

Thus, we see that $\mathcal{M}_{I=0,p^2}^{\not{C}} = 0$ and $\mathcal{M}_{I=2,p^2}^{\not{C}} = -\sqrt{3}/2\mathcal{M}_{p^2}^{\not{C}}$. This demonstrates that the order p^2 amplitude only contributes to the $I=2$ final state. Actually, it is well known that the lowest-order nonzero $I=0$ amplitude first appears at order p^6 [35]. To better understand the $I=0$ case, we also investigate the order p^6 ChPT operators that contribute to the $I=0$ final state. Among the numerous order p^6 operators, we find that all operators with three $\bar{U}$ yield a vanishing $I=0$ amplitude, whereas the following operators with five $\bar{U}$ have a nonzero contribution:

$$\mathcal{L}_{p^6}^{\not{C}P} = \frac{ivg}{M_W^2}\frac{1}{\Lambda^2}c_{ij}[f_1\langle\lambda\partial_\mu\partial_\nu\partial_\rho\bar{U}^\dagger U\lambda_L\partial^\mu\partial^\nu\bar{U}^\dagger U\partial^\rho\bar{U}^\dagger + \lambda^\dagger\partial_\mu\partial_\nu\partial_\rho\bar{U}U^\dagger\lambda_R\partial^\mu\partial^\nu\bar{U}U^\dagger\partial^\rho\bar{U} - \text{H.c.}\rangle$$
$$+f_2\langle\lambda\partial_\mu\partial_\nu\partial_\rho\bar{U}^\dagger U\partial^\mu\partial^\nu\bar{U}^\dagger\lambda_R U\partial^\rho\bar{U}^\dagger + \lambda^\dagger\partial_\mu\partial_\nu\partial_\rho\bar{U}U^\dagger\partial^\mu\partial^\nu\bar{U}\lambda_L U^\dagger\partial^\rho\bar{U} - \text{H.c.}\rangle$$
$$+f_3\langle\lambda\partial_\mu\partial_\nu\partial_\rho\bar{U}^\dagger\partial^\mu\partial^\nu U\bar{U}^\dagger\partial^\rho U\lambda_L\bar{U}^\dagger + \lambda^\dagger\partial_\mu\partial_\nu\partial_\rho\bar{U}\partial^\mu\partial^\nu U^\dagger\bar{U}\partial^\rho U^\dagger\lambda_R\bar{U} - \text{H.c.}\rangle$$
$$+f_4\langle\lambda\partial_\mu\partial_\nu\partial_\rho\bar{U}^\dagger\partial^\mu\partial^\nu U\bar{U}^\dagger\lambda_R\partial^\rho U\bar{U}^\dagger + \lambda^\dagger\partial_\mu\partial_\nu\partial_\rho\bar{U}\partial^\mu\partial^\nu U^\dagger\bar{U}\lambda_L\partial^\rho U^\dagger\bar{U} - \text{H.c.}\rangle$$
$$+f_5\langle\lambda\partial_\mu\partial_\nu\partial_\rho\bar{U}^\dagger U\lambda_L\partial^\mu\partial^\nu\bar{U}^\dagger\partial^\rho U\bar{U}^\dagger + \lambda^\dagger\partial_\mu\partial_\nu\partial_\rho\bar{U}U^\dagger\lambda_R\partial^\mu\partial^\nu\bar{U}\partial^\rho U^\dagger\bar{U} - \text{H.c.}\rangle$$
$$+f_6\langle\lambda\partial_\mu\partial_\nu\partial_\rho\bar{U}^\dagger U\partial^\mu\partial^\nu\bar{U}^\dagger\lambda_R\partial^\rho U\bar{U}^\dagger + \lambda^\dagger\partial_\mu\partial_\nu\partial_\rho\bar{U}U^\dagger\partial^\mu\partial^\nu\bar{U}\lambda_L\partial^\rho U^\dagger\bar{U} - \text{H.c.}\rangle$$
$$+f_7\langle\lambda\partial_\mu\partial_\nu\partial_\rho\bar{U}^\dagger U\partial^\mu\partial^\nu\bar{U}^\dagger\partial^\rho U\lambda_L\bar{U}^\dagger + \lambda^\dagger\partial_\mu\partial_\nu\partial_\rho\bar{U}U^\dagger\partial^\mu\partial^\nu\bar{U}\partial^\rho U^\dagger\lambda_R\bar{U} - \text{H.c.}\rangle], \quad (19)$$

where f_i are of mass-dimension-one. Note that these p^6 operators contribute to both $I = 0$ and $I = 2$ final states after expanding the $\bar{U}$ field. The corresponding interaction vertex relevant to the $I = 0$ final state is

$$\frac{ivg}{M_W^2}\frac{1}{\Lambda^2 F_0^4} 8\mathcal{N}_{p^6}\epsilon_{IJK}(\partial_\mu\partial_\nu\partial_\rho\pi^I)(\partial^\mu\partial^\nu\pi^J)(\partial^\rho\pi^K)\eta, \quad (20)$$

where $I, J, K = +, -, 0$ and

$$\mathcal{N}_{p^6} = \sqrt{\frac{2}{3}}(c_{uu} - c_{ud} - c_{du} + c_{dd}) \times (f_1 + f_2 + f_3 - f_4 - f_5 - f_6 + f_7). \quad (21)$$

The structure of Eq. (20) is, in fact, well known, e.g., Ref. [35]; however, it is derived here from SMEFT and ChPT for the first time to the best of our knowledge. The resultant order $p^6 I = 0$ amplitude is expressed as

$$\mathcal{M}_{p^6}^{\not{C}P} = i\frac{vg}{M_W^2}\frac{1}{\Lambda^2 F_0^4}\mathcal{N}_{p^6}(s-t)(u-s)(t-u) \equiv i\beta(s-t)(u-s)(t-u), \quad (22)$$

where β is of mass-dimension −6.

In Eqs. (16) and (22), the effective coefficients α and β depend on the new physics scale Λ. Thus, determining α and β from experimental observables helps to estimate the potential new physics scale. The total amplitude square that is directly related to the experimental measurements can be written as

$$|\mathcal{M}(s,t,u)|^2 = |\mathcal{M}^C(s,t,u)|^2 + 2\mathrm{Re}[\mathcal{M}^C(s,t,u)\cdot\mathcal{M}^{\not{C}}(s,t,u)^*] + \mathcal{O}(\alpha^2,\beta^2), \quad (23)$$

where $\mathcal{M}^C$ is the $I = 1$ C-conserving amplitude and $\mathcal{M}^{\not{C}}$ indicates the C-violating amplitudes. The second term is the interference of the C-conserving and C-violating amplitudes that contributes to C and CP violation of $\eta \to \pi^+\pi^-\pi^0$ decay.

III. DETERMINING α AND β FROM EXPERIMENTS

In this section, we describe our procedures to determine α and β from experimental observables. The experimental Dalitz plot distribution of $\eta \to \pi^+\pi^-\pi^0$ is usually described by variables X and Y defined as

$$X \equiv \sqrt{3}\frac{T_{\pi^+} - T_{\pi^-}}{Q_\eta} = \frac{\sqrt{3}}{2m_\eta Q_\eta}(u-t), \quad (24)$$

$$Y \equiv \frac{3T_{\pi^0}}{Q_\eta} - 1 = \frac{3}{2m_\eta Q_\eta}[(m_\eta - m_{\pi^0})^2 - s] - 1, \quad (25)$$

where $Q_\eta = T_{\pi^+} + T_{\pi^-} + T_{\pi^0} = m_\eta - 2_m\pi^+ - m_{\pi^0}$ and T_{π^i} is the π^i's kinetic energy in the η rest frame. Since $X, Y \in (0,1)$, the amplitude squared can be expanded in X and Y as

$$|\mathcal{M}(s,t,u)|^2 = N_0(1 + aY + bY^2 + cX + dX^2 + eXY + fY^3 + gX^2Y + hXY^2 + lX^3 + \cdots), \quad (26)$$

where N_0 is a normalization factor and $a, b, c, \ldots$ are called Dalitz plot parameters. Since the C transformation on the amplitude switches t with u, which is equivalent to switching X with $-X$, the nonzero coefficients of terms with X in odd power, i.e., c, e, h, and l in Eq. (26), would imply C and CP violation in this decay. Reference [32] constructs the $I = 0$ and $I = 2$ amplitudes by reassembling the C-conserving $I = 1$ amplitude from next-to-leading order (NLO) ChPT and fitting the mock data generated using the C-violating parameters c, e, h, and l reported by KLOE-2 [71] to determine the decay pattern. In comparison, the work of Ref. [56] uses dispersion theory to form the amplitudes and fits the whole Dalitz plot distribution of $\eta \to \pi^+\pi^-\pi^0$. In this work, we construct the C-violating amplitudes from SMEFT and ChPT without any phenomenological input, and we adopt two procedures to determine α and β from the $\eta \to \pi^+\pi^-\pi^0$ data. One is to fit the left-right asymmetric distribution directly, and the other is to use the integrated asymmetries A_Q and A_S. We first explain the fitting procedure in what follows.

KLOE-2 [71] and BESIII [72] provide the binned Dalitz plot distribution in (X, Y) space. The left-right asymmetric distribution of the Dalitz plot can be obtained through subtracting each of the binned data at the $X > 0$ side by the other side with opposite X and the same Y, i.e.,

$$N_i^{\not{C}}(X_i, Y_i) = \frac{1}{2}[N_i(X_i, Y_i) - N_i(-X_i, Y_i)] \quad (X_i > 0), \quad (27)$$

where (X_i, Y_i) is the coordinate of the center of the ith bin and $N_i(X_i, Y_i)$ represents the number of events in bin i. The asymmetric events $N_i^{\not{C}}(X_i, Y_i)$ can be related to the theoretical amplitudes as

$$\frac{N_i^{\not{C}}}{N_{\mathrm{tot}}} = \frac{\int_i 2\mathrm{Re}[\mathcal{M}^C(X,Y)\cdot\mathcal{M}^{\not{C}}(X,Y)^*]dXdY}{\int|\mathcal{M}^C(X,Y)|^2 dXdY}, \quad (28)$$

where N_{tot} represents the total number of events in the whole phase space and $\int_i$ means that the integral region is within the ith bin. For the denominator on the right-hand side, since the integral region is the whole phase space, only the C-conserving part of the amplitude square survives, and

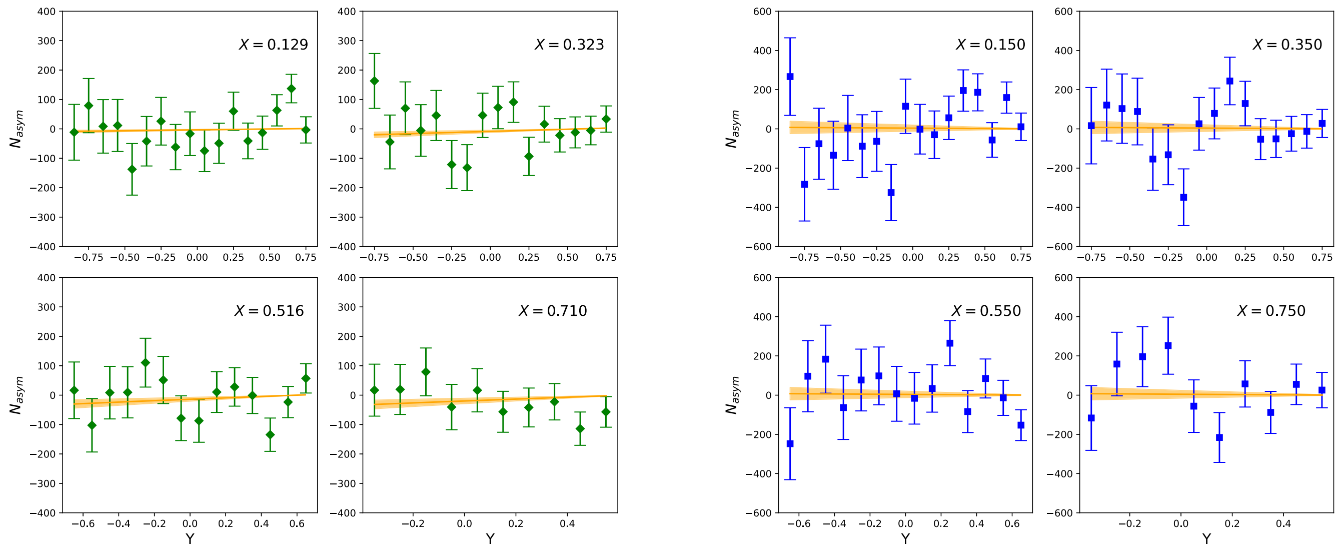

FIG. 1. The comparison of the experimental left-right asymmetric distribution with the theoretical result using the order p^2 amplitude in Eq. (16), with α determined from fitting the KLOE-2 [71] (left panel) and the BESIII [72] (right panel) data separately.

we apply the NLO ChPT form [43] as done in Ref. [32] in our fit.

We use Eq. (28) to fit the KLOE-2 [71] and BESIII [72] data. For $\mathcal{M}^{\not{C}}$ in Eq. (28), we either use the order $p^2\, I=2$ amplitude only or the order p^2 amplitude together with the order $p^6\, I=0$ amplitude. Adding the order $p^6\, I=0$ amplitude to the order p^2 amplitude is in principle not proper since the order p^4 amplitude has not been included. However, the order p^6 amplitude is a higher-order correction, and we include it to see roughly what the $I=0$ effects might be. Figure 1 illustrates the fitting with the p^2 amplitude only for different selected X. The data points are $N_i^{\not{C}}$ obtained from the experimental data using Eq. (27), and the yellow line represents the fitted theoretical C-asymmetric distribution with the band indicating the uncertainty at $\pm 1\sigma$. One can see that both the KLOE-2 data (left panel) and the BES-III (right-panel) data are basically consistent with zero and that the BES-III data have larger errors, which is understandable since they involve smaller η decay samples. The fitted lines are also consistent with zero within two sigma. All the fitting results together with the χ^2/d.o.f. are listed in Table I. Similar to what we have learned from Fig. 1, the values of α and β are consistent with zero within two sigma. In this case, their uncertainties are more significant than the central values and can be treated as the upper limits of the experimental constraint. Again, the errors of α and β from the BESIII data are about 3 times those from KLOE-2, which is as expected since the statistics of BESIII are about $1/10$ of KLOE-2. The results from BESIII and KLOE-2 are consistent within errors, so the joint fitting is feasible. One can also conclude that adding the order p^6 amplitude has little effect on the α values, which means the order p^6 amplitude does indeed behave as a higher-order effect. The large uncertainties in β mean that the experimental data provide much less constraint on the decay to the $I=0$ final state.

TABLE I. Determined values of α and β together with the χ^2/d.o.f. by fitting the left-right asymmetric distribution of KLOE-2 [71] and BESIII [72].

	KLOE-2	BESIII	KLOE-2 + BESIII
α/GeV^{-2}	−0.031(14)	0.009(36)	−0.026(13)
χ^2/d.o.f	1.00	0.97	0.99
α/GeV^{-2}	−0.033(15)	0.011(37)	−0.027(14)
β/GeV^{-6}	−7(12)	14(29)	−4(11)
χ^2/d.o.f	1.00	0.97	0.99

Now we switch to the second procedure in which we use the integrated charge asymmetries A_Q and A_S. The quadrant asymmetry A_Q and the sextant asymmetry A_S of the $\eta\to\pi^+\pi^-\pi^0$ decay are defined as

$$A_Q=\frac{N_A+N_C-N_B-N_D}{N_A+N_C+N_B+N_D}, \tag{29}$$

$$A_S=\frac{N_I+N_{III}+N_V-N_{II}-N_{IV}-N_{VI}}{N_I+N_{III}+N_V+N_{II}+N_{IV}+N_{VI}}, \tag{30}$$

where the different partitions are illustrated in Fig. 2. We can use A_Q to deduce α and A_S to determine β, respectively. KLOE-2 [71] and BESIII [72] report their measurements of A_Q and A_S as $A_Q^{\mathrm{KLOE}}=(+1.8\pm 4.5^{+4.8}_{-2.3})\times 10^{-4}$, $A_S^{\mathrm{KLOE}}=(-0.4\pm 4.5^{+3.1}_{-3.5})\times 10^{-4}$, $A_Q^{\mathrm{BES}}=(-3.5\pm 13.1\pm 1.1)\times 10^{-4}$, and $A_S^{\mathrm{BES}}=(-7.0\pm 13.1\pm 0.9)\times 10^{-4}$. Accordingly, we use the expression

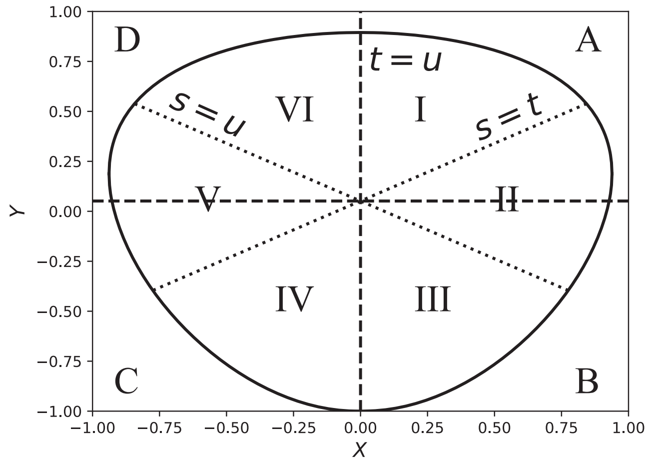

FIG. 2. Different partitions of the Dalitz plot data used in the quadrant A_Q and sextant A_S asymmetries defined in Eqs. (29) and (30), respectively.

$$N_X = \int_X |\mathcal{M}(X,Y)|^2 dXdY, \tag{31}$$

where X denotes the partitions, and Eqs. (29) and (30) are used to calculate the theoretical asymmetries with α and β to be determined. The resultant α and β are shown in Table II. One can see that the α values are consistent with those from fitting the left-right asymmetric distribution within errors, though the uncertainties are slightly larger in this procedure. Determining β from A_S can, in principle, result in a better signal since only the order p^6 amplitude is included. However, as we see, the resultant value is not better constrained. This again suggests that the present data cannot yet provide a precise constraint on the $I = 0$ channel.

We have applied two procedures to determine α and β. We use the values determined by jointly fitting the KLOE-2 and BESIII left-right asymmetric distribution as our primary results, i.e., $\alpha = -0.027(14)\ \mathrm{GeV}^{-2}$ and $\beta = -4(11)\ \mathrm{GeV}^{-6}$. For the BSM scale estimation in the next section, we estimate the upper limits of their absolute vales at 90% confidence level (C.L.) as

$$|\alpha| \lesssim 0.05\ \mathrm{GeV}^{-2}, \tag{32}$$

$$|\beta| \lesssim 22\ \mathrm{GeV}^{-6}. \tag{33}$$

Given the limitations of our current data in determining a constraint on $I = 0$ BSM effects, we pause to consider how else these effects could be constrained. Here we note the possibility of constraining the $I = 0$ sector through the study of $\eta \to \pi^0 \ell^+ \ell^-$ decay, though the current experimental limits on the latter give comparable constraints [101].

TABLE II. Values of α and β determined by A_S and A_Q from KLOE-2 and BESIII, respectively.

	KLOE-2	BESIII
α/GeV^{-2} (A_Q)	$-0.007^{+0.019}_{-0.023}$	0.013 ± 0.048
β/GeV^{-6} (A_S)	$-1.05^{+14.93}_{-14.61}$	18.33 ± 33.66

IV. NEW PHYSICS SCALE AND IMPACT STUDY

With the upper limits of $|\alpha|$ and $|\beta|$, we utilize NDA for a rough order-of-magnitude estimate of the new physics scale. When matching the dimension-seven quark-level operator in Eq. (11) to meson-level ones, an effective low energy constant multiplies each operator, and according to NDA [102–107] its natural size is expected to be

$$F_0^4 \Lambda_\chi^3 \frac{1}{F_0^m} \frac{1}{\Lambda_\chi^n}, \tag{34}$$

where m is the number of meson fields and n denotes the number of derivatives in the ChPT operator. Thus, for the order p^2 operators in Eq. (12), the coefficient g_i has the order of $F_0^4 \Lambda_\chi$. Analogously, for the operators at order p^6 in Eq. (19), $f_i \sim F_0^4/\Lambda_\chi^3$. Assuming there is no unexpected fine-tuning in the ultraviolet completion of SMEFT, c_{ij} should have an order of unity. Therefore, from Eqs. (15) and (21), we have $\mathcal{N}_{p^2} \sim F_0^4 \Lambda_\chi$ and $\mathcal{N}_{p^6} \sim F_0^4/\Lambda_\chi^3$. Based on Eqs. (16) and (22), the new physics scale is finally related to α and β as

$$\Lambda_{p^2} \sim \left(\frac{vg\Lambda_\chi}{M_W^2} \frac{1}{|\alpha|} \right)^{1/2}, \tag{35}$$

$$\Lambda_{p^6} \sim \left(\frac{vg}{M_W^2 \Lambda_\chi^3} \frac{1}{|\beta|} \right)^{1/2}, \tag{36}$$

where Λ_{p^2} and Λ_{p^6} represent the new physics scale derived from the coefficients of p^2 and p^6 amplitudes, respectively. Note that our resulting Λ is proportional to $-1/2$ power of the amplitude coefficients which is different from the LEFT work of Ref. [60]. In Sec. III we estimate the upper limits of $|\alpha|$ and $|\beta|$ at 90% C.L. as $|\alpha| \lesssim 0.05\ \mathrm{GeV}^{-2}$ and $|\beta| \lesssim 22\ \mathrm{GeV}^{-6}$. Utilizing Eqs. (35) and (36) we have

$$\Lambda_{p^2} \gtrsim 0.8\ \mathrm{GeV}, \tag{37}$$

$$\Lambda_{p^6} \gtrsim 0.03\ \mathrm{GeV}, \tag{38}$$

where we use $v = 246.22$ GeV, $M_W = 80.37$ GeV, $g = 0.653$, and $F_0 = 92.28$ MeV as reported in Ref. [108], noting that their empirical errors are unneeded since we care only about the order of magnitude. It is understandable that Λ_{p^6} is very small, because as mentioned in Sec. III, the current experimental data have little constraint on β. The scale of the p^2 coefficient is much larger than that of the p^6 coefficient, but still obviously lower than the expected new physics scale, which implies that the current experiments are not yet precise enough to provide a reasonable estimate of the new physics scale. However, our analysis is valid to estimate the possibility that the C and CP violation of $\eta \to \pi^+\pi^-\pi^0$ could be observed by the upcoming η experiments. An impact study from the order p^2 coefficient is performed as follows, from which one can see that the low limits of Λ are driven by the relevant low statistics of present data as shown in Fig. 3.

On account that the value of α is statistically consistent with zero and the uncertainty indicates the experimental precision, we use its upper limit using its uncertainty as $|\alpha| \lesssim 0.1\ \mathrm{GeV}^{-2}$ in the impact study. Thus, the current estimation of the new physics scale would be $\Lambda \gtrsim 1.7$ GeV. Since the scale is just a rough order-of-magnitude approximation from NDA, it is reasonable to consider a ± 1 order-of-magnitude deviation as its uncertainty. Suppose, e.g., Λ_{p^2} should increase from 10 GeV to 1 TeV; then, according to Eq. (35), the upper limit of α, which is estimated by its uncertainty, should decrease by a factor of 10^{-4}. Since the uncertainty of α is proportional to the experimental statistics, it should improve by 10^8—at least. Generally, if we think Λ should increase by 10^n times, the experimental statistics would need to improve by 10^{4n} times. Note that, as pointed out in the first section, for a pure CP-violating process, rather than the interference effect we study in $\eta \to \pi^+\pi^-\pi^0$ decay, the required experimental improvement would be 10^{8n} times.

The data we use in this analysis are from the KLOE-2 Collaboration [71] with a sample of 4.7×10^6 $\eta \to \pi^+\pi^-\pi^0$ decays and BESIII Collaboration [72] with a sample size of

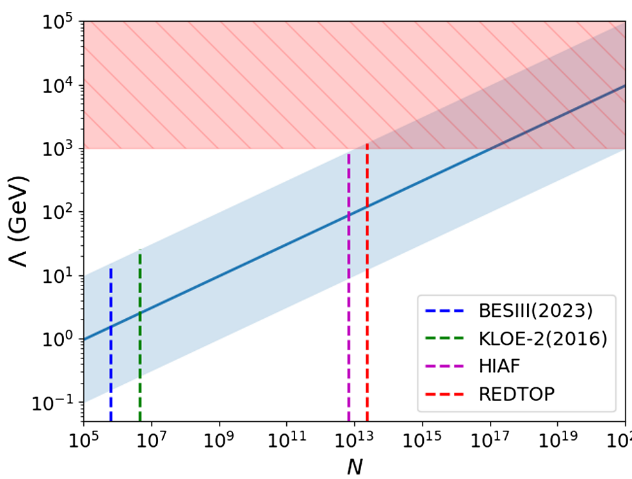

FIG. 3. The predicted relation between estimates of the new physics scale and experimental statistics for $\eta \to \pi^+\pi^-\pi^0$ decay, where the pink area with slashes means the energy region where new physics could occur, estimated to be $\Lambda > 1$ TeV, and the blue band is our theoretical result with uncertainty. The blue, green, purple, and red dashed lines indicate the statistics from the BESIII [72], KLOE-2 [71], HIAF [80], and REDTOP [77–79] experiments, respectively.

6.3×10^5 events. Supposing that the new physics lives at a scale above 1 TeV, it means future experiments need to reach a sample size of at least 10^{13} to observe C and CP violation in $\eta \to \pi^+\pi^-\pi^0$ decay. We illustrate the relation between experimental statistics and the estimated new physics scale in Fig. 3, in which the pink area with slashes means the energy region where new physics could occur and the blue band is our theoretical result with uncertainty. Future experiments are expected to be made at the JLab Eta Factory(JEF) of Jefferson Lab [73,75,76], by the REDTOP Collaboration [77–79], and by the collaboration from the HIAF eta factory [80], etc. Among them, the REDTOP Collaboration plans to have as much as 10^{14} η events in three years of running, or about 2.3×10^{13} $\eta \to \pi^+\pi^-\pi^0$ samples. The HIAF Collaboration [80] plans to have over 10^{13} η events per year, which is about 6.9×10^{12} $\eta \to \pi^+\pi^-\pi^0$ events for a three-year period of running. We draw the statistics of BESIII [72], KLOE-2 [71], HIAF [80], and REDTOP [77–79] from left to right, respectively, in Fig. 3. One can see that the new physics potential of studies of C and CP violation is promising and could possibly be observed in the upcoming HIAF and REDTOP experiments.

V. SUMMARY AND OUTLOOK

The decay $\eta \to \pi^+\pi^-\pi^0$ is an ideal process in which to study flavor-conserving C- and CP-violating physics BSM. We have shown the key procedures involved in deriving the quark-level operators pertinent to C and CP violation in $\eta \to \pi^+\pi^-\pi^0$ decay, originating from the dimension-six SMEFT operators. Then, besides the order p^2 meson-level C- and CP-odd operators corresponding exclusively to the $I = 2$ final state, the order p^6 operators producing the nonvanishing $I = 0$ amplitude at lowest order are presented for the first time to the best of our knowledge. By directly fitting the asymmetric distribution of the Dalitz plot of $\eta \to \pi^+\pi^-\pi^0$ from KLOE-2 [71] and BESIII [72] Collaborations, the upper limits of the order p^2 and p^6 amplitude coefficients α and β from using chiral effective theory are determined without theoretical model uncertainties. Based on NDA, we perform an order-of-magnitude estimate of the new physics scale. A corresponding impact study of future η experiments is also carried out and shown in Fig. 3. Considering future experiments with much higher statistics, such as planned by the REDTOP Collaboration [77–79] and at the HIAF eta factory [80], it seems promising that C and CP violation in $\eta \to \pi^+\pi^-\pi^0$ decay could be observed in the near future.

ACKNOWLEDGMENTS

We are grateful to Xiaodong Ma, Yi Liao, Hakan Akdag, Tobias Isken, Bastian Kubis, Shuang-Shi Fang, Xiao-Lin Kang, Xu-Rong Chen, and Qian Wang for helpful discussions. This work is partially supported by Guangdong Major Project of Basic and Applied Basic Research under Grant No. 2020B0301030008. J. S. is supported by the Natural Science Foundation of China under Grant No. 12105108. J. L. is supported by the Natural Science Foundation of China under Grants No. 12175073 and No. 12222503, and the Natural Science Foundation of Basic and Applied Basic Research of Guangdong Province under Grant No. 2023A1515012712. S. G. acknowledges partial support from the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under Contract No. DE-FG02-96ER40989.

[1] A. D. Sakharov, Violation of CP Invariance, C asymmetry, and baryon asymmetry of the universe, Pis'ma Zh. Eksp. Teor. Fiz. **5**, 32 (1967).
[2] N. Aghanim *et al.* (Planck Collaboration), Planck 2018 results. VI. Cosmological parameters, Astron. Astrophys. **641**, A6 (2020); **652**, C4(E) (2021).
[3] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, The Electroweak phase transition: A Nonperturbative analysis, Nucl. Phys. **B466**, 189 (1996).
[4] F. Csikor, Z. Fodor, and J. Heitger, Endpoint of the hot electroweak phase transition, Phys. Rev. Lett. **82**, 21 (1999).
[5] G. R. Farrar and M. E. Shaposhnikov, Baryon asymmetry of the universe in the minimal Standard Model, Phys. Rev. Lett. **70**, 2833 (1993); **71**, 210(E) (1993).
[6] G. R. Farrar and M. E. Shaposhnikov, Baryon asymmetry of the universe in the standard electroweak theory, Phys. Rev. D **50**, 774 (1994).
[7] P. Huet and E. Sather, Electroweak baryogenesis and standard model CP violation, Phys. Rev. D **51**, 379 (1995).
[8] M. B. Gavela, P. Hernandez, J. Orloff, O. Pene, and C. Quimbay, Standard model CP violation and baryon asymmetry. Part 2: Finite temperature, Nucl. Phys. **B430**, 382 (1994).
[9] G. Elor, M. Escudero, and A. Nelson, Baryogenesis and Dark Matter from B Mesons, Phys. Rev. D **99**, 035031 (2019).
[10] A. E. Nelson and H. Xiao, Baryogenesis from B meson oscillations, Phys. Rev. D **100**, 075002 (2019).
[11] A. B. Carter and A. I. Sanda, CP violation in cascade decays of B mesons, Phys. Rev. Lett. **45**, 952 (1980).
[12] Y. Grossman and M. P. Worah, CP asymmetries in B decays with new physics in decay amplitudes, Phys. Lett. B **395**, 241 (1997).
[13] S. Laplace, Z. Ligeti, Y. Nir, and G. Perez, Implications of the CP asymmetry in semileptonic B decay, Phys. Rev. D **65**, 094040 (2002).
[14] A. K. Giri and R. Mohanta, Direct CP violation and new physics effects in the decay mode $B^+ \to \phi K^+$, Mod. Phys. Lett. A **19**, 1903 (2004).
[15] A. Kundu, S. Nandi, and J. P. Saha, New physics in $b \to s\bar{s}s$ decay, Phys. Lett. B **622**, 102 (2005).
[16] F. J. Botella and J. P. Silva, Reparametrization invariance of B decay amplitudes and implications for new physics searches in B decays, Phys. Rev. D **71**, 094008 (2005).
[17] M. K. Mohapatra, Inspection of new physics in $B_s \to K^+K^-$ decay mode, Phys. Rev. D **101**, 075033 (2020).
[18] R. Aaij *et al.* (LHCb Collaboration), Observation of CP violation in charm decays, Phys. Rev. Lett. **122**, 211803 (2019).
[19] A. Dery and Y. Nir, Implications of the LHCb discovery of CP violation in charm decays, J. High Energy Phys. 12 (2019) 104.
[20] R. Bause, H. Gisbert, M. Golz, and G. Hiller, Exploiting CP-asymmetries in rare charm decays, Phys. Rev. D **101**, 115006 (2020).
[21] Y. Grossman and Y. Nir, $K_L \to \pi^0\nu\nu$ beyond the standard model, Phys. Lett. B **398**, 163 (1997).
[22] G. Buchalla and A. J. Buras, The rare decays $K \to \pi\nu\nu-$, $B \to X\nu\nu-$ and $B \to l^+l^-$: An update, Nucl. Phys. **B548**, 309 (1999).
[23] S. K. Lamoreaux and R. Golub, Experimental searches for the neutron electric dipole moment, J. Phys. G **36**, 104002 (2009).
[24] C. Abel *et al.*, Measurement of the permanent electric dipole moment of the neutron, Phys. Rev. Lett. **124**, 081803 (2020).
[25] V. Andreev *et al.* (ACME Collaboration), Improved limit on the electric dipole moment of the electron, Nature (London) **562**, 355 (2018).
[26] T. S. Roussy *et al.*, An improved bound on the electron's electric dipole moment, Science **381**, adg4084 (2023).
[27] R. Alarcon *et al.*, Electric Dipole Moments and the Search for New Physics, in Snowmass 2021 (2022), arXiv:2203.08103.
[28] W. Dekens, J. de Vries, M. Jung, and K. K. Vos, The phenomenology of electric dipole moments in models of scalar leptoquarks, J. High Energy Phys. 01 (2019) 069.
[29] C. Idegawa and E. Senaha, Electron electric dipole moment and electroweak baryogenesis in a complex singlet extension of the Standard Model with degenerate scalars, Phys. Lett. B **848**, 138332 (2024).
[30] L. Di Luzio, H. Gisbert, and P. Sørensen, On the oscillating electric dipole moment induced by axion-fermion couplings, J. High Energy Phys. 04 (2024) 076.
[31] T. D. Lee, Possible C-noninvariant effects in the 3π Decay Modes of η^0 and ω^0, Phys. Rev. **139**, B1415 (1965).
[32] S. Gardner and J. Shi, Patterns of CP violation from mirror symmetry breaking in the $\eta \to \pi^+\pi^-\pi^0$ Dalitz plot, Phys. Rev. D **101**, 115038 (2020).
[33] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Evidence for the 2π decay of the K_2^0 meson, Phys. Rev. Lett. **13**, 138 (1964).

[34] T. D. Lee and L. Wolfenstein, Analysis of CP noninvariant interactions and the K0(1), K0(2) System, Phys. Rev. **138**, B1490 (1965).

[35] J. Prentki and M. J. G. Veltman, Possibility of CP violation in semistrong interactions, Phys. Lett. **15**, 88 (1965).

[36] M. Nauenberg, The $\eta \to \pi^+\pi^0\pi^-$ decay with C-violation, Phys. Lett. **17**, 329 (1965).

[37] B. Barrett, M. Jacob, M. Nauenberg, and T. N. Truong, Consequences of C-violating interactions in η^0 and X^0 decays, Phys. Rev. **141**, 1342 (1966).

[38] D. G. Sutherland, Current algebra and the decay $\eta \to 3\pi$, Phys. Lett. **23**, 384 (1966).

[39] J. S. Bell and D. G. Sutherland, Current algebra and $\eta \to 3\pi$, Nucl. Phys. **B4**, 315 (1968).

[40] K. G. Wilson, Nonlagrangian models of current algebra, Phys. Rev. **179**, 1499 (1969).

[41] R. Baur, J. Kambor, and D. Wyler, Electromagnetic corrections to the decays $\eta \to 3\pi$, Nucl. Phys. **B460**, 127 (1996).

[42] C. Ditsche, B. Kubis, and U.-G. Meissner, Electromagnetic corrections in $\eta \to 3\pi$ decays, Eur. Phys. J. C **60**, 83 (2009).

[43] J. Gasser and H. Leutwyler, $\eta \to 3\pi$ to one loop, Nucl. Phys. **B250**, 539 (1985).

[44] A. V. Anisovich and H. Leutwyler, Dispersive analysis of the decay $\eta \to 3\pi$, Phys. Lett. B **375**, 335 (1996).

[45] D. J. Gross, S. B. Treiman, and F. Wilczek, Light quark masses and isospin violation, Phys. Rev. D **19**, 2188 (1979).

[46] P. Langacker and H. Pagels, Light quark mass spectrum in quantum chromodynamics, Phys. Rev. D **19**, 2070 (1979).

[47] H. Leutwyler, The ratios of the light quark masses, Phys. Lett. B **378**, 313 (1996).

[48] J. Bijnens and J. Gasser, Eta decays at and beyond p^4 in chiral perturbation theory, Phys. Scr. T **99**, 34 (2002).

[49] J. Bijnens and K. Ghorbani, $\eta \to 3\pi$ at two loops in chiral perturbation theory, J. High Energy Phys. 11 (2007) 030.

[50] G. Colangelo, S. Lanz, and E. Passemar, A new dispersive analysis of $\eta \to 3\pi$, Proc. Sci., CD09 (**2009**) 047 [arXiv:0910.0765].

[51] K. Kampf, M. Knecht, J. Novotny, and M. Zdrahal, Analytical dispersive construction of $\eta \to 3\pi$ amplitude: first order in isospin breaking, Phys. Rev. D **84**, 114015 (2011).

[52] P. Guo, I. V. Danilkin, D. Schott, C. Fernández-Ramírez, V. Mathieu, and A. P. Szczepaniak, Three-body final state interaction in $\eta \to 3\pi$, Phys. Rev. D **92**, 054016 (2015).

[53] M. Albaladejo and B. Moussallam, Extended chiral Khuri-Treiman formalism for $\eta \to 3\pi$ and the role of the $a_0(980)$, $f_0(980)$ resonances, Eur. Phys. J. C **77**, 508 (2017).

[54] G. Colangelo, S. Lanz, H. Leutwyler, and E. Passemar, $\eta \to 3\pi$: Study of the Dalitz plot and extraction of the quark mass ratio Q, Phys. Rev. Lett. **118**, 022001 (2017).

[55] G. Colangelo, S. Lanz, H. Leutwyler, and E. Passemar, Dispersive analysis of $\eta \to 3\pi$, Eur. Phys. J. C **78**, 947 (2018).

[56] H. Akdag, T. Isken, and B. Kubis, Patterns of C- and CP-violation in hadronic η and η' three-body decays, J. High Energy Phys. 02 (**2022**) 137; 12 (2022) 156(E).

[57] J. Shi, Theoretical studies of C and *CP* violation in $\eta \to \pi^+\pi^-\pi^0$ decay, Ph.D. thesis, Kentucky U., 2020.

[58] S. Gardner and J. Shi (to be published).

[59] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, Dimension-six terms in the standard model lagrangian, J. High Energy Phys. 10 (**2010**) 085.

[60] H. Akdag, B. Kubis, and A. Wirzba, C and CP violation in effective field theories, J. High Energy Phys. 06 (2023) 154.

[61] J. G. Layter, J. A. Appel, A. Kotlewski, W.-Y. Lee, S. Stein, and J. J. Thaler, Measurement of the charge asymmetry in the decay $\eta \to \pi^+\pi^-\pi^0$, Phys. Rev. Lett. **29**, 316 (1972).

[62] C. Baltay, Search for C Violation in $\eta \to \pi^+\pi^-\pi^0$, Phys. Rev. **149**, 1044 (1966).

[63] M. Gormley, E. Hyman, W. Lee, T. Nash, J. Peoples, C. Schultz, and S. Stein, Experimental Test of C Invariance in $\eta \to \pi^+\pi^-\pi^0$, Phys. Rev. Lett. **21**, 402 (1968).

[64] M. Gormley, E. Hyman, W.-Y. Lee, T. Nash, J. Peoples, C. Schultz, and S. Stein, Experimental determination of the dalitz-plot distribution of the decays $\eta \to \pi^+\pi^-\pi^0$ and $\eta \to \pi^+\pi^-\gamma$, and the branching ratio $\frac{\eta \to \pi^+\pi^-\gamma}{\eta \to \pi^+\pi^-\pi^0}$, Phys. Rev. D **2**, 501 (1970).

[65] A. Larribe *et al.*, Test of C invariance in the 3π decay mode of the η meson, Phys. Lett. **23**, 600 (1966).

[66] J. G. Layter, J. A. Appel, A. Kotlewski, W.-Y. Lee, S. Stein, and J. J. Thaler, Study of dalitz-plot distributions of the decays $\eta \to \pi^+\pi^-\pi^0$ and $\eta \to \pi^+\pi^-\gamma$, Phys. Rev. D **7**, 2565 (1973).

[67] M. R. Jane *et al.*, A measurement of the charge asymmetry in the decay $\eta \to \pi^+\pi^-\pi^0$, Phys. Lett. **48B**, 260 (1974).

[68] F. Ambrosino *et al.* (KLOE Collaboration), Determination of $\eta \to \pi^+\pi^-\pi^0$ Dalitz plot slopes and asymmetries with the KLOE detector, J. High Energy Phys. 05 (2008) 006.

[69] P. Adlarson *et al.* (WASA-at-COSY Collaboration), Measurement of the $\eta \to \pi^+\pi^-\pi^0$ Dalitz plot distribution, Phys. Rev. C **90**, 045207 (2014).

[70] M. Ablikim *et al.* (BESIII Collaboration), Measurement of the matrix elements for the decays $\eta \to \pi^+\pi^-\pi^0$ and $\eta/\eta\prime \to \pi^0\pi^0\pi^0$, Phys. Rev. D **92**, 012014 (2015).

[71] A. Anastasi *et al.* (KLOE-2 Collaboration), Precision measurement of the $\eta \to \pi^+\pi^-\pi^0$ Dalitz plot distribution with the KLOE detector, J. High Energy Phys. 05 (2016) 019.

[72] M. Ablikim *et al.* (BESIII Collaboration), Precision measurement of the matrix elements for $\eta \to \pi^+\pi^-\pi^0$ and $\eta \to \pi^0\pi^0\pi^0$ decays, Phys. Rev. D **107**, 092007 (2023).

[73] L.-P. Gan and A. Gasparian, Search of new physics via η rare decays, Proc. Sci., CD09 (**2009**) 048.

[74] L. Gan, Probes for fundamental QCD symmetries and a dark gauge boson via light meson decays, Proc. Sci., CD15 (**2015**) 017.

[75] L. Gan, Test fundamental symmetries via precision measurements of π^0, η, and $\eta\prime$ decays, J. Phys. Soc. Jpn. Conf. Proc. **13**, 020063 (2017).

[76] A. Somov, Jlab eta factory experiment in hall D,, Proc. Sci., CD2021 (**2024**) 029.

[77] C. Gatto, B. Fabela Enriquez, and M. I. Pedraza Morales (REDTOP Collaboration), The REDTOP project: Rare Eta

decays with a TPC for optical photons, Proc. Sci., ICHEP2016 (**2016**) 812.

[78] C. Gatto (REDTOP Collaboration), The REDTOP experiment, arXiv:1910.08505.

[79] J. Elam *et al.* (REDTOP Collaboration), The REDTOP experiment: Rare $\eta/\eta\prime$ decays to probe new physics, arXiv:2203.07651.

[80] X.-R. Chen *et al.*, A plan for a super η factory at Huizhou accelerator complex, arXiv:2407.00874.

[81] S. Weinberg, Baryon and lepton nonconserving processes, Phys. Rev. Lett. **43**, 1566 (1979).

[82] Y. Liao, X.-D. Ma, and Q.-Y. Wang, Extending low energy effective field theory with a complete set of dimension-7 operators, J. High Energy Phys. 08 (2020) 162.

[83] I. B. Khriplovich, What do we know about T odd but P even interaction?, Nucl. Phys. **B352**, 385 (1991).

[84] R. S. Conti and I. B. Khriplovich, New limits on T odd, P even interactions, Phys. Rev. Lett. **68**, 3262 (1992).

[85] J. Engel, P. H. Frampton, and R. P. Springer, Effective Lagrangians and parity conserving time reversal violation at low-energies, Phys. Rev. D **53**, 5112 (1996).

[86] M. J. Ramsey-Musolf, Electric dipole moments and the mass scale of new T violating, P conserving interactions, Phys. Rev. Lett. **83**, 3997 (1999); **84**, 5681(E) (2000).

[87] A. Kurylov, G. C. McLaughlin, and M. J. Ramsey-Musolf, Constraints on T odd, P even interactions from electric dipole moments, revisited, Phys. Rev. D **63**, 076007 (2001).

[88] J. Gasser and H. Leutwyler, , Chiral perturbation theory to one loop, Ann. Phys. (N.Y.) **158**, 142 (1984).

[89] J. Gasser and H. Leutwyler, Chiral perturbation theory: Expansions in the mass of the strange quark, Nucl. Phys. **B250**, 465 (1985).

[90] M. L. Graesser, An electroweak basis for neutrinoless double β decay, J. High Energy Phys. 08 (2017) 099.

[91] Y. Liao, X.-D. Ma, and H.-L. Wang, Effective field theory approach to lepton number violating decays $K^\pm \to \pi^\mp l^\pm l^\pm$: Short-distance contribution, J. High Energy Phys. 01 (2020) 127.

[92] C. Rosenzweig, J. Schechter, and C. G. Trahern, Is the effective lagrangian for QCD a sigma model?, Phys. Rev. D **21**, 3388 (1980).

[93] P. Di Vecchia and G. Veneziano, Chiral dynamics in the large n limit, Nucl. Phys. **B171**, 253 (1980).

[94] E. Witten, Large N chiral dynamics, Ann. Phys. (N.Y.) **128**, 363 (1980).

[95] K. Kawarabayashi and N. Ohta, , The problem of η in the large N limit: Effective lagrangian approach, Nucl. Phys. **B175**, 477 (1980).

[96] H. Leutwyler, On the 1/N expansion in chiral perturbation theory, Nucl. Phys. B, Proc. Suppl. **64**, 223 (1998).

[97] P. Herrera-Siklody, J. I. Latorre, P. Pascual, and J. Taron, Chiral effective Lagrangian in the large N(c) limit: The Nonet case, Nucl. Phys. **B497**, 345 (1997).

[98] R. Kaiser and H. Leutwyler, Large N(c) in chiral perturbation theory, Eur. Phys. J. C **17**, 623 (2000).

[99] P. Bickert, P. Masjuan, and S. Scherer, $\eta - \eta\prime$ mixing in large-Nc chiral perturbation theory, Phys. Rev. D **95**, 054023 (2017).

[100] M. Gaspero, B. Meadows, K. Mishra, and A. Soffer, Isospin analysis of D^0 decay to three pions, Phys. Rev. D **78**, 014015 (2008).

[101] H. Akdag, B. Kubis, and A. Wirzba, Correlations of C and CP violation in $\eta \to \pi^0\ell^+\ell^-$ and $\eta\prime \to \eta\ell^+\ell^-$, J. High Energy Phys. 03 (2024) 059.

[102] A. Manohar and H. Georgi, Chiral quarks and the nonrelativistic quark model, Nucl. Phys. **B234**, 189 (1984).

[103] S. Weinberg, Larger higgs exchange terms in the neutron electric dipole moment, Phys. Rev. Lett. **63**, 2333 (1989).

[104] H. Georgi, Generalized dimensional analysis, Phys. Lett. B **298**, 187 (1993).

[105] E. E. Jenkins, A. V. Manohar, and M. Trott, Naive dimensional analysis counting of gauge theory amplitudes and anomalous dimensions, Phys. Lett. B **726**, 697 (2013).

[106] B. M. Gavela, E. E. Jenkins, A. V. Manohar, and L. Merlo, Analysis of general power counting rules in effective field theory, Eur. Phys. J. C **76**, 485 (2016).

[107] A. V. Manohar, Effective field theories, Lect. Notes Phys. **479**, 311 (1997).

[108] R. L. Workman *et al.* (Particle Data Group), Review of particle physics, Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).