Forestry: An International Journal of Forest Research, 2024, **97**, 625–634
https://doi.org/10.1093/forestry/cpae001
Advance access publication date 24 January 2024
Original Article

Modeling basal area yield using simultaneous equation systems incorporating uncertainty estimators

Simón Sandoval [1,2,*], Cristián R. Montes [3], Bronson P. Bullock [3]

[1]Departamento de Manejo de Bosques y Medio Ambiente, Facultad de Ciencias Forestales, Universidad de Concepción, Concepción, Chile
[2]Laboratorio de Análisis y Modelamiento de Geoinformación, Facultad de Ciencias Forestales, Universidad de Concepción, Concepción 4070386, Chile
[3]Warnell School of Forestry & Natural Resources, University of Georgia. Director Plantation Management Research Cooperative (PMRC), Athens, GA 30602, United States

*Corresponding author. E-mail: simonsandoval@udec.cl

Abstract

Over the last three decades, many growth and yield systems developed for the southeast USA have incorporated methods to create a compatible basal area (BA) prediction and projection equation. This technique allows practitioners to calibrate BA models using both measurements at a given arbitrary age, as well as the increment in BA when time series panel data are available. As a result, model parameters for either prediction or projection alternatives are compatible. One caveat of this methodology is that pairs of observations used to project forward have the same weight as observations from a single measurement age, regardless of the projection time interval. To address this problem, we introduce a variance–covariance structure giving different weights to predictions with variable intervals. To test this approach, prediction and projection equations were fitted simultaneously using an *ad hoc* matrix structure. We tested three different error structures in fitting models with (i) homoscedastic errors described by a single parameter (Method 1); (ii) heteroscedastic errors described with a weighting factor w_t (Method 2); and (iii) errors including both prediction ($\breve{\varepsilon}$) and projection errors ($\tilde{\varepsilon}$) in the weighting factor w_t (Method 3). A rotation-age dataset covering nine sites, each including four blocks with four silvicultural treatments per block, was used for model calibration and validation, including explicit terms for each treatment. Fitting using an error structure which incorporated the combined error term ($\breve{\varepsilon}$ and $\tilde{\varepsilon}$) into the weighting factor w_t (Method 3), generated better results according to the root mean square error with respect to the other two methods evaluated. Also, the system of equations that incorporated silvicultural treatments as dummy variables generated lower root mean square error (RMSE) and Akaike's index values (AIC) in all methods. Our results show a substantial improvement over the current prediction-projection approach, resulting in consistent estimators for BA.

Keywords: prediction and projection models; weighted regression; dummy variables; silvicultural treatments; errors propagation; yield models

Introduction

In forest management, growth and yield models are considered essential tools for forest planning, providing insights into future forest conditions (Vanclay 1994, Fortin and Langevin 2011). These models aim to describe growth dynamics over the life of a given stand, using mathematical relations between state variables (dominant height, mortality, and basal area (BA)), whose parameters are calibrated using statistical methods. A large variety of growth and yield models exist that aim at explaining stand-level transition functions, diameter class, or individual tree changes over time (Burkhart and Tomé 2012). Over the last two decades, there have been several attempts to summarize existing studies on the moderation of forest growth and yield, with the aim of consolidating models and adjustment methods for different site conditions and species and generalizing their implementation in simulation systems (Monserud 2003, Pretzsch et al. 2008). Most approaches for forest modeling are grouped into three types: empirical, process-based, and hybrid models (Sun et al. 2007). One common objective of these models is to quantify some metric related to stand productivity. As such, stand BA (the sum of cross-sectional areas of stems at 1.37 meters on a per ha basis) and its growth prediction are an essential part of stand-level equations due to their ease of measurement and their strong correlation with volume and forest growth (Sun et al. 2007, Burkhart et al. 2019). BA growth summarizes a big portion of forest dynamics over time (i.e. growth, mortality, reproduction, and associated changes at the stand level). This approach is widely used in forest management for its ability to update inventories, predict future yield, and to explore different management alternatives (Gao et al. 2018).

Along with the rapid development of advanced mathematical statistics and computing technologies, parameter estimation for stand-level BA models have seen several critical developments: Sullivan and Clutter (1972) used ordinary least squares (OLS) to estimate the parameters for a volume projection equation that had a BA projection equation embedded in it, and these estimated parameter values were then used to obtain numerically consistent parameter estimates for the implied BA projection equation. Eerikäinen (2002) implemented simultaneous equation methods, while other researchers used difference-equation models (Carson et al. 1999, Corona et al. 2002, García and Ruiz 2003, Bravo-Oviedo et al. 2004); artificial neural network techniques (Chuangmin Liu et al. 2003), linear/nonlinear regression models (Nyland et al. 2000, Fang et al. 2001, Sharma et al. 2002,Wang 2003) and models

Handling editor: Tzeng Yih Lam
Received 8 May 2023. **Revised** 28 December 2023. **Accepted** 4 January 2024

expressed in matrix form to describe the system of equations for modeling forest growth and yield (Stanton 2001, Hao et al. 2005, Zhao et al. 2005).

One desirable property of a growth and yield system is the compatibility between equations describing relations between state variables at a given point in time and dynamic equations that project these state variables forward. Compatibility between prediction and projection equations was first introduced by Clutter (1963) in the form of a derivative-integral relationship such that the growth function, when integrated over a time interval, would equal the yield predicted by the yield equation. Matney and Sullivan (1982) selected compatible stand-level projection equations similar to those presented by Clutter (1963), later refined by Sullivan and Clutter (1972) because of their demonstrated applicability. Burkhart and Sprinz (1984) used simultaneous estimations to derive compatible volume and BA projection equations invariant of projection length. Later, Borders and Bailey (1986) reviewed parameter estimation procedures for systems of interrelated linear equations. They used restricted three-stage least squares as a theoretically sound estimation procedure in their development of a system of compatible growth and yield equations. These authors used three-stage least squares methods to predict a compatible system of growth and yield equations for slash pine. Appropriate ways to find parameters for such a system involve fitting the equations at the same time, ensuring a common error structure (non-zero covariances), even if there are no endogenous variables on the right-hand side (Zellner 1962).

Depending on variable interrelationships and model structures, simultaneous estimation of the parameters of the models might be necessary in order to provide estimates that are consistent and efficient. Hasenauer et al. (1998) compared an individual tree BA increment model, a height increment model, and a crown ratio model separately using OLS and simultaneously by applying two and three-stage least square. The results indicated the simultaneous models fitted were more efficient, while the separately determined OLS estimates were biased. More recently, Scolforo et al. (2019) developed a stand-level growth and yield model system using the simultaneous approach, while accounting for water availability. They used a linear algebraic technique to simultaneously fit a compatible set of prediction and projection BA equations.

The simultaneous prediction-projection algorithm, designed to calibrate compatible BA models, ensures compatibility. However, the method doesn't inherently account for the fact that longer projection intervals might introduce higher variances. In its formulation, it assigns weights to projections with different lengths similar to those applied to one-time observations, potentially introducing bias into the final model. Therefore, we hypothesize that assigning different weights to prediction and projection parts of the system might be better for the overall prediction, correctly weighting observations from a given year and those projected from variable length time intervals. Thus, in this research, we evaluated the difference in the estimated coefficients and goodness-of-fit of independently fitted and simultaneously fitted BA prediction and projection models. We also tested three different error structures in fitting models with (i) homoscedastic errors described by a single parameter (Method 1); (ii) heteroscedastic errors described with a weighting factor w_t (Method 2); and (iii) errors including both prediction ($\breve{\varepsilon}$) and projection errors ($\tilde{\varepsilon}$) in one parameter (Method 3). The database used for this analysis included silvicultural treatments; therefore, here we illustrate the use of each method, while including silvicultural treatment effects through a dummy variable approach and compare their performance in relation to a base model which does not include the effect of silvicultural treatments.

Methods

Data

For this analysis, we used nine study sites established in 1987 throughout the state of Georgia in the southeastern USA. The trial was installed by the Consortium for Accelerated Pine Production Studies (CAPPS) and maintained by the Plantation Management Research Cooperative (PMRC) to investigate site responses to intensive silvicultural regimes. The study corresponds to a randomized complete block design, with four 0.15-hectare treatment plots with an interior 0.05-hectare measurement plot. There was a range of four to six blocks per site. Within each site, blocks were established in different years. Sites spanned two physiographic regions: coastal plain and piedmont in the southeast USA. All sites had an initial planting density of 1679 trees per hectare (2.44 × 2.44 m). The PMRC field crew collected data annually for the first 20 growing seasons, biennial measurements began after age 21. Tree level measurements include diameter at breast height, total stem height, and age. At the study establishment, there were 80 trees in the interior measurement plots. At the plot level, trees were summarized for total BA per hectare, dominant height in meters, and the number of surviving trees per hectare. To calculate the dominant height, local height equations were developed to estimate heights from unmeasured trees within each plot. The dominant and co-dominant trees were averaged to estimate the dominant height. The silvicultural treatments include a Control (C), Fertilization (F), Competition Control (H), and the combination of Fertilization with Competition Control (HF). The fertilization treatment was carried out in the spring adding 280 kg ha^{-1} diammonium phosphate, 112 kg ha^{-1} potassium chloride, a summer application of 56 kg ha^{-1} of ammonium nitrate for the first two growing seasons, followed by early to mid-spring application of 150 kg ha^{-1} ammonium nitrate in growing seasons 3–9. Age 10 treatment was 336 kg ha^{-1} of ammonium nitrate and 140 kg ha^{-1} triple super phosphate. Age 11 treatments included 560 kg ha^{-1} super rainbow with added micronutrients, and 168 kg ha^{-1} of ammonium nitrate in early spring. Growing seasons 12 and onwards received 336 kg ha^{-1} of ammonium nitrate in the early spring. For the competition control treatment, there was repeated herbicide application to control herbaceous and woody plants on the plots.

Model development

We formulated the system of compatible BA prediction and projection equations using the base models proposed by Pienaar et al. (1985) and reformulated by Pienaar and Harrison (1989), who incorporated the compatibility constraints. The model is defined by two equations, one for within year prediction and one for between years projection. We adopted a new notation to discriminate between BA estimates derived from predicted values and from the BA estimates derived from projected values; this separation acknowledges the different errors resulting from either measured or projected values:

$$\breve{B}_t = e^{(\beta_0+\beta_1/A)} H_t^{\beta_2} N_t^{\beta_3} \tag{1}$$

$$\tilde{B}_t = B_{t-1}{}^{\frac{A_{t-1}}{A_t}} \cdot e^{\beta_0\left(1-\frac{A_{t-1}}{A_t}\right)} \left(\frac{H_t}{H_{t-1}{}^{A_{t-1}/A_t}}\right)^{\beta_2} \left(\frac{N_t}{N_{t-1}{}^{A_{t-1}/A_t}}\right)^{\beta_3} \tag{2}$$

Equations (1) and (2) correspond to the predicted ($\breve{B}$) and projected ($\tilde{B}$) BA estimates under a nonlinear model structure, respectively. A is the age in years, H is the dominant height in meters, and N is the stand density at either the reference time (t) or at the previous time ($t-1$) in trees per hectare. $\beta_1, \beta_2, \ldots, \beta_{(\cdot)}$ correspond to the set of parameters to be estimated. Using a log transformation, both equations are linearized and can be localized using generalized least squares:

$$\ln\left(\breve{B}_t\right) = \beta_0 + \beta_1 A^{-1} + \beta_2 \ln\left(H_t\right) + \beta_3 \ln\left(N_t\right) + \breve{\varepsilon} \quad (3)$$

$$\ln\left(\tilde{B}_t\right) = \frac{A_{t-1}}{A_t}\ln\left(B_{t-1}\right) + \beta_0\left(1 - \frac{A_{t-1}}{A_t}\right) + \beta_2 \ln\left(\frac{H_t}{{H_{t-1}}^{A_{t-1}/A_t}}\right) + \beta_3 \ln\left(\frac{N_t}{{N_{t-1}}^{A_{t-1}/A_t}}\right) + \tilde{\varepsilon}, \quad (4)$$

These equations were considering as "base model" refering to models proposed by Pienaar and Harrison (1989). Here, $\breve{\varepsilon}$ is the prediction equation error term and $\tilde{\varepsilon}$ the projection equation error term. Since we have common terms, and a linear system, we can utilize a matrix to solve it. To calibrate either model simultaneously using a linear system, we created a matrix that include both prediction and projection systems. We defined the dependent variable as the list of left-hand side values from equation 3 for each observation in the database as $\ln\left(B_{t-1}\right)$, and for the projection variable we used the left-hand side of equation (4) minus the first term from the same equation. An example for such vector for a single plot and using non-overlapping increments looks as follows:

$$Y = \begin{pmatrix} \ln\left(B_1\right) \\ \ln\left(B_2\right) \\ \vdots \\ \ln\left(B_t\right) \\ \ln\left(B_2\right) - \frac{A_1}{A_2}\ln\left(B_1\right) \\ \ln\left(B_3\right) - \frac{A_2}{A_3}\ln\left(B_2\right) \\ \vdots \\ \ln\left(B_t\right) - \frac{A_{t-1}}{A_t}\ln\left(B_{t-1}\right) \end{pmatrix}$$

The right-hand side matrix for the same example data looks as follows:

$$X = \begin{pmatrix} 1 & 1/A_1 & \ln\left(H_1\right) & \ln\left(N_1\right) \\ 1 & 1/A_2 & \ln\left(H_2\right) & \ln\left(N_2\right) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 1/A_t & \ln\left(H_t\right) & \ln\left(N_t\right) \\ 1 - A_1/A_2 & 0 & \ln\left(H_2\right) - A_1/A_2 \ln\left(H_1\right) & \ln\left(N_2\right) - A_1/A_2 \ln\left(N_1\right) \\ 1 - A_2/A_3 & 0 & \ln\left(H_3\right) - A_2/A_3 \ln\left(H_2\right) & \ln\left(N_3\right) - A_2/A_3 \ln\left(N_2\right) \\ \vdots & \vdots & \vdots & \vdots \\ 1 - A_{t-1}/A_t & 0 & \ln\left(H_t\right) - A_{t-1}/A_t \ln\left(H_{t-1}\right) & \ln\left(N_t\right) - A_{t-1}/A_t \ln\left(N_{t-1}\right) \end{pmatrix}$$

The precision matrix W is equal to:

$$\hat{W} = \sum^{-1} = \frac{1}{\sigma^2} I,$$

with I the square identity matrix with size $m \times m$.

The model was calibrated using a two-step least squares using the following expression.

$$\hat{\beta}^{GLS} = \left(X^T \hat{W} X\right)^{-1} X^T \hat{W} y \quad (5)$$

Where $\hat{\beta}^{GLS}$ are the estimated parameters, $\hat{W}$ is a precision matrix with correlations between the different elements in the linear system. For the general case reported by Pienaar and Shiver (1984) correlation between different observations was not accounted for, therefore, the off-diagonal elements of the $\hat{W}$ matrix were left as zero.

One inconveniance of this system is the assumption that both prediction and projection errors share the same precision. However, it is expected that the observation error, when propagated over time, will be at least proportional to the linear transformation. On top of that, there should be an error associated with projection length due to random perturbations that are not accounted for by the projection equation.

To address this problem, we implemented a variance–covariance matrix as part of a generalized least squares estimation that defines different variance depending on the projection length. Since we have two sources of error, $\breve{\varepsilon}$ and $\tilde{\varepsilon}$, we need to specify them accordingly inside the variance–covariance matrix. If we assume the observation errors to be the same, we can estimate them on a first step, using only the observed values at time t. The prediction error $\breve{\varepsilon}_t$ is dependent on the allometric relation between the independent predictor's age, height, and number of trees per hectare. Age is assumed to be known precisely, and height and number of trees per hectare help to improve the relationship as auxiliary variables. For its part, process error $\tilde{\varepsilon}$ depends on the variance between measurement periods $t-1$ as well. Therefore, a weight function w_t should consider both conditions; resulting in:

$$w_t = \begin{cases} 1/\sigma_t^2, & \breve{\beta} = f\left(\hat{\beta}, t\right) \\ 1/\left(\sigma_t^2 + \sigma_{t-1}^2\right), & \tilde{\beta} = f\left(\hat{\beta}, t, t-1\right) \end{cases} \quad (6)$$

In this way, the simultaneous fit incorporating weighting factor w_t for both prediction $\breve{\beta}$ and projection $\tilde{\beta}$ models allows modeling the BA and the uncertainty of the estimates in each phase. Models (Burkhart et al. 2019) and (Gao et al. 2018) are the prediction equations and models (Sullivan and Clutter 1972) and (Eerikäinen 2002) are projection equations incorporating silvicultural treatments. Equations (8) and (10) are the linear transformations of equations (7) and (9), respectively, and equations (8) and (10) were further modified to accommodate the treatment effects using a dummy variable coding (0,1) in a multiplicative way indicating the presence of a given treatment.

$$\breve{B}_t = e^{(\beta_0 + \beta_1/A_t)} H_t^{\beta_2} N_t^{\beta_3} \cdot {\phi_C}^{I_C} \cdot {\phi_F}^{I_F} \cdot {\phi_H}^{I_H} \cdot {\phi_{HF}}^{I_{HF}} \quad (7)$$

$$\ln\left(\breve{B}_t\right) = \beta_0 + \beta_1/A_t + \beta_2 \ln H_t + \beta_3 \ln N_t + \phi'_C I_C + \phi'_F I_F + \phi'_H I_H + \phi'_{HF} I_{HF} + \breve{\varepsilon}_t \quad (8)$$

$$\tilde{B}_t = {B_{t-1}}^{\frac{A_{t-1}}{A_t}} \cdot e^{\beta_0\left(1 - \frac{A_{t-1}}{A_t}\right)} \left(\frac{H_t}{{H_{t-1}}^{A_{t-1}/A_t}}\right)^{\beta_2} \left(\frac{N_t}{{N_{t-1}}^{A_{t-1}/A_t}}\right)^{\beta_3} e^{\phi_C I_C\left(1 - \frac{A_{t-1}}{A_t}\right)} e^{\phi_F I_F\left(1 - \frac{A_{t-1}}{A_t}\right)} e^{\phi_H I_H\left(1 - \frac{A_{t-1}}{A_t}\right)} e^{\phi_{HF} I_{HF}\left(1 - \frac{A_{t-1}}{A_t}\right)} \quad (9)$$

$$\ln\left(\hat{B}_t\right) = \frac{A_{t-1}}{A_t}\ln\left(B_{t-1}\right) + \beta_0\left(1-\frac{A_{t-1}}{A_t}\right) + \beta_2\ln\left(\frac{H_t}{H_{t-1}{}^{A_{t-1}/A_t}}\right) + \beta_3\ln\left(\frac{N_t}{N_{t-1}{}^{A_{t-1}/A_t}}\right) + \phi_C\left(1-\frac{A_{t-1}}{A_t}\right)I_C + \phi_F\left(1-\frac{A_{t-1}}{A_t}\right)I_F + \phi_H\left(1-\frac{A_{t-1}}{A_t}\right)I_H + \phi_{HF}\left(1-\frac{A_{t-1}}{A_t}\right)I_{HF} + \tilde{\varepsilon} \quad (10)$$

For the prediction equation $\phi'_i = \ln\phi_i$. In the equations (7)–(10), *C*, *F*, *H*, and *FH* correspond to dummy treatment effects for presence or absence (0,1) in Control, Fertilization, Competition Control, and both factors, respectively. The full model without dummy coefficients represents the Fertilization with Competition Control treatments. Thus, dummy variable for the treatment Fertilization with Competition Control $I_{HF} = 1$ when $I_C = I_F = I_H = 0$, therefore the parameter ϕ_{HF} has no estimation required.

Model calibration and evaluation

The approach of compatible prediction and BA projection models has been frequently used since they were proposed by Pienaar and Harrison (1989). Their main advantage is the simplicity of their implementation and the consistency of the resulting models; however, the method does not incorporate the effects of autocorrelation and heteroscedasticity of the information. Both prediction and projection equations in the compatible system were fit simultaneously using three different error structures: Method 1: homoscedastic errors with a single variance parameter; Method 2: heteroscedastic errors using a time dependent weighting factor $w_t = 1/\sigma_t^2$; and Method 3: a weighting function that propagates variances at times $t-1$ and combines them with the variance at time t, $w_t = 1/\left(\sigma_t^2 + \sigma_{t-1}^2\right)$. The Method 1 assumes homoscedasticity in the information, which is generally true only in projection models, but not in prediction models. The Method 2 uses a traditional weighting factor in a two-phase generalized least square adjustment that considers the error only in the prediction instance t, assuming that there is no autocorrelation of errors. The Method 3 incorporates a weighting factor using the error from the prediction and projection models. This allows incorporating the concept of error propagation as an element in the estimator and implicitly considers the autocorrelation of errors at time $t-1$.

Furthermore, for each model, the addition of silvicultural treatments was also tested and compared with the base model, which was described by Pienaar and Harrison (1989) and does not incorporate the effect of silvicultural treatments. Parameter calibration was done solving the $\hat{\beta}^{GLS}$ using matrix algebra in two steps. First solving for the least square errors for prediction, next doing it for the projection, and finally combining both. The data were divided into a training set and a validation set, corresponding to 80 and 20%, respectively, considering the complete series of measurements for each plot, and this process was iteratively performed 1000 times to determine the variation on model parameters. Model performance comparison was evaluated using the root mean square error (RMSE), Akaike's index (AIC), and BIAS evaluated in the validation dataset.

$$RMSE_k = \sqrt{\sum_{i=1}^{n}\frac{\left(y_i-\hat{y}_i\right)^2}{n-p}}$$

$$AIC_k = n\ln\left(sse_k/n\right) + 2p$$

$$BIAS_k = -\frac{100\%}{n}\sum_{i=1}^{n}\frac{y_i-\hat{y}_i}{y_i}$$

Where y_i and $\hat{y}_i$ are the BA observed and predicted, n is the sample number, p is the number of parameter of models. *sse* is the sum square error define as $\sum_{i=1}^{n}\left(y_i-\hat{y}_i\right)^2$. In $RMSE_k$, AIC_k, and $BIAS_k$ the subscript denote the kth iteration out of a total of 1000. Thus, the results of these indicators were the average of the iterations. All calibrations were implemented in R (R-CoreTeam 2022).

Results

Base model (no silvicultural treatments)

The base model with no silvicultural treatments showed an improvement in fit statistics after the incorporation of the weighting factor w_t using Method 3 (Table 1). This effect was observed for both the prediction only and the simultaneous model formulations. In the independent fit for the prediction model, the RMSE shows values of 7.55, 7.17, and 7.08 m^2 ha^{-1} for Methods 1, 2, and 3, respectively. The projection model shows RMSE values of 7.54, 7.50, and 7.49 m^2 ha^{-1} for each respective method. As expected, the simultaneous fit model underperformed with respect to the independent fits (prediction and projection functions fitted independently) for the three methods tested. However, the performance improvement over the validation data was noticeable. Constraining the parameters in this case introduced a sub-optimal result from the error standpoint. In this phase of simultaneous fit, the RMSE obtained were 9.87, 9.69, and 8.12 m^2 ha^{-1} for the estimation model in fit Methods 1, 2, and 3, respectively. For the projection model the RMSE was 9.60, 9.47, and 9.45 m^2 ha^{-1} for each respective method. These results combined with the estimates of BIAS show that the performance of BA model using the error in a weighting factor w_t generates more consistent predictions in both prediction and projection functions.

Parameters estimated using these three modeling techniques were similar when comparing the coefficients obtained from the prediction model and from the simultaneous fit (Table 1). In the simultaneous fit, parameters $\boldsymbol{\beta}_0$, $\boldsymbol{\beta}_1$, and $\boldsymbol{\beta}_2$ have a lower value compared with the independent estimation function; contrasting with parameter $\boldsymbol{\beta}_3$ that showed an increase. This is evidenced in Methods 1 and 2, which correspond to the fit that assumed homoscedasticity and heteroscedastic with weighting factor $w_t = 1/\sigma_t^2$, respectively. Method 3 generates very similar coefficient values. Weighting with the function $w_t = 1/\left(\sigma_t^2+\sigma_{t-1}^2\right)$ generates a better approximation to the independent estimation function, since at each time t of the estimation, the weight function decreases the weight at projection between ages.

Method 3 incorporates the weighting factor $1/\left(\sigma_t^2+\sigma_{t-1}^2\right)$, which reduced the RMSE and AIC in relation to Methods 1 and 2 (Table 1). Here, the weighting factor incorporates in its calculation the variance of the prediction at time t and for the projection stage the variances at time t and $t-1$. Overall, the weighting w_t results in an increase in accuracy, this effect being even greater in the transition model, where the weighting factor significantly corrects the error when projecting from very early ages. The greatest uncertainty occurs when projecting BA growth across wide age ranges. Thus, Method 3, which considers the sources of error in the prediction and projection, contributes to reduce the BIAS observed with Methods 1 and 2.

Model incorporating silvicultural treatments

The incorporation of silvicultural treatments via dummy variables resulted in an increase in model precision fitting only in the projection models (Table 2). In the prediction model, there was low precision, and this effect is evidenced in both independent and simultaneous fit phases. From Table 2, the most accurate model is the one that uses the fit technique that incorporates

Table 1. Estimated parameters and goodness-of-fit indicators obtained in the models with the validation dataset in the three methods without incorporation of silvicultural treatment effects. All values presented in the table correspond to the average of 1000 iterations.

Method	Function	Estimated parameters				RMSE (m ^{2}/ha)		AIC		BIAS (%)	
		β_0	β_1	β_2	β_3	Ind.	Sim.	Ind.	Sim.	Ind.	Sim.
1	Pred.	−4.5032	4.0782	1.8063	0.6032	7.55	9.87	9758.9	10698.4	−2.97	−3.01
		(0.1902)	(0.1822)	(0.0295)	(0.0252)						
	Proj.	1.0258		0.2269	0.647	7.54	9.60	9661.1	10522.8	−0.62	0.58
		(0.1566)		(0.0309)	(0.0191)						
	Sim.	−4.2758	3.2385	1.7278	0.6188						
		(0.1789)	(0.1722)	(0.0281)	(0.0233)						
2	Pred.	−4.483	4.0724	1.8035	0.6003	7.17	9.69	9582.8	10635.1	−2.96	−3.03
		(0.2001)	(0.1829)	(0.0299)	(0.027)						
	Proj.	1.0181		0.2282	0.6477	7.50	9.47	9645.3	10474.8	−0.61	0.57
		(0.1584)		(0.0316)	(0.0189)						
	Sim.	−4.2560	3.225	1.7249	0.6163						
		(0.1871)	(0.1700)	(0.0283)	(0.0248)						
3	Pred.	−4.4988	4.086	1.8058	0.6023	7.08	8.12	9539.5	9559.4	−2.72	−2.76
		(0.1961)	(0.1828)	(0.0294)	(0.0261)						
	Proj.	0.9690		0.2592	0.6172	7.49	9.45	5409.7	10305.0	−0.01	0.48
		(0.2069)		(0.0405)	(0.0300)						
	Sim.	−4.4889	4.0562	1.8025	0.6027						
		(0.2258)	(0.2360)	(0.0329)	(0.0297)						

Pred: prediction function, Proj: projection function, Ind: denotes the independent fit, Sim: denotes the simultaneous fit. All the parameters were significant at an alpha-level of 0.05. The standard deviation is denoted in parenthesis.

Table 2. Estimated parameters and goodness-of-fit indicators obtained in the models with the validation dataset over the three methods techniques incorporating the silvicultural treatment effects as dummy variables. All values presented in the table correspond to the average of 1000 iterations.

Method	Function	Estimated parameters							RMSE (m ^{2}/ha)		AIC		BIAS (%)	
		β_0	β_1	β_2	β_3	ϕ_1	ϕ_2	ϕ_3	Ind.	Sim.	Ind.	Sim.	Ind.	Sim.
1	Pred.	−3.8187	2.173	1.6002	0.6313	−0.1538	−0.0504	−0.0259	1.97	1.99	8620.1	9597.2	−2.67	−2.88
		(0.2263)	(0.2449)	(0.0368)	(0.0273)	(0.0085)	(0.007)	(0.006)						
	Proj.	0.9102		0.1967	0.6828	0.1326	0.3529	−0.1229	3.98	2.98	10827.0	10982.2	−0.45	0.36
		(0.1433)		(0.0298)	(0.0166)	(0.0219)	(0.0268)	(0.0172)						
	Sim.	−3.5579	1.2206	1.5093	0.6519	−0.1733	−0.0549	−0.0371						
		(0.2127)	(0.2306)	(0.0349)	(0.0251)	(0.0083)	(0.0067)	(0.0059)						
2	Pred.	−3.8085	2.1548	1.5963	0.6319	−0.1548	−0.0508	−0.0268	1.97	1.99	8422.9	9533.3	−2.68	−2.90
		(0.2418)	(0.2475)	(0.0381)	(0.0299)	(0.0086)	(0.0069)	(0.0058)						
	Proj.	0.9129		0.1966	0.6823	0.1311	0.3521	−0.1229	3.98	2.98	10822.0	10962.8	−0.45	0.37
		(0.1500)		(0.0314)	(0.0172)	(0.0218)	(0.0275)	(0.0173)						
	Sim.	−3.5450	1.1989	1.5057	0.6518	−0.1739	−0.0551	−0.0378						
		(0.2260)	(0.2355)	(0.0364)	(0.0271)	(0.0083)	(0.0067)	(0.0056)						
3	Pred.	−3.8139	2.1707	1.5984	0.6313	−0.154	−0.0503	−0.0257	1.97	0.99	8535.9	8460.8	−2.66	−2.68
		(0.2375)	(0.2448)	(0.0381)	(0.0287)	(0.0085)	(0.0072)	(0.0058)						
	Proj.	0.9646		0.1996	0.6575	0.1221	0.2631	−0.1077	3.02	2.17	7316.3	10760.2	−0.01	0.45
		(0.1817)		(0.0339)	(0.0243)	(0.0266)	(0.0377)	(0.0219)						
	Sim.	−3.8011	2.1282	1.5938	0.6323	−0.1548	−0.0504	−0.0260						
		(0.2365)	(0.2487)	(0.038)	(0.0293)	(0.0091)	(0.0076)	(0.0069)						

Pred: prediction function, Proj: projection function, Ind: denotes the independent fit, Sim: denotes the simultaneous fit. All the parameters were significant at an alpha-level of 0.05. The standard deviation is denoted in parenthesis.

the weighting factor w_t (Method 3), when incorporating the silvicultural treatment effect. Method 3 remains the most accurate compared with Methods 1 and 2 (Table 2). In Method 3, for the independent fit phase projection model the RMSE decreases from 7.49 to 3.02 m^2 ha^{-1} and in the simultaneous fit phase it decreases from 9.45 to 2.17 m^2 ha^{-1}. For the same method, in the case of the prediction model, the RMSE decreases from 7.08 to 1.97 and from 8.12 to 0.99 m^2 ha^{-1} in the independent and simultaneous phase fit of the model, respectively. As in the base models without the incorporation of the effects of silvicultural treatments, a significant decrease in the BIAS of the estimates was observed.

As with the base model, the effect of the weighting factor w_t on the Method 3 resulted in a lower estimation error and better parsimony, as evaluated by the RMSE and AIC (Table 2). The improvement is greater in the projection model, in contrast to the prediction model, where it is observed that the RMSE of the model fit in the independent phase is 3.98, 3.98, and 3.02 m^2 ha^{-1} in the Methods 1, 2, and 3, respectively; in that same order, in the simultaneous fit phase, the RMSE is 2.98, 2.98, and 2.17 m^2 ha^{-1}.

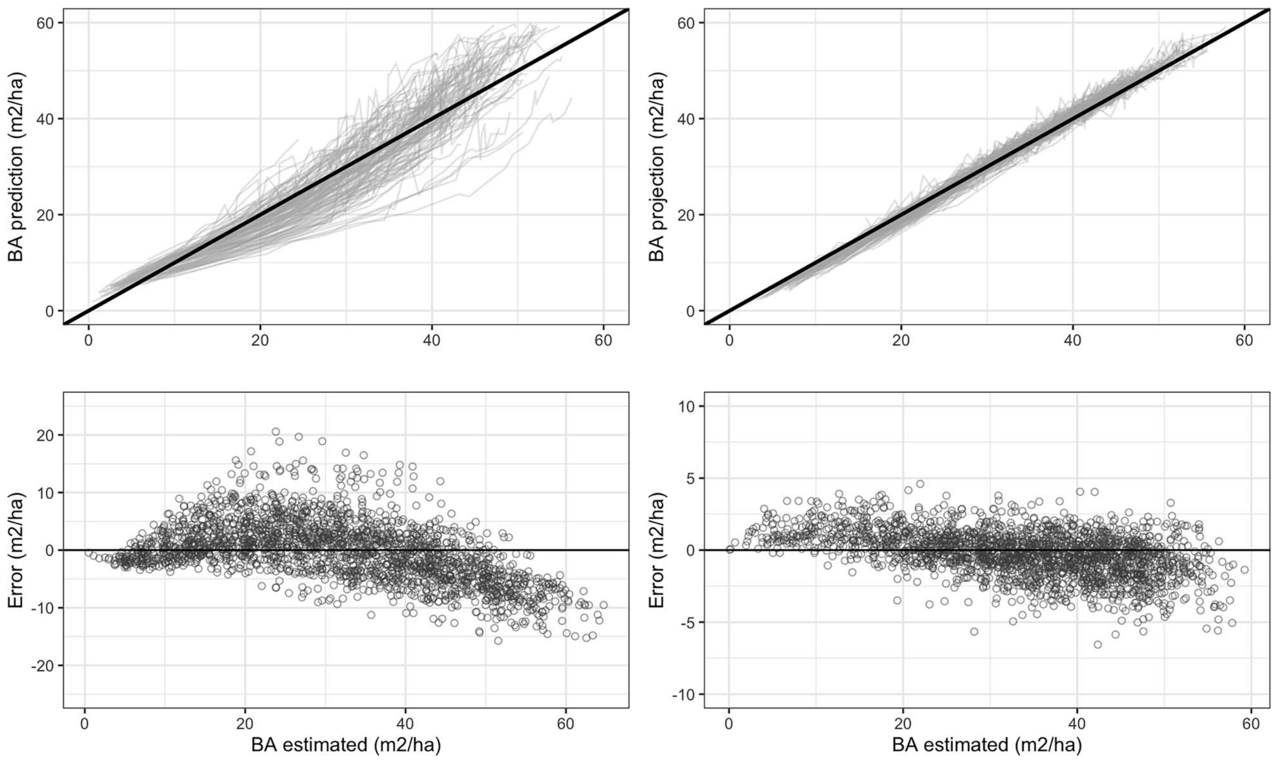

Figure 1. Relationship between predicted-observed BA and projected-observed BA for Method 3, both in simultaneous fitting incorporating silvicultural treatments.

In the case of the prediction model, there is no clear tendency to increase the precision in the fit of any method, we only observed changes in the value of the RMSE for Method 3 (0.99 m^2 ha^{-1}) in contrast to other two methods (1.99 m^2 ha^{-1}).

The simultaneous fit phase model generates consistent estimates with respect to the independent phase-adjusted model (Fig. 1). In the estimation model, an increase in the variation of BA with increasing values of BA is observed relative to the 1:1 line, showing more variation in relation to the transition model, indicating heteroscedasticity in the model. Here, the model fit in both independent and simultaneous phases shows a slight tendency to underestimate the BA between the range of 20 to 40 m^2 ha^{-1}, and to overestimate the BA after 40 m^2 ha^{-1}. Meanwhile, in the transition model, note that the 1:1 relationship between the observed and estimated BA in two estimation phases (independent and simultaneous phases) shows high accuracy and homoscedastic variance.

When incorporating the effect of silvicultural treatments in the BA models, the distribution of parameters increases among the three methods (Figs 2 and 3). In the prediction model, the distribution of parameters is similar among the three methods when comparing the fitting with and without the incorporation of the silvicultural treatment variables. On the other hand, in the projection model and in the simultaneous fitting, the distribution of parameters changes when incorporating the effect of silvicultural treatments. In the prediction model, Method 3 is where the largest change in the distribution of parameters was observed. Here, without incorporating silvicultural treatments, the distribution of parameters $\boldsymbol{\beta_0}$, $\boldsymbol{\beta_2}$, and $\boldsymbol{\beta_3}$ varied in relation to Methods 1 and 2, while when incorporating the effect of silvicultural treatments, only the distribution of parameter $\boldsymbol{\beta_3}$ showed substantial changes. In the simultaneous fit, changes in parameters $\boldsymbol{\beta_0}$, $\boldsymbol{\beta_1}$, and $\boldsymbol{\beta_2}$ were observed in Method 3, while the distribution of parameter $\boldsymbol{\beta_3}$ is stable between methods and when incorporating the effect of silvicultural treatments. In this comparison, changes in the distribution of parameters $\boldsymbol{\beta_0}$, $\boldsymbol{\beta_1}$, and $\boldsymbol{\beta_2}$ were observed to be greater when incorporating the effect of silvicultural treatments.

Discussion

Our study implemented a system of compatible equations similar to the one proposed by Clutter (1963) almost 60 years ago. We introduce a novel way to link prediction and projection variances in a system of simultaneous equations that weights projections differently, correctly addressing cumulative errors over longer projection intervals. The benefit from our system rests in the possibility to calculate long-term projection uncertainty as well as to better localize equation parameters. Practitioners have relied on the Clutter (1963) system for many years, with numerous examples in the USA, such as the work by Burkhart and Sprinz (1984),Clutter and Jones (1980), in loblolly pine, and Pienaar and Shiver (1984), Pienaar et al. (1985) in slash pine. Other examples can be found in Spain (Palahí et al. 2002), New Zealand (Woollons and Hayward 1985), Finland, South Africa, and Portugal (Soares et al. 1995). Bailey and Ware (1983) introduced the concept of fitting both prediction and projection simultaneously, further improving compatibility. Borders and Bailey (1986) introduced a full system of equations using linear, and later, nonlinear systems. Our model includes the addition of silvicultural treatments as part of the base model as shown by Ramirez et al. (2022), showing a consistent way to add these effects as they affect either the asymptote or the slope of the relationship, resulting in a reduction of RMSE and AIC with respect to the base model. The system from Fang et al. (2001), which evaluated a simultaneous equation system, similar to the one proposed by Borders and Bailey (1986), also incorporated silvicultural treatments

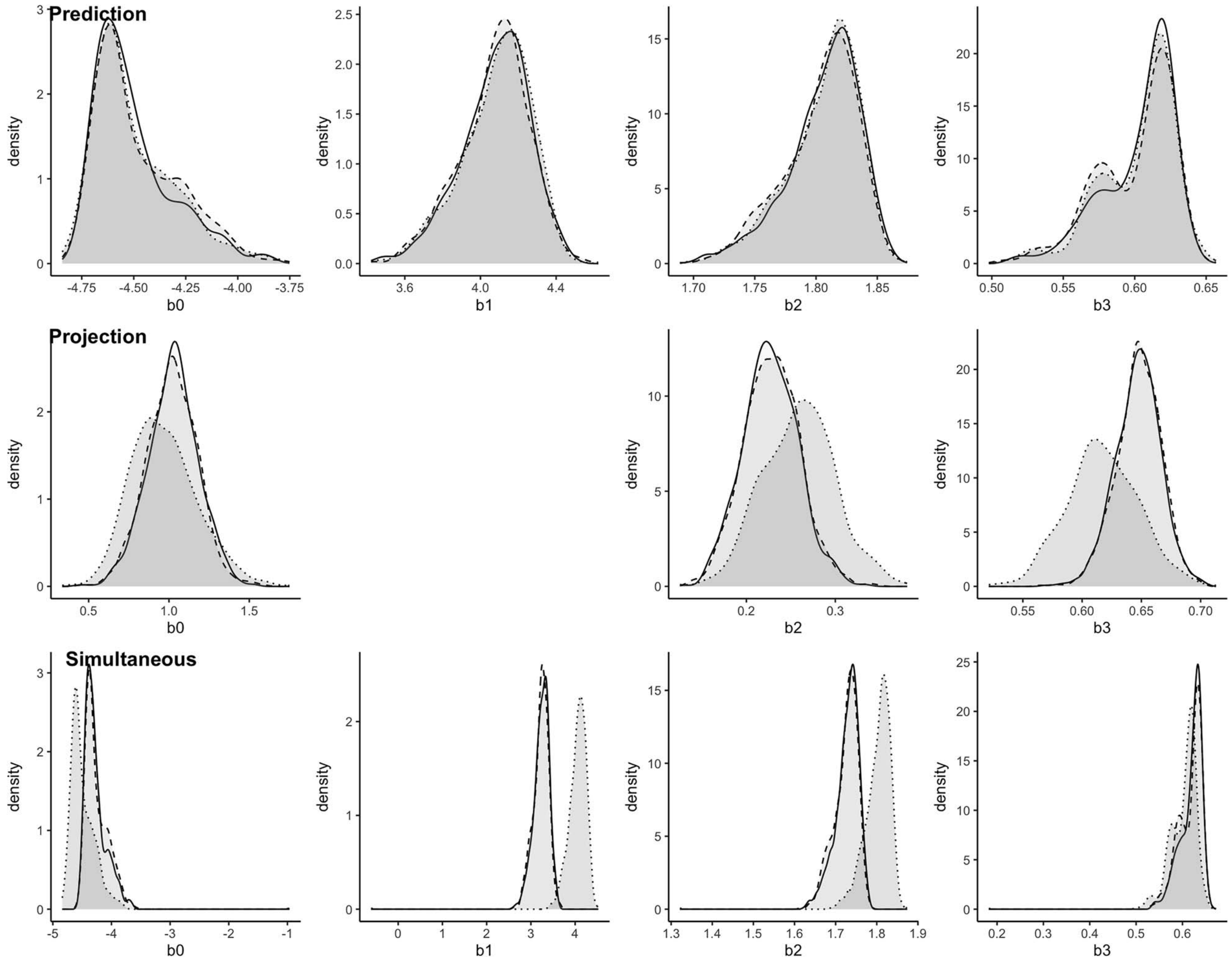

Figure 2. Distributions of estimated parameters in base models without incorporating silvicultural treatments. The line type, solid, long-dashed, and short-dashed, denotes Methods 1, 2, and 3, respectively. The distribution of the estimated parameters generated from the 1000 iterations performed.

(chopping, burning, fertilizer, bedding, or herbicide) using different types of soils as dummy variables inside of a mixed effects model for slash pine. However, bias in prediction values was not addressed in their study. Other studies that include compatible terms between prediction and projection values beyond the original work from Pienaar and Shiver (1984) and the work by Pienaar et al. (1985) include McTague and Bailey (1987) for loblolly pine plantations, and Pienaar and Harrison (1989) for slash pine in Brazil.

Studies have shown that BA per hectare yield increases asymptotically with age and for its modeling the incorporation of site quality is frequently included to improve estimates. In these empirical models, algebraic differences have been incorporated to improve the precision of the BA projection (Ochi and Cao 2003). Other approaches have been developed from the projection of the diameter distribution and its relationship with the BA (Zhang and Duan 2004). Some approaches have incorporated climatic variables to improve the estimates and projections of BA. Woollons et al. (1997) reported improvements of 10% in the precision of the model using climatic variables. Snowdon et al. (1999) included climatic variables in a temporal and spatial variation for *Pinus radiata* in Australia. Makela et al. (2000) mentioned that the development of BA yield projection models should go toward hybrid models that incorporate climatic and empirical variables at the tree level. Recently, Scolforo et al. (2019) incorporated information on the water deficit related to the yield in BA, improving the estimates with respect to the base model using the same base model structure used in this study.

Due to the algebraic constraints on the parameters, the increased RMSE of the simultaneous fitting system compared with fitting the BA equations independently is not surprising. It was expected to see a reduction in fit statistics when imposing constraints to the model that require more than one equation to be fit with additional parameters. This loss in precision was already noted by several authors, but LeMay (1990) and Zhang and Duan (2004) indicated that the improvements in stand level model consistency offset the loss in model precision. That effect has been previously reported and most authors stress that the loss of precision is compensated by allowing the system of compatible equations to generate consistent projections (Fang et al. 2001).

Method 3, which incorporates the weight factor $w_t = 1/\left(\sigma_t^2 + \sigma_{t-1}^2\right)$ to improve projection values, showed a reduction in RMSE in comparison with Methods 1 and 2. These results were the same for every regression model tested, and all models were further improved through the incorporation of silvicultural treatment effects (Table 2). Method 2, which incorporates the weight factor $w_t = 1/\sigma_t^2$ that accounts for variance heteroscesedacity, did not significantly reduce the RMSE when compared with Method 1 that assumes homoscedasticity. Using the $w_t = 1/\left(\sigma_t^2 + \sigma_{t-1}^2\right)$ weighting factor was the best choice to

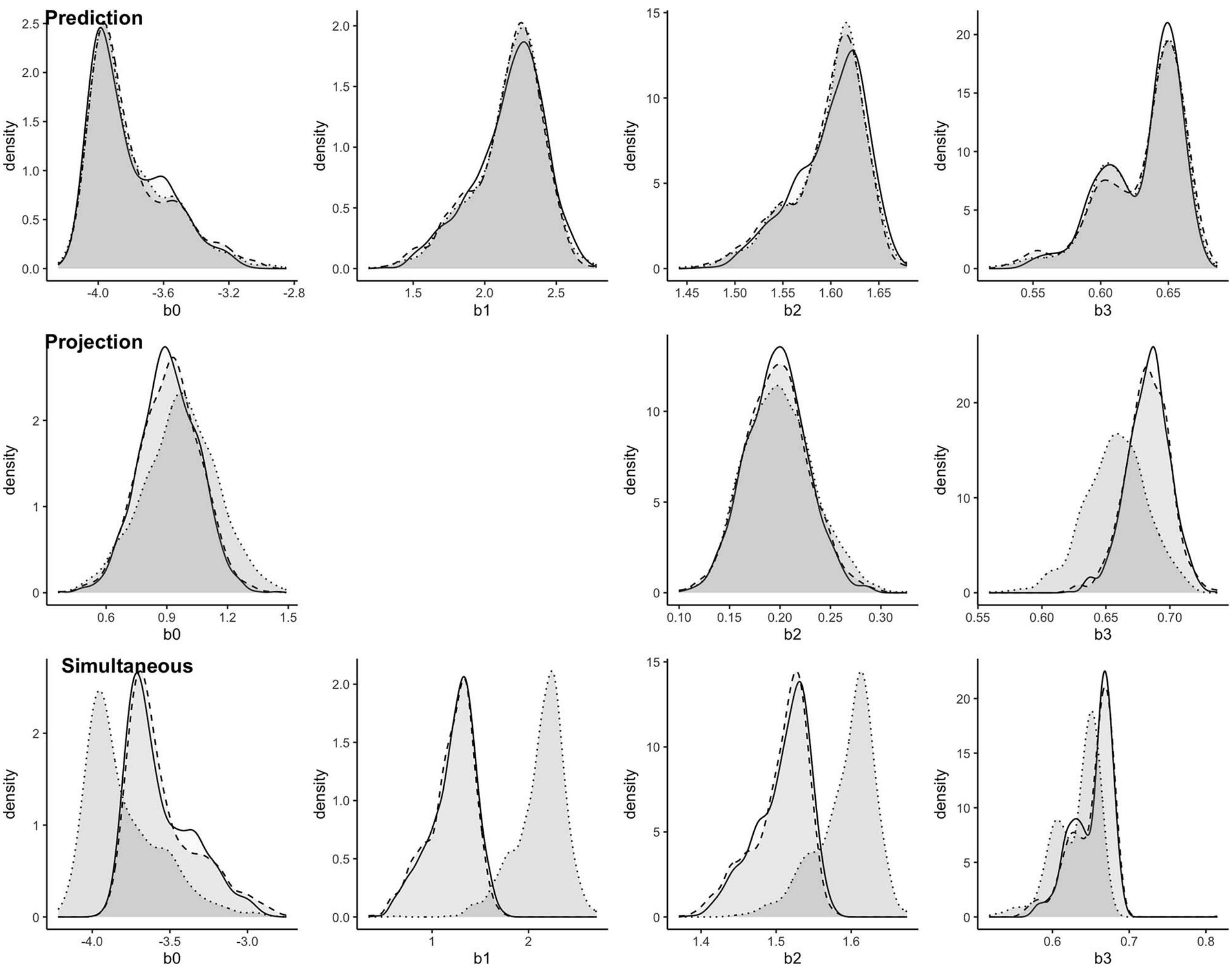

Figure 3. Distributions of estimated parameters in models incorporating silvicultural treatments. The line type, solid, long-dashed, and short-dashed, denotes Methods 1, 2, and 3, respectively. The distribution of the estimated parameters generated from the 1000 iterations performed.

correct variance heteroscedasticity as it includes both prediction ($\breve{\varepsilon}$) and projection variances ($\tilde{\varepsilon}$). Our system does not explicitly account for time series autocorrelation. There are contrasting views about the subject. Meng and Huang (2010) indicated that growth and yield models should include an autocorrelation structure due to the repeated measurements nature of the data. LeMay (1990) indicate that a system of equations calibrated from permanent sampling plots will produce a set of heteroscedastic errors because BA corresponds to the expression of multiple factors that cannot independently be measured, generating an accumulated effect. Hall and Clutter (2004) indicated that a system of equations fitted simultaneously should include a term for time series autocorrelation in the covariance matrix. However, error estimation using such a matrix is complex and depends strongly on the structure of the correlation between each equation. Fortin et al. (2007) indicate that an autocorrelation term in the variance–covariance matrix improves the autocorrelation problem, however this alone does not solve the problem of heteroscedasticity in the prediction and the projection equation.

The compatible system estimates parameters all at once, ensuring compatibility between the prediction and the projection equations. This method, proposed by Bailey and Ware (1983), had been used several times to derive BA equations. All these equations had implemented the required constraints to ensure compatibility, however according to Sun et al. (2007), these methods have only increased the complexity, but the error structure term has not been the subject of study. In fact, Wilson et al. (2019) citing other authors indicate that the projection error has seldom been part of further consideration. One exception would be Meng and Huang (2010) who generated a BA model including fixed and random effects using a mixed effect modeling approach. Results from this study show that the incorporation of fixed and random effects does reduce the overall error, with small effect in the parameters from the model when compared with the fixed effects only methods. The results of our research show, on one hand, a simple methodology to incorporate the errors in a compatible fitting system between simultaneously fitted prediction and projection models. Furthermore, we demonstrate the effect of incorporating the estimation error at time $t-1$ and its effect at time t, which can be considered as a recognition of the effect of error propagation in a projection system.

In our study, we also incorporated a distribution analysis of the estimated parameters for the three methods evaluated. In general, Methods 1 and 2 showed a similar distribution of parameters in all cases of analysis. That is, in the models fitted independently and the models fitted simultaneously. On the other hand, the distribution of the parameters of Method 3 varied with respect to Methods 1 and 2. Evidently, the difference was generated by the incorporation of the weighting factor $1/\left(\sigma_t^2+\sigma_{t-1}^2\right)$, which produces effects on the weighting of each of the parameters. Method

3, in the simultaneous fit without the incorporation of dummy variables (Fig. 2), parameters $\boldsymbol{\beta_1}$, and $\boldsymbol{\beta_2}$ are higher with respect to their estimation in Methods 1 and 2. This indicates that Age and Dominant Height are more significant, thus correcting the bias that was observed in Methods 1 and 2. A similar trend is observed with the distribution of the parameters in the fit incorporating the dummy variables (Fig. 3). Here, the parameter $\boldsymbol{\beta_0}$ also decreases with respect to Methods 1 and 2, which makes the BA prediction curve fit better to the observed information. Another relevant change occurs in parameter $\boldsymbol{\beta_3}$, which showed changes only in the projection model when compared between the three methods. Here, this parameter defines the mortality rate between two time periods, and using the weighting factor $1/\left(\sigma_t^2+\sigma_{t-1}^2\right)$ in Method 3, it was observed that this variable becomes more significant in the BA projection process.

Method 3 generates the weight matrix (W_t) independently from the prediction and the projection equation. In the simultaneous fitting approach, the W_t for the prediction equation used the reciprocal for the prediction variance ($1/\sigma_t^2$) using the error generated with the observation at time t, while the projection factor w_t used the weighting factor $1/\left(\sigma_t^2+\sigma_{t-1}^2\right)$ that depends on the projection error at time $t-1$. Doing this, the weighting factor now includes two sources of error. The result of this reduces the effect of the projection values as the elapsed time between two consecutive measurements increases, giving more weight to the observation instead of the projection. This error propagation method has been studied before, using other techniques. For example, McGarrigle et al. (2013) proposed a different type of dynamic model using copulas derived from nearest neighbor imputation; this method was called "informed random walks". Wilson et al. (2019) produced projections up to 40 years in length for *Pseudotsuga menziesii* using a Bayesian probabilistic modeling approach. According to Wilson et al. (2019), this technique allows the inclusion of error propagation using Bayes's theorem. However, it is important to note the limitations of the Markov Chain Monte Carlo algorithm for parameter calibration when the number of series increase. Our results present a simpler framework that can be easily implemented to estimate compatible equations using a matrix formulation that is solved using simple two-stage least squares or a generalized least squares formulation.

Conclusion

Incorporating the effects of silvicultural treatments as dummy variables in the models improved the RMSE and AIC fit statistics relative to the base model. This improvement was observed over all three methods evaluated in this research. As expected, the more restrictive simultaneous fit method of the prediction and projection equations generated higher RMSE values on the calibration data, showing that this type of fit generates a loss of flexibility in the model. However it does ensure compatibility between prediction and projection functions. Our Method 3, which incorporates the weighting factor w_t, generated the best results according to the RMSE in relation to the other two methods evaluated in both independent and simultaneous fitting phases. In Method 3, the W_t-weighted matrix was generated independently for the prediction and projection functions. Thus, Method 3 proposed in this study showed advantages with respect to the other two, because it considers the prediction and projection error to generate the weighting factor. This method allows correcting the effect of the autocorrelation generated in the modeling of serially correlated information. In this way, the W_t-weighted matrix integrated the error sources of the prediction and projection process, in addition to the σ_{t-1}^2 error assimilation term. The error assimilation term decreases the weighting of the w_t factor when the BA yield projections are made between an initial age very distant from the final projection age, and this effect was observed with improved RMSE values in Method 3. Our results show the importance of incorporating the error terms in the fitting system of simultaneously compatible and fitted BA models. In addition to improving the accuracy and BIAS indicators in the projections, the main advantage of this methodology is that it is very simple to implement, which only requires setting up the predictor matrices in a weighted least squares approach.

Acknowledgements

The authors like to express appreciation for the support from the Plantation Management Research Cooperative (PMRC) at the University of Georgia, its cooperative members, and the PMRC Field Crew who, over many years, collected the data used in this research. This work was also supported by McIntire-Stennis Project GEOZ0202-MS.

Conflict of interest: None declared.

Funding

This work was supported by the Plantation Management Research Cooperative (PMRC) at the University of Georgia of United States.

Data availability

The data underlying this article will be shared upon reasonable request.

References

Bailey RL, Ware KD. Compatible basal-area growth and yield model for thinned and unthinned stands. *Can J For Res* 1983;**13**:563–71. https://doi.org/10.1139/x83-082.

Borders BE, Bailey RL. A compatible system of growth and yield equations for slash pine fitted with restricted three-stage least squares. *For Sci* 1986;**32**:185–201.

Bravo-Oviedo A, Río MD, Montero G. Site index curves and growth model for Mediterranean maritime pine (*Pinus pinaster* Ait.) in Spain. *For Ecol Manage* 2004;**201**:187–97. https://doi.org/10.1016/j.foreco.2004.06.031.

Burkhart HE, Sprinz PT. Compatible cubic volume and basal area projection equations for thinned old-field loblolly pine plantations. *For Sci* 1984;**30**:86–93.

Burkhart HE, Tomé M *Modeling Forest Trees and Stands*. Dordrecht, New York: Springer, 2012, 457. https://doi.org/10.1007/978-90-481-3170-9.

Burkhart HE, Avery TE, Bullock BP. *Forest Measurements*. 6th edition. Waveland Pr Inc, 2019, 434.

Carson SD, Garcia O, Hayes JD. Realized gain and prediction of yield with genetically improved *Pinus radiata* in New Zealand. *For Sci* 1999;**45**:186–200.

Chuangmin Liu LZ, Davis CJ, Solomon DS. *et al.* Comparison of neural networks and statistical methods in classification of ecological habitats using FIA data. *For Sci* 2003;**49**:619–31.

Clutter JL. Compatible growth and yield models for loblolly pine. *For Sci* 1963;**9**:354–71.

Clutter JL, Jones EP. *Prediction of Growth after Thinning in Old-Field Slash Pine Plantations*. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station, Asheville, N.C, 1980; 19.

Corona P, Antonio Marziliano P, Scotti R. Top-down growth modelling: a prototype for poplar plantations in Italy. *For Ecol Manage* 2002;**161**:65–73. https://doi.org/10.1016/S0378-1127(01)00481-9.

Eerikäinen K. A site dependent simultaneous growth projection model for Pinus kesiya plantations in Zambia and Zimbabwe. *For Sci* 2002;**48**:518–29.

Fang Z, Bailey RL, Shiver BD. A multivariate simultaneous prediction system for stand growth and yield with fixed and random effects. *For Sci* 2001;**47**:550–62.

Fortin M, Langevin L. Stochastic or deterministic single-tree models: is there any difference in growth predictions? *Ann For Sci* 2011;**69**: 271–82.

Fortin M, Daigle G, Ung C-H. *et al.* A variance-covariance structure to take into account repeated measurements and heteroscedasticity in growth modeling. *Eur J For Res* 2007;**126**:573–85. https://doi.org/10.1007/s10342-007-0179-1.

Gao W-Q, Liu J-F, Xue Z-M. *et al.* Geographical patterns and drivers of growth dynamics of Quercus variabilis. *For Ecol Manage* 2018;**429**: 256–66. https://doi.org/10.1016/j.foreco.2018.07.024.

García O, Ruiz F. A growth model for eucalypt in Galicia, Spain. *For Ecol Manage* 2003;**173**:49–62. https://doi.org/10.1016/S0378-1127(01)00817-9.

Hall DB, Clutter M. Multivariate multilevel nonlinear mixed effects models for timber yield predictions. *Biometrics* 2004;**60**:16–24. https://doi.org/10.1111/j.0006-341X.2004.00163.x.

Hao Q, Meng F, Zhou Y. *et al.* Determining the optimal selective harvest strategy for mixed-species stands with a transition matrix growth model. *New For* 2005;**29**:207–19. https://doi.org/10.1007/s11056-005-5656-0.

Hasenauer H, Monserud RA, Gregoire TG. Using simultaneous regression techniques with individual-tree growth models. *For Sci* 1998;**44**:87–95.

LeMay VM. MSLS: a linear least squares technique for fitting a simultaneous system of equations with a generalized error structure. *Can J For Res* 1990;**20**:1830–9. https://doi.org/10.1139/x90-246.

Makela A, Landsberg J, Ek AR. *et al.* Process-based models for forest ecosystem management: current state of the art and challenges for practical implementation. *Tree Physiol* 2000;**20**:289–98. https://doi.org/10.1093/treephys/20.5-6.289.

Matney TG, Sullivan AD. Compatible stand and stock tables for thinned and unthinned loblolly pine stands. *For Sci* 1982;**28**: 161–71.

McGarrigle E, Kershaw JA, Ducey MJ. *et al.* A new approach to modeling stand-level dynamics based on informed random walks: influence of bandwidth and sample size. *Forestry* 2013;**86**:377–89. https://doi.org/10.1093/forestry/cpt008.

McTague JP, Bailey RL. Simultaneous total and merchantable volume equations and a compatible taper function for loblolly pine. *Can J For Res* 1987;**17**:87–92. https://doi.org/10.1139/x87-015.

Meng SX, Huang S. Incorporating correlated error structures into mixed forest growth models: prediction and inference implications. *Can J For Res* 2010;**40**:977–90. https://doi.org/10.1139/X10-032.

Monserud R. Evaluating forest models in a sustainable forest management context. *For Biometry Model Inf Sci* 2003;**1**:35–47.

Nyland RD, Ray DG, Yanai RD. *et al.* Early cohort development following even-aged reproduction method cuttings in New York northern hardwoods. *Can J For Res* 2000;**30**:67–75. https://doi.org/10.1139/x99-190.

Ochi N, Cao Q. A comparison of compatible and annual growth models. *For Sci* 2003;**49**:285–90.

Palahí M, Miina J, Tomé M. *et al.* Stand-level yield models for Scots pine (*Pinus sylvestris* L.) in north-east Spain. *Investigación agraria Sistemas y recursos forestales* 2002;**11**:409–24.

Pienaar LV, Harrison WM. Simultaneous growth and yield prediction equations for Pinus elliottii plantations in Zululand. *South Afr For J* 1989;**149**:48–53. https://doi.org/10.1080/00382167.1989.9628992.

Pienaar LV, Shiver BD. An analysis and models of basal area growth in 45-year-old unthinned and thinned slash pine plantation plots. *For Sci* 1984;**30**:933–42.

Pienaar LV, Shiver BD, Grider GE. Predicting basal area growth in thinned slash pine plantations. *For Sci* 1985;**31**:731–41.

Pretzsch H, Grote R, Reineking B. *et al.* Models for forest ecosystem management: a European perspective. *Ann Bot* 2008;**101**: 1065–87.

Ramirez L, Montes CR, Bullock BP. Long-term term effect of bedding and vegetation control on dominant height of slash pine plantations in the southeastern United States. *For Ecol Manage* 2022;**522**:120479. https://doi.org/10.1016/j.foreco.2022.120479.

R-Core Team. R: *A Language and Environment for Statistical Computing*. R Version 4.0.1 Ed. Vienna, Austria: R Foundation for Statistical Computing, 2022.

Scolforo HF, McTague JP, Burkhart H. *et al.* Stand-level growth and yield model system for clonal eucalypt plantations in Brazil that accounts for water availability. *For Ecol Manage* 2019;**448**:22–33. https://doi.org/10.1016/j.foreco.2019.06.006.

Sharma M, Oderwald RG, Amateis RL. A consistent system of equations for tree and stand volume. *For Ecol Manage* 2002;**165**:183–91. https://doi.org/10.1016/S0378-1127(01)00616-8.

Snowdon P, Jovanovic T, Booth TH. Incorporation of indices of annual climatic variation into growth models for Pinus radiata. *For Ecol Manage* 1999;**117**:187–97. https://doi.org/10.1016/S0378-1127(98)00463-0.

Soares P, Tomé M, Skovsgaard JP. *et al.* Evaluating a growth model for forest management using continuous forest inventory data. *For Ecol Manage* 1995;**71**:251–65. https://doi.org/10.1016/0378-1127(94)06105-R.

Stanton BJ. Clonal variation in basal area growth patterns during stand development in hybrid poplar. *Can J For Res* 2001;**31**: 2059–66. https://doi.org/10.1139/x01-150.

Sullivan AD, Clutter JL. A simultaneous growth and yield model for loblolly pine. *For Sci* 1972;**18**:76–86.

Sun H-G, Zhang J-G, Duan A-G. *et al.* A review of stand basal area growth models. *For Stud China* 2007;**9**:85–94. https://doi.org/10.1007/s11632-007-0014-2.

Vanclay J. *Modeling Forest Growth and Yield. Applications to Mixed Tropical Forests*. Wallingford UK, CABI Publishing Series, 1994; 312.

Wang GG. Early regeneration and growth dynamics of Populus tremuloides suckers in relation to fire severity. *Can J For Res* 2003;**33**:1998–2006. https://doi.org/10.1139/x03-123.

Wilson D, Monleon V, Weiskittel A. Quantification and incorporation of uncertainty in forest growth and yield projections using a Bayesian probabilistic framework: a demonstration for plantation coastal Douglas-fir in the Pacific Northwest, USA. *MathComput For Nat-Resour Sci* 2019;**11**:264–85.

Woollons RC, Hayward WJ. Revision of a growth and yield model for radiata pine in New Zealand. *For Ecol Manage* 1985;**11**:191–202. https://doi.org/10.1016/0378-1127(85)90026-X.

Woollons RC, Snowdon P, Mitchell ND. Augmenting empirical stand projection equations with edaphic and climatic variables. *For Ecol Manage* 1997;**98**:267–75. https://doi.org/10.1016/S0378-1127(97)00090-X.

Zellner A. An efficient method of estimating seemingly unrelated regressions and tests for aggregation bias. *J Am Stat Assoc* 1962;**57**: 348–68. https://doi.org/10.1080/01621459.1962.10480664.

Zhang J, Duan A. *Study on Theoretical Growth Equation and Diameter Structure Model*. Beijing: Life Science Press, 2004; 2–7.

Zhao D, Borders B, Wilson M. A density-dependent matrix model for bottomland hardwood stands in the Lower Mississippi Alluvial Valley. *Ecol Model* 2005;**184**:381–95. https://doi.org/10.1016/j.ecolmodel.2004.11.002.