Radiative Corrections to Superallowed β Decays in Effective Field Theory

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The accuracy of V_{ud} determinations from superallowed β decays critically hinges on control over radiative corrections. Recently, substantial progress has been made on the single-nucleon, universal corrections, while nucleus-dependent effects, typically parametrized by a quantity δ_{NS}, are much less well constrained. Here, we lay out a program to evaluate this correction from effective field theory (EFT), highlighting the dominant terms as predicted by the EFT power counting. Moreover, we compare the results to a dispersive representation of δ_{NS} and show that the expected momentum scaling applies even in the case of low-lying intermediate states. Our EFT framework paves the way toward *ab initio* calculations of δ_{NS} and thereby addresses the dominant uncertainty in V_{ud}.

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Introduction—A precise and robust determination of V_{ud}, the first element of the Cabibbo–Kobayashi–Maskawa (CKM) matrix [1,2], is a critical input for the unitarity test of the first row of the CKM matrix

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1. \tag{1}$$

At the moment, Eq. (1) displays a tension at the level of 2.8σ [3]. While a separate tension among V_{us} determinations from kaon decays could potentially be resolved by future measurements at NA62 [3], V_{ud} has also come under increased scrutiny in recent years, mainly in view of the increased tension that followed from a reevaluation of universal radiative corrections (RC) associated with γW box diagrams [4–10]. Such a violation of CKM unitarity could point to a wide range of possible beyond-the-standard-model scenarios [11,12], including vectorlike quarks [13–16] and leptons [17,18], or could be interpreted as a modification of the Fermi constant [19,20], the violation of lepton flavor universality [21–26], or, more generally, in the context of standard-model EFT [27–30]. It is thus paramount to consolidate the evaluation of V_{ud} and potentially even improve its precision.

The current best determination arises from superallowed $0^+ \to 0^+$ transitions [31], for which the average over a large number of different isotopes ultimately yields the gain in precision compared to other probes. In those cases, the resulting precision of V_{ud} is limited by experimental uncertainties: for neutron decay, recent years have witnessed impressive progress for the lifetime τ_n [32] and the decay parameter λ [33], but at least another factor of 2 in the latter is required for a competitive determination, especially in view of the tension with Ref. [34]. An extraction from pion β decay would be theoretically even more pristine [35–37] yet experimentally challenging [38], forming a key physics goal of the PIONEER experiment [39].

In contrast, the challenges in the interpretation of superallowed β decays are of theoretical nature. In the formula for the decay half-life t [31,40]

$$\frac{1}{t} = \frac{G_F^2 |V_{ud}|^2 m_e^5}{\pi^3 \log 2}(1+\Delta_R^V)(1+\delta_R')(1+\delta_{\mathrm{NS}}-\delta_C)\times f, \tag{2}$$

f is a phase-space factor that includes the Fermi function, due to the Coulomb interaction of the outgoing electron in the nuclear field, the nuclear electroweak (EW) form factor, nuclear recoil, atomic electron screening, and atomic overlap [31,40]. The other terms denote purely theoretical input due to isospin-breaking and non-Coulomb RC. δ_C denotes the deviation of the Fermi matrix element $M_{\mathrm{F}} = \langle f|\tau^+|i\rangle = M_{\mathrm{F}}^{(0)}(1-\delta_C/2)$ from its isospin-limit value

$M_{\rm F}^{(0)} = \sqrt{2}$. The so-called outer correction δ'_R encodes all infrared-sensitive RC not included in the Fermi function. At $\mathcal{O}(\alpha)$, these include the Sirlin function [41]. The precise extraction of V_{ud} requires control of corrections of $\mathcal{O}(\alpha^2 Z)$ and higher [42–44]. The remaining RC are collectively denoted as the inner correction and are usually split into the single-nucleon correction Δ_R^V and the nuclear-structure-dependent term $\delta_{\rm NS}$. The latter arises from the phase-space average of a correction that in general depends on the positron energy E_e, as pointed out recently in Refs. [45,46].

Currently, the largest uncertainties reside in δ_C and $\delta_{\rm NS}$. First, control over δ_C has long been a concern [47,48], and Refs. [49–54] provide recent studies and strategies for improvements. Second, while the single-nucleon, universal RC from γW box diagrams have reached a good level of maturity, including a comprehensive analysis in EFT [55] and a first lattice-QCD evaluation [56], the same cannot be said for the nucleus-dependent effects of the same diagrams. The nuclear correction called $\delta_{\rm NS}$ dominates the resulting uncertainty in V_{ud} [45]. The formalism for an evaluation using dispersion relations has been put forward in Refs. [40,46], including subtleties that arise in the case of low-lying intermediate states, such as the 3^+ and 1^+ levels of ^{10}B in the $^{10}{\rm C} \to {}^{10}{\rm B}$ $0^+ \to 0^+$ transition [57].

In this Letter, we lay out a program to evaluate $\delta_{\rm NS}$ in an EFT framework. We first set up the EFT power counting, identify the leading contributions, and discuss the impact of low-lying nuclear states in the EFT and the dispersive representation of Refs. [40,46]. We then discuss in detail the leading nuclear-structure-dependent contribution $\delta_{\rm NS}$. In particular, we analyze which contact terms are required as well as possible strategies for their determination, as has proved critical in the case of neutrinoless double-β decay [58–64].

Effective field theory—The RC to nuclear β decay involve several widely separated energy scales. These range from the EW scale (M_W) to the very low-energy scale $q_{\rm ext}$ of order of the reaction $\mathcal{Q}_{\rm EC}$ value and the electron mass m_e. The matrix element of the product of EW and electromagnetic (EM) currents in nuclear states brings in two additional scales: the hadronic scale set by the nucleon mass m_N (comparable to the breakdown scale of chiral perturbation theory, Λ_χ) and the typical nuclear scales, $\gamma \simeq R^{-1} \simeq M_\pi \simeq \mathcal{O}(100~{\rm MeV})$, with binding momentum γ, nuclear radius R, and pion mass M_π.

In the spirit of EFT we exploit the hierarchy

$$q_{\rm ext} \ll M_\pi \ll \Lambda_\chi \ll M_W \tag{3}$$

to systematically expand the β decay amplitude in the ratios of scales probed by the virtual photon. Besides the ratio $G_F q_{\rm ext}^2$ that sets the overall scale, these are

$$\epsilon_{\rm recoil} = \mathcal{O}\left(\frac{q_{\rm ext}}{\Lambda_\chi}\right), \quad \epsilon_{\not\pi} = \mathcal{O}\left(\frac{q_{\rm ext}}{M_\pi}\right), \quad \epsilon_\chi = \mathcal{O}\left(\frac{M_\pi}{\Lambda_\chi}\right), \tag{4}$$

scaling roughly as $\simeq 0.005$, $\simeq 0.05$, and $\simeq 0.1$, respectively. Our goal is to catalog all corrections to superallowed β decays at the permille level. This requires keeping $\mathcal{O}(\alpha\epsilon_\chi)$ and $\mathcal{O}(\alpha\epsilon_{\not\pi})$ corrections, which are the focus of this Letter. Terms that are subleading in α but enhanced by the nuclear charge Z or large logarithms, e.g., $\mathcal{O}(Z\alpha^2)$ or $\mathcal{O}(\alpha^2 \log r)$, with r a ratio of the scales in Eq. (3), are also relevant and discussed in detail in Ref. [65], as is the potential role of $\mathcal{O}(\alpha\epsilon_\chi^2)$ corrections, which are not yet included at present.

The presence of multiple scales requires the use of a tower of EFTs, as done in the single nucleon sector [55,66]. Between the EW scale and the hadronic scale, the relevant EFT is given by the Fermi theory obtained by integrating out the heavy standard-model particles. The resulting semileptonic operators are evolved using the renormalization group (RG) to the hadronic scale, where they are matched onto an EFT written in terms of nucleons, pions, light leptons, and photons [66], according to the symmetries of low-energy EW interactions, QED, and QCD.

In terms of the heavy-baryon nucleon $N^T = (p, n)$ isodoublet, the nucleon four-velocity v_μ and spin S_μ, and isospin Pauli matrices τ^a [67,68], the leading-order (LO) EW one-body (1b) Lagrangian is

$$\mathcal{L}_W^{\rm 1b} = -\sqrt{2} G_F V_{ud} \bar{e}_L \gamma_\mu \nu_L \bar{N}(g_V v^\mu - 2 g_A S^\mu)\tau^+ N + \cdots, \tag{5}$$

where the ellipsis denotes omitted terms involving pion fields or of higher order in ϵ_χ. The effects of hard photons with virtuality $Q^2 \geq \Lambda_\chi^2$ are captured in the deviation of the vector coupling g_V from one (and g_A from $g_A^{\rm QCD}$ [66]); see Ref. [65] for explicit expressions. Hard photons also generate EW two-body (2b) contact operators. We can write two S-wave operators relevant for superallowed β decays that connect $^1S_0$ to $^1S_0$ states, with isospin $I = 1$ and $I = 2$, given by

$$\mathcal{L}_W^{\rm 2b} = -\sqrt{2} e^2 G_F V_{ud} \bar{e}_L \not{v} \nu_L \big(g_{V1}^{NN} N^\dagger \tau^+ N N^\dagger N + g_{V2}^{NN} N^\dagger \tau^+ N N^\dagger \tau^3 N\big) + \cdots. \tag{6}$$

Weinberg power counting based on naive dimensional analysis would indicate that $g_{V1,V2}^{NN} = \mathcal{O}(\Lambda_\chi^{-3})$, but the requirement that the final nuclear amplitude be independent of the regulator promotes the low-energy constants (LECs) to $\mathcal{O}(\Lambda_\chi^{-1} F_\pi^{-2})$, where $F_\pi = 92.3$ MeV is the pion decay constant. The values of $g_{V1,V2}^{NN}$ are not known, but we will discuss strategies to obtain them below.

Within this chiral EFT with dynamical photons and leptons we compute EW transition amplitudes involving multiple nucleons; see Fig. 1 for some of the topologies relevant for nuclear decays. In the presence of more than one nucleon, the photon four-momentum can be in three regions; see, e.g., Refs. [69,70]: (1) soft: $q_\gamma^0 \simeq |\mathbf{q}_\gamma| \simeq M_\pi$, (2) ultrasoft: $q_\gamma^0 \simeq |\mathbf{q}_\gamma| \simeq q_{\rm ext}$, (3) potential: $q_\gamma^0 \simeq \mathbf{q}_\gamma{}^2/m_N \simeq q_{\rm ext}$, $|\mathbf{q}_\gamma| \simeq M_\pi$.

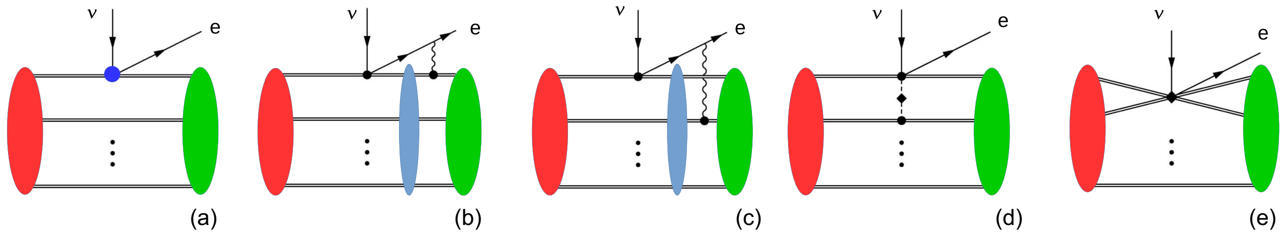

FIG. 1. Representative diagrams for RC to superallowed β decays up to $\mathcal{O}(\alpha\epsilon_\chi)$ and $\mathcal{O}(\alpha\epsilon_{\not\pi})$. Leptons, nucleons, photons, and pions are denoted by plain, double, wavy, and dashed lines, respectively. A blue circle denotes the insertion of the EW current, including $\mathcal{O}(\alpha)$ corrections from hard photon exchange. Black circles denote 1b EW and EM currents. The red and green ovals denote the wave functions of the initial and final nuclei; the blue oval represents the iteration of the nuclear interaction. See main text for the discussion of diagrams (a)–(e).

In the nuclear EFT, potential and soft modes are integrated out and give rise to an EW transition operator, analogous to the pion-exchange potential in the strong sector. In addition, hard photons also contribute to short-range transition operators proportional to $g^{NN}_{V1,V2}$ in Eq. (6); see Fig. 1(e). Because the exchange of a hard, soft, or potential photon leaves the intermediate nuclear state far off shell, these contributions can be calculated by taking the matrix element of the transition operator between the wave functions of the initial and final state. On the other hand, ultrasoft photons are sensitive to nuclear excitations and to the spectrum of intermediate states that are connected to the initial and final state by EW and EM currents.

We now discuss the contributions from each region.

Ultrasoft modes—Ultrasoft modes contribute at $\mathcal{O}(\alpha)$ through the LO photon-nucleon coupling. Through topologies such as those shown in Fig. 1(b) and 1(c) (and real emission topologies that we have omitted), ultrasoft modes give rise to the Sirlin function [41] and reproduce the $\mathcal{O}(\alpha)$ expansion of the Fermi [71] function, with the correct nuclear charge Z; see Ref. [65] for details. Using known results, terms to all orders in αZ, including logarithmically enhanced terms that start at $\mathcal{O}(\alpha^2 Z^2 \log \varepsilon_{\not\pi})$, as well as terms at $\mathcal{O}(\alpha^2 Z \log \varepsilon_{\not\pi})$ and $\mathcal{O}(\alpha^2 \log \varepsilon_{\not\pi})$, can be captured; see Refs. [42–44,72–74] and, in an EFT formalism, Refs. [55,75–78]. Subleading interactions, such as the interactions of the photon with the nucleon magnetic moment, are proportional to the ultrasoft momentum and appear at $\mathcal{O}(\alpha\epsilon_{\rm recoil})$ beyond the order at which we work.

Potential modes—Through the topology shown in Fig. 1(c), potential modes give rise to $\mathcal{O}(\alpha\epsilon_{\not\pi})$ and $\mathcal{O}(\alpha\epsilon_\chi)$ corrections to $\delta_{\rm NS}$. The former depend on the electron energy (E_e) and are induced by diagrams with the EW vector current and the EM charge density. The latter are E_e independent and are induced by the axial current and the nucleon magnetic moments or recoil corrections to the vector current. Three-body (3b) potentials contribute at $\mathcal{O}(\alpha\epsilon_\chi^2)$ and are not shown.

Soft modes—Beyond tree level, the potentials receive corrections from one-loop diagrams involving soft pions and photons. By power counting, these first contribute to $\delta_{\rm NS}$ at $\mathcal{O}(\alpha\epsilon_\chi^2)$ and $\mathcal{O}(\alpha^2)$.

Hard modes—Hard modes give $\mathcal{O}(\alpha)$ corrections to g_V [55] and generate the $\mathcal{O}(\alpha\epsilon_\chi)$ two-nucleon counterterms ($g^{NN}_{V1,V2}$) needed for renormalization. In addition, they produce $\mathcal{O}(\alpha\epsilon_{\not\pi}, \alpha\epsilon_\chi)$ effects in $\delta_{\rm NS}$ through the electromagnetic pion mass splitting in pion-mediated 2b currents; see Fig. 1(d). The pion-mass splitting corrections are the nuclear analogs of the pion-induced RC in neutron decay [66].

The implication of this analysis is that in chiral EFT the dominant contribution to $\delta_{\rm NS}$ comes from the matrix element of appropriate EW potentials between the initial and final nuclear states. Some contributions (from pion exchange) do not arise from nuclear γW box diagrams. Sensitivity of $\delta_{\rm NS}$ to intermediate nuclear states, including low-lying levels, arises from ultrasoft contributions that start to $\mathcal{O}(\alpha\epsilon_{\rm recoil})$. This result is seemingly at odds with a recent dispersive analysis [40,46] in which some individual contributions scale as $\mathcal{O}(\alpha\sqrt{\epsilon_{\rm recoil}})$ and thus enhanced compared to the identified EFT scalings. We, therefore, turn next to a detailed comparison to the dispersive representation.

Dispersive representation—In the current-algebra framework [73] for EW RC, $\delta_{\rm NS}$ arises from the γW box diagram, in which a virtual photon is exchanged between the electron and the hadronic system. The relevant dynamical quantity is the Compton tensor

$$T^{\mu\nu}(q;p',p) = \frac{1}{2}\int \mathrm{d}^4x e^{iq\cdot x}\langle f(p')|T\{J^\mu_{\rm em}(x)J^\nu(0)\}|i(p)\rangle, \tag{7}$$

involving the matrix element of an EW and an EM current between the initial and final states with momentum p and p', respectively. The E_e-independent part of $\delta_{\rm NS}$ is induced by the axial-vector component $T^{\mu\nu}_A$ [46]. Ignoring recoil corrections, the relevant amplitude is expressed as the forward limit

$$T^{\mu\nu}_A(p,q) = \frac{i\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{2M\nu}T_3(\nu,Q^2), \tag{8}$$

where $\nu = p\cdot q/M = q^0$, $Q^2 = -q^2 = -\nu^2 + \mathbf{q}^2$, and $M_i = M_f \equiv M$ has been assumed. Setting $m_e = 0$, the correction relative to $M_{\mathrm{F}}^{(0)}$ becomes [46]

$$\Box_{\gamma W} = -\frac{e^2}{M_{\mathrm{F}}^{(0)}}\int \frac{\mathrm{d}^4 q}{(2\pi)^4}\frac{M_W^2}{Q^2+M_W^2} \times \frac{T_3(\nu,Q^2)}{(p_e-q)^2 Q^2}\frac{Q^2 + M\nu\frac{p_e\cdot q}{p\cdot p_e}}{M\nu}, \quad (9)$$

where p_e is the momentum of the positron. The nucleus-dependent correction is finally determined by subtracting the single-nucleon contribution, i.e., [46]

$$\delta_{\mathrm{NS}} = 2\big(\Box_{\gamma W}^{\mathrm{nucl}} - \Box_{\gamma W}^{n}\big). \quad (10)$$

One way to perform the loop integral in Eq. (9) relies on a Wick rotation $\nu \to i\nu_E$, which is advantageous when the Compton tensor is expressed via a dispersion relation. However, as pointed out in Ref. [40], such a Wick rotation is not always possible. In the case of low-lying nuclear states, as does happen in the $^{10}\mathrm{C} \to {}^{10}\mathrm{B}$ decay, the additional pole can move into the third quadrant and thus must be subtracted explicitly. It was found that the resulting residue contribution becomes singular for $E_e \to 0$, which could lead to a numerical enhancement. Such an enhancement should be reflected by the momentum scaling and a different region in the EFT analysis.

To clarify the role of such low-lying states, we consider a simple example that displays all the relevant features:

$$\frac{iT_3^{\mathrm{toy}}(\nu,Q^2)}{M\nu} = \frac{M}{m_N}\frac{g_A g_M}{s-\bar{M}^2+i\epsilon}, \quad (11)$$

where $s = M^2 + \nu^2 - \mathbf{q}^2 + 2M\nu$ and $M^2 - \bar{M}^2 = 2M\Delta$. Here, g_A and g_M parameterize the matrix elements for the interaction with the EW and EM current, respectively. We focus on a single intermediate state with mass $\bar{M}$, with $\Delta > 0$ corresponding to a low-lying state. The prefactor has been chosen to match the corresponding EFT expression [65], counting the binding energy $\Delta \simeq q_{\mathrm{ext}}$ as before.

We can evaluate the integral by collecting all three residues in the upper half plane; see Ref. [65], which gives

$$\Box_{\gamma W}^{\mathrm{toy},\Delta} = \frac{3 g_A g_M}{4 M_{\mathrm{F}}^{(0)}}\frac{\alpha}{\pi}\frac{\Delta}{m_N}\log\frac{2\Delta}{M} + \mathcal{O}(\Delta^2), \quad (12)$$

where we have only displayed the corrections to the $M \to \infty$ limit. As expected from the ultrasoft region in the EFT analysis, the result scales with $\mathcal{O}(\alpha\epsilon_{\mathrm{recoil}})$.

In the dispersive approach, the presence of a low-lying state impedes a straightforward Wick rotation, and its residue needs to be subtracted whenever the pole lies in the first or third quadrant. This gives rise to the residue contribution

$$\Box_{\gamma W}^{\mathrm{toy,res}} = \frac{g_A g_M}{M_{\mathrm{F}}^{(0)}}\sqrt{\frac{M}{m_N}}\frac{\alpha}{\pi}\sqrt{\frac{2\Delta}{m_N}} + \mathcal{O}(\Delta^{3/2}), \quad (13)$$

which is again finite for $E_e \to 0$, but, contrary to Eq. (12), scales as $\mathcal{O}(\alpha\sqrt{\epsilon_{\mathrm{recoil}}})$ and could thus be enhanced numerically. The solution to this apparent mismatch is that the Wick-rotated integral also involves terms scaling with $\sqrt{\Delta}$, and one can show explicitly that [65]

$$\Box_{\gamma W}^{\mathrm{toy}} = \Box_{\gamma W}^{\mathrm{toy,Wick}} - \Box_{\gamma W}^{\mathrm{toy,res}}. \quad (14)$$

This demonstrates that no contributions of $\mathcal{O}(\alpha\sqrt{\epsilon_{\mathrm{recoil}}})$ appear in the dispersive representation even in the case of low-lying states, confirming the EFT scalings.

Leading contributions to δ_{NS}—In Ref. [65] we derive the nuclear decay rate in the EFT framework, while here we focus on the implications for δ_{NS}. Potential modes induce an effective Hamiltonian of the form

$$H_\beta = \sqrt{2}G_F V_{ud}\bar{e}_L[\gamma^0(\mathcal{V}^0 + E_0\mathcal{V}_E^0) + m_e\mathcal{V}_{m_e} + \ldots]\nu_L, \quad (15)$$

where E_0 is the end-point energy, and the ellipsis denotes higher powers of lepton energy or m_e. The functions $\mathcal{V}^0$, $\mathcal{V}_E^0$, and $\mathcal{V}_{m_e}$ have a chiral expansion in ϵ_χ. The LO contributions to H_β arise from diagrams such as those in Figs. 1(c)–1(e). We first consider Fig. 1(c). Because the LO 1b vector and axial currents are momentum independent, the LO potential is odd in the photon three-momentum $\mathbf{q}_\gamma$ and vanishes between 0^+ states. To get a nonvanishing correction we need to retain the lepton momenta. Similarly, the pion-exchange diagram [Fig. 1(d)] involving LO vertices requires an insertion of an external lepton momentum, leading to the only nonvanishing LO contributions

$$\mathcal{V}_E^0 = \frac{1}{3}\left(\frac{1}{2}+\frac{4E_e}{E_0}\right)\mathcal{V}_E + \mathcal{V}_E^\pi, \qquad \mathcal{V}_{m_e} = \frac{1}{2}\mathcal{V}_E + \mathcal{V}_{m_e}^\pi, \quad (16)$$

with the explicit expressions for $\mathcal{V}_E$, $\mathcal{V}_E^\pi$, and $\mathcal{V}_{m_e}^\pi$ given in Ref. [65]. The latter two depend on the pion mass splitting, $M_{\pi^\pm}^2 - M_{\pi^0}^2 = 2e^2F_\pi^2 Z_\pi$, encoding effects of hard photons. These potentials are energy dependent and affect both the spectral shape and the total decay rate at $\mathcal{O}(\alpha\epsilon_{\not{\pi}})$.

Additional contributions arise from Figs. 1(c) and 1(d) when using subleading vertices instead of inserting a lepton momentum. One order down in the chiral expansion at $\mathcal{O}(\alpha\epsilon_\chi)$ we obtain potentials that are independent of the lepton momenta and thus contribute to $\mathcal{V}^0$. These $\mathcal{O}(\alpha\epsilon_\chi)$ terms can be further decomposed as

$$\mathcal{V}^0 = \mathcal{V}_0^{\mathrm{mag}} + \mathcal{V}_0^{\mathrm{rec}} + \mathcal{V}_0^{\mathrm{CT}}, \quad (17)$$

corresponding to Figs. 1(c) and 1(d) via magnetic, recoil, and contact-term contributions [65]. Beyond tree level, the potentials (16) and (17) receive corrections from soft pions and photons at $\mathcal{O}(\alpha\epsilon_\chi^2)$ and $\mathcal{O}(\alpha^2)$, beyond the accuracy of this work. At this order we also expect effects from 3b potentials.

It is important to notice that $\mathcal{V}_0^{\rm mag}$ has a Coulombic, $\mathbf{q}^{-2}$, scaling. Such a potential, when inserted into $^1S_0$ chiral EFT wave functions, gives rise to nuclear matrix elements that depend logarithmically on the applied regulator [58,79]. This regulator dependence signals sensitivity to hard-photon exchange between nucleons which, in chiral EFT, are captured by the short-range operators in Eq. (6). The corresponding LECs absorb the regulator dependence and after renormalization are enhanced over naive dimensional analysis as anticipated below Eq. (6). This is analogous to the short-range operators identified for neutrinoless double-β decay [58,59]. The short-range terms give an $\mathcal{O}(\alpha\epsilon_\chi)$ contribution

$$\mathcal{V}_0^{\rm CT} = e^2\big(g_{V1}^{NN} O_1 + g_{V2}^{NN} O_2\big), \tag{18}$$

where

$$O_1 = \sum_{j\neq k} \tau^{+(j)} \mathbb{1}_k, \qquad O_2 = \sum_{j<k} \Big[\tau^{+(j)}\tau_3^{(k)} + (j \leftrightarrow k)\Big]. \tag{19}$$

$\mathcal{V}_0^{\rm CT}$ depends on two unknown LECs and corresponds to genuine new 2b contributions arising from high-momentum photon exchange. It is an intrinsic two-nucleon effect that cannot be obtained from one-nucleon processes. Below we compute the contributions of $g_{V1,V2}^{NN}$ by using the scaling discussed below Eq. (6) for the LECs and treating the result as an overall uncertainty [65].

In the EFT approach, up to the order considered, $\delta_{\rm NS}$ is entirely determined by matrix elements of appropriate potentials [see Eq. (15)] between the initial and final states without dependence on intermediate nuclear states. The EFT power counting indicates that $\delta_{\rm NS}$ receives a LO E_e-independent contribution of $\mathcal{O}(\alpha\epsilon_\chi)$, $\delta_{\rm NS}^{(0)}$ and an E_e-dependent contribution of $\mathcal{O}(\alpha\epsilon_{\not\pi})$, $\delta_{\rm NS}^{E}$. In the case of $\delta_{\rm NS}^{(0)}$ we also found an $\mathcal{O}(\alpha^2)$ potential $\mathcal{V}_+$ that needs to be included for $\mathcal{O}(10^{-4})$ precision [65].

The two currently unknown LECs $g_{V1,V2}^{NN}$ can be determined in the future both from theory and experiment. First, one can envision a matching calculation to the underlying theory, performed in lattice QCD or within the Cottingham-like approach [60,61]. Second, the LECs can be extracted from experimental data, based on the observations that (i) there are $\mathcal{O}(10)$ very precisely measured superallowed β decays [31], connecting members of $I=1$ triplets with initial $m_I=-1$ or $m_I=0$; (ii) the LECs contribute to $\delta_{\rm NS}$ through the combinations $g_{V1}^{NN}\langle f||O_1||i\rangle \mp \sqrt{3/5}g_{V2}^{NN}\times \langle f||O_2||i\rangle$, depending on whether $m_I=-1$ or $m_I=0$, and $\langle f||O_{1,2}||i\rangle$ are reduced matrix elements that depend on the decaying nucleus and can be computed with *ab initio* nuclear methods. It is then possible to perform a global fit to extract values of $g_{V1,V2}^{NN}$ and V_{ud} simultaneously from the set of superallowed β decay measurements.

Based on the EFT framework described here, we have derived a master formula for the decay rate and performed first numerical calculations for $\delta_{\rm NS}$ in the decay $^{14}{\rm O} \to {}^{14}{\rm N}$ with quantum Monte Carlo methods, confirming the expectations from the EFT power counting [65]. As an illustration, we extract V_{ud} from the ^{14}O decay, finding $V_{ud}[^{14}{\rm O}] = 0.97364(56)$, with uncertainty dominated by our ignorance of the LECs, $(\delta V_{ud})_{g_V^{NN}} = 4.3\times10^{-4}$. Eliminating this uncertainty would result in $\delta V_{ud} = 3.6\times10^{-4}$. This is to be compared with $V_{ud}[^{14}{\rm O}] = 0.97405(37)$ from Ref. [31], with uncertainty dominated by $\delta_{\rm NS}$, $(\delta V_{ud})_{\delta_{\rm NS}} = 3.1\times10^{-4}$ and with $V_{ud} = 0.97373(31)$ obtained by a global analysis of the $0^+ \to 0^+$ decays [31]. These considerations show that there is a clear path toward reaching $\delta V_{ud} \simeq 3\times10^{-4}$, once the LECs are determined following the strategies outlined above. We expect that a few decays of light nuclei, combined with nuclear-structure calculations, should suffice to obtain a competitive determination of V_{ud}, including a robust estimate of the nuclear-structure uncertainties.

Discussion and outlook—We have performed a first study of RC to superallowed nuclear β decays in an EFT framework that bridges the EW scale to nuclear scales. We have identified the leading nuclear-structure-dependent corrections $\delta_{\rm NS}$ as arising from matrix elements of EW transition operators of $\mathcal{O}(G_F\alpha\epsilon_{\not\pi}, G_F\alpha\epsilon_\chi)$ between initial and final nuclear wave functions. Several terms, such as the magnetic and recoil pieces of $\delta_{\rm NS}$, already appear in the seminal work [80], while others are new. Most strikingly, we identified novel pion-exchange and short-range corrections that affect $\delta_{\rm NS}$ at the same order as the usually considered corrections. Furthermore, we have sketched a strategy using global fits to superallowed β decays to empirically determine the contact operators' Wilson coefficients.

To map these EFT considerations onto a dispersive approach for $\delta_{\rm NS}$ [45,46], we first showed that the only contributions that scale with $q_{\rm ext}$ arise in the potential region and thus do not depend on the properties of individual states. This remains true in the presence of low-lying levels. Second, while the leading $\mathcal{O}(\alpha\epsilon_\chi)$ effects are energy independent, $\mathcal{O}(\alpha\epsilon_{\not\pi})$ energy-dependent corrections are predicted by the EFT, related to $\delta_{\rm NS}^{E}$ in the dispersive approach.

In conclusion, the EFT approach presented in this Letter allows one to derive corrections in a systematic way and thereby opens up new avenues to control the theoretical uncertainties in superallowed nuclear β decays. This enables first-principles nuclear many-body calculations of structure-dependent corrections, whose uncertainty currently dominates the extraction of V_{ud}, to further sharpen precision tests of the standard model and potentially reveal hints of physics beyond.

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