Robust PID Control against Nonlinear Uncertainties in Modern Power Systems with Microgrids

Shuo Yuan, Caisheng Wang, Feng Lin, Le Yi Wang

***Abstract*—This paper investigates the problem of robust PID control for nonlinear power systems with uncertainties. In this paper, we analyze the challenges posed by nonlinearities and uncertainties in power systems, as well as the limitations of traditional linearization methods. On this basis, a PID control design method is proposed, which utilizes only the Lipschitz constant information of the nonlinear uncertainties to achieve global stability control for the power systems. The paper provides a rigorous mathematical foundation to demonstrate the effectiveness and robustness of the controller. Finally, simulation studies are conducted to validate and evaluate the performance of the proposed model, algorithm, and controller design.**

***Index Terms*—Modern power system, PID control, parameter design, convergence, robustness.**

I. Introduction

As an indispensable infrastructure, modern power systems (MPSs) provide essential support for economic development and everyday activities [1], [2]. With the continuous growth of society's demand for energy and the rapid development of technology, the complexity and scale of power systems are correspondingly increasing. As distributed energy systems, microgrids can be interconnected with the power grid or operate independently. Microgrids integrate various energy sources, including wind power [3], solar power [4], and battery storage systems [5], and are equipped with flexible load control technologies. Through this configuration, microgrids not only enhance the reliability and resilience of local energy supplies but also optimize energy usage efficiency, reduce dependence on long-distance transmission, and contribute to the flexibility and sustainability of power systems.

In power systems, controlling and maintaining system stability are of paramount importance. Effective control not only enhances the reliability, resilience, and performance of MPS but also ensures continuous power supply and protects the integrity of critical infrastructure. However, power systems, being practical systems, exhibit nonlinear and uncertain characteristics due to the nonlinear properties of equipment, load fluctuations, external noise, and other factors [6]–[8]. These characteristics make the power system a complex and uncertain network, complicating the study of control problems [9], [10]. Furthermore, the validation of controller effectiveness often relies on experimental evidence, lacking rigorous theoretical foundations.

The conventional approach in existing literature for dealing with nonlinear systems typically linearizes nonlinear dynamic systems around nominal operating points [11], [12]. However, this approach has several limitations. For instance, linearization typically occurs near the operating point, and accuracy decreases as the system operates further from these points. Simplifying nonlinear behavior may fail to accurately describe the system's dynamic characteristics, leading to and even invalid control strategies. Additionally, linearized models are effective only within a limited range; significant changes in operating conditions can render the linearized model invalid, resulting in suboptimal control performance. Furthermore, important nonlinear characteristics of power systems are often overlooked during linearization, which may lead to insufficient system responses in practical applications. Nonlinear systems usually exhibit complex dynamic responses that linear models cannot capture, potentially causing unexpected behavior in real-world applications. Lastly, linearized control systems tend to have poor robustness when facing model uncertainties and external disturbances, making them susceptible to performance degradation due to changes in system parameters and external perturbations.

Therefore, designing robust controllers to deal with nonlinear uncertainties in power systems and rigorously ensuring the effectiveness and robustness of the controllers from a mathematical theory perspective is the starting point and the motivation of our research. Due to the simplicity and ease of implementation of PID control, it has been widely used in engineering applications [13]. Additionally, PID has the capability to handle nonlinear uncertainties [14]. Therefore, this paper employs PID control to address the control problems of nonlinear power systems with uncertainties.

The main contributions of this paper are as follows:

1) It investigates the PID control problem for nonlinear uncertain power systems.
2) It proposes a parameter design method for PID controllers that only utilizes the nonlinear Lipschitz constant information, enabling the design of PID controllers that achieve global stability and control objectives for uncertain nonlinear power systems.
3) It provides a rigorous mathematical theoretical foundation for the effectiveness and robustness of the proposed

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controller.

4) It validates and evaluates the robustness of the proposed model, controller design, convergence characteristics, and algorithms through simulation studies.

The rest of this paper is organized as follows. Section II introduces the notations of this paper. Section III presents the nonlinear state space model of power systems. Section IV discusses the robust PID control for nonlinear uncertain systems. Section V presents performance evaluation case studies. Finally, we conclude the paper with some remarks in Section VI.

II. Notations

Denote $\mathbb{R}$ as $(-\infty,\infty)$. Denote $\mathbb{R}^n$ as the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ as the space of $m\times n$ real matrices. Denote I_n as the identity matrix of dimension $n\times n$. For a vector $x\in\mathbb{R}^n$, its Euclidean norm is denoted by $\|x\|$. For a matrix $M\in\mathbb{R}^{m\times n}$, its norm is denoted by $\|M\| = \sup_{x\in\mathbb{R}^n,\|x\|=1}\|Mx\|$, and its transpose is denoted by M'. Let $C^k(\mathbb{R}^n,\mathbb{R}^m)$ be the space of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$ with k-times continuous partial derivatives.

III. Nonlinear dynamic Models of Power Systems with Microgrids

Microgrids are localized grids that can disconnect from the traditional grid to operate autonomously, enhancing reliability and resilience. They incorporate various sources of power generation and storage, including traditional generators, wind turbines, solar panels, battery storage systems, and controllable loads. The integration of these diverse power sources and storage systems allows microgrids to operate flexibly and efficiently, providing reliable power even in the event of disturbances or disconnections from the main grid.

In [15], [16], buses are divided into two types: dynamic bus and non-dynamic bus. For dynamic Bus i, denote z_i^d as the local state variable, z_i^- as the neighboring variables, v_i^d as the local control input, and ℓ_i^d as the local load. Then, dynamic Bus i is characterized by the state model,

$$\dot{z}_i^d = f_i(z_i^d, z_i^-, v_i^d, \ell_i^d). \quad (1)$$

For non-dynamic Bus j, denote z_j^{nd} as the local system variable, z_j^- as the neighboring variables, v_j^{nd} as the local control input, and ℓ_j^{nd} as the local load. Then, the non-dynamic Bus j is in a steady state or pseudo-steady state that is characterized by an implicit algebraic relationship,

$$0 = g_j(z_j^{nd}, z_j^-, v_j^{nd}, \ell_j^{nd}). \quad (2)$$

Denote z^d, v^d, and ℓ^d as the state variables, control variables, and loads of dynamic buses, respectively. Denote z^{nd}, v^{nd}, and ℓ^{nd} as the system variables, control variables, and loads of non-dynamic buses, respectively. From (2) for all non-dynamic buses, the equation has a unique solution such that

$$z^{nd} = H(z^d, v^{nd}, \ell^{nd}). \quad (3)$$

Then, by using (1) and substituting (3), we obtain the dynamic system

$$\dot{z}^d = F^0(z^d, v^d, v^{nd}, \ell^d, \ell^{nd}). \quad (4)$$

The control objective of the power system is to control the system state to the nominal operating point, that is, the equilibrium point. Given the steady-state loads $\bar{\ell}^d, \bar{\ell}^{nd}$, and inputs $\bar{v}^d, \bar{v}^{nd}$, the steady-state $\bar{z}^d$ (which represents the equilibrium point or the nominal operating condition) is determined as the solution to $F^0(\bar{z}^d, \bar{v}^d, \bar{v}^{nd}, \bar{\ell}^d, \bar{\ell}^{nd}) = 0$. Denote the perturbation variables from the nominal operating point as $x = z^d - \bar{z}^d, u = v^d - \bar{v}^d, u^n = v^{nd} - \bar{v}^{nd}, \zeta = \ell^d - \bar{\ell}^d, \zeta^n = \ell^{nd} - \bar{\ell}^{nd}$, by (4), the dynamics can be written as the following nonlinear dynamic system,

$$\dot{x} = F(x, u, u^n, \zeta, \zeta^n). \quad (5)$$

The nonlinear state equation (1) is general in representing a bus system' dynamic. For instance, it can represent a swing equation that is a common dynamic model for synchronous generators,

$$M_i\dot{\omega}_i + h_i(\omega_i) = P_i^{in} - P_i^L - P_i^{out}, \quad (6)$$

where δ_i is the electric angle, $\omega_i = \dot{\delta}_i$. M_i is the equivalent electric-side inertia. $h_i(\cdot)$ is the damping effect, which is usually taken as its approximate value near the equilibrium point, denoted as $h_i(\cdot) = b_i\omega_i$ with $b_i > 0$. P_i^{out} represents the total transmitted power from Bus i to its neighboring buses. P_i^{in} represents the equivalent electric-side real power input that is considered as the local control input. P_i^L represents the local real power load on Bus i that is considered as a disturbance.

For both dynamic and non-dynamic buses, the interaction between local variables and neighboring buses follows standard power flow relationships. For an AC power microgrid, voltages and currents will be represented by their phasors $\vec{V} = V\angle\delta$ and $\vec{I} = I\angle\gamma$. Consider the transmission line between Bus i and Bus j with impedance $Z_{ij}\angle\theta_{ij}$. The current through the line is

$$I_{ij}\angle\gamma = \frac{V_i\angle\delta_i - V_j\angle\delta_j}{Z_{ij}\angle\theta_{ij}} = \frac{V_i}{Z_{ij}}\angle(\delta_i-\theta_{ij}) - \frac{V_j}{Z_{ij}}\angle(\delta_j-\theta_{ij}).$$

Denote $\delta_{ij} = \delta_i - \delta_j$. The complex power flow from Bus i to Bus j at Bus i is

$$S_{ij} = V_i\angle\delta_i \times I_{ij}\angle(-\gamma) = \frac{V_i^2}{Z_{ij}}\angle\theta_{ij} - \frac{V_iV_j}{Z_{ij}}\angle(\theta_{ij}+\delta_{ij}),$$

indicating that the transmitted real and reactive powers at Bus i are

$$P_{ij} = \frac{V_i^2}{Z_{ij}}\cos(\theta_{ij}) - \frac{V_iV_j}{Z_{ij}}\cos(\theta_{ij}+\delta_{ij}),$$

$$Q_{ij} = \frac{V_i^2}{Z_{ij}}\sin(\theta_{ij}) - \frac{V_iV_j}{Z_{ij}}\sin(\theta_{ij}+\delta_{ij}).$$

Then, $P_i^{out} = \sum_{j\in\mathcal{N}_i} P_{ij}$.

We now illustrate the general dynamic modeling approach discussed in this section with a case study.

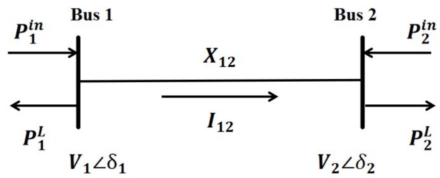

Fig. 1. Two-Bus system

Example 3.1: Consider the two-Bus system shown in Fig. 1, with dynamic equation (6). Suppose both Bus 1 and Bus 2 are dynamic dispatchable buses, and the transmission line is lossless, namely the angle of impedance $\theta_{12} = 90^o$. Suppose $h_i(\omega_1) = b_i\omega_i$, $b_i > 0$, $i = 1, 2$. Denote $\beta = V_1V_2/Z_{12}$. Then, the system has dynamics

$$\begin{cases} \dot{\delta}_1 = \omega_1, \\ \dot{\omega}_1 = -\frac{b_1\omega_1}{M_1} + \frac{1}{M_1}(-\beta\sin(\delta_1 - \delta_2) + P_1^{in} - P_1^L), \\ \dot{\delta}_2 = \omega_2, \\ \dot{\omega}_2 = -\frac{b_2\omega_2}{M_2} + \frac{1}{M_2}(\beta\sin(\delta_1 - \delta_2) + P_2^{in} - P_2^L). \end{cases}$$

In order to derive the dynamic equation (5), we given $v_1^d = P_1^{in}$, $v_2^d = P_2^{in}$, $\ell_1^d = P_1^L$, $\ell_2^d = P_2^L$, and the equilibrium point is $\bar{\omega}_1 = 0$, $\bar{\omega}_2 = 0$, and $\bar{\delta} = \bar{\delta}_1 - \bar{\delta}_2 = \sin^{-1}\left(\frac{P_1^{in} - P_1^L}{\beta}\right)$.

Denote states $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $x_1 = \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} = \begin{bmatrix} \delta_1 - \bar{\delta}_1 \\ \delta_2 - \bar{\delta}_2 \end{bmatrix}$, $x_2 = \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} = \begin{bmatrix} \omega_1 - \bar{\omega}_1 \\ \omega_2 - \bar{\omega}_2 \end{bmatrix}$, and control $u = \begin{bmatrix} P_1^{in} - \bar{P}_1^{in} \\ P_2^{in} - \bar{P}_2^{in} \end{bmatrix}$. Then, the system dynamics can be rewritten as

$$\begin{cases} \dot{x}_1 = & x_2, \\ \dot{x}_2 = & \begin{bmatrix} -\frac{b_1}{M_1}x_{21} - \frac{\beta}{M_1}\sin(x_{11} - x_{12} + \bar{\delta}) \\ -\frac{b_2}{M_2}x_{22} + \frac{\beta}{M_2}\sin(x_{11} - x_{12} + \bar{\delta}) \end{bmatrix} \\ & + \begin{bmatrix} \frac{1}{M_1} & 0 \\ 0 & \frac{1}{M_2} \end{bmatrix} u + \begin{bmatrix} \frac{1}{M_1}(\bar{P}_1^{in} - P_1^L) \\ \frac{1}{M_2}(\bar{P}_2^{in} - P_2^L) \end{bmatrix}. \end{cases}$$

The equilibrium point of the above system is 0.

IV. Robust PID control of nonlinear uncertain systems

Linearization near the nominal operating point is a common method to solve the control problem of the nonlinear power system. Although linearization facilitates simplified analysis and control design, it also brings some problems.

Firstly, it reduces accuracy as linearization is typically valid only near specific operating points, failing to accurately model system behavior across broader operational ranges. Secondly, linearized models have limited applicability, as they may not adequately represent the nonlinear dynamics of power grid components under varying conditions. Additionally, linearization may overlook critical nonlinear effects, which are crucial in realistic system responses. This approach also lacks robustness compared to nonlinear control methods, making it more susceptible to disturbances and uncertainties in practical applications.

Therefore, in order to avoid the problems of accuracy, applicability and robustness may caused by linearization, this paper designs a PID controller for the nonlinear power system, which has a simple structure, is easy to implement, and provides a theoretical foundation for its reliability.

In this section, we describe PID controller design procedures. Some methods recently introduced in [14] are utilized.

Consider the PID-controlled system

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = f(x_1, x_2, u), \end{cases} \tag{7}$$

$$u(t) = k_i \int_0^t e(s)ds + k_p e(t) + k_d \dot{e}(t), \tag{8}$$

where $x_1, x_2 \in \mathbb{R}^n$ are the system states, $u \in \mathbb{R}^n$ is the PID controller with the control error $e(t) = x^* - x_1(t)$ and the control target x^*, and $f(\cdot) \in C^2(\mathbb{R}^{3n}, \mathbb{R}^n)$ is an unknown nonlinear function. PID parameters k_p, k_i, k_d are positive constants that should be designed.

$f(\cdot)$ is an uncertain function. We introduce the following assumption to quantitatively measure the size of the uncertainty and to develop a rigorous mathematical framework.

Assumption 4.1: The nonlinear function $f(\cdot)$ satisfies the Lipschitz condition

$$|f(x, u) - f(y, u)| \leq L|x - y|, \quad \forall x, y \in \mathbb{R}^{2n}, u \in \mathbb{R}^n,$$

where L is a known positive constant. $f(\cdot)$ has continuous partial differential w.r.t u, and satisfies

$$\frac{\partial f(x, u)}{\partial u} \geq \underline{b} I_n > 0, \quad \forall x \in \mathbb{R}^{2n},\ u \in \mathbb{R}^n,$$

where $\underline{b}$ is a known positive constant.

We will show that the PID parameters can be designed based on the Lipschitz constant of the uncertain nonlinear function $f(\cdot)$, such that the overall system can be stabilized globally and achieves the control objective.

Theorem 4.1: Consider the uncertain PID controlled system (7) and (8) with the unknown nonlinear function $f(\cdot)$ satisfying Assumption 4.1. Suppose that the positive PID parameters k_p, k_i, k_d are taken from the following set

$$\Omega = \big\{(k_p, k_i, k_d) \big|\ k_p^2 - 2k_ik_d > 2L(k_p + k_d)/\underline{b}, \\ k_d^2 - k_p/\underline{b} > 2L(k_p + k_d)/\underline{b}\big\}.$$

Then, for any initial states $(x_1(0), x_2(0)) \in \mathbb{R}^{2n}$, the closed-loop system will achieve

$$\|x_1(t) - x^*\|^2 \to 0,\ \|x_2(t)\|^2 \to 0,\ \text{as } t \to \infty,$$

exponentially.

The above theorem is a special case of Theorem 1 in [14] under the deterministic case. We briefly outline the proof here.

Proof: Firstly, denote $y = [y_0', y_1', y_2']'$,

$$y_0(t) = \int_0^t \big(x_1(s) - x^*\big)ds + k_i^{-1}u^*, \\ y_1(t) = x_1(t) - x^*, \quad y_2(t) = x_2(t), \\ \hat{y}(t) = k_iy_0(t) + k_py_1(t) + k_dy_2(t),$$

where u^* is the unique solution of $f(x^*, 0, u^*) = 0$. Then, we can rewrite the PID control (8) as

$$u(t) = -\hat{y}(t) + u^*.$$

The closed-loop system (7) and (8) turns into

$$\begin{cases} \dot{y}_0 = y_1, \\ \dot{y}_1 = y_2, \\ \dot{y}_2 = f(y_1 + x^*, y_2, -\hat{y} + u^*). \end{cases} \tag{9}$$

Secondly, define the matrix

$$P = \frac{1}{2}\begin{bmatrix} 2\underline{b}k_i k_p & 2\underline{b}k_i k_d & k_i \\ 2\underline{b}k_i k_d & 2\underline{b}k_p k_d - k_i & k_p \\ k_i & k_p & k_d \end{bmatrix}.$$

From Lemma 3 in [14], P is positive definite if $(k_p, k_i, k_d) \in \Omega$.

Next, we consider the Lyapunov candidate $V = y' P y$, which can be verified to be positive definite. Then, calculating $\dot{V}$ along (9), and using the conditions in Assumption 4.1, from Theorem 1 in [14], it can be verified that there exists some positive constant λ such that $\dot{V} \le -\lambda V$. Therefore, we obtain that $\|y(t)\|^2 \to 0$ as $t \to \infty$. Hence, the theorem is proved. □

The following corollary provides a straightforward method for parameter selection, and shows that Ω is a nonempty unbounded open set.

Corollary 4.1: For any given $k_i > 0$, as long as $k_p = k_d \ge 2k_i + (4L+1)/\underline{b}$, then $(k_i, k_p, k_d) \in \Omega$.

Remark 4.1: Theorem 4.1 demonstrates that PID control has large-scale robustness against system nonlinear uncertainties and the selection of PID gains. This contributes to the widespread applicability of PID control. On the one hand, we do not need to know the specific parameter values of the unknown nonlinear dynamics of the power system, nor do we need to utilize its specific structural equations; we only need to use its Lipschitz constant. On the other hand, PID parameters can be selected arbitrarily within an unbounded open set.

Remark 4.2: From Theorem 4.1, we know that if the nonlinear dynamics (5) of the bus system satisfy Assumption 4.1, then we can design a PID controller to achieve stability and achieve the control objectives. In fact, in the frequency regulation, the dynamic system of a synchronous machine is described by the swing equation (6). As shown in the previous section, it can be verified that its nonlinear dynamics are Lipschitz continuous. Therefore, we can use a PID controller to regulate this nonlinear system, and theoretically ensure that it achieves the desired regulation objective.

V. Case Study

In this section, we consider the two-Bus system in Example 3.1. We assume the loads are fixed, namely, $P_1^L = \bar{\ell}_1^d, P_2^L = \bar{\ell}_2^d$. Then, the system can be written as

$$\begin{cases} \dot{x}_1 = & x_2, \\ \dot{x}_2 = & \begin{bmatrix} -\frac{b_1}{M_1}x_{21} - \frac{\beta}{M_1}\sin(x_{11} - x_{12} + \bar{\delta}) \\ -\frac{b_2}{M_2}x_{22} + \frac{\beta}{M_2}\sin(x_{11} - x_{12} + \bar{\delta}) \end{bmatrix} \\ & + \begin{bmatrix} \frac{1}{M_1} & 0 \\ 0 & \frac{1}{M_2} \end{bmatrix} u + \begin{bmatrix} \frac{1}{M_1}(\bar{P}_1^{in} - \bar{\ell}_1^d) \\ \frac{1}{M_2}(\bar{P}_2^{in} - \bar{\ell}_2^d) \end{bmatrix}, \end{cases}$$

where $\bar{P}_1^{in}, \bar{P}_2^{in}, \bar{\ell}_1^d, \bar{\ell}_2^d$ are given, $(M_1, M_2, b_1, b_2, \beta, \bar{\delta})$ are unknown system parameter set and $\bar{\delta} = \sin^{-1}\left(\frac{\bar{P}_1^{in} - \bar{\ell}_1^d}{\beta}\right)$. The control objective is to control the electric angle δ_1 and δ_2 to the normal point, i.e, the system state $x_1 = \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} = \begin{bmatrix} \delta_1 - \bar{\delta}_1 \\ \delta_2 - \bar{\delta}_2 \end{bmatrix}$ to zero (i.e., equilibrium point). We use PID controller

$$u(t) = -k_i \int_0^t e(s)ds - k_p e(t) - k_d \dot{e}(t),$$

where $e(t) = -x_1(t)$.

Assume $\bar{P}_1^{in} = 100, \bar{P}_2^{in} = 50, \bar{\ell}_1^d = 70, \bar{\ell}_2^d = 80$. We take 3 different systems with parameter set $(M_1, M_2, b_1, b_2, \beta, \bar{\delta})$ as $(1, 1.5, 0.2, 0.3, 50, 0.6435)$ for system 1, $(1.5, 1.5, 0.25, 0.2, 80, 0.3844)$ for system 2, and $(1.5, 2, 0.3, 0.25, 100, 0.3047)$ for system 3. The initial states for system 1 is $[0.6, 0.4, 0.5, 0.2]'$, for system 2 is $[0.4, 0.3, 0.1, 0.5]'$, for system 3 is $[0.5, 0.6, 0.2, 0.7]'$, are also randomly taken. It can be verified that all the system parameter sets satisfy Assumption 4.1 with $L = 119, \underline{b} = 0.45$. By calculating

$$\begin{aligned} \frac{\partial f}{\partial x_1} = & \begin{bmatrix} -\frac{\beta}{M_1}\cos(x_{11} - x_{12} + \bar{\delta}) & \frac{\beta}{M_1}\cos(x_{11} - x_{12} + \bar{\delta}) \\ \frac{\beta}{M_2}\cos(x_{11} - x_{12} + \bar{\delta}) & -\frac{\beta}{M_2}\cos(x_{11} - x_{12} + \bar{\delta}) \end{bmatrix} \\ = & \beta\cos(x_{11} - x_{12} + \bar{\delta}) \begin{bmatrix} -\frac{1}{M_1} & \frac{1}{M_1} \\ \frac{1}{M_2} & -\frac{1}{M_2} \end{bmatrix}, \end{aligned}$$

$$\frac{\partial f}{\partial x_2} = \begin{bmatrix} -\frac{b_1}{M_1} & 0 \\ 0 & -\frac{b_2}{M_2} \end{bmatrix}, \quad \frac{\partial f}{\partial u} = \begin{bmatrix} \frac{1}{M_1} & 0 \\ 0 & \frac{1}{M_2} \end{bmatrix}.$$

Then,

$$\begin{aligned} & \|\frac{\partial f}{\partial [x_1', x_2']}\| \le \|\frac{\partial f}{\partial x_1}\| + \|\frac{\partial f}{\partial x_2}\| \\ \le & \max\{\beta \| \begin{bmatrix} -\frac{1}{M_1} & \frac{1}{M_1} \\ \frac{1}{M_2} & -\frac{1}{M_2} \end{bmatrix} \| + \| \begin{bmatrix} -\frac{b_1}{M_1} & 0 \\ 0 & -\frac{b_2}{M_2} \end{bmatrix} \|\} \\ = & \max\{85.1837, 106.8333, 118.0511\} \\ = & 118.0511 < L, \end{aligned}$$

and

$$\min\{\frac{1}{M_1}, \frac{1}{M_2}\} = 0.5 > \underline{b}.$$

Note that when designing the PID control parameters, we only need to use the values of L and $\underline{b}$, rather than knowing the specific values of the system parameter set. The values of L and $\underline{b}$ can be obtained by using the range of the system parameter sets.

According to Theorem 4.1 and Corollary 4.1, we take PID parameters $(k_p, k_i, k_d) = (1062, 1, 1062)$ in Ω. Then, Figs. 2-4 shows the trajectories of x_{11}, x_{12} in three different systems, which shows that the designed PID controller can achieve the control objective in all systems. Therefore, the result illustrates the robustness and control capabilities of PID.

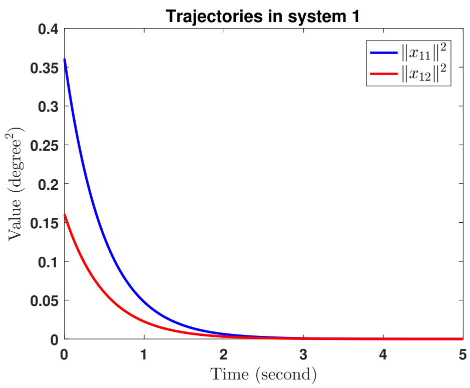

Fig. 2. Trajectories of x_{11}, x_{12} in system 1

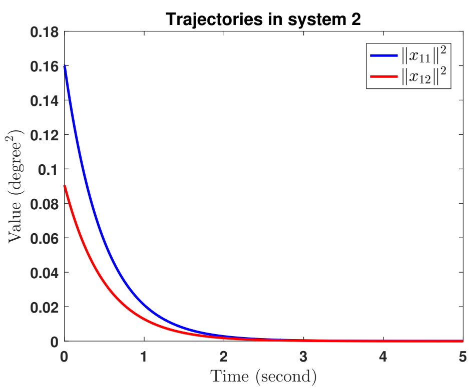

Fig. 3. Trajectories of x_{11}, x_{12} in system 2

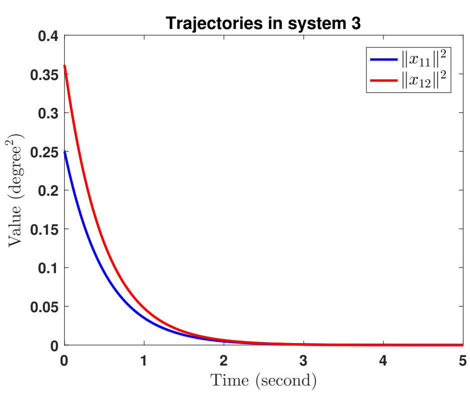

Fig. 4. Trajectories of x_{11}, x_{12} in system 3

VI. Conclusions

This paper proposes an effective control strategy for robust PID control in nonlinear uncertain power systems. Different from traditional linearization methods, this approach only utilizes the Lipschitz constant to design controllers without linearization, ensuring global stability and robustness of the system. Through rigorous mathematical analysis and simulation studies, the effectiveness of this method in power grid environments is validated. This research not only provides new theoretical and methodological support for power system control but also ensures system stability and reliability in practical applications. Future research can further optimize control algorithms, improve system response speed, and enhance disturbance rejection capabilities.

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