2024 10th International Conference on Control, Decision and Information Technologies
CoDIT 2024 | Valletta, Malta / July 01-04, 2024 Technically co-sponsored by IEEE & IFAC

2024 10th International Conference on Control, Decision and Information Technologies (CoDIT) | 979-8-3503-7397-4/24/$31.00 ©2024 IEEE | DOI: 10.1109/CoDIT62066.2024.10708228

Optimality Conditions for Distributed Primary Control in Energy State Space

Dan Wu*,†, Rupamathi Jaddivada†, Riley Lawson†, and Marija Ilic†

***Abstract*—This paper addresses the problem of designing optimal distributed control in the changing electric energy systems. We aim to improve the aggregated reactive power inefficiency of inverter-controlled and power-coupled electrical systems. To achieve this, we formulate an optimal control problem aimed at optimizing reactive power and interface mismatch using the energy state space model. Despite the general complexity of solving an optimal control problem, we demonstrate that our proposed model in energy state space admits a linear optimality condition in the variational form. We demonstrate that this condition can be decomposed into local state variables and measurements from adjacent connected components, facilitating a distributed approach to optimal control design. The variables exchanged with the neighboring components are technology agnostic and can be interpreted as costate variables with intuitive physical meaning. We illustrate the conditions and results of the distributed optimal control using a small system with controllable source serving a time-varying power load. Additionally, we explore voltage regulation in the proposed control design, which can be implemented using fast switching power electronics.**

***Index Terms*—Energy state space, optimality conditions, distributed control**

I. Introduction

This paper is motivated by immediate industry needs to control power electronically-switched equipment, such as batteries, wind and solar power plants, and other forms of fast storage so that fast control induced system stability (CISS) problems are avoided [1]. These problems are quite challenging because they cannot be modeled by making typical assumptions, such as sub-transient dynamics in wires and reactive storage components being instantaneous and stable [2]. Also, one cannot assume real-reactive power decoupling. Eliminating these two assumptions poses a major challenge to today's modeling and control of fast primary dynamics in the changing industry. On the other hand, solving the problem is quite relevant because the CISS operating problems are already seen in actual operations and are known to create real roadblocks to the deployment and utilization of fast varying intermittent resources [3].

Distributed control provides a promising way to handle fast varying component behaviors without the communication bottlenecks of centralized controllers, or the lack of robust performance with only local control [4]. A number of distributed control algorithms have been proposed for power systems, but lack provable performance, or require very strong conditions on the system due to the nonlinear nature of components. The work in [5] requires certain constraints on the underlying communication network. Reference [6] presents optimal control conditions that requires the communication of large data sets between components to achieve global control objectives. Other works make certain weak coupling assumptions on the interactions between components [7].

As a way forward, we build on our earlier introduced modeling in energy space [8]–[10]. The approach is fundamentally based on representing each component module with technology-specific internal state variables, and commonly shared interaction variables at the interfaces. Two port components, like wires and transformers which are typically modeled as constant impedance are key to capturing the very fast sub-transients created by fast controlling inverter control. At the same time, the component interactions represented using these aggregate variables are shown to be linear.

The utilized aggregate dynamic modeling in energy space is introduced in Section II. The optimal control problem with a performance objective being a combination of maximizing physical efficiency and minimizing dynamic mismatches at the component interfaces is shown in Section III, and it is shown that the derived optimal conditions can be implemented in an entirely distributed manner and support stable distributed optimal control of sub-transient dynamics in the changing electric energy systems. Physical intepretations of the derived control laws is provided in Section IV. Proof-of-concept simulations are given by considering an inverter-controlled voltage source providing time varying power by means of an RLC delivery circuit in Section V. Concluding remarks are provided in Section VI.

II. Basic Modeling in Energy State Space

In this section we recall the basic modeling of general interconnected multi-agent systems in the energy state space [8], [9].

A. Definitions in Energy State Space

Consider a general multi-agent system shown in Fig. 1. For each individual agent-i, its dynamics are determined by

$$\dot{x}_i = h_i(x_i, u_i, m_i, r_i) \tag{1a}$$
$$y_i = g_i(x_i, u_i, m_i, r_i) \tag{1b}$$
$$x_i(0) = x_{i,0} \tag{1c}$$

*: Huazhong University of Science and Technology, Wuhan, China
†: Laboratory for Information and Decision Systems, Massachusetts Institute of Technology, Cambridge, MA, danwumit@mit.edu, rjaddiva@mit.edu, lawsonri@mit.edu, ilic@mit.edu

where x_i are the state variables; u_i are the control inputs; m_i are the exogenous disturbances; r_i are the adjacent interactions from agent-j; y_i are the outputs.

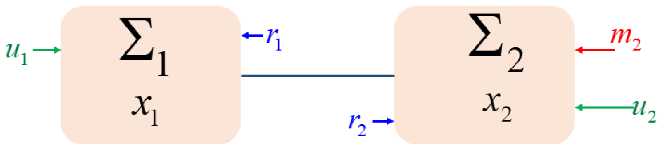


Fig. 1: Interconnected system comprising two components Σ_1 and Σ_2 with local controllable input u_1, u_2 and disturbances m_2

Now let's define the basic energy state variables that will be used in the modeling based on the effort variables ψ_i, flow variables ϕ_i, and states x_i in Agent-i [11].

1) Power Injection P_i to Agent-i:

$$P_i := \psi_i^T \phi_i \tag{2}$$

2) Internal Stored Energy E_i of Agent-i:

$$E_i := \frac{1}{2} x_i^T \ H_i \ x_i \tag{3}$$

3) Internal Stored Energy $E_{t,i}$ in Tangent Space:

$$E_{t,i} := \frac{1}{2} \dot{x}_i^T \ H_i \ \dot{x}_i \tag{4}$$

4) Internal Dissipation Energy D_i of Agent-i:

$$D_i := \frac{1}{2} x_i^T \ B_i \ x_i \tag{5}$$

5) Time Constant τ_i of Agent-i [12]:

$$\tau_i := \frac{E_i}{D_i} \tag{6}$$

6) Reactive Power Injection Q_i to Agent-i:

$$Q_i := \int \left(\psi^T \dot{\phi}_i - \dot{\psi}_i^T \phi_i \right) dt \tag{7}$$

Note that the definition of reactive power Q_i follows from [13], which is consistent with the ordinary reactive power defined as the imaginary part of the complex power for the AC system. Our definition here accounts for more general systems with non-sinusoidal voltage and current variables.

B. Interaction Model in Energy State Space

Assume that stored energy E_i is a strict convex function, i.e. $H_i \succ 0$, and dissipation energy D_i is a strict convex function, i.e. $B_i \succ 0$. Then, agent-i's interactions with the rest of the system can be described as follows.

$$\dot{E}_i = P_i^{r,out} - \frac{E_i}{\tau_i} := p_i \tag{8a}$$

$$\dot{p}_i = 4E_{t,i} - \dot{Q}_i^{r,out} \tag{8b}$$

$\dot{z}_i^{r,out} = [P_i^{r,out}, \dot{Q}_i^{r,out}]^T$ represents the components interaction with the rest of the system by virtue of its local energy conversion dynamics. These quantities in steady state after

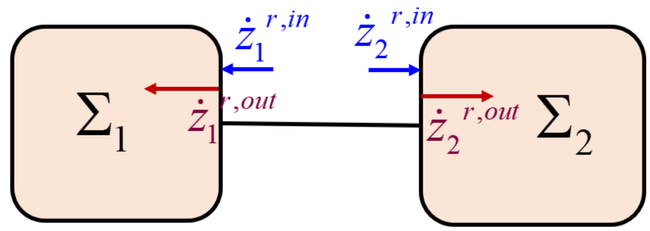


Fig. 2: Representation for the interconnected system: Incoming interactions are shown with blue arrows while the outgoing ones by virtue of local energy conversion dynamics are shown in brown

energy dynamics settle i.e. when $\dot{E}_i = 0, \dot{p}_i = 0$ are defined as the output variables of interest.

$$y_{z,i} = \begin{bmatrix} P_2^{r,t,\prime} \\ \dot{Q}_2^{r,t,\prime} \end{bmatrix} = \begin{bmatrix} \frac{E_i}{\tau_i} \\ 4E_{t,i} \end{bmatrix} \tag{9}$$

When the component is connected to rest of the system, the energy conversion dynamics in connected components results in power flows into the component, which can be defined using the Equations (2) and (7) and are denoted using the vector $\dot{z}_i^{r,in} = [P_i^{r,in}, \dot{Q}_i^{r,in}]^T$.

It is required for the output variables of interest $y_{z,i}$ to converge to the incoming interaction $\dot{z}_i^{r,in}$ for a feasible interconnected system [14], [15].

III. Optimality Conditions for Distributed Control in Energy State Space

In this section we focus on designing optimal control laws for interconnected multi-agent systems in the energy state space. Specifically, we consider a two-agent system for simplicity (shown in Fig. 2), and present its optimal control law based on the interaction variables between these two agents. We then present an extension of the derived conditions to a general multi-agent system.

A. Optimizing Physical Efficiency and Long-Term Interface Mismatch

As discussed in the previous section, the interaction variable $\dot{Q}_i^{r,out}$ can be interpreted as physical efficiency for the interconnected multi-agent system [9]. A smaller $|\dot{Q}_i^{r,out}|$ during transient dynamics means a higher physical efficiency of the system. Hence, it is natural to seek an optimal control design which minimizes the accumulated effects of $|\dot{Q}_i^{r,out}|$ during transient dynamics. On the other hand, the output interaction variables $y_{z,i}$ of agent-i should follow the incoming interaction variable $\dot{z}^{r,in}$ when energy dynamics settle. Therefore, their accumulated difference should also be minimized. The overall objective function is given as follows.

$$J = \int_{t=0}^{T_{hor}} \sum_{i=1}^{2} \alpha_i (\dot{Q}_i^{r,out})^2 + \beta_i (y_{z,i} - \dot{z}_i^{r,in})^T (y_{z,i} - \dot{z}_i^{r,in}) dt \tag{10}$$

where $t \in \mathbb{R}$ is the time variable; $T_{hor} \in \mathbb{R}$ is a large time horizon; α_i and β_i are the weights associated with the physical efficiency and the interface mismatch.

It is subject to the system dynamical equations in the energy state space

$$\frac{d}{dt}\begin{bmatrix} E_i \\ p_i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} E_i \\ p_i \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix}\dot{Q}_i^{r,out} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} E_{t,i} \quad (11)$$

where subscript $i \in \{1, 2\}$; the initial conditions are given by $E_i(0) = E_i^0$ and $p_i(0) = p_i^0$.

If we further define

$$A_{ii} := \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ B_{ii} := [0\ \ -1]^T,\ L_{ii} := [0\ 4]^T \quad (12a)$$

$$x_{z,i} := [E_i\ p_i]^T,\ u_{z,i} := \dot{Q}_i^{r,out},\ f_i := E_{t,i} \quad (12b)$$

$$x_z := [x_{z,1}^T\ x_{z,2}^T]^T,\ u_z := [u_{z,1}\ u_{z,2}]^T,\ f := [f_1\ f_2]^T \quad (12c)$$

where $i \in \{1, 2\}$, and consider the integral form of the dynamical constraints, then the overall optimal control problem becomes a standard state regulation problem with the initial boundary condition as follows.

$$\min \int_0^{T_{hor}} u_z^T D u_z + x_z^T M x_z + C u_z + f^T K f\ dt \quad (13a)$$

$$\text{s.t. } x_z(t) - x_z(0) - \int_0^t A x_z + B u_z + L f\ ds = 0 \quad (13b)$$

$$x_z(0) = x_{z,0} \quad (13c)$$

where $C = 8[\beta_2 f_2 \quad \beta_1 f_1], K = 16\begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix}, A = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix}, B = \begin{bmatrix} B_{11} & 0 \\ 0 & B_{22} \end{bmatrix}, L = \begin{bmatrix} L_{11} & 0 \\ 0 & L_{22} \end{bmatrix}, D = \begin{bmatrix} \alpha_1 + \beta_2 & 0 \\ 0 & \alpha_2 + \beta_1 T \end{bmatrix}$, and

$$M = \begin{bmatrix} \frac{\beta_1+\beta_2}{\tau_1^2} & \frac{\beta_2}{\tau_1} & \frac{\beta_1+\beta_2}{\tau_1\tau_2} & \frac{\beta_1}{\tau_1} \\ \frac{\beta_2}{\tau_1} & \beta_2 & \frac{\beta_2}{\tau_2} & 0 \\ \frac{\beta_1+\beta_2}{\tau_1\tau_2} & \frac{\beta_2}{\tau_2} & \frac{\beta_1+\beta_2}{\tau_2^2} & \frac{\beta_1}{\tau_2} \\ \frac{\beta_1}{\tau_1} & 0 & \frac{\beta_1}{\tau_2} & \beta_1 \end{bmatrix} \quad (14)$$

B. Optimality Conditions in Variational Form

Now let's consider the Lagrangian function of (13).

$$\mathcal{L}(x_z, u_z, \lambda) = \int_0^{T_{hor}} u_z^T D u_z + x_z^T M x_z + C u_z + f^T K f\ dt - \int_0^{T_{hor}} \lambda^T \left(x_z - x_{z,0} - \int_0^t A x_z + B u_z + L f\ ds \right) dt \quad (15)$$

where $f = f(t)$ are assumed to be explicitly given; λ is the vector of Lagrangian multipliers which are L_2-functions of time.

Define the costate of the system by $w(t) := \int_t^{T_{hor}} \lambda(\eta)\ d\eta$, with $w(T_{hor}) = 0$. Substituting this into (15) yields

$$\mathcal{L}(x_z, u_z, w, \dot{w}) = \int_0^{T_{hor}} u_z^T D u_z + x_z^T M x_z + C u_z + f^T K f dt + \int_0^{T_{hor}} \dot{w}^T (x_z - x_{z,0}) dt + \int_0^{T_{hor}} w^T \big(A x_z + B u_z + L f \big) dt \quad (16)$$

By forcing the partial derivatives of the Lagrangian function $\mathcal{L}$ with respect to x_z and u_z to 0 we obtain the optimality conditions in the variational form[1].

$$\frac{\partial \mathcal{L}}{\partial x_z} = \dot{w} + A^T w + 2M^T x_z = 0 \quad (17a)$$

$$\frac{\partial \mathcal{L}}{\partial u_z} = 2D^T u_z + C^T + B^T w = 0 \quad (17b)$$

Solving Eqt (17b) we reach the optimal control law of u_z,

$$u_z = \frac{-1}{2} D^{-T} (B^T w + C^T) \quad (18)$$

which, in the explicit form, is

$$u_{z,1} = \frac{1}{2(\alpha_1 + \beta_2)} w_2 - \frac{4\beta_2}{\alpha_1 + \beta_2} f_2 \quad (19a)$$

$$u_{z,2} = \frac{1}{2(\alpha_2 + \beta_1)} w_4 - \frac{4\beta_1}{\alpha_2 + \beta_1} f_1 \quad (19b)$$

where w_2 and w_4 are the second and fourth entries of the costate vector w.

C. Distributed Optimal Control Law

From Eqt (19) one may notice that the optimal control law is only related to the costate w and the external input f. In general, the costates w and the states x_z are coupled together in the ordinary differential equation (ODE) form with two-boundary conditions. Substituting Eqt (19) into Eqt (11), and stacking Eqt (17a) below we have

$$\frac{d}{dt}\begin{bmatrix} x_z \\ w \end{bmatrix} = \begin{bmatrix} A & \frac{-1}{2} B D^{-T} B^T \\ -2M^T & -A^T \end{bmatrix}\begin{bmatrix} x_z \\ w \end{bmatrix} + \begin{bmatrix} F \\ 0 \end{bmatrix} f \quad (20a)$$

$$x_z(0) = x_{z,0} \quad (20b)$$

$$w(T_{hor}) = 0 \quad (20c)$$

where

$$F = \begin{bmatrix} 0 & 4 & 0 & \frac{4\beta_1}{\alpha_2+\beta_1} \\ 0 & \frac{4\beta_2}{\alpha_1+\beta_2} & 0 & 4 \end{bmatrix}^T$$

However, due to the special sparse linear form A of the energy state space dynamical equations, we may avoid solving the two-boundary problem (20) directly, and can express the costates w in terms of states x_z.

Considering Eqt (17a), denote the k-th column of matrix M by m_k, we have, for $i \in \{1, 2\}$

$$\dot{w}_{2i-1} = -2m_{2i-1}^T x_z \quad (21a)$$

$$\dot{w}_{2i} = -w_{2i-1} - 2m_{2i}^T x_z \quad (21b)$$

Hence, w_2 and w_4 can be expressed in terms of x_z as follows.

$$w_2(t) = 2\int^t \int^s m_1^T x_z(\eta) d\eta ds - 2\int^t m_2^T x_z(s) ds \quad (22a)$$

$$w_4(t) = 2\int^t \int^s m_3^T x_z(\eta) d\eta ds - 2\int^t m_4^T x_z(s) ds \quad (22b)$$

[1]In the objective function t appears as an explicit variable in the given function f. But the partial derivative w.r.t. t does not alter other partial derivatives. Thus, we omit it here.

Substituting Eqt (22) into Eqt (19) we obtain the optimal control law in terms of states x_z.

$$u_{z,1} = \frac{1}{\alpha_1 + \beta_2}\left(\int\int m_1^T x_z d\eta ds - \int m_2^T x_z ds - 4\beta_2 f_2\right) \tag{23a}$$

$$u_{z,2} = \frac{1}{\alpha_2 + \beta_1}\left(\int\int m_3^T x_z d\eta ds - \int m_4^T x_z ds - 4\beta_1 f_1\right) \tag{23b}$$

D. Extension to General Multi-Agent Systems

Following the same derivation detailed above in a two-agent system for one with instead n-agents having arbitrary interconnections results in slight deviations in the optimal control law. In place of the control being dependent on only a component's internal states and those of its neighbors, with multi-port components, the control law additionally depends on parameters from components which share neighbors. In sparsely connected systems, such as in microgrids, this does not greatly increase the amount of information needed to be transmitted. However, in densely connected systems, this leads to needing knowledge of parameters for nearly all components. This further dependency is due to the nature of the controller which aims to balance the interactions of components. As such, each component must have knowledge of the interactions on the other ports of the component for the optimal control to be successful. This, however, does not change what information must be exchanged, just how many devices must communicate.

IV. Physical Interpretation and Simplification of Optimal Control Law

In this section we discuss a few properties and interpretations of the optimal control law which has been derived in Section III.

A. Asymptotic Behavior of Optimal Control Law

Consider the optimal control law in terms of the costates in Eqt (19). In the finite time horizon $t \in [0, T_{hor}]$ we recall that $w(T_{hor}) = 0$. Thus, the optimal control law should have a boundary condition.

$$u_1(T_{hor}) = \frac{-4\beta_2}{\alpha_1 + \beta_2} f_2(T_{hor}) \tag{24a}$$

$$u_2(T_{hor}) = \frac{-4\beta_1}{\alpha_2 + \beta_1} f_1(T_{hor}) \tag{24b}$$

Now let's consider when the time horizon goes to infinity, e.g. $T_{hor} \to +\infty$. Then,

$$\lim_{T_{hor}\to+\infty} w(T_{hor}) = \lim_{T_{hor}\to+\infty} \int_{T_{hor}}^{T_{hor}} \lambda(\eta)\ d\eta = 0 \tag{25}$$

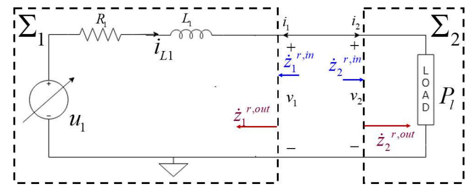


Fig. 3: Two-Agent Test System Configuration

Therefore, the asymptotic behavior of the optimal control law is

$$\lim_{T_{hor}\to+\infty} u_1(T_{hor}) = \lim_{T_{hor}\to+\infty} \frac{-4\beta_2}{\alpha_1 + \beta_2} f_2(T_{hor}) \tag{26a}$$

$$\lim_{T_{hor}\to+\infty} u_2(T_{hor}) = \lim_{T_{hor}\to+\infty} \frac{-4\beta_1}{\alpha_2 + \beta_1} f_1(T_{hor}) \tag{26b}$$

If we further assume that $\alpha_1 = \alpha_2 = 0$, namely, no requirement on the minimal control efforts. Then, by abusing the limit notation, we have

$$u_1(+\infty) = -4f_2(+\infty) \tag{27a}$$

$$u_2(+\infty) = -4f_1(+\infty) \tag{27b}$$

Eqt (27) suggests that when we only consider long-term interface mismatch in the objective function, the terms in the objective function relating to the sum of $\dot{Q}$ and $4E_t$ asymptotically converge to 0.

B. Physical Interpretations of Costates in Energy State Space

From the expressions of control derived for each agent in Eqt (19), we can write in physical variables the control for the first agent as

$$\begin{aligned} u_1 &= \tfrac{1}{2(\alpha_1+\beta_2)} w_2 - \tfrac{4\beta_2}{\alpha_1+\beta_2} E_{t,2} \\ \dot{w}_2 &= -w_1 - 2\beta_2({P_2}^{r,t,\prime} - P_2^{r,in}) \\ \dot{w}_1 &= \tfrac{-2\beta_1}{\tau_1}({P_1}^{r,t,\prime} - P_1^{r,in}) - \tfrac{2\beta_2}{\tau_1}({P_2}^{r,t,\prime} - P_2^{r,in}) \end{aligned} \tag{28}$$

Notice that the control is dependent on the co-state variable, which depends on the instantaneous power mismatch measured at the interface of each component, and on the stored energy in tangent space of the second component.

From this physical variable level of the controller, we can determine a minimal set of information needed to be transmitted between the two components to implement this optimal controller. First, each component must have knowledge of the instantaneous real power mismatch on the terminals of each device. Second, each component must transmit its measurement of energy stored in tangent space to the other component. Thus, only two pieces of locally measurable information need to be exchanged between the components.

V. Numerical Simulations

A. Two-Agent Test System Configurations

The test system used in this paper (shown in Fig. 3) comprises a controllable source, an RLC circuit, and a variable power load, a challenging control problem, though feasibility issues were encountered when the load real power exceeded certain levels [16]. The source is controllable in the sense that its output power can be adjusted according to the demand variations. This kind of source can be achieved by a DC battery with fast-switching power electronics converters.

B. Optimal Controller

Our optimal controller is designed based on the interaction variables between these two agents. The model utilized for simulations is shown below:

$$\begin{aligned}
&\frac{di_{L1}}{dt} = -\frac{R_1}{L_1} i_{L1} + \frac{1}{L_1}(u_1 - v_1) && i_{L1}(0) = i_{L1,0} \\
&\frac{dv_1}{dt} = \frac{1}{2i_1}\left(\dot{P}_1^{r,in} - \dot{Q}_1^{r,in}\right) && v_1(0) = v \\
&v_2 = \frac{P_L}{i_2} && \\
&\frac{di_2}{dt} = \frac{1}{2v_2}\left(\dot{P}_2^{r,in} + \dot{Q}_2^{r,in}\right) && i_2(0) = i_{2,0}
\end{aligned} \tag{29}$$

The incoming interactions driving each of the agents are given as

$$\begin{aligned}
\dot{z}_1^{r,in} &= -\dot{z}_2^{r,out} = -\begin{bmatrix} {P_L}^{\star} \\ 2\frac{{P_L}^{\star}}{i_2}\frac{di_2}{dt} - \dot{P}_L^{\star} \end{bmatrix} \\
\dot{z}_2^{r,in} &= -\dot{z}_1^{r,out} = -\begin{bmatrix} p_1 + \frac{E_1}{\tau_1} \\ 4E_{t,1} - \frac{dp_1}{dt} \end{bmatrix} \\
&= -\begin{bmatrix} L_1 i_{L1}\frac{di_{L1}}{dt} + R_1 {i_{L1}}^2 \\ 2L_1\left(\frac{di_{L1}}{dt}\right)^2 - \frac{d}{dt}\left(L_1 i_{L1}\frac{di_{L1}}{dt}\right) \end{bmatrix}
\end{aligned} \tag{30}$$

The energy space optimal control $u_{z,1} = \dot{Q}_1^{r,out}$ is mapped back to the physical controllable voltage source u_1 shown in Fig. 3 through a dynamical map shown below [15]:

$$\frac{du_1}{dt} = \frac{1}{i_1}\left(-u_1\frac{di_1}{dt} + 4E_{t,1} - {P_1}^{r,out} + \frac{p_1}{\tau_1} - u_{z,1}\right) \tag{31}$$

where we specify the initial condition at $u_1(0) = u_{1,0}$.

C. Numerical Results for Stabilization

In this section, we demonstrate the controlling performance of our proposed optimal controller in stabilizing the system of Fig. 3 in response to the time varying power load in Fig. 4. The simulation results are plotted in Fig. 5. Since the controller is designed in the energy state space, it responds directly to power variations, leaving voltage unregulated. Hence, the controllers' dynamics show large voltage drift across the capacitor. That said, the controller does successfully stabilize the system over the entire operating time. Additionally, the optimal controller consumes low levels of reactive power, as measured in Volt-Amperes reactive (VAr), in accomplishing stabilization.

D. Optimal Controller with Voltage Regulation

From the above simulation results shown in Fig. 5 as was discussed prior, the voltage across the capacitor experiences high levels of drift as the voltage is left unregulated. To further regulate the voltage performance, in this part we introduce a voltage-regulated optimal controller. The change is only in terms of the mapping from the energy space control to the physical control. Let $v_1^{\star}$ denote the reference voltage. Then,

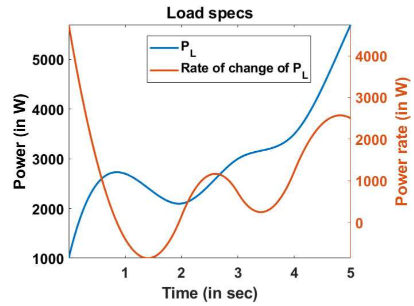
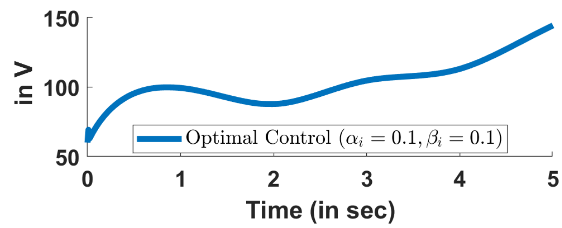
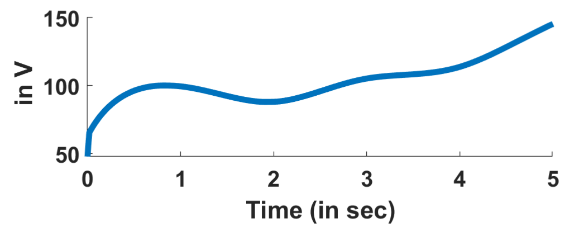


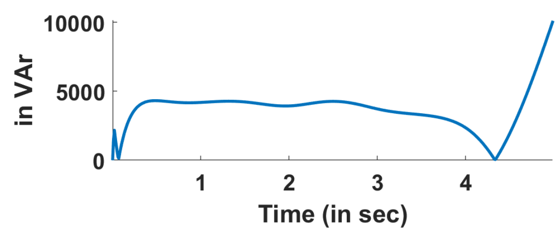
Fig. 4: Time-Varying Load



(a) Control Voltage u_1



(b) Voltage across capacitor



(c) IntVar $Q_1^{r,out}$

Fig. 5: Control Performance for Stabilization at Time-Varying Power Load

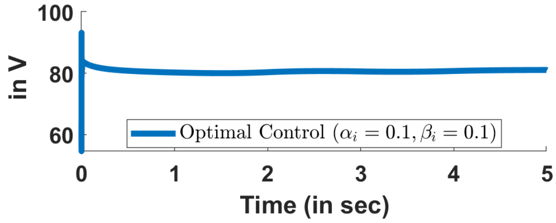
the mapping resulting in a regulating optimal control is given as in [15]:

$$\frac{du_1}{dt} = \frac{1}{i_1}\begin{pmatrix} -u_1\frac{di_1}{dt} + 4E_{t,1} - {P_1}^{r,out} \\ +\frac{p_1}{\tau_1} - u_{z,1} + \frac{{v_1}^{\star}}{{v_1}^2}\frac{dv_1}{dt}{P_1}^{r,in} \end{pmatrix} \tag{32}$$

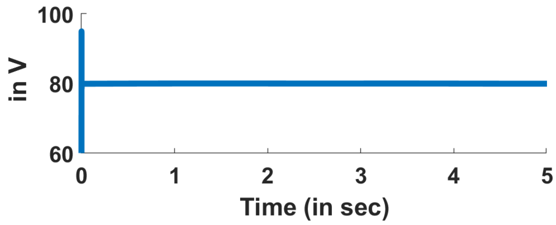
where we specify initial condition $u_1(0) = u_{1,0}$. This controller approaches the original optimal controller when $v_1 \to v_1^{\star}$.

E. Numerical Results for Regulation

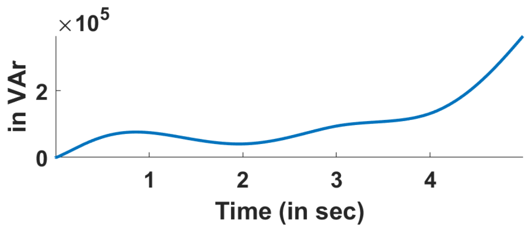
In this part, a voltage reference of $v^{\star} = 80V$ is utilized. We demonstrate our proposed voltage-regulated optimal controller in the time-varying load scenario in Fig. 4. The simulation results are plotted in Fig. 6. The voltage dynamics across



(a) Control Voltage u_1



(b) Voltage across capacitor



(c) IntVar $Q_1^{r,out}$

Fig. 6: Control Performance for Regulation at Time-Varying Power Load

the capacitor show very limited drifting phenomenon, validating the well-functioning voltage regulation of the proposed controller. Additionally, the controller consumes low levels of reactive power, though more than was required in the unregulated case indicating higher levels of controller effort are required to achieve regulation.

VI. Conclusion

This paper focused on designing optimal distributed control for multi-agent energy systems. We aimed at minimizing an aggregated objective of reactive power inefficiency and interface mismatch. Instead of working in the traditional state space, we explored the coupled real-reactive power dynamics in the energy state space, and derived the optimality conditions in the variational form. Due to the locally interactive nature among adjacent agents of the energy state space modeling, the optimal control design from these conditions can be naturally implemented in a distributed manner. The controller was implemented on a controllable source, RLC circuit system with a variable power load. Simulation results show the controller is able to stabilize the system with minimal reactive power consumption, though with capacitor voltage drift. To maintain the voltage level across the internal capacitor, we further introduced voltage regulation. In this paper, only the two-agent case was implemented. A future research direction is implementing the optimal control law in a larger system to demonstrate the communication needs in a more interconnected system.

Acknowledgement

This work has been funded in part by the NSF EAGER project ECCS-2002570 entitled EAGER: Fundamentals of Modeling and Control for the Evolving Electric Power System Architectures, by the DOE SETO project entitled SECURE MONITORING AND CONTROL OF SOLAR PHOTOVOLTAIC SYSTEMS THROUGH DYNAMIC WATERMARKING, by the NSF CNS-1735463, and is based upon work supported by the National Science Foundation Graduate Research Fellowship Program under Grant No. 2141064. The authors greatly appreciate the support.

References

[1] ERCOT, "Dynamic stability assessment of high penetration of renewable generation in the ercot grid," 2018.

[2] Y. Lin, J. H. Eto, B. B. Johnson, J. D. Flicker, R. H. Lasseter, H. N. Villegas Pico, G.-S. Seo, B. J. Pierre, and A. Ellis, "Research roadmap on grid-forming inverters," National Renewable Energy Lab.(NREL), Golden, CO (United States), Tech. Rep., 2020.

[3] A. Ellis, R. Nelson, E. Von Engeln, R. Walling, J. McDowell, L. Casey, E. Seymour, W. Peter, C. Barker, and B. Kirby, "Reactive power interconnection requirements for pv and wind plants-recommendations to nerc," *Sandia National Laboratories, Albuquerque, New Mexico*, vol. 87185, 2012.

[4] E. Espina, J. Llanos, C. Burgos-Mellado, R. Cardenas-Dobson, M. Martinez-Gomez, and D. Saez, "Distributed control strategies for microgrids: An overview," *IEEE Access*, vol. 8, pp. 193 412–193 448, 2020.

[5] A. Bidram, F. L. Lewis, and A. Davoudi, "Distributed control systems for small-scale power networks: Using multiagent cooperative control theory," *IEEE Control systems magazine*, vol. 34, no. 6, pp. 56–77, 2014.

[6] S. M. Mohiuddin and J. Qi, "Optimal distributed control of ac microgrids," *Microgrids: Theory and Practice*, pp. 287–306, 2024.

[7] D. D. Siljak, *Decentralized control of complex systems*. Courier Corporation, 2011.

[8] M. D. Ilić and R. Jaddivada, "Multi-layered interactive energy space modeling for near-optimal electrification of terrestrial, shipboard and aircraft systems," *Annual Reviews in Control*, vol. 45, pp. 52–75, 2018.

[9] M. D. Ilic and R. Jaddivada, "Fundamental modeling and conditions for realizable and efficient energy systems," in *2018 IEEE Conference on Decision and Control (CDC)*. IEEE, 2018, pp. 5694–5701.

[10] M. Ilic and R. Jaddivada, "Unified value-based feedback, optimization and risk management in complex electric energy systems," *Optimization and Engineering*, pp. 1–57, 2020.

[11] D. Jeltsema and J. M. Scherpen, "Multidomain modeling of nonlinear networks and systems," *IEEE Control Systems Magazine*, vol. 29, no. 4, pp. 28–59, 2009.

[12] D. A. Hill, M. T. Ma, A. R. Ondrejka, B. F. Riddle, M. L. Crawford, and R. T. Johnk, "Aperture excitation of electrically large, lossy cavities," *IEEE transactions on Electromagnetic Compatibility*, vol. 36, no. 3, pp. 169–178, 1994.

[13] J. Wyatt and M. Ilic, "Time-domain reactive power concepts for nonlinear, nonsinusoidal or nonperiodic networks," in *IEEE international symposium on circuits and systems*. IEEE, 1990, pp. 387–390.

[14] R. Jaddivada, "A unified modeling for control of reactive power dynamics in electrical energy systems," Ph.D. dissertation, Massachusetts Institute of Technology, 2020.

[15] R. Jaddivada and M. D. Ilic, "A feasible and stable distributed interactive control design in energy state space," in *2021 60th IEEE Conference on Decision and Control (CDC)*. IEEE, 2021, pp. 4950–4957.

[16] P. Monshizadeh, J. E. Machado, R. Ortega, and A. van Der Schaft, "Power-controlled hamiltonian systems: Application to electrical systems with constant power loads," *Automatica*, vol. 109, p. 108527, 2019.