Algebraic Geometry 11 (2) (2024) 290–295
doi:10.14231/AG-2024-009

Prill's problem

Aaron Landesman and Daniel Litt

Abstract

We solve Prill's problem, originally posed by David Prill in the late 1970s and popularized in Arbarello, Cornalba, Griffiths and Harris's "Geometry of Algebraic Curves." That is, for any curve Y of genus 2, we produce a finite étale degree 36 connected cover $f\colon X \to Y$ where, for every point $y \in Y$, the preimage $f^{-1}(y)$ moves in a pencil.

1. Introduction

Throughout, we work over the complex numbers. Let $f\colon X \to Y$ be a dominant map of smooth projective connected algebraic curves, where $g(Y)$, the genus of Y, is at least 2. In this case, the Riemann–Hurwitz formula yields $\deg(f) < g(X)$, so one would not expect that $f^{-1}(y)$ moves in a pencil for a general $y \in Y$. That is, one expects $h^0\big(X, \mathscr{O}_X\big(f^{-1}(y)\big)\big) = 1$ for a general $y \in Y$. Of course, there are many cases where $f^{-1}(y)$ moves in a pencil for special $y \in Y$, but in the late 1970s, David Prill raised the following question.

Question 1.1 (Prill's problem, [ACGH85, Exercise VI.D]). Given any curve Y of genus $g \geqslant 2$ and a finite covering $f\colon X \to Y$, does $h^0\big(X, \mathscr{O}_X\big(f^{-1}(y)\big)\big) = 1$ for a general $y \in Y$?

Due to its elementary nature, Prill's problem garnered much attention in the early 1980s. Various special cases of Prill's problem were answered affirmatively, such as those summarized in [ACGH85, BB05, Moh07].

We say that a finite cover $f\colon X \to Y$ of smooth proper geometrically connected curves is *Prill exceptional* if $h^0\big(X, \mathscr{O}_X\big(f^{-1}(y)\big)\big) \geqslant 2$ for every point $y \in Y$. The general belief, up until this point, was that no Prill exceptional covers should exist. Our main result unexpectedly resolves Prill's problem by showing that Prill exceptional covers do, in fact, exist. Even more surprisingly, our construction gives a Prill exceptional cover of *any* genus 2 curve.

Theorem 1.2. *If Y is any smooth proper connected curve of genus 2 over the complex numbers, there is a finite étale cover $f\colon X \to Y$ which is Prill exceptional.*

The idea of the proof of Theorem 1.2 is to relate Prill's problem to a problem in Hodge theory. Inspired by recent work of Marković [Mar22], we employ a construction of Bogomolov and Tschinkel [BT02, BT04]. This yields a finite étale cover $f\colon X \to Y$ of degree 36 such

Received 2 January 2023, accepted in final form 9 June 2023.
2020 Mathematics Subject Classification 14D07, 14H10.
Keywords: covers of curves, Hodge theory, generically globally generated vector bundles, Hesse pencil, Prill's problem.

Landesman was supported by the National Science Foundation under Award No. DMS-2102955. Litt was supported by an NSERC Discovery Grant, "Anabelian methods in arithmetic and algebraic geometry."

that the Jacobian Pic^0_X has an isogeny factor which is independent of the complex structure on Y. Analyzing the infinitesimal variation of Hodge structure associated to $H^1(X, \mathbb{Q})$ yields that $f_*\omega_X$ is not generically globally generated, and a routine calculation shows that f is Prill exceptional.

Remark 1.3. If $\psi\colon X' \to Y$ is a finite cover such that ψ factors through a Prill exceptional cover $f\colon X \to Y$, then ψ is also Prill exceptional, as there is an injection $H^0\big(X, \mathscr{O}\big(f^{-1}(y)\big)\big) \to H^0\big(X', \mathscr{O}\big(g^{-1}(y)\big)\big)$. Thus Theorem 1.2 can be used to construct Prill exceptional covers of arbitrary degree, by composing with an arbitrary map $X' \to X$.

Remark 1.4. Although Theorem 1.2 solves Prill's problem, it would still be extremely interesting to know if there any Prill exceptional covers where Y has genus $g > 2$.

Remark 1.5. In this paper, we give a short solution to Prill's problem, a simple to state question, which has been open since the 1970s. In a companion paper, [LL22a, Remark 5.7], we give a much more involved proof of a slightly weaker result, yielding a Prill exceptional cover of a general genus 2 curve. That proof relies on heavier machinery, but we decided to also include it in [LL22a] as it is nearly automatic from the tools developed there.

2. The proof of Theorem 1.2

To start, we rephrase the condition that $f\colon X \to Y$ is Prill exceptional in terms of $f_*\omega_X$ not being generically globally generated, meaning that all of its global sections lie in a proper subbundle. This lemma is known to experts, but we recall it for completeness.

LEMMA 2.1. *Let $f\colon X \to Y$ be a finite étale morphism of smooth proper connected curves. Then $f_*\omega_X$ is not generically globally generated if and only if f is Prill exceptional.*

Proof. We wish to show that $f_*\omega_X$ is not generically globally generated if and only if

$$h^0\big(X, \mathscr{O}_X\big(f^{-1}(y)\big)\big) = h^0(Y, (f_*\mathscr{O}_X)(y)) > 1$$

for every point y in Y. Note that $f_*\omega_X$ is not generically globally generated if and only if, for a general $y \in Y$, we have an exact sequence

$$\begin{aligned} 0 \longrightarrow H^0(Y, f_*\omega_X(-y)) \longrightarrow H^0(Y, f_*\omega_X) \xrightarrow{\beta} H^0(Y, f_*\omega_X|_y) \longrightarrow \\ \longrightarrow H^1(Y, f_*\omega_X(-y)) \longrightarrow H^1(Y, f_*\omega_X) \longrightarrow 0\,, \end{aligned} \tag{2.1}$$

where β is not surjective. Since $h^1(Y, f_*\omega_X) = h^1(X, \omega_X) = 1$, the map β is not surjective precisely when $h^1(Y, f_*\omega_X(-y)) > 1$. By Serre duality, this is equivalent to $h^0(Y, f_*\mathscr{O}_X(y)) > 1$ for a general point $y \in Y$. This is equivalent to the statement that $h^0(Y, f_*\mathscr{O}_X(y)) > 1$ for all points $y \in Y$ by the upper semicontinuity of sheaf cohomology. □

Throughout the remainder of the proof, we work in the following setup.

NOTATION 2.2. Consider a diagram of the form

$$\begin{array}{ccc} \mathscr{X} & \xrightarrow{\quad h \quad} & \mathscr{Y} \\ & {\scriptstyle \pi'} \searrow \quad \swarrow {\scriptstyle \pi} & \\ & \mathscr{M}\,, & \end{array} \tag{2.2}$$

where π is a relative smooth proper curve of genus $g \geqslant 2$ with geometrically connected fibers, h is finite étale, and π' is a smooth proper curve of genus g' with geometrically connected fibers. Suppose that the map $\mathscr{M} \to \mathscr{M}_g$ induced by π is dominant étale. Fix $m \in \mathscr{M}$ and let $X = \mathscr{X}_m$, $Y = \mathscr{Y}_m$, and $f = h|_X$. We refer to the data of (2.2) as a *versal family of covers of curves of genus g*.

The next proposition shows that, in the above setup, in order to construct Prill exceptional curves, it is enough to produce an isotrivial isogeny factor in $\mathrm{Pic}^0_{\mathscr{X}/\mathscr{M}}$.

Proposition 2.3. *With notation as in Notation 2.2, suppose that the Jacobian $\mathrm{Pic}^0_{\mathscr{X}/\mathscr{M}}$ has an isotrivial isogeny factor. Then, $f_*\omega_X$ is not generically globally generated.*

Proof. Let $\mathbb{V} = R^1\pi'_*\mathbb{Q}$ and $\mathscr{V} = \mathbb{V} \otimes \mathscr{O}_{\mathscr{M}}$. We study the infinitesimal variation of Hodge structure associated to $\mathbb{V}$. The Hodge filtration on $\mathscr{V}$ satisfies

$$F^1\mathscr{V} = \pi'_*\omega_{\mathscr{X}/\mathscr{M}} \quad \text{and} \quad \mathscr{V}/F^1\mathscr{V} = R^1\pi'_*\mathscr{O}_{\mathscr{X}}\,.$$

The Gauss–Manin connection $\nabla\colon \mathscr{V} \to \mathscr{V} \otimes \Omega^1_{\mathscr{M}}$ induces an $\mathscr{O}_{\mathscr{M}}$-linear map

$$\overline{\nabla}\colon \pi'_*\omega_{\mathscr{X}/\mathscr{M}} = F^1\mathscr{V} \to \mathscr{V}/F^1\mathscr{V} \otimes \Omega^1_{\mathscr{M}} = R^1\pi'_*\mathscr{O}_{\mathscr{X}} \otimes \Omega^1_{\mathscr{M}}\,.$$

The given isotrivial isogeny factor of $\mathrm{Pic}^0_{\mathscr{X}/\mathscr{M}}$ yields a nonzero isotrivial sub-$\mathbb{Q}$-Hodge structure $\mathbb{W} \subset \mathbb{V}$. Hence $\overline{\nabla}$ has nontrivial kernel: the kernel contains $F^1(\mathbb{W} \otimes \mathscr{O}_{\mathscr{M}})$.

Restricting $\overline{\nabla}$ to the fiber over $m \in \mathscr{M}$, we obtain a map

$$\overline{\nabla}_m\colon H^0(X, \omega_X) \to H^1(X, \mathscr{O}_X) \otimes \Omega^1_{\mathscr{M},m}\,.$$

As $\mathscr{M} \to \mathscr{M}_g$ is étale, there is a natural identification of $\Omega^1_{\mathscr{M},m}$ with $\Omega^1_{\mathscr{M}_g,[Y]} \simeq H^0\big(Y, \omega_Y^{\otimes 2}\big)$. Applying Serre duality, we may view $\overline{\nabla}_m$ as a map

$$H^0(X, \omega_X) \to \mathrm{Hom}\left(H^0(X, \omega_X), H^0\big(Y, \omega_Y^{\otimes 2}\big)\right),$$

or equivalently, as a map

$$H^0(Y, f_*\omega_X) \to \mathrm{Hom}\left(H^0(Y, f_*\omega_X), H^0\big(Y, \omega_Y^{\otimes 2}\big)\right). \tag{2.3}$$

Then [LL22b, Theorem 5.1.6 and Lemma A.1.8], applied where the local system $\mathbb{V}$ in [LL22b, Notation 5.1.1] is taken to be $h_*\mathbb{Q}$, shows that (2.3) is induced by the map

$$\begin{aligned} \alpha\colon f_*\omega_X &\to \underline{\mathrm{Hom}}\big(f_*\omega_X, \omega_Y^{\otimes 2}\big) \\ \eta &\mapsto \big(q_\eta\colon \nu \mapsto \mathrm{tr}_{X/Y}(\eta \otimes \nu)\big) \end{aligned}$$

by taking global sections. Note that α is injective since, by [LL22b, Theorem 5.1.6], it is obtained from the isomorphism $\beta\colon f_*\mathscr{O}_X \to \underline{\mathrm{Hom}}(f_*\mathscr{O}_X, \mathscr{O}_Y)$ (which corresponds to the self-duality of the regular representation) by tensoring β with powers of ω_Y to obtain the map α as the composition of injective maps

$$f_*\omega_X \simeq f_*\mathscr{O}_X \otimes \omega_Y \to \underline{\mathrm{Hom}}\big(f_*\mathscr{O}_X \otimes \omega_Y, \omega_Y^{\otimes 2}\big) \simeq \underline{\mathrm{Hom}}\big(f_*\omega_X, \omega_Y^{\otimes 2}\big)\,.$$

As $\overline{\nabla}$ has nontrivial kernel, the same is true for $\overline{\nabla}_m$. That is, there exists a nonzero $\eta \in H^0(Y, f_*\omega_X)$ such that the nonzero map $q_\eta\colon f_*\omega_X \to \omega_Y^{\otimes 2}$ induces the zero map on global sections. Hence any global section of $f_*\omega_X$ lies in the kernel of q_η. Said another way, $f_*\omega_X$ is not generically globally generated. □

It remains to show that there are versal families of covers of curves of genus 2 so that $\mathrm{Pic}^0_{\mathscr{X}/\mathscr{M}}$ has an isotrivial isogeny factor. We now do this carefully but note that it can also be extracted from [BT04, § 3], culminating in [BT04, Example 3.7 and Proposition 3.8]. We include a proof for completeness, following [BT04].

PROPOSITION 2.4. *There exists a versal family $\mathscr{X} \xrightarrow{h} \mathscr{Y} \xrightarrow{\pi} \mathscr{M}$ of covers of curves of genus 2 with $\pi' = h \circ \pi$ as in Notation 2.2, such that h has degree 36, the Jacobian $\mathrm{Pic}^0_{\mathscr{X}/\mathscr{M}}$ has an isotrivial isogeny factor, and the map $\mathscr{M} \to \mathscr{M}_2$ induced by π is surjective and étale.*

Remark 2.5. In fact, Proposition 2.4 has a straightforward generalization to higher genus: Let $\mathscr{H}_g$ denote the moduli stack of hyperelliptic curves of genus g. There exists a family $\mathscr{X} \xrightarrow{h} \mathscr{Y} \xrightarrow{\pi} \mathscr{M}$ such that π is a family of smooth proper genus g hyperelliptic curves with geometrically connected fibers, $\pi \circ h$ is a family of smooth proper curves with geometrically connected fibers, h is finite étale of degree 36, the Jacobian $\mathrm{Pic}^0_{\mathscr{X}/\mathscr{M}}$ has an isotrivial isogeny factor, and $\mathscr{M} \to \mathscr{H}_g$ is a surjective étale map.

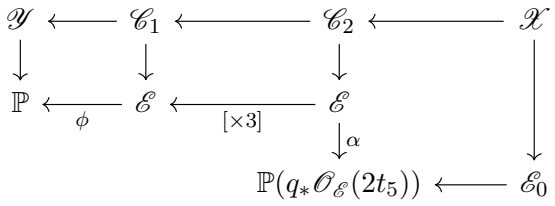

FIGURE 1. A diagram depicting the relevant curves in the proof of Proposition 2.4

Proof of Proposition 2.4. First, we construct a particular scheme $\mathscr{M}$, which has a surjective étale map to $\mathscr{M}_2$. Let $\mathscr{M}'$ be the S_6-cover of $\mathscr{M}_2$ parametrizing orderings of Weierstrass points on the universal curve, and let $\psi' \colon \mathscr{Y}' \to \mathscr{M}'$ be the pullback of the universal curve to $\mathscr{M}'$. Let $\mathbb{P}' := \mathbb{P}(\psi'_* \omega_{\mathscr{Y}'/\mathscr{M}'})$, so that there is a natural 2-to-1 map $\mathscr{Y}' \to \mathbb{P}'$, ramified over the images of six disjoint sections $s'_1, \dots, s'_6 \colon \mathscr{M}' \to \mathscr{Y}'$. The statement of Proposition 2.4 is insensitive to replacing $\mathscr{M}'$ by a Zariski-open cover, and replacing $\mathscr{Y}'$, $\mathbb{P}'$, s'_i by their pullbacks to this cover, and we do so freely. Zariski-locally on $\mathscr{M}'$, we may construct the double cover $p' \colon \mathscr{E}' \to \mathbb{P}'$ branched over the images of $s'_1, \dots, s'_4$ in $\mathbb{P}'$. Now, let $\mathscr{M} := (p')^{-1}(s'_5(\mathscr{M}'))$ denote the finite étale double cover of $\mathscr{M}'$ where one additionally marks a point of $\mathscr{E}'$ mapping to the image of s'_5 under p'. Let $\mathbb{P}$, $\mathscr{Y}$, $\mathscr{E}$ denote the pullbacks of $\mathbb{P}'$, $\mathscr{Y}'$, $\mathscr{E}'$ along $\mathscr{M} \to \mathscr{M}'$, and let $q \colon \mathscr{E} \to \mathscr{M}$ be the natural map. By construction, q has a section, call it t_5, whose image lies over the image of s'_5. We consider $(\mathscr{E}, t_5)$ as an elliptic curve with identity section $t_5 \colon \mathscr{M} \to \mathscr{E}$.

The next several steps in the proof construct a sequence of three finite étale covers of $\mathscr{Y}$, the last of which maps to an isotrivial elliptic curve $\mathscr{E}_0$, as in Figure 1. Let $\mathscr{C}_1$ be the normalization of the fiber product $\mathscr{Y} \times_{\mathbb{P}} \mathscr{E}$. We claim that $\mathscr{C}_1$ is finite étale over $\mathscr{Y}$. To see this, observe that $\mathscr{Y} \to \mathbb{P}$ is branched to order 2 at every point in the branch locus of the map $\phi \colon \mathscr{E} \to \mathbb{P}$ obtained by pulling back p'. Therefore, $\mathscr{C}_1 \to \mathscr{Y}$ is finite étale by a relative version of Abhyankar's lemma [GR71, Exposeé XIII, Proposition 5.5].

Next, define $\mathscr{C}_2 := \mathscr{C}_1 \times_{\mathscr{E}, [\times 3]} \mathscr{E}$, where the map $[\times 3] \colon \mathscr{E} \to \mathscr{E}$ is multiplication by 3 on the relative elliptic curve, and where we use t_5 as the identity section of the elliptic curve $\mathscr{E}$. Because $[\times 3]$ is finite étale, $\mathscr{C}_2$ is finite étale over $\mathscr{C}_1$, hence over $\mathscr{Y}$.

We next construct one further finite étale cover $\mathscr{X}$ of $\mathscr{C}_2$. Let $\alpha\colon \mathscr{E} \to \mathbb{P}(q_*\mathscr{O}_{\mathscr{E}}(2t_5))$ denote the map induced by the complete linear system associated to $2t_5$. The map α is a double cover ramified along the 2-torsion of the relative elliptic curve $(\mathscr{E}, t_5)$ over $\mathscr{M}$. Let $\mathscr{D} = \alpha(\mathscr{E}[3] \setminus \mathrm{im}(t_5))$, and note that $\mathscr{D}$ is finite étale of degree 4 over $\mathscr{M}$. Zariski-locally on $\mathscr{M}$, we can and do construct the double cover $\mathscr{E}_0 \to \mathbb{P}(q_*\mathscr{O}_{\mathscr{E}}(2t_5))$, branched over $\mathscr{D}$, which is a family of genus 1 curves. We replace $\mathscr{M}$ with the above Zariski cover used to construct $\mathscr{E}_0$. Then, $\mathscr{X}$, defined as the normalization of $\mathscr{E}_0 \times_{\mathbb{P}(q_*\mathscr{O}_{\mathscr{E}}(2t_5))} \mathscr{C}_2$, has a dominant map to $\mathscr{E}_0$. To conclude the proof, it is enough to show that $\mathscr{E}_0$ is isotrivial and $\mathscr{X} \to \mathscr{C}_2$ is finite étale of degree 36. Indeed, since $\mathscr{X} \to \mathscr{E}_0$ is a surjective map, $\mathrm{Pic}^0_{\mathscr{X}/\mathscr{M}}$ has $\mathscr{E}_0$ as an isogeny factor, which we will show to be isotrivial.

First, h is a composite of three maps of degrees 2, 9, and 2, so h has degree 36.

Next, we claim that $\mathscr{X} \to \mathscr{C}_2$ is finite étale. Since $\mathscr{C}_1 \to \mathscr{E}$ is branched to order 2 over t_5, the map $\mathscr{C}_2 \to \mathscr{E}$ is branched to order 2 over $\mathscr{E}[3]$. Hence $\mathscr{C}_2 \to \mathbb{P}(q_*\mathscr{O}_{\mathscr{E}}(2t_5))$ is evenly branched over $\mathscr{D}$, meaning that each preimage of any point in $\mathscr{D}$ has even ramification order. Therefore, $\mathscr{X} \to \mathscr{C}_2$ is finite étale by relative Abhyankar's lemma [GR71, Expose XIII, Proposition 5.5].

It remains to show that $\mathscr{E}_0$ is isotrivial. This follows from the computation preceding [BT04, Lemma 4.3], which we now recall. Consider the Hesse pencil

$$E_\lambda\colon x^3 + y^3 + z^3 + \lambda xyz = 0\,,$$

where we view E_λ as a family of elliptic curves with identity point $[1:-1:0] \in E_\lambda \subset \mathbb{P}^2$. Projecting E_λ away from the point $[1:-1:0]$, we obtain a double cover $E_\lambda \to \mathbb{P}^1$, given as the quotient $E_\lambda \to E_\lambda/\{\pm 1\} \simeq \mathbb{P}^1$. Since $E_\lambda[3]$ is precisely the base locus of the Hesse pencil, the image of $E_\lambda[3]$ is independent of λ.

We next claim that any elliptic curve over $\mathbb{C}$ is isomorphic to E_λ for some $\lambda \in \mathbb{C}$. Note that the family $(E_\lambda)_{\lambda \in \mathbb{P}^1}$ defines a relative curve over $\mathbb{P}^1_\lambda$, whose fiber at $\lambda = \infty$ is the reducible nodal curve $xyz = 0$. The other singular members of the Hesse pencil are also nodal curves with three irreducible components. This family therefore corresponds to a map $\upsilon\colon \mathbb{P}^1 \to \overline{\mathscr{M}}_{1,1}$, where $\overline{\mathscr{M}}_{1,1}$ is the moduli stack of elliptic curves. The map is surjective because it induces a nonconstant map from $\mathbb{P}^1$ to the coarse moduli space of $\overline{\mathscr{M}}_{1,1}$. Therefore, every smooth elliptic curve over the complex numbers is isomorphic to E_λ for some $\lambda \in \mathbb{C}$.

Since any two elliptic curves E and E' appear as members of the Hesse pencil, the images $E[3] \to E/\{\pm 1\} \simeq \mathbb{P}^1$ and $E'[3] \to E'/\{\pm 1\} \simeq \mathbb{P}^1$ have the same cross ratios. Hence any two fibers of the pair $(\mathbb{P}(q_*\mathscr{O}_{\mathscr{E}}(2t_5)), \mathscr{D})$ are isomorphic, so any two fibers of $\mathscr{E}_0$ are isomorphic. Hence $\mathscr{E}_0$ is isotrivial. □

We now straightforwardly combine the above results to prove Theorem 1.2.

Proof of Theorem 1.2. By Proposition 2.4, there is a family $h\colon \mathscr{X} \to \mathscr{Y}$ of degree 36 finite étale covers of genus 2 curves over $\mathscr{M}$, where the induced map $\mathscr{M} \to \mathscr{M}_2$ is surjective étale and $\mathrm{Pic}^0_{\mathscr{X}/\mathscr{M}}$ has an isotrivial isogeny factor. For any point $m \in \mathscr{M}$, let $f\colon X \to Y$ be the fiber of $h\colon \mathscr{X} \to \mathscr{Y}$ over m. By Proposition 2.3, the sheaf $f_*\omega_X$ is not generically globally generated. By Lemma 2.1, $X \to Y$ is Prill exceptional. □

Acknowledgements

We especially thank Anand Deopurkar for alerting us to Prill's problem, and Rob Lazarsfeld for encouraging us to write this short note and for giving us useful comments on it. We would like to

thank Ravindra Girivaru, Joe Harris, Neithalath Mohan Kumar and Rob Lazarsfeld for telling us about their previous work on Prill's problem. We thank Mihnea Popa for a helpful discussion.

References

ACGH85 E. Arbarello, M. Cornalba, P. A. Griffiths and J. Harris, *Geometry of algebraic curves. Vol. I*, Grundlehren math. Wiss., vol. 267 (Springer, New York, 1985); doi:10.1007/978-1-4757-5323-3.

BB05 I. Biswas and D. C. Butler, *On Prill's problem*, Comm. Algebra **33** (2005), no. 1, 323–330; doi:10.1081/AGB-200041017.

BT02 F. Bogomolov and Y. Tschinkel, *On curve correspondences*, Communications in arithmetic fundamental groups (Kyoto, 1999/2001), Sūrikaisekikenkyūsho Kōkyūroku (2002), no. 1267, 157–166.

BT04 ———, *On curve correspondences*, 16 February 2004, available at https://www.math.nyu.edu/~tschinke/papers/yuri/02genram/genram.pdf.

GR71 A. Grothendieck and M. Raynaud, *Revêtements étales et groupe fondamental (Séminaire de Géométrie Algébrique du Bois Marie 1960–1961 (SGA 1))*, Lecture Notes in Math., vol. 224 (Springer, Berlin, 1971); doi:10.1007/BFb0058656.

LL22a A. Landesman and D. Litt, *Applications of the algebraic geometry of the Putman–Wieland conjecture*, 2022, arXiv:2209.00718v2.

LL22b ———, *Canonical representations of surface groups*, 2022, arXiv:2205.15352v2.

Mar22 V. Marković, *Unramified correspondences and virtual properties of mapping class groups*, Bull. Lond. Math. Soc. **54** (2022), no. 6, 2324–2337; doi:10.1112/blms.12696.

Moh07 N. Mohan Kumar, *Some remarks on Prill's problem*, in *Affine algebraic geometry* (Osaka Univ. Press, Osaka, 2007), 289–294.

Aaron Landesman aaronl@mit.edu
Department of Mathematics, MIT, 182 Memorial Dr, Cambridge, MA 02142, USA

Daniel Litt daniel.litt@utoronto.ca
Department of Mathematics, University of Toronto, Bahen Centre, 40 St. George St., Toronto, Ontario, M5S 2E4, Canada