PROCEEDINGS OF SCIENCE

Theoretical results for top-pair and top+W production

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I present theoretical results for top-pair production as well as for the associated production of top quarks with W bosons. Soft-gluon corrections from resummation are calculated through approximate N^3LO and added to fixed-order QCD results, and electroweak corrections are included at NLO. Top-quark transverse-momentum and rapidity distributions are also presented. In all cases the higher-order corrections are large, they reduce the scale dependence, and they improve agreement with recent data.

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PoS(ICHEP2024)371

1. Introduction

The contributions of higher-order corrections to top-quark processes, such as $t\bar{t}$ production, tW production, and $t\bar{t}W$ production are significant and improve the precision of theoretical predictions. Soft-gluon corrections are important for all top-quark processes that have been studied, and they approximate known exact results at NLO and NNLO very well. For a $2 \rightarrow n$ process with $p_a + p_b \rightarrow p_t + p_2 + \cdots + p_n$, where p_t is the top-quark momentum, we define the threshold variable $s_4 = (p_2 + \cdots + p_n + p_g)^2 - (p_2 + \cdots + p_n)^2$ where an extra gluon with momentum p_g is emitted. At partonic threshold $p_g \rightarrow 0$ and, thus, $s_4 \rightarrow 0$. The soft corrections involve logarithms of the form $[\ln^k(s_4/m_t^2)/s_4]_+$ with m_t the top-quark mass and $k \leq 2n-1$ for the order α_s^n corrections. We resum these corrections in single-particle-inclusive kinematics for the double-differential cross section and use them to derive approximate NNLO (aNNLO) and approximate N^{3}LO (aN^{3}LO) predictions for total cross sections and top-quark differential distributions.

2. Soft-gluon corrections

The factorized cross section for a general top-quark production process can be written as

$$d\sigma_{pp\rightarrow tX} = \sum_{a,b} \int dx_a \, dx_b \, \phi_{a/p}(x_a, \mu_F) \, \phi_{b/p}(x_b, \mu_F) \, d\hat{\sigma}_{ab\rightarrow tX}(s_4, \mu_F) \tag{1}$$

where the ϕ's are parton distribution functions (pdf), μ_F is the factorization scale, and $\hat{\sigma}$ is the partonic cross section. We take Laplace transforms $d\tilde{\hat{\sigma}}_{ab\rightarrow tX}(N) = \int (ds_4/s) \, e^{-Ns_4/s} d\hat{\sigma}_{ab\rightarrow tX}(s_4)$ and $\tilde{\phi}(N) = \int_0^1 e^{-N(1-x)} \, \phi(x) \, dx$ with transform variable N, and we write the parton-parton cross section as

$$d\tilde{\sigma}_{ab\rightarrow tX}(N) = \tilde{\phi}_{a/a}(N_a, \mu_F) \, \tilde{\phi}_{b/b}(N_b, \mu_F) \, d\tilde{\hat{\sigma}}_{ab\rightarrow tX}(N, \mu_F) \, . \tag{2}$$

A further refactorization for the cross section,

$$d\tilde{\sigma}_{ab\rightarrow tX}(N) = \tilde{\psi}_a(N_a, \mu_F) \, \tilde{\psi}_b(N_b, \mu_F) \, \mathrm{tr} \left\{ H_{ab\rightarrow tX} \left(\alpha_s(\mu_R)\right) \, \tilde{S}_{ab\rightarrow tX} \left(\frac{\sqrt{s}}{N\mu_F} \right) \right\} , \tag{3}$$

is given in terms of new functions ψ for collinear emission from incoming partons, a hard function $H_{ab\rightarrow tX}$, and a soft function $S_{ab\rightarrow tX}$ for noncollinear soft gluons. The hard and soft functions are matrices in the space of color flow. In the case of collinear emission from any massless final-state particles we also need a function J, but this does not apply to the processes studied here.

From Eqs. (2) and (3) we, thus, have the formula

$$d\tilde{\hat{\sigma}}_{ab\rightarrow tX}(N, \mu_F) = \frac{\tilde{\psi}_{a/a}(N_a, \mu_F) \, \tilde{\psi}_{b/b}(N_b, \mu_F)}{\tilde{\phi}_{a/a}(N_a, \mu_F) \, \tilde{\phi}_{b/b}(N_b, \mu_F)} \, \mathrm{tr} \left\{ H_{ab\rightarrow tX} \left(\alpha_s(\mu_R)\right) \, \tilde{S}_{ab\rightarrow tX} \left(\frac{\sqrt{s}}{N\mu_F} \right) \right\} . \tag{4}$$

$S_{ab\rightarrow tX}$ satisfies the renormalization-group equation, with μ_R the renormalization scale,

$$\left(\mu_R \frac{\partial}{\partial \mu_R} + \beta(g_s) \frac{\partial}{\partial g_s} \right) S_{ab\rightarrow tX} = -\Gamma^{\dagger}_{S\,ab\rightarrow tX} \, S_{ab\rightarrow tX} - S_{ab\rightarrow tX} \, \Gamma_{S\,ab\rightarrow tX} \, . \tag{5}$$

The soft anomalous dimension $\Gamma_{S\,ab\rightarrow tX}$ controls the evolution of the soft function via the above equation and, thus, gives the exponentiation of logarithms of N [1, 2].

PoS(ICHEP2024)371

Renormalization-group evolution leads to resummation, and we find the expression [1, 2]

$$d\tilde{\sigma}^{\rm resum}_{ab\to tX}(N) = \exp\left[\sum_{i=a,b} E_i(N_i)\right] \exp\left[\sum_{i=a,b} 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu}\gamma_{i/i}(N_i)\right] \times {\rm tr}\left\{H_{ab\to tX}\left(\alpha_s(\sqrt{s})\right)\right.$$
$$\left.\times \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/N} \frac{d\mu}{\mu}\Gamma^{\dagger}_{S\,ab\to tX}\left(\alpha_s(\mu)\right)\right] \tilde{S}_{ab\to tX}\left(\alpha_s\left(\frac{\sqrt{s}}{N}\right)\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/N} \frac{d\mu}{\mu}\Gamma_{S\,ab\to tX}\left(\alpha_s(\mu)\right)\right]\right\}. \quad (6)$$

The resummed cross section is then expanded at fixed-order through N^{3}LO and transformed back to physical space. Many top processes have been studied with calculations of corrections for total and differential cross sections. These include top-pair production: $t\bar{t}$ in the SM through aN^{3}LO, and $t\bar{t}$ in SMEFT at aNNLO; top-pair+X: $t\bar{t}\gamma$ at aNNLO and $t\bar{t}W$ at aN^{3}LO; single top: t- and s-channel and tW through aN^{3}LO; single-top+X: tqH, $tq\gamma$, and tqZ through aNNLO; single-top BSM: $t\gamma$, tZ, tZ', tg through aNNLO, and tH^- through aN^{3}LO; see Ref. [3] for reviews of the theory of soft-gluon corrections used for all these processes and additional references.

3. $t\bar{t}$ production

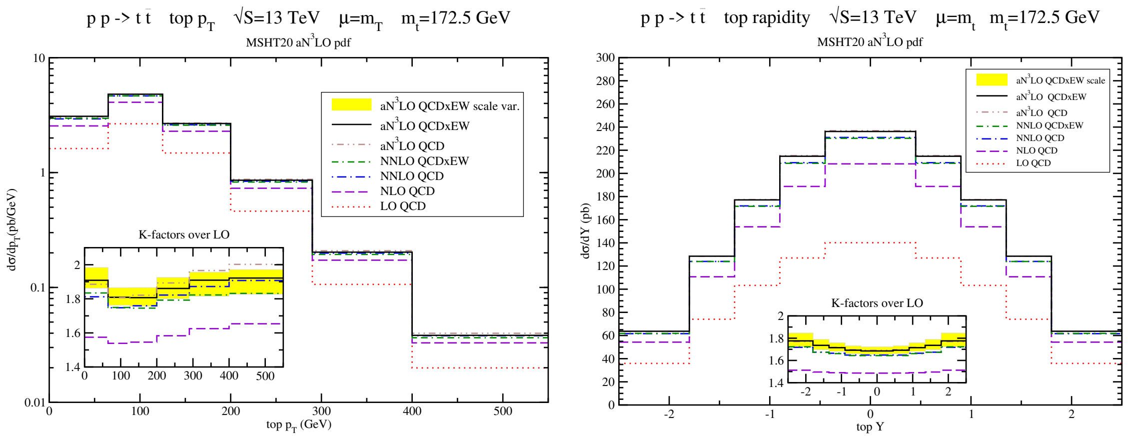

Figure 1: Top-quark p_T (left) and rapidity (right) distributions in top-pair production at 13 TeV LHC energy.

Calculations of soft-gluon corrections for top-quark differential cross sections go back thirty years [4]. The soft anomalous dimension matrix is 2×2 for the $q\bar{q}\to t\bar{t}$ channel and 3×3 for the $gg\to t\bar{t}$ channel; they were calculated at one loop in the mid-90's [1] and at two loops fifteen years ago [2]. There are partial results at three loops [3], and the four-loop massive cusp anomalous dimension (determined from its asymptotics in Ref. [5]) contributes to the four-loop result.

The NLO expansions of the resummed cross section agree with exact NLO results very well, and the NNLO expansions provide aNNLO results that predicted the exact NNLO to very high accuracy (percent or per mille) for total cross sections and top-quark p_T and rapidity distributions. By further adding the third-order soft-gluon corrections to the exact NNLO result, we obtain aN^{3}LO predictions [6] which are the state of the art, and to which we also add electroweak corrections [7].

The aN^{3}LO QCD + NLO EW cross section with $\mu = m_t$ and scale and pdf uncertainties is [7] with MSHT20 aN^{3}LO pdf [8] at 13 TeV, $802^{+22}_{-17}{}^{+16}_{-17}$ pb, and at 13.6 TeV, $886^{+24}_{-19}{}^{+18}_{-20}$ pb; with

MSHT20 NNLO pdf [9] at 13 TeV, $836^{+23\,+17}_{-18\,-11}$ pb, and at 13.6 TeV, $925^{+25\,+18}_{-20\,-12}$ pb; with CT18 NNLO pdf [10] at 13 TeV, $842^{+23\,+18}_{-18\,-16}$ pb, and at 13.6 TeV, $932^{+25\,+20}_{-20\,-18}$ pb; and with NNPDF4.0 NNLO pdf [11] at 13 TeV, $816^{+23\,+5}_{-18\,-4}$ pb, and at 13.6 TeV, $904^{+25\,+5}_{-20\,-5}$ pb. Furthermore, with PDF4LHC21 NNLO pdf [12] the result at 13 TeV is $837^{+23\,+20}_{-18\,-16}$ pb, and at 13.6 TeV it is $926^{+25\,+22}_{-20\,-17}$ pb.

Figure 1 displays theoretical results through aN^{3}LO QCD $\times$ EW for the top-quark p_T and rapidity distributions in $t\bar{t}$ production [7] at 13 TeV LHC energy using MSHT20 aN^{3}LO pdf [8].

4. tW production

Calculations of soft-gluon corrections for tW production have been available for close to two decades [13, 14], and the soft anomalous dimension for tW production is known to three loops [13].

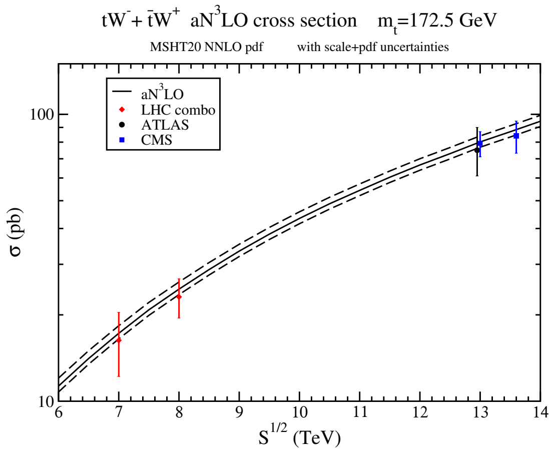

Figure 2: $tW^- + \bar{t}W^+$ cross section at aN^{3}LO with scale and pdf uncertainties at LHC energies.

Figure 2 shows the theoretical $tW^- + \bar{t}W^+$ production cross sections at aN^{3}LO [14] at LHC energies and compares them with LHC data at 7, 8, 13, and 13.6 TeV [15–17].

The aN^{3}LO cross section for $tW^- + \bar{t}W^+$ production with $\mu = m_t$ and scale and pdf uncertainties with MSHT20 NNLO pdf [9] at 13 TeV is $79.5^{+1.9\,+2.0}_{-1.8\,-1.4}$ pb, and at 13.6 TeV it is $87.6^{+2.0\,+2.1}_{-1.9\,-1.5}$ pb; with MSHT20 aN^{3}LO pdf [8] at 13 TeV it is $77.3^{+1.9\,+2.0}_{-1.8\,-2.1}$ pb, and at 13.6 TeV it is $85.6^{+2.0\,+2.2}_{-1.9\,-2.3}$ pb; and with PDF4LHC21 pdf [12] at 13 TeV it is $79.3^{+1.9\,+2.2}_{-1.8\,-2.2}$ pb, and at 13.6 TeV it is $87.9^{+2.0\,+2.4}_{-1.9\,-2.4}$ pb.

5. $t\bar{t}W$ production

The observation of $t\bar{t}W$ events at the LHC has shown that the measurements are significantly higher than theoretical predictions at NLO. The QCD corrections at NLO are large while the electroweak corrections are smaller but significant. Further improvement in theoretical accuracy can be obtained by the inclusion of higher-order soft-gluon corrections [18] using the formalism for $2 \to 3$ processes [19]. We calculate aN^{3}LO QCD corrections and further add to them NLO electroweak corrections to provide state-of-the-art theoretical predictions.

Figure 3 shows the theoretical predictions for the total $t\bar{t}W$ cross section using MSHT20 NNLO pdf [9] and a comparison with data at 8 and 13 TeV from ATLAS [20] and CMS [21]. We note the large K-factors and the improved agreement with data at aN^{3}LO.

PoS(ICHEP2024)371

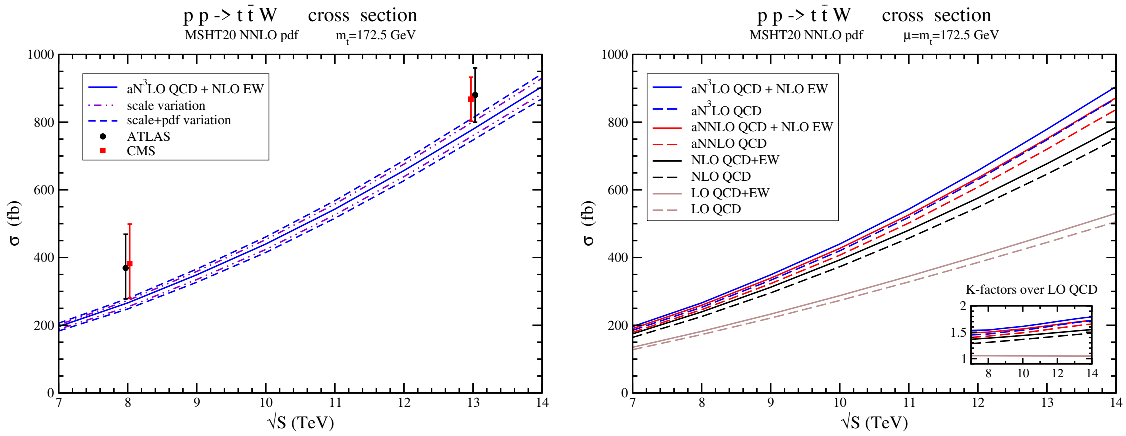

Figure 3: Cross sections through aN^{3}LO QCD+NLO EW for $t\bar{t}W$ production at LHC energies.

At 13.6 TeV with $\mu = m_t$, the NLO QCD corrections increase the LO result by 47%, the aNNLO QCD corrections by a further 17% (and consistent with the partial NNLO result in [22]), and the aN^{3}LO QCD corrections by an extra 6%, while the electroweak NLO corrections provide 7%. Thus, the total aN^{3}LO QCD+NLO EW cross section is 78% bigger than LO QCD.

In the comparison with 8 and 13 TeV CMS and ATLAS data, the NLO and even aNNLO results are not sufficient; we need aN^{3}LO corrections to describe the data. At 8 TeV, the measured cross section from CMS is 382^{+117}_{-102} fb and from ATLAS it is 369^{+100}_{-91} fb. The theoretical prediction at aN^{3}LO QCD + NLO EW with central scale $\mu = m_t$ is $266^{+7}_{-12}{}^{+6}_{-6}$ fb.

At 13 TeV, CMS finds 868 ± 65 fb, with 553 ± 42 fb for $t\bar{t}W^+$ and 343 ± 36 fb for $t\bar{t}W^-$, while ATLAS finds 880 ± 80 fb, with 583 ± 58 fb for $t\bar{t}W^+$ and 296 ± 40 fb for $t\bar{t}W^-$. The theoretical prediction at aN^{3}LO QCD + NLO EW with central scale $\mu = m_t$ is $779^{+22}_{-19}{}^{+12}_{-13}$ fb, with $517^{+14}_{-12}{}^{+8}_{-9}$ fb for $t\bar{t}W^+$ and $262^{+8}_{-7}{}^{+4}_{-4}$ fb for $t\bar{t}W^-$.

Top-quark p_T and rapidity distributions in $t\bar{t}W$ production at 13 and 13.6 TeV have also been calculated in [18]; the K-factors decrease at larger top p_T while they increase at larger rapidities.

Finally, we note that soft-gluon corrections are important for the related processes of $t\bar{t}Z$ production.

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