

Search for the baryon number and lepton number violating decays $\tau^- \rightarrow \Lambda\pi^-$ and $\tau^- \rightarrow \bar{\Lambda}\pi^-$ at Belle II

I. Adachi¹⁰, L. Aggarwal¹⁰, H. Ahmed¹⁰, H. Aihara¹⁰, N. Akopov¹⁰, A. Aloisio¹⁰, N. Althubiti¹⁰, N. Anh Ky¹⁰, D. M. Asner¹⁰, H. Atmacan¹⁰, T. Aushev¹⁰, V. Aushev¹⁰, M. Aversano¹⁰, R. Ayad¹⁰, V. Babu¹⁰, H. Bae¹⁰, S. Bahinipati¹⁰, P. Bambade¹⁰, Sw. Banerjee¹⁰, S. Bansal¹⁰, M. Barrett¹⁰, J. Baudot¹⁰, A. Baur¹⁰, A. Beaubien¹⁰, F. Becherer¹⁰, J. Becker¹⁰, J. V. Bennett¹⁰, F. U. Bernlochner¹⁰, V. Bertacchi¹⁰, M. Bertemes¹⁰, E. Bertholet¹⁰, M. Bessner¹⁰, S. Bettarini¹⁰, B. Bhuyan¹⁰, F. Bianchi¹⁰, L. Bierwirth¹⁰, T. Bilka¹⁰, D. Biswas¹⁰, A. Bobrov¹⁰, D. Bodrov¹⁰, J. Borah¹⁰, A. Boschetti¹⁰, A. Bozek¹⁰, P. Branchini¹⁰, T. E. Browder¹⁰, A. Budano¹⁰, S. Bussino¹⁰, Q. Campagna¹⁰, M. Campajola¹⁰, G. Casarosa¹⁰, C. Cecchi¹⁰, J. Cerasoli¹⁰, M.-C. Chang¹⁰, R. Cheaib¹⁰, P. Cheema¹⁰, B. G. Cheon¹⁰, K. Chilikin¹⁰, K. Chirapatpimol¹⁰, H.-E. Cho¹⁰, K. Cho¹⁰, S.-J. Cho¹⁰, S.-K. Choi¹⁰, S. Choudhury¹⁰, J. Cochran¹⁰, L. Corona¹⁰, J. X. Cui¹⁰, E. De La Cruz-Burelo¹⁰, S. A. De La Motte¹⁰, G. De Nardo¹⁰, G. De Pietro¹⁰, R. de Sangro¹⁰, M. Destefanis¹⁰, S. Dey¹⁰, R. Dhamija¹⁰, A. Di Canto¹⁰, F. Di Capua¹⁰, J. Dingfelder¹⁰, Z. Doležal¹⁰, I. Domínguez Jiménez¹⁰, T. V. Dong¹⁰, M. Dorigo¹⁰, K. Dort¹⁰, D. Dossett¹⁰, S. Dubey¹⁰, G. Dujany¹⁰, P. Ecker¹⁰, D. Epifanov¹⁰, J. Eppelt¹⁰, P. Feichtinger¹⁰, T. Ferber¹⁰, T. Fillinger¹⁰, C. Finck¹⁰, G. Finocchiaro¹⁰, A. Fodor¹⁰, F. Forti¹⁰, A. Frey¹⁰, B. G. Fulsom¹⁰, A. Gabrielli¹⁰, E. Ganiev¹⁰, M. Garcia-Hernandez¹⁰, G. Gaudino¹⁰, V. Gaur¹⁰, A. Gaz¹⁰, A. Gellrich¹⁰, G. Ghevondyan¹⁰, D. Ghosh¹⁰, H. Ghumaryan¹⁰, G. Giakoustidis¹⁰, R. Giordano¹⁰, P. Gironella¹⁰, A. Glazov¹⁰, B. Gobbo¹⁰, R. Godang¹⁰, P. Goldenzweig¹⁰, W. Gradl¹⁰, E. Graziani¹⁰, D. Greenwald¹⁰, Z. Gruberová¹⁰, K. Gudkova¹⁰, I. Haide¹⁰, S. Halder¹⁰, K. Hara¹⁰, C. Harris¹⁰, H. Hayashii¹⁰, S. Hazra¹⁰, C. Hearty¹⁰, M. T. Hedges¹⁰, A. Heidelbach¹⁰, I. Heredia de la Cruz¹⁰, M. Hernández Villanueva¹⁰, T. Higuchi¹⁰, M. Hoek¹⁰, M. Hohmann¹⁰, R. Hoppe¹⁰, P. Horak¹⁰, C.-L. Hsu¹⁰, T. Humair¹⁰, T. Iijima¹⁰, K. Inami¹⁰, N. Ipsita¹⁰, A. Ishikawa¹⁰, R. Itoh¹⁰, M. Iwasaki¹⁰, W. W. Jacobs¹⁰, D. E. Jaffe¹⁰, E.-J. Jang¹⁰, Q. P. Ji¹⁰, S. Jia¹⁰, Y. Jin¹⁰, H. Junkerkalefeld¹⁰, J. Kandra¹⁰, K. H. Kang¹⁰, G. Karyan¹⁰, T. Kawasaki¹⁰, F. Keil¹⁰, C. Kiesling¹⁰, D. Y. Kim¹⁰, J.-Y. Kim¹⁰, K.-H. Kim¹⁰, Y.-K. Kim¹⁰, K. Kinoshita¹⁰, P. Kodyš¹⁰, T. Koga¹⁰, S. Kohani¹⁰, K. Kojima¹⁰, A. Korobov¹⁰, S. Korpar¹⁰, E. Kovalenko¹⁰, R. Kowalewski¹⁰, P. Križan¹⁰, P. Krokovny¹⁰, T. Kuhr¹⁰, R. Kumar¹⁰, K. Kumara¹⁰, A. Kuzmin¹⁰, Y.-J. Kwon¹⁰, S. Lacaprara¹⁰, Y.-T. Lai¹⁰, K. Lalwani¹⁰, T. Lam¹⁰, L. Lanceri¹⁰, J. S. Lange¹⁰, M. Laurenza¹⁰, K. Lautenbach¹⁰, R. Leboucher¹⁰, M. J. Lee¹⁰, P. Leo¹⁰, D. Levit¹⁰, P. M. Lewis¹⁰, C. Li¹⁰, L. K. Li¹⁰, W. Z. Li¹⁰, Y. Li¹⁰, Y. B. Li¹⁰, J. Libby¹⁰, J. Lin¹⁰, M. H. Liu¹⁰, Q. Y. Liu¹⁰, Z. Q. Liu¹⁰, D. Liventsev¹⁰, S. Longo¹⁰, T. Lueck¹⁰, C. Lyu¹⁰, Y. Ma¹⁰, M. Maggiora¹⁰, S. P. Maharana¹⁰, R. Maiti¹⁰, S. Maity¹⁰, G. Mancinelli¹⁰, R. Manfredi¹⁰, E. Manoni¹⁰, M. Mantovano¹⁰, D. Marcantonio¹⁰, S. Marcello¹⁰, C. Marinas¹⁰, C. Martellini¹⁰, A. Martens¹⁰, A. Martini¹⁰, T. Martinov¹⁰, L. Massaccesi¹⁰, M. Masuda¹⁰, T. Matsuda¹⁰, D. Matvienko¹⁰, S. K. Maurya¹⁰, J. A. McKenna¹⁰, R. Mehta¹⁰, F. Meier¹⁰, M. Merola¹⁰, C. Miller¹⁰, M. Mirra¹⁰, S. Mitra¹⁰, S. Mondal¹⁰, S. Moneta¹⁰, H.-G. Moser¹⁰, R. Mussa¹⁰, I. Nakamura¹⁰, M. Nakao¹⁰, Y. Nakazawa¹⁰, M. Naruki¹⁰, D. Narwal, Z. Natkaniec¹⁰, A. Natochii¹⁰, M. Nayak¹⁰, G. Nazaryan¹⁰, M. Neu¹⁰, C. Niebuhr¹⁰, S. Nishida¹⁰, S. Ogawa¹⁰, H. Ono¹⁰, P. Pakhlov¹⁰, E. Paoloni¹⁰, S. Pardi¹⁰, J. Park¹⁰, K. Park¹⁰, S.-H. Park¹⁰, B. Paschen¹⁰, A. Passeri¹⁰, S. Patra¹⁰, T. K. Pedlar¹⁰, R. Peschke¹⁰, R. Pestotnik¹⁰, M. Piccolo¹⁰, L. E. Piilonen¹⁰, P. L. M. Podesta-Lerma¹⁰, T. Podobnik¹⁰, S. Pokharel¹⁰, C. Praz¹⁰, S. Prell¹⁰, E. Prencipe¹⁰, M. T. Prim¹⁰, H. Purwar¹⁰, G. Raeuber¹⁰, S. Raiz¹⁰, N. Rauls¹⁰, M. Reif¹⁰, S. Reiter¹⁰, M. Remnev¹⁰, L. Reuter¹⁰, I. Ripp-Baudot¹⁰, G. Rizzo¹⁰, J. M. Roney¹⁰, A. Rostomyan¹⁰, N. Rout¹⁰, S. Sandilya¹⁰, L. Santelj¹⁰, V. Savinov¹⁰, B. Scavino¹⁰, M. Schnepf¹⁰, C. Schwanda¹⁰, Y. Seino¹⁰, A. Selce¹⁰, K. Senyo¹⁰, J. Serrano¹⁰, M. E. Sevior¹⁰, C. Sfienti¹⁰, W. Shan¹⁰, C. Sharma¹⁰, C. P. Shen¹⁰, X. D. Shi¹⁰, T. Shillington¹⁰, T. Shimasaki¹⁰, J.-G. Shiu¹⁰, D. Shtol¹⁰, B. Shwartz¹⁰, A. Sibidanov¹⁰, F. Simon¹⁰, J. B. Singh¹⁰, J. Skorup¹⁰, R. J. Sobie¹⁰, M. Sobotzik¹⁰, A. Soffer¹⁰, A. Sokolov¹⁰, E. Solovieva¹⁰, W. Song¹⁰, S. Spataro¹⁰, B. Spruck¹⁰, M. Starič¹⁰, P. Stavroulakis¹⁰, S. Stefkova¹⁰, R. Stroili¹⁰, Y. Sue¹⁰, M. Sumihama¹⁰, K. Sumisawa¹⁰, W. Sutcliffe¹⁰, N. Suwonjandee¹⁰, H. Svidras¹⁰, M. Takahashi¹⁰, M. Takizawa¹⁰, U. Tamponi¹⁰, K. Tanida¹⁰, F. Tenchini¹⁰, O. Tittel¹⁰, R. Tiwary¹⁰, D. Tonelli¹⁰, E. Torassa¹⁰, K. Trabelsi¹⁰, I. Ueda¹⁰, K. Unger¹⁰, Y. Unno¹⁰, K. Uno¹⁰, S. Uno¹⁰, P. Urquijo¹⁰, S. E. Vahsen¹⁰, R. van Tonder¹⁰, K. E. Varvell¹⁰, M. Veronesi¹⁰, V. S. Vismaya¹⁰, L. Vitale¹⁰, V. Vobbillisetti¹⁰, R. Volpe¹⁰, M. Wakai¹⁰, S. Wallner¹⁰, E. Wang¹⁰, M.-Z. Wang¹⁰, Z. Wang¹⁰, A. Warburton¹⁰, S. Watanuki¹⁰, C. Wessel¹⁰, E. Won¹⁰, X. P. Xu¹⁰, B. D. Yabsley¹⁰, S. Yamada¹⁰, W. Yan¹⁰, S. B. Yang¹⁰, J. Yelton¹⁰, J. H. Yin¹⁰, K. Yoshihara¹⁰, C. Z. Yuan¹⁰, L. Zani¹⁰, B. Zhang¹⁰, J. S. Zhou¹⁰, Q. D. Zhou¹⁰, V. I. Zhukova¹⁰, and R. Žlebčík¹⁰

(Belle II Collaboration)



(Received 8 July 2024; accepted 5 November 2024; published 3 December 2024)

We present a search for the baryon number B and lepton number L violating decays $\tau^- \rightarrow \Lambda\pi^-$ and $\tau^- \rightarrow \bar{\Lambda}\pi^-$ produced from the $e^+e^- \rightarrow \tau^+\tau^-$ process, using a 364 fb^{-1} data sample collected by the Belle II experiment at the SuperKEKB collider. No evidence of signal is found in either decay mode, which have $|\Delta(B - L)|$ equal to 2 and 0, respectively. Upper limits at 90% credibility level on the branching fractions of $\tau^- \rightarrow \Lambda\pi^-$ and $\tau^- \rightarrow \bar{\Lambda}\pi^-$ are determined to be 4.7×10^{-8} and 4.3×10^{-8} , respectively.

DOI: 10.1103/PhysRevD.110.112003

Baryon number violation (BNV) is necessary to explain a dynamical generation of the asymmetry of matter and antimatter in the universe [1–4]. For standard model (SM) processes, at perturbative level, lepton number L and baryon number B are conserved. However, nonperturbative effects, such as sphaleron processes, could lead to violations of B and L separately at high temperatures, while conserving their difference $B - L$ [5–8]. Several beyond-the-SM theories predict BNV, such as supersymmetry models with R-parity violation [9], a black hole model [10], superstring models [11], and grand unification models [12]. Most of the models require $B - L$ conservation, while $|\Delta(B - L)| = 2$ is allowed in some scenarios [12–14]. Thus, BNV processes can be accompanied by lepton number violation. Discoveries of such BNV processes would reveal the existence of physics beyond the SM and shed light on matter-antimatter asymmetry.

Over the last few decades, several experiments have searched for BNV, but no evidence has been found [15]. Proton decays have been extensively studied by Super-Kamiokande [16], with lower limits on the lifetime of the proton on the order of 10^{33} years at the 90% confidence level, while baryon-number-violating decays of charmed and bottom hadrons, τ leptons, and Z bosons were studied by the CLEO [17,18], BABAR [19,20], OPAL [21], Belle [22,23], and LHCb collaborations [24], with upper limits on the branching fractions in the range $10^{-8} - 10^{-5}$ at the 90% confidence level.

We use a sample of e^+e^- collisions collected at a center-of-mass (c.m.) energy \sqrt{s} of 10.58 GeV corresponding to the mass of the $\Upsilon(4S)$ resonance to search for BNV decays $\tau^- \rightarrow \Lambda\pi^-$ and $\tau^- \rightarrow \bar{\Lambda}\pi^-$. The data, recorded by the Belle II detector [25] operating at the SuperKEKB [26] asymmetric-energy collider, have an integrated luminosity \mathcal{L} of $(364 \pm 2) \text{ fb}^{-1}$ [27]. Searches for $\tau^- \rightarrow \Lambda\pi^-$ and $\bar{\Lambda}\pi^-$ were performed by Belle [22] using a 154 fb^{-1} data sample yielding upper limits of 0.72×10^{-7} and 1.4×10^{-7} , respectively, at the 90% confidence level. With an upgraded

detector and a larger data sample, the Belle II experiment can investigate these decays with improved sensitivity.

We select $e^+e^- \rightarrow \tau^+\tau^-$ events in which one of the two τ leptons decays into three charged particles (3-prong) and the other into one charged particle (1-prong) and one or more neutrinos. We search for a signal on the 3-prong (signal) side via the decay $\tau^- \rightarrow \Lambda(\rightarrow p\pi^-)\pi^-$ or $\tau^- \rightarrow \bar{\Lambda}(\rightarrow \bar{p}\pi^+)\pi^-$, while we require that the other τ (tag) decays into $e^+\nu_e\bar{\nu}_\tau$, $\mu^+\nu_\mu\bar{\nu}_\tau$, $\pi^+\bar{\nu}_\tau$, or $\pi^+\pi^0\bar{\nu}_\tau$ final states. Inclusion of charge-conjugate states is always implied. A search region is defined in a plane formed by the energy difference $\Delta E = E(\Lambda\pi) - \sqrt{s}/2$ and the invariant mass $M(\Lambda\pi)$, where Λ means Λ or $\bar{\Lambda}$, and signal is identified as an excess over background expectations.

In the following, all variables are defined in the e^+e^- c.m. system unless otherwise specified. The analysis is optimized using simulated events, and data events outside the signal region, before examining data in the signal region.

The Belle II detector has a cylindrical geometry with the axis of symmetry along the beamline (z axis) [25]. The innermost component is a two-layer silicon-pixel detector surrounded by a four-layer double-sided silicon-strip detector and a 56-layer central drift chamber (CDC). These detectors reconstruct trajectories of charged particle (tracks). Only one sixth of the second layer of the silicon-pixel detector was installed for the data analyzed here. Surrounding the CDC, which also provides dE/dx energy-loss measurements, is a time-of-propagation detector in the central region and an aerogel-based ring-imaging Cherenkov detector in the forward region. These detectors provide particle identification (PID) for charged hadrons. Surrounding them is an electromagnetic calorimeter (ECL) composed of CsI(Tl) crystals that primarily provides energy and timing measurements for photons and electrons. Outside of the ECL is a superconducting solenoid magnet. Its flux return is instrumented with resistive-plate chambers and plastic scintillator modules to detect muons, K_L^0 mesons, and neutrons. The solenoid magnet provides a 1.5 T magnetic field parallel to the z axis. The longitudinal direction and the polar angle θ are defined with respect to the z axis, whose positive direction is that of the electron beam.

Simulated samples are used to estimate signal efficiencies and the number of expected background events [28]. We use the KKMC software package to generate 2×10^7

$e^+e^- \rightarrow \tau^+\tau^-(\gamma)$ signal events [29]. The subsequent τ decays are simulated by the TAUOLA software package [30], with one τ decaying to $\Lambda\pi$ or $\bar{\Lambda}\pi$ according to a phase-space model and the other decaying according to known branching fractions. [15,29]. We use KKMC and TAUOLA also to simulate backgrounds.

The KKMC generator is also used for $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ and $q\bar{q}$ processes, where q indicates u , d , s , or c quarks, with the $q\bar{q}$ pair fragmentation being simulated by the PYTHIA8 software package [31]. The PYTHIA8 and EvtGen [32] software packages are used to simulate the $e^+e^- \rightarrow b\bar{b}$ process. The BabaYaga@NLO software package is used to simulate the $e^+e^- \rightarrow e^+e^-(\gamma)$ processes [33–37]; the AAFH [38–40] and TREPS [41] software packages are used for simulations of $e^+e^- \rightarrow \ell^+\ell^-\ell^+\ell^-$ and $e^+e^-h^+h^-$ processes, respectively, where ℓ indicates an electron, muon, or τ lepton, and h indicates a pion, kaon, or proton.

The size of each simulated sample is four times the size of the corresponding component in data, except for the $e^+e^- \rightarrow \ell^+\ell^-\ell^+\ell^-$ and $e^+e^-h^+h^-$ samples, which are the same size as expected in data, and the $e^+e^- \rightarrow e^+e^-(\gamma)$ sample, which is 10% of that expected in data. EvtGen is used to simulate the decay of hadrons [32]. The PHOTOS software package is used for the simulation of final state radiation [42]. The Belle II software [43] uses the Geant4 [44] software package to simulate the response of the detector to the interactions of particles.

The hardware trigger relies on energy deposits (clusters with energy larger than 100 MeV) and their topologies in the ECL, and on the number of reconstructed tracks in the CDC. Events are required to satisfy one of the following criteria: three ECL clusters with a topology inconsistent with a Bhabha process, where at least one cluster has an energy greater than 500 MeV; the sum of clusters exceeding 1 GeV; at least one charged particle with a momentum greater than 0.7 GeV/c . The trigger efficiency for the $\tau^- \rightarrow \Lambda\pi^-$ ($\tau^- \rightarrow \bar{\Lambda}\pi^-$) channel, estimated on simulated signal samples, is 99.5% (99.6%).

Both charged and neutral particles are required to be within the acceptance of the CDC, i.e., $-0.866 < \cos\theta < 0.956$. The transverse momentum of each charged particle is required to exceed 0.1 GeV/c . Photons are identified as clusters with energies greater than 0.1 GeV not associated to any track. The thrust axis \hat{t} is defined such that the value of

$$T = \frac{\sum_i |\vec{p}_i \cdot \hat{t}|}{\sum_i |\vec{p}_i|} \quad (1)$$

is maximized. Here, \vec{p}_i is the momentum of the i th particle and the sum runs over all tracks and clusters. Events are geometrically split into two opposite hemispheres with a plane perpendicular to the thrust axis. The hemispheres corresponding to the signal and tag sides are required to contain exactly three tracks and one track, respectively. In

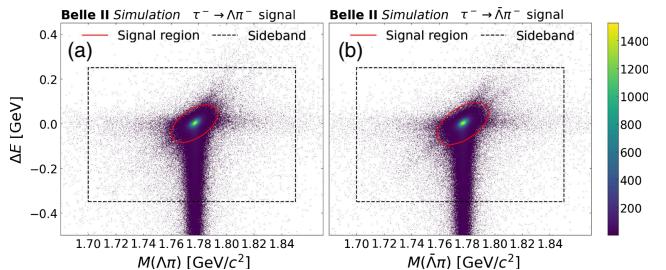


FIG. 1. Distributions of ΔE as a function of $M(\Lambda\pi)$ for simulated (a) $\tau^- \rightarrow \Lambda\pi^-$ and (b) $\tau^- \rightarrow \bar{\Lambda}\pi^-$ signal samples. The red solid ellipses identify the signal regions, while the areas between the dashed black boxes and the corresponding ellipses are the sideband regions.

the following, we refer to the pion from the Λ or $\bar{\Lambda}$ decays as π_Λ , the pion from the signal τ decays as π_τ , and the pion from the tag τ decays as π_{tag} . We require that the numbers of photons N_γ^{sig} and N_γ^{tag} on the signal and tag sides each be less than four, and that the energy E_γ^{sig} of the most energetic photon on the signal side be less than 1.0 GeV. These requirements take into account the possibility of photons radiated from the initial state and photons from π^0 decays on the tag side.

A signal region is defined in the plane of the ΔE versus $M(\Lambda\pi)$, in which signal is expected to peak at $\Delta E \approx 0.0$ and at $M(\Lambda\pi) \approx 1.78 \text{ GeV}/c^2$. The signal region, shown in Fig. 1, has an elliptical shape whose size is defined so as to include 90% of all reconstructed signal events. The distribution of signal in the ΔE versus $M(\Lambda\pi)$ plane is broadened by detector resolution and radiative effects. Photons radiated from the initial state lead to a tail at low values of ΔE . A sideband region is also defined as the complements of the signal regions enclosed in the ranges $1.70 \text{ GeV}/c^2 < M(\Lambda\pi) < 1.85 \text{ GeV}/c^2$ and $-0.35 \text{ GeV} < \Delta E < 0.25 \text{ GeV}$, indicated by the rectangular boxes in Fig. 1. Only candidates falling within the rectangular boxes in Fig. 1 are analyzed.

We require that charged particles on the signal and tag sides be identified by combining information from various subdetectors to form the likelihood \mathcal{L}_i for particle species i . The ratio $\mathcal{R}_i = \mathcal{L}_i / \sum_j \mathcal{L}_j$ is used to identify protons and muons, where i indicates proton or muon, and j indicates electron, muon, pion, kaon, proton, and deuteron. We require $\mathcal{R}_p > 0.5$ and $\mathcal{R}_\mu > 0.9$, with efficiencies of 98.5% and 95%, respectively; 3.4% (6.5%) probability for misidentifying a proton as a pion (kaon); and 4.1% for misidentifying a muon as a pion. The electron identification relies on a boosted decision tree classifier trained with information from all subdetectors except the silicon vertex detectors. We require the boosted decision tree output $\mathcal{P}^e > 0.9$, with an efficiency of 99.4%, and the rate for misidentifying an electron as a pion is 0.7%. Pion identification relies on the ratio for particle types m and m' ,

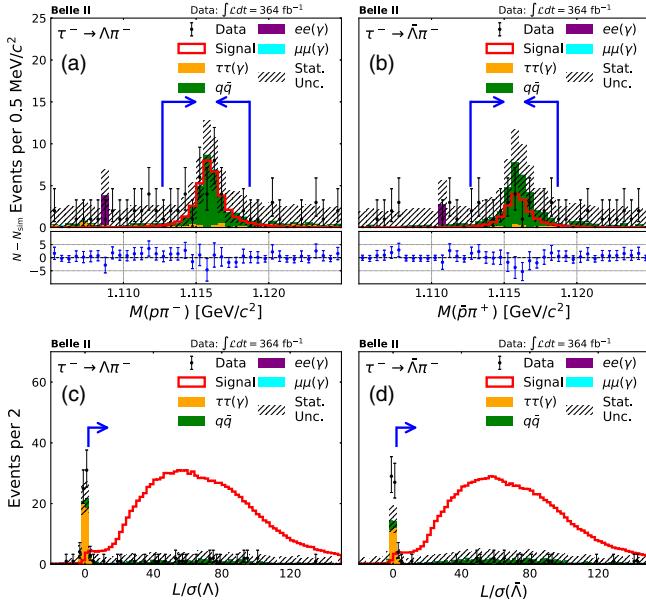


FIG. 2. Distributions of (a) $M(p\pi^-)$ for $\tau^- \rightarrow \Lambda\pi^-$ candidates, (b) $M(\bar{p}\bar{\pi}^+)$ for $\tau^- \rightarrow \bar{\Lambda}\pi^-$ candidates, (c) L/σ of Λ candidates for $\tau^- \rightarrow \Lambda\pi^-$, and (d) L/σ of $\bar{\Lambda}$ candidates for $\tau^- \rightarrow \bar{\Lambda}\pi^-$. The red open histograms show the simulated signal distributions, the filled histograms are stacked to show the simulated background distributions, with statistical uncertainties displayed as hatched areas, and the points with error bars show the distributions of the data in the sideband regions. The simulated signal distribution is arbitrarily scaled. The blue vertical lines and arrows indicate the selected ranges. Beneath (a) and (b) we show the difference between the observed number of candidates N and the expectation from simulation N_{sim} in each bin.

$\mathcal{R}(m|m') = \mathcal{L}_m / (\mathcal{L}_m + \mathcal{L}_{m'})$, where m and m' represent proton, pion, and kaon. For π_τ and π_{tag} , we require $\mathcal{R}(p|\pi) < 0.6$ and $\mathcal{R}(K|\pi) < 0.4$, with identification efficiencies of 98.2% and 99.9%, and the rates for misidentifying a pion as a proton or a pion as a kaon are 9.4% and 8.6%, respectively. The charged particle on the tag side is required to be identified as e , μ , or π .

We reconstruct Λ and $\bar{\Lambda}$ candidates by combining an identified p or \bar{p} with an oppositely charged particle for which no particle identification is required. The resulting $p\pi^-$ and $\bar{p}\bar{\pi}^+$ invariant mass distributions are shown in Figs. 2(a) and 2(b). Candidates within a 6 MeV/c^2 range around the known Λ mass are selected [15]. The flight significance of Λ and $\bar{\Lambda}$ candidates, defined as the ratio L/σ of the flight distance L to its uncertainty σ , is required to be larger than 2.0, as shown in Figs. 2(c) and 2(d). After applying these selections, 83% (88%) of the remaining candidates are correctly identified as Λ ($\bar{\Lambda}$), while the remaining 17% (12%) are due to random combinations of tracks, according to simulation.

The missing momentum p_{miss} is defined as the difference between the total initial momentum and the sum of the momenta of all charged particles and photons. To suppress

backgrounds from $e^+e^- \rightarrow e^+e^-(\gamma)$, $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$, $e^+e^- \rightarrow \ell^+\ell^-\ell^+\ell^-$, and $e^+e^- \rightarrow e^+e^-h^+h^-$ processes, the cosine of the angle between p_{miss} and the track on the tag side is required to be greater than 0.15. The angle $\theta_{\Lambda-\pi_\tau}$ between the momenta of the Λ candidates and π_τ is required to be greater than 0.1 radians. We also require $T_{\text{max}} > 0.9$, where T_{max} is the maximized value of T which was defined in Eq. (1). After these selections are applied, contributions from $e^+e^- \rightarrow \ell^+\ell^-\ell^+\ell^-$ and $e^+e^- \rightarrow e^+e^-h^+h^-$ processes are negligible. At this stage of the analysis, 99.2% (99.3%) of the background is removed, with a signal efficiency of 74.3% (74.7%) for the $\tau^- \rightarrow \Lambda(\bar{\Lambda})\pi^-$ decay modes. The remaining background is dominated by $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$ and $e^+e^- \rightarrow q\bar{q}$ processes.

To further suppress the remaining background, a gradient boosted decision tree (GBDT) classifier is used [45]. We use 15 discriminating observables defined at the event level, on the signal side, and on the tag side as inputs to the classifier. The observables at event level are the sum of the energies of all visible particles E_{vis} ; the magnitude of the missing momentum; the square of the missing mass $M_{\text{miss}}^2 = E_{\text{miss}}^2 - p_{\text{miss}}^2$, where E_{miss} is the difference between the total initial energy and E_{vis} ; the cosine of the polar angle of the missing momentum; the cosine of the angle between the missing momentum and \hat{t} ; and the cosine of the angle between the missing momentum and the $\Lambda\pi$ ($\bar{\Lambda}\pi$) system. The observables related to the signal side are N_γ^{sig} ; E_γ^{sig} ; $\theta_{\Lambda-\pi_\tau}$; the angle between the momenta of π_τ and π_Λ ; and the momentum of the $\Lambda\pi$ ($\bar{\Lambda}\pi$) system. The observables related to the tag side are N_γ^{tag} ; the energy of the most energetic photon on the tag side; the mass of the system recoiling against the track on the tag side; and an identification code assigned to the track's particle-identification information to distinguish between e , μ , and π . We train the GBDT classifier with samples of simulated signal and background events satisfying the selections described above. The GBDT for the two decay channels are trained separately. Signal and background events are divided into training and test samples in a 1:4 ratio. For training the GBDT, we use 360800 (360900) signal and 3900 (5400) background events for the $\tau^- \rightarrow \Lambda\pi^-$ ($\tau^- \rightarrow \bar{\Lambda}\pi^-$) decay mode.

The distributions of the GBDT outputs are shown in Fig. 3. To optimize the requirements on the GBDT output values, we minimize the expected upper limits at 90% credibility level [46] on $\mathcal{B}(\tau^- \rightarrow \Lambda\pi^-)$ and $\mathcal{B}(\tau^- \rightarrow \bar{\Lambda}\pi^-)$ (as described later in the paper) estimated with a simulated sample independent of those used for training and testing. The optimized GBDT selection results in a signal efficiency of 78.8% (82.3%) and background rejection rate of 98.9% (98.8%) for the $\tau^- \rightarrow \Lambda\pi^-$ ($\tau^- \rightarrow \bar{\Lambda}\pi^-$) decay mode. These values are consistent for both the training and test samples. The final signal efficiencies are 9.5% and 9.9% for $\tau^- \rightarrow \Lambda\pi^-$ and $\tau^- \rightarrow \bar{\Lambda}\pi^-$ decays, respectively,

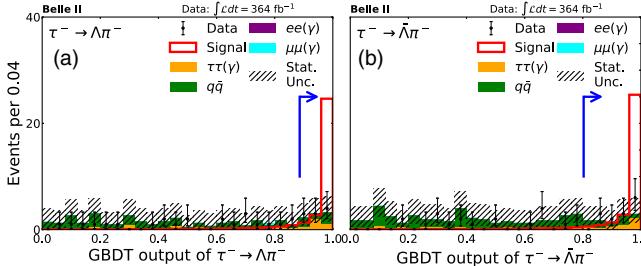


FIG. 3. Distributions of GBDT outputs for (a) $\tau^- \rightarrow \Lambda\pi^-$ and (b) $\tau^- \rightarrow \bar{\Lambda}\pi^-$ samples. The red open histograms show the simulated signal distributions, the filled histograms are stacked to show the simulated background distributions, with statistical uncertainties displayed as hatched areas, and the points with error bars show the distributions of the data in the sideband regions. The simulated signal distribution is arbitrarily scaled. The blue vertical lines and arrows indicate the selected ranges.

which include corrections for PID, trigger, and Λ selection, as described below.

The branching fraction of the $\tau^- \rightarrow \Lambda\pi^-$ decay channel is calculated using

$$\mathcal{B}(\tau^- \rightarrow \Lambda\pi^-) = \frac{N_{\text{sig}}}{2\epsilon_{\text{sig}} \mathcal{L} \sigma_{\tau\tau} \mathcal{B}(\Lambda \rightarrow p\pi^-)}, \quad (2)$$

where N_{sig} is the signal yield, ϵ_{sig} is the signal efficiency, \mathcal{L} is the integrated luminosity, $\sigma_{\tau\tau} = 0.919 \pm 0.003$ nb is the cross section of $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$ [48], and $\mathcal{B}(\Lambda \rightarrow p\pi^-) = 0.641 \pm 0.005$ is the known branching fraction of $\Lambda \rightarrow p\pi^-$ [15]. This formula also applies to the $\tau^- \rightarrow \bar{\Lambda}\pi^-$ decay mode.

Several corrections and systematic uncertainties affect the terms in Eq. (2) and therefore the determination of the branching fraction. The sources are from signal efficiency, luminosity, $\sigma_{\tau\tau}$, $\mathcal{B}(\Lambda \rightarrow p\pi^-)$, and signal yield.

Sources of systematic uncertainties for the signal efficiency include uncertainties in the efficiencies of the Λ selection, GBDT selection, tracking, PID, and trigger. The uncertainty due to the requirement on the flight significance for the Λ selection is studied using a $\Lambda_c^+ \rightarrow \Lambda\pi^+$ control channel. The ratio of Λ selection efficiencies in data and simulation is 0.977 ± 0.002 (0.976 ± 0.002) for $\tau^- \rightarrow \Lambda(\bar{\Lambda})\pi^-$ channel. These values are used to correct the signal efficiencies, and the corresponding uncertainties are taken as systematic uncertainties. The systematic uncertainties on the signal efficiency from the GBDT selection are studied by splitting the training samples into two equal size parts and training the GBDTs separately. The differences between the resulting signal efficiencies after applying the same requirements on the output values as in the nominal analysis are both 0.50% for the two decay channels, which are taken as systematic uncertainties due to the GBDT. The uncertainty due to tracking efficiency is 0.24% per track, estimated with $e^+e^- \rightarrow \tau^+\tau^-$ events,

leading to a systematic uncertainty of 0.96%. The systematic uncertainties from charged-particle identification are studied in $\Lambda \rightarrow p\pi$, $D^{*+} \rightarrow D^0(\rightarrow K^-\pi^+)\pi^+$, and $K_S^0 \rightarrow \pi^+\pi^-$ decays for protons and pions, and in $J/\psi \rightarrow e^+e^-(\gamma)$, $\mu^+\mu^-(\gamma)$, $e^+e^-e^+e^-$, and $e^+e^-\mu^+\mu^-$ processes for muons and electrons. Correction factor to the signal efficiency for the $\tau^- \rightarrow \Lambda(\bar{\Lambda})\pi^-$ decay mode is 0.953 ± 0.021 (0.954 ± 0.022) for the $\tau^- \rightarrow \Lambda(\bar{\Lambda})\pi^-$ channel. The quadratic sum of the individual charged-particle contributions is taken at the systematic uncertainty from charged particle identification. The systematic uncertainty on the trigger efficiency is studied using $e^+e^- \rightarrow \tau^+\tau^-$ events with $\tau^- \rightarrow \pi^-\pi^+\pi^-\nu_\tau$ and τ^+ decaying into the same final states as in the tag side of this analysis [49]. The trigger efficiency measured in data differs by 0.7% from that in simulation; this difference is taken as the systematic uncertainty due to the trigger.

The uncertainty in the branching fraction $\mathcal{B}(\Lambda \rightarrow p\pi^-)$ is 0.78% [15]. The uncertainty of the $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$ cross section, estimated with KKMC, is 0.33% [48]. The uncertainty in the integrated luminosity is 0.60%, determined from the large-angle Bhabha scattering process [50]. The quadratic sum of all the above uncertainties, summarized in Table I, is $\sigma = 2.77\%$ (2.82%) for the $\tau^- \rightarrow \Lambda(\bar{\Lambda})\pi^-$ channel.

The estimate of N_{sig} depends on the expected background yield. Systematic uncertainties affecting the expected background yield and due to data-simulation discrepancies are evaluated from the sideband regions. There are $N_{\text{SB}}^{\text{data}} = 7$ (6) data events in the sideband region for the $\tau^- \rightarrow \Lambda(\bar{\Lambda})\pi^-$ channel, while the corresponding number of events observed in simulated background samples is $N_{\text{SB}}^{\text{sim}} = 3.2^{+1.7}_{-1.2}$ ($5.5^{+2.1}_{-1.6}$), where the uncertainties are due to the limited size of the simulated samples. The data-simulation ratios $f_{\text{bkg}} = N_{\text{SB}}^{\text{data}}/N_{\text{SB}}^{\text{sim}}$ are used as correction factors for the background estimated in the signal region: they are $2.2^{+1.7}_{-1.2}$ ($1.1^{+0.8}_{-0.5}$) for the $\tau^- \rightarrow \Lambda(\bar{\Lambda})\pi^-$ channels, where the uncertainties are statistical only. The relative uncertainties in f_{bkg} are treated as systematic

TABLE I. Summary of fractional systematic uncertainties.

Source	Uncertainty (%)	
	$\tau^- \rightarrow \Lambda\pi^-$	$\tau^- \rightarrow \bar{\Lambda}\pi^-$
Λ selection	0.20	0.20
GBDT selection	0.50	0.50
Tracking efficiency	0.96	0.96
Particle identification	2.21	2.28
Trigger efficiency	0.70	0.70
$\mathcal{B}(\Lambda \rightarrow p\pi^-)$	0.78	0.78
τ -pair cross section	0.33	0.33
Luminosity	0.60	0.60
Total	2.77	2.82

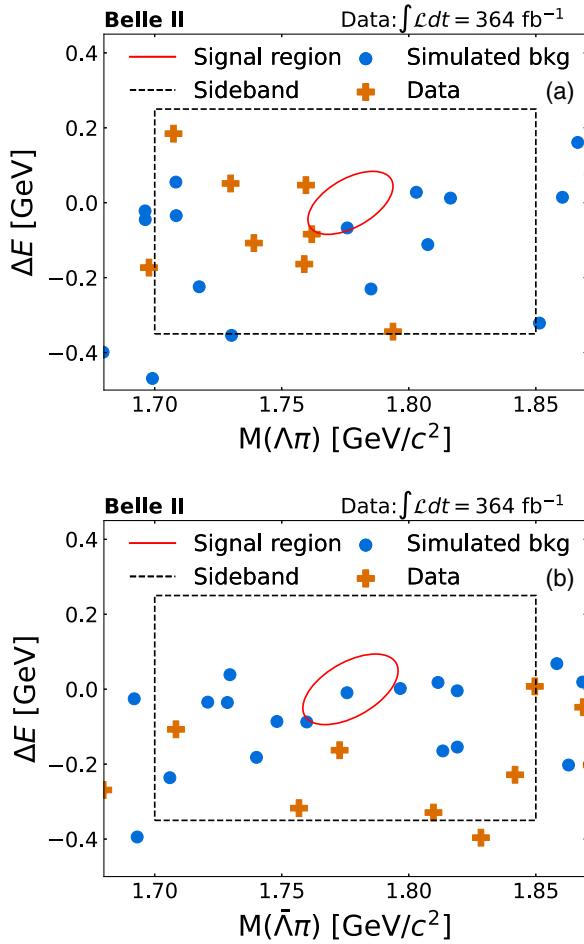


FIG. 4. Distributions of the events for data and simulated background in the ΔE versus $M(\Lambda\pi)$ plane for (a) $\tau^- \rightarrow \Lambda\pi^-$ and (b) $\tau^- \rightarrow \bar{\Lambda}\pi^-$ candidates after applying all selections.

uncertainties on the expected background yield, being $^{+77\%}_{-55\%}$ ($^{+73\%}_{-45\%}$) for the $\tau^- \rightarrow \Lambda(\bar{\Lambda})\pi^-$ channel. Some of the sources of uncertainty listed in Table I also affect the expected background yield. Their contributions are negligible compared to those affecting the correction factors estimated in the sidebands and are not taken into account.

Figure 4 shows the two-dimensional ΔE vs $M(\Lambda\pi)$ distributions for both Belle II data and simulation. The signal excess is calculated by comparing the data yield in the signal region over the expected yield from simulation which is corrected by the data/simulation ratio in the sideband region. From simulation, the expected background yields for $\tau^- \rightarrow \Lambda\pi^-$ and $\tau^- \rightarrow \bar{\Lambda}\pi^-$ within the signal region, after applying the correction factor f_{bkg} , are $N_{\text{exp}} = 1.0^{+1.3}_{-1.1}$ and 0.5 ± 0.6 , respectively, where the uncertainties include both statistical and systematic uncertainties. In the data sample, no events are observed in the signal region in both modes.

Since no signal is observed, we compute upper limits on the signal yields for the $\tau^- \rightarrow \Lambda\pi^-$ and $\tau^- \rightarrow \bar{\Lambda}\pi^-$ channels. We assume that the signal yields N_{sig} and the expected

background yields N_{exp} in the signal region follow a Poisson distribution. We use the prior probability density function $1/\sqrt{N_{\text{sig}} + N_{\text{exp}}}$ to estimate the expected upper limits of signal yields based on a Bayesian approach [51]. The uncertainty in N_{exp} and σ are treated as two independent parameters when estimating the upper limits on the signal yields [51]. The upper limits on the signal yields at 90% credibility level are found to be 1.92 and 1.81, for the $\tau^- \rightarrow \Lambda\pi^-$ and $\tau^- \rightarrow \bar{\Lambda}\pi^-$ channels, respectively. The corresponding upper limits on the branching fractions are 4.7×10^{-8} and 4.3×10^{-8} , respectively, while the expected upper limits based on background simulated samples are 7.2×10^{-8} and 5.5×10^{-8} , respectively. If we use the flat prior probability density function, although it is not advised in Ref. [51], the corresponding upper limits on the branching fractions will be 5.7×10^{-8} and 5.5×10^{-8} . These upper limits do not appreciably change if we do not include systematic uncertainties.

In summary, we present a search for the BNV and lepton number violation decays $\tau^- \rightarrow \Lambda\pi^-$ and $\tau^- \rightarrow \bar{\Lambda}\pi^-$ using a 364 fb^{-1} data sample collected by the Belle II experiment at $\sqrt{s} = 10.58 \text{ GeV}$. No evidence of signal is observed, and upper limits at 90% credibility level on the branching fractions $\mathcal{B}(\tau^- \rightarrow \Lambda\pi^-)$ and $\mathcal{B}(\tau^- \rightarrow \bar{\Lambda}\pi^-)$ are estimated to be 4.7×10^{-8} and 4.3×10^{-8} , respectively. These are the most stringent constraints to date on the branching fraction of $\tau^- \rightarrow \Lambda\pi^-$ with $|\Delta(B - L)| = 2$ and $\tau^- \rightarrow \bar{\Lambda}\pi^-$ with $|\Delta(B - L)| = 0$.

This work, based on data collected using the Belle II detector, which was built and commissioned prior to March 2019, was supported by Higher Education and Science Committee of the Republic of Armenia Grant No. 23LCG-1C011; Australian Research Council and Research Grants No. DP200101792, No. DP210101900, No. DP210102831, No. DE220100462, No. LE210100098, and No. LE230100085; Austrian Federal Ministry of Education, Science and Research, Austrian Science Fund No. P 31361-N36 and No. J4625-N, and Horizon 2020 ERC Starting Grant No. 947006 “InterLeptons”; Natural Sciences and Engineering Research Council of Canada, Compute Canada and CANARIE; National Key R&D Program of China under Contract No. 2022YFA1601903, National Natural Science Foundation of China and Research Grants No. 11575017, No. 11761141009, No. 11705209, No. 11975076, No. 12135005, No. 12150004, No. 12161141008, No. 12405102, and No. 12175041, and Shandong Provincial Natural Science Foundation Project ZR2022JQ02; the Czech Science Foundation Grant No. 22-18469S; European Research Council, Seventh Framework PIEF-GA-2013-622527, Horizon 2020 ERC-Advanced Grants No. 267104 and No. 884719, Horizon 2020 ERC-Consolidator Grant No. 819127, Horizon 2020 Marie Skłodowska-Curie

Grant Agreements No. 700525 “NIOBE” and No. 101026516, and Horizon 2020 Marie Skłodowska-Curie RISE project JENNIFER2 Grant Agreement No. 822070 (European grants); L’Institut National de Physique Nucléaire et de Physique des Particules (IN2P3) du CNRS and L’Agence Nationale de la Recherche (ANR) under Grant No. ANR-21-CE31-0009 (France); BMBF, DFG, HGF, MPG, and AvH Foundation (Germany); Department of Atomic Energy under Project Identification No. RTI 4002, Department of Science and Technology, and UPES SEED funding Programs No. UPES/R&D-SEED-INFRA/17052023/01 and No. UPES/R&D-SOE/20062022/06 (India); Israel Science Foundation Grant No. 2476/17, U.S.-Israel Binational Science Foundation Grant No. 2016113, and Israel Ministry of Science Grant No. 3-16543; Istituto Nazionale di Fisica Nucleare and the Research Grants BELLE2; Japan Society for the Promotion of Science, Grant-in-Aid for Scientific Research Grants No. 16H03968, No. 16H03993, No. 16H06492, No. 16K05323, No. 17H01133, No. 17H05405, No. 18K03621, No. 18H03710, No. 18H05226, No. 19H00682, No. 20H05850, No. 20H05858, No. 22H00144, No. 22K14056, No. 22K21347, No. 23H05433, No. 26220706, and No. 26400255, the National Institute of Informatics, and Science Information NETwork 5 (SINET5), and the Ministry of Education, Culture, Sports, Science, and Technology (MEXT) of Japan; National Research Foundation (NRF) of Korea Grants No. 2016R1D1A1B02012900, No. 2018R1A2B3003643, No. 2018R1A6A1A06024970, No. 2019R1I1A3A01058933, No. 2021R1A6A1A03043957, No. 2021R1F1A1060423, No. 2021R1F1A1064008, No. 2022R1A2C1003993, and No. RS-2022-00197659, Radiation Science Research Institute, Foreign Large-Size Research Facility Application Supporting project, the Global Science Experimental Data Hub Center of the Korea Institute of Science and Technology Information and KREONET/GLORIAD; Universiti Malaya RU grant, Akademi Sains Malaysia, and Ministry of Education Malaysia; Frontiers of Science Program Contracts

No. FOINS-296, No. CB-221329, No. CB-236394, No. CB-254409, and No. CB-180023, and SEP-CINVESTAV Research Grant No. 237 (Mexico); the Polish Ministry of Science and Higher Education and the National Science Center; the Ministry of Science and Higher Education of the Russian Federation and the HSE University Basic Research Program, Moscow; University of Tabuk Research Grants No. S-0256-1438 and No. S-0280-1439 (Saudi Arabia); Slovenian Research Agency and Research Grants No. J1-9124 and No. P1-0135; Agencia Estatal de Investigacion, Spain Grant No. RYC2020-029875-I and Generalitat Valenciana, Spain Grant No. CIDEgent/2018/020; National Science and Technology Council, and Ministry of Education (Taiwan); Thailand Center of Excellence in Physics; TUBITAK ULAKBIM (Turkey); National Research Foundation of Ukraine, Project No. 2020.02/0257, and Ministry of Education and Science of Ukraine; the U.S. National Science Foundation and Research Grants No. PHY-1913789 and No. PHY-2111604, and the U.S. Department of Energy and Research Awards No. DE-AC06-76RLO1830, No. DE-SC0007983, No. DE-SC0009824, No. DE-SC0009973, No. DE-SC0010007, No. DE-SC0010073, No. DE-SC0010118, No. DE-SC0010504, No. DE-SC0011784, No. DE-SC0012704, No. DE-SC0019230, No. DE-SC0021274, No. DE-SC0021616, No. DE-SC0022350, No. DE-SC0023470; and the Vietnam Academy of Science and Technology (VAST) under Grants No. NVCC.05.12/22-23 and No. DL0000.02/24-25. These acknowledgements are not to be interpreted as an endorsement of any statement made by any of our institutes, funding agencies, governments, or their representatives. We thank the SuperKEKB team for delivering high-luminosity collisions; the KEK cryogenics group for the efficient operation of the detector solenoid magnet; the KEK computer group and the NII for onsite computing support and SINET6 network support; and the raw-data centers at BNL, DESY, GridKa, IN2P3, INFN, and the University of Victoria for off-site computing support.

- [1] A. D. Sakharov, *Pis’ma Zh. Eksp. Teor. Fiz.* **5**, 32 (1967).
- [2] S. Weinberg, *Phys. Rev. D* **26**, 287 (1982).
- [3] V. A. Rubakov and M. E. Shaposhnikov, *Usp. Fiz. Nauk* **164**, 493 (1996).
- [4] D. E. Morrissey and M. J. Ramsey-Musolf, *New J. Phys.* **14**, 125003 (2012).
- [5] V. Q. Phong, P. H. Khiem, N. P. D. Loc, and H. N. Long, *Phys. Rev. D* **101**, 116010 (2020).
- [6] A. Papaefstathiou, S. Plätzer, and K. Sakurai, *J. High Energy Phys.* **12** (2019) 017.
- [7] R. Zhou, L. Bian, and H. K. Guo, *Phys. Rev. D* **101**, 091903 (2020).
- [8] D. L. J. Ho and A. Rajantie, *Phys. Rev. D* **102**, 053002 (2020).
- [9] T. Sjöstrand and P. Z. Skands, *Nucl. Phys.* **B659**, 243 (2003).
- [10] V. De Luca, G. Franciolini, A. Kehagias, and A. Riotto, *Phys. Lett. B* **819**, 136454 (2021).
- [11] G. Lazarides, C. Panagiotakopoulos, and Q. Shafi, *Nucl. Phys.* **B278**, 657 (1986).

[12] A. de Gouvea, J. Herrero-Garcia, and A. Kobach, *Phys. Rev. D* **90**, 016011 (2014).

[13] Y. A. Kamyshkov, *AIP Conf. Proc.* **533**, 84 (2000).

[14] F. Wilczek and A. Zee, *Phys. Lett.* **88B**, 311 (1979).

[15] S. Navas *et al.* (Particle Data Group), *Phys. Rev. D* **110**, 030001 (2024).

[16] H. Nishino *et al.* (Super-Kamiokande Collaboration), *Phys. Rev. Lett.* **102**, 141801 (2009).

[17] P. Rubin *et al.* (CLEO Collaboration), *Phys. Rev. D* **79**, 097101 (2009).

[18] R. Godang *et al.* (CLEO Collaboration), *Phys. Rev. D* **59**, 091303 (1999).

[19] P. del Amo Sanchez *et al.* (BABAR Collaboration), *Phys. Rev. D* **83**, 091101 (2011).

[20] J. P. Lees *et al.* (BABAR Collaboration), *Phys. Rev. D* **84**, 072006 (2011).

[21] G. Abbiendi *et al.* (OPAL Collaboration), *Phys. Lett. B* **447**, 157 (1999).

[22] Y. Miyazaki *et al.* (Belle Collaboration), *Phys. Lett. B* **632**, 51 (2006).

[23] S. Maity *et al.* (Belle Collaboration), *Phys. Rev. D* **109**, L031101 (2024).

[24] R. Aaij *et al.* (LHCb Collaboration), *Phys. Lett. B* **724**, 36 (2013).

[25] T. Abe *et al.* (Belle II Collaboration), arXiv:1011.0352.

[26] K. Akai, K. Furukawa, and H. Koiso, *Nucl. Instrum. Methods Phys. Res., Sect. A* **907**, 188 (2018).

[27] J. S. Zhou *et al.* (Belle II Collaboration), *Chin. Phys. C* (2024).

[28] X. Y. Zhou, S. X. Du, G. Li, and C. P. Shen, *Comput. Phys. Commun.* **258**, 107540 (2021).

[29] S. Jadach, B. F. L. Ward, and Z. Was, *Comput. Phys. Commun.* **130**, 260 (2000).

[30] S. Jadach, J. H. Kühn, and Z. Was, *Comput. Phys. Commun.* **64**, 275 (1991).

[31] T. Sjöstrand, S. Ask, J. R. Christiansen, R. Corke, N. Desai, P. Ilten, S. Mrenna, S. Prestel, C. O. Rasmussen, and P. Z. Skands, *Comput. Phys. Commun.* **191**, 159 (2015).

[32] D. J. Lange, *Nucl. Instrum. Methods Phys. Res., Sect. A* **462**, 152 (2001).

[33] G. Balossini, C. M. Carloni Calame, G. Montagna, O. Nicrosini, and F. Piccinini, *Nucl. Phys. B* **758**, 227 (2006).

[34] G. Balossini, C. Bignamini, C. M. Carloni Calame, G. Montagna, O. Nicrosini, and F. Piccinini, *Phys. Lett. B* **663**, 209 (2008).

[35] C. M. Carloni Calame, G. Montagna, O. Nicrosini, and F. Piccinini, *Nucl. Phys. B, Proc. Suppl.* **131**, 48 (2004).

[36] C. M. Carloni Calame, *Phys. Lett. B* **520**, 16 (2001).

[37] C. M. Carloni Calame, C. Lunardini, G. Montagna, O. Nicrosini, and F. Piccinini, *Nucl. Phys. B* **584**, 459 (2000).

[38] F. A. Berends, P. H. Daverveldt, and R. Kleiss, *Nucl. Phys. B* **253**, 421 (1985).

[39] F. A. Berends, P. H. Daverveldt, and R. Kleiss, *Nucl. Phys. B* **253**, 441 (1985).

[40] F. A. Berends, P. H. Daverveldt, and R. Kleiss, *Comput. Phys. Commun.* **40**, 285 (1986).

[41] S. Uehara, arXiv:1310.0157.

[42] E. Barberio, B. van Eijk, and Z. Was, *Comput. Phys. Commun.* **66**, 115 (1991).

[43] T. Kuhr, C. Pulvermacher, M. Ritter, T. Hauth, and N. Braun, *Comput. Software Big Sci.* **3**, 1 (2019).

[44] S. Agostinelli *et al.* (GEANT4 Collaboration), *Nucl. Instrum. Methods Phys. Res., Sect. A* **506**, 250 (2003).

[45] P. Speckmayer and A. Höcker, J. Stelzer, and H. Voss, *J. Phys. Conf. Ser.* **219**, 032057 (2010).

[46] For Bayesian upper limits, it is now standard to quote credibility intervals rather than confidence levels, which are appropriate for frequentist analysis [47].

[47] K. Gray *et al.*, *J. Mod. Appl. Stat. Methods* **14**, 43 (2015), <https://digitalcommons.wayne.edu/jmasm/vol14/iss1/8/>.

[48] S. Banerjee, B. Pietrzyk, J. Michael Roney, and Z. Was, *Phys. Rev. D* **77**, 054012 (2008).

[49] I. Adachi *et al.* (Belle II Collaboration), *J. High Energy Phys.* **09** (2024) 062.

[50] F. Abudinen *et al.* (Belle II Collaboration), *Chin. Phys. C* **44**, 021001 (2020).

[51] Y. S. Zhu, *Nucl. Instrum. Methods Phys. Res., Sect. A* **578**, 322 (2007).