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Novel Representations of the Experiences of Calculus I Students' Participation in the Parallel Spaces of Coursework and Complementary Instruction

Karmen Yu, Ph.D. & Steven Greenstein, Ph.D.

Montclair State University, USA

Abstract: Calculus has long been known for its "gatekeeping" role in postsecondary students' pursuit of STEM careers. In addressing this pressing issue, researchers at Montclair State University developed a model of peer-led complementary instruction to engage Calculus I students in small-group, collaborative problem solving on inquiry-oriented, groupworthy tasks. This work comes from a multiple-case study that sought to address the question, "*How do undergraduate students experience and navigate their calculus learning in the parallel spaces of coursework and inquiry-oriented complementary instruction?*" The analytic representations that were constructed to represent the findings of that study are presented here. Those findings include characterizations of the different forms of Calculus I students' agentive participation and the figured worlds of class and complementary instruction. The analytic representations depict those findings in the form of word clouds and Venn diagrams. The analytical representations of "Victor's" participation are presented and discussed here, and an argument is made for their particular representational power and efficiency. As such, this work seeks to make a methodological contribution to education research that seeks to characterize the nature of participation by the actors in figured worlds.

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Introduction

Historically, Calculus has functioned as a barrier in postsecondary education for students' ambitions to pursue a STEM degree. In fact, Calculus is the leading cause of students' decisions to abandon their pursuit of a career in a STEM field (Hagman et al., 2017). In an effort to transform calculus education to be "lean and lively," calculus reform in the 1990s emphasized the need for fewer topics and an active and engaging approach to learning and teaching (Johnson et al., 2014). After twenty years and very little progress, similar recommendations were made by the President's Council of Advisors on Science and Technology (2012) that calculus instruction should focus on providing students the time they need to develop robust understandings of mathematical concepts in order to succeed. Then, in 2015, the Mathematical Association of America (MAA) published its own Insights and Recommendations (Bressoud et al., 2015) for the design of "effective calculus programs that promote rather than inhibit students' continuation into successful careers in science and engineering" (p. v). Despite the continuing reform effort, Calculus has maintained its gate-keeping status. Informed by these recommendations, the math department at Montclair State University designed and implemented a model of peer-led, inquiry-based complementary instruction called Inquiry-Based Instructional Support (IBIS). We call this model "complementary," because it runs parallel to the Calculus I course and it has its own curriculum and objectives, which are distinct from course objectives. In peer-facilitated (Roth et al., 2001) IBIS workshops, students enrolled in Calculus I engage in small-group, collaborative problem solving on problem sets that consist of non-routine, inquiry-oriented (Buell et al., 2016), groupworthy tasks (Cohen & Lotan, 2014) that promote students' conceptual understanding of calculus concepts.

Following Buell et al. (2016), we conceive of an inquiry orientation as "an approach to knowing and understanding mathematics that draws on and builds upon learners' current knowledge by exploring the mathematical world, asking questions, solving problems, testing conjectures, validating ideas, and explaining relationships" (p. 78). The problems are conceivably realistic and require non-routine thinking so that students explore and develop connections promote conceptual understanding and lay the groundwork for procedural fluency. They also have a low threshold and high ceiling, meaning that they provide students with multiple entry points and allow for multiple solution strategies that can be represented in a variety of ways. Moreover, the problems are thought-revealing, thereby affording opportunities for teachers to interpret and respond to their thinking and students to share their ideas and strategies with one another.

The effectiveness of peer-led cooperative learning models such as IBIS on postsecondary students' academic achievement in various mathematics courses is well documented in the literature (e.g., Altomare & Moreno-Gongora, 2018; Liou-Mark et al., 2015; Trenshaw et al., 2019). However, this literature is dominated by studies that evaluate effectiveness using quantitative methods. What's missing from that literature are insights into the *why*, *how*, and *what* of peer-led cooperative learning models that contribute to these successful outcomes. The study from which this work is drawn used qualitative methods to address this research gap and generate these new insights. The following question framed the inquiry: *How do undergraduate students experience and navigate their calculus learning in the parallel spaces of coursework and inquiry-oriented complementary instruction?* Novel representations of the findings of that study were developed to efficiently depict those experiences. These representations have been well received by those who have read this work and by others who have attended presentations of it. Accordingly, the purpose of this paper is to share these representations with the broader STEM education research community.

Perspectives and Methods

In order for the reader to be prepared to interpret these representations and appreciate their summative power, it is necessary to provide the theoretical framing of the study as well as its methods. Grounded in a situated perspective (Lave & Wenger, 1991) and leveraging its theory of "learning as participation," the study utilized the concept of figured worlds (Holland et al., 1998) to examine changes in students' agentive participation and identity formation (Vågan, 2011). Figured worlds are "socially and culturally constructed realms of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others" (Holland et al., 1998, p. 52). In other words, figured worlds are constructed through people's participation and social engagement with one another. What this means in the context of this study is that the norms and expectations for participation in the figured worlds of class and workshop can be inferred from the ways that students, instructors, and peer leaders interact with each other, with the mathematics, and with all the material artifacts that mediate these interactions (e.g., curricular tasks and graphing calculators).

To answer the research question, an exploratory (Yin, 2003) multiple-case study (Merriam, 1998) approach was taken. The participants were two cohorts of four Calculus I undergraduate students who attended IBIS as a part of their course requirement in fall 2022. Each cohort included four participants from the same class taught by the same instructor, and attended the same workshop facilitated by the same peer leader. Each cohort was taught by a different instructor and worked with a different workshop peer leader. A peer leader was an undergraduate or graduate student who had succeeded in Calculus I and was nominated by faculty members in the Mathematics Department. Video recordings and field notes were taken for all 24 classes, six workshops, and three focus group interviews (Creswell, 2012). Focus group interviews allowed for opportunities to member check interpretations of the observation data. All of the observation data was transcribed and analyzed using the grounded theory analytical approach (Corbin & Strauss, 2014). Specifically, we used Saldaña's (2013) streamlined codes-to-theory model, where explicit segments of data are coded, codes are refined and reduced, and then they are organized into meaningful categories. Subsequently, themes are assigned to a collection of categories to represent an overarching idea. The codes that were applied to the transcripts throughout this analysis and that emerged from it are inferences from observations of students' participation in class and in workshop. As such, they are second-hand accounts of their experiences as opposed to the first-hand accounts that can be generated from participants' self-reported experiences. Intercoder reliability was calculated using percentage of agreement. Since three coders participated in the analysis, each coder was compared to one another in a pairwise manner. Thus, every coding decision had a total number of three pairs to check for agreement. The number of agreements was noted, and ultimately divided by the total number of possible agreements in order to calculate the percentage of agreement. The data presented here had a percentage of agreement of .89, well within the standard put forth by Neuendorf (2002).

The findings in "Victor's" (a pseudonym) case will be presented here. We selected Victor's case with which to demonstrate the power of the representations, because Victor enacted a greater variety of agentive participations in both instructional spaces than any other participant. At the time of data collection, Victor was a sophomore majoring in Exercise Science and Data Science, although he shared that he was planning on switching his major to Mathematics with a minor in Computer Science.

Representations of Findings

The grounded theory approach yielded a long list of codes (e.g., *explaining reasoning*, *sharing ideas*, and *independent work*) and categories (e.g., levels of high, moderate, and nominal interactivity) of participatory actions enacted by each participant. These codes are organized in the table in Figure 1. Each code contains 1, 2, or 3 levels. Level 1 codes appear on the left side of the diagram; their corresponding Level 2 and 3 codes are then organized by the space in which they were enacted and by the cohort who enacted them. For example, for the Level 1 code "sharing," which appears in the first row of the table in Figure 1, the Level 2 codes include those that depict sharing that was "voluntary" or "upon request." Level 3 codes include those that depict the voluntary sharing of "work," an "idea," or an "answer." These participation actions were further organized into three categories of interactivity: *high*, *moderate*, and *nominal* activity. These categories describe the level of students' participatory interactions with one

another and with material resources or tasks. The codes highlighted in purple are codes that the participants had enacted in both class and workshop. While the participation actions enacted by each cohort during class are listed separately under the "Class A" and "Class B" columns, the Level 2 and 3 participation actions enacted by both cohorts in workshop are listed together under the "Cohorts A & B Workshop" column since the workshop space for both cohorts consist of the same norms and expectations for small-group, collaborative problem solving.

Figure 1

Table of Participation Actions Observed Across Both Instructional Spaces

Categories of Interactivity	Level 1 Codes	Cohorts A & B		
		Class A	**Class B**	**Cohorts A & B IBIS Workshop**
High	Sharing	(Voluntary *[Answer] [Idea] [Resources] [Work]*) (Upon request *[Answer] [Idea] [Resources] [Work]*) (Solicit *[Answer]* ***[Resources]*** *[Work]*)		(Voluntary *[Answer^{A}] [Idea^{A}] [Resources^{A}] [Work^{A}]*) (Upon request *[Answer^{A}] [Idea^{A}] [Resources^{A}] [Work^{A}]*) (Solicit *[Answer^{A}]* ***[Idea]*** *[Work^{A}]*) (Offer ***[Work]*** ***[Idea]***)
	Inquiring	(Conceptual) (Procedure)	(Procedure)	(Conceptual^{A}) (Other mathematical) (Procedure^{A})
	Scaffolding			(Scaffolding)
	Explaining	(Concept *[Representation]*) (Mistake *[Peer's]*) (Procedure) (Struggle) (Task) (Technicality)	(Mistake *[Instructor's]*) (Procedure) (Reasoning)	(Concept *[Definition]* *[Representation^{A}]*) (Mistake *[Facilitator's]* *[Peer's^{A}]* *[Self]*) (Notation) (Procedure^{AB}) (Provide Example) (Reasoning^{B} *[Realistic]*) (Struggle^{A}) (Task^{A}) (Technicality^{A})
Moderate	Independent work	(Student initiated *[Task]* ***[Review]*** *[Homework]*) (Instructor initiated)	(Student initiated *[Task]* ***[Review]*** *[Homework]*) (Instructor initiated)	(Student initiated *[Task^{AB}]*) (Facilitator initiated)
	Seeking	(Confirmation) (Help) (Resources) (Time) (Clarification *[About something]* *[For someone]*)	(Confirmation) (Help) (Clarification *[About something]*)	(Confirmation^{AB}) (Help^{AB}) (Resources^{A}) (Time^{A}) (Clarification *[About something^{AB}]* *[For someone^{A}]*)
	Responding	(Agree/Disagree) (Answer) (Confirm) (Respond to help request) (Private) (Uncertain) (Unfamiliar)	(Agree/Disagree) (Answer) (Confirm) (Private) (Unfamiliar)	(Agree/Disagree^{AB}) (Answer^{AB}) (Confirm^{AB}) (Respond to help request^{A}) (Uncertain^{A}) (Unfamiliar^{AB})
	Check-in	(Peer) (Self)	(Self)	(Peer^{AB}) (Self^{AB})
	Check (and revise)	(Compare) (Other's) (Self)	(Self)	(Compare^{A}) (Other's^{A}) (Self^{AB})
	Accessing resources	(Lesson) (Notes) (Online resources)	(Homework) (Notes) (Online resources) (Textbook)	(Homework^{B}) (Notes^{AB}) (Online resources^{AB})
Nominal	Agency request unfulfilled	(Public) (Private)	(Public)	(Private^{A})
	Refraining	(Refraining)	(Refraining)	
	(Re)launches task	(Read aloud)	(Read aloud)	(Read aloud^{AB}) (Recite info) (Invitation to work on problem)
	Emoting	(Affirmation) (Confusion) (Frustration) (Success)	(Affirmation) (Confusion) (Frustration) (Success)	(Affirmation^{AB}) (Confusion^{AB}) (Frustration^{AB}) (Relief) (Success^{AB})
	Note-taking	(Note-taking)	(Note-taking)	(Note-taking^{AB})
	General coursework	(Give) (Seeking)	(Seeking)	(Give^{AB}) (Seeking^{AB})
	Non-participation	(Non-participation)	(Non-participation)	(Non-participation^{AB})

(Lvl 2 code *[Lvl 3 code] [Lvl 3 code]*)
(Lvl 2 code *[Lvl 3 code^{ClassA}] [Lvl 3 code$^{Class\ B}$]*) = Occurred in both spaces

The table along the left side of Figure 1 lists the various forms of participation enacted in class and workshop by all of the participants in the study. Each row consists of a Level 1 participation code (e.g., *sharing*). A 3-column chart appears next to the table. The columns contain the enacted Level 2 participation codes enclosed with parentheses and any corresponding Level 3 participation codes enclosed with brackets. For example, the (*voluntary* [*answer*]) in Class A's column in the *sharing* row indicates *how* a participant in Cohort A shared (*voluntary*, Level 2 code) and *what* they shared (*answer*, Level 3 code) with one or more of their peers. These columns organize these codes by cohort and instructional space (i.e., Class A, Class B, and Cohorts A and B in Workshop). In so doing, they convey what the figured worlds were like in each space. As the contrasting codes in the diagram convey, Class A and Class B operated somewhat differently. Owing mainly to the roles assumed by their instructors, their students took on somewhat different participatory roles. Since the same roles for students and peer leaders were endorsed across all

workshops – i.e., peer-facilitated support for small-group, inquiry-oriented, collaborative problem solving – the workshop column contains Levels 2 and 3 participation codes enacted by all participants in both workshops.

Also, in Figure 1 the shading of rows containing codes in these categories corresponds to these levels of interactivity: *high* codes are yellow, *moderate* codes are orange, and *nominal* codes are pink. These interactivity categories describe students' participatory interactions with one another and with material resources or tasks. The *high interactivity* category refers to agentive participation that exhibits a high level of interaction among students, such as *explaining*, *inquiring*, and *sharing* with one another. The agentive participation actions in this category are often conceptual practices (Pickering, 1995) involving high cognitive demand tasks (Stein et al., 2000) that call on problem solvers to engage in practices such as making associations and connections between mathematical concepts by *explaining concepts*, *inquiring about conceptual* understanding, and *scaffolding* to support each other's problem solving. The agentive participation actions in the *moderate interactivity* category involve moderate-level interactions, such as *working independently* to execute procedures, *seeking confirmation* of *ideas* and the accuracy of answers, and providing brief *responses* to low-level questions (e.g., yes/no or right/wrong). This category includes participation actions involving disciplinary agency (Pickering, 1995), which refers to executing well-established procedures. Lastly, the *nominal interactivity* category describes agentive participation involving limited interactions with peers or material resources, such as *note-taking* and *(re)launching tasks*. Agentive participation in these two categories involves low cognitive demand tasks since they entail memorizing and executing procedures without connecting them to facts or ideas (Stein et al., 2000).

The list of Level 1, 2, and 3 codes was extensive, despite the efforts to reduce in the analysis. As such, when it came time to present the "participation profiles" of each participant's enacted codes in each space over the course of one semester, it became necessary to find a means to present them in a way that clearly and efficiently depicted these profiles and that did not overwhelm the reader. Moreover, these representations needed to serve a second purpose. In order to answer the research question, which asks about students' experiences in each instructional space, these representations needed to depict changes (if any) in students' agentive participation over time by comparing and contrasting their participation from one "round" to the next. [A round is a sequence of observations that concludes with a workshop and includes observations of all of the classes leading up to it.] *Novel uses of Venn diagrams and Word Clouds were designed to solve this problem*. Student-level Venn diagrams, specific to each round of student participation, were designed to depict the evolution of students' participation over time. These diagrams are introduced first. Word clouds were designed to depict a summary overview of each participant's enacted agentive participation in class and in workshop. These will be presented later on in the paper.

Analytic Representations of Victor's Participation

In this section, we introduce these novel forms of Venn diagrams and word clouds. We describe their features and explain the role they played in the analysis. Then we demonstrate their analytic value in the context of Victor's case, where they were used to portray the qualities of his agentive participation both in class and in workshop.

Venn Diagrams: Representing the Evolution of Student Participation

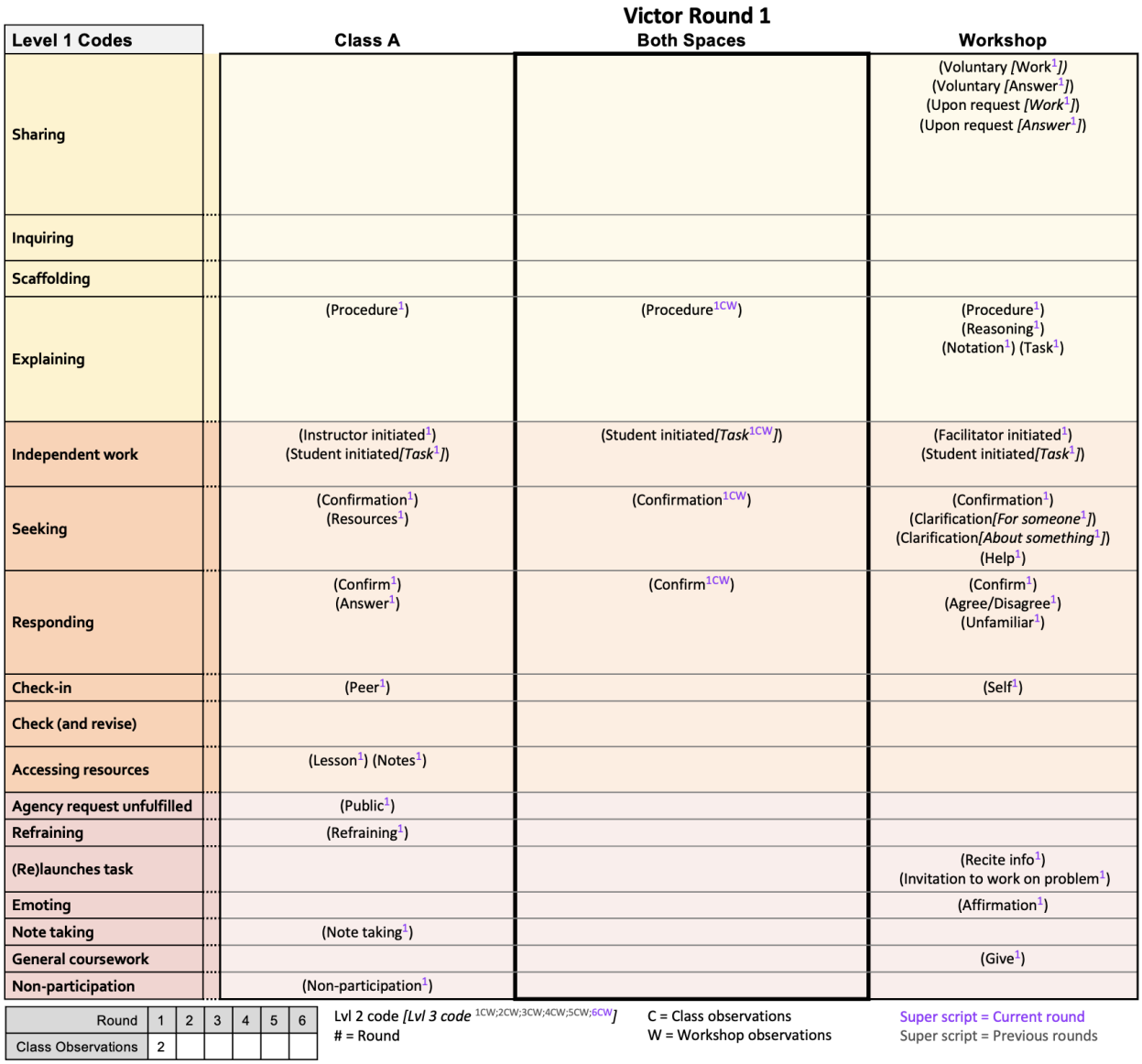
Venn diagrams were designed to illustrate each student's trajectory of participation over the semester. One Venn diagram was drawn for each round of observations. Since there were six workshop observations, each participant has a set of six Venn diagrams. Victor's six Venn diagrams appear in Figures 2 through 7. Figure 2 shows Victor's Venn diagram from the first round of observation. It appears below and will be used for reference as its component parts are explained. Victor's second through sixth-round diagrams appear in Figures 3 through 7 below.

In contrast to the table in Figure 1, which provides all of the participation codes enacted by all participants in each instructional space, student-level Venn diagrams were constructed to depict an overall summary of each student's trajectory of participation across the semester. As such, these Venn diagrams are used in the analysis to address the "navigation" component of the research question.

Figure 2 shows Victor's Venn diagram from the first round of observation. These Venn diagrams' Levels 1, 2, and 3 codes follow the same structure as the table in Figure 1. In addition, a table in the legend at the lower left corner of each Venn diagram shows the number of class sessions that were included in each round of observations. The legend in Figure 2 shows that in the first round of observations there were two class sessions that took place before the first workshop session. The number of class sessions in each round of observations can vary depending on the school calendar. Furthermore, to denote the round and class session in which each Level 2 and 3 code was enacted, a superscript is added to those codes. In the superscript, the number represents the observation round number, the letter *C* represents class sessions, and the letter *W* represents a workshop session. For example, the code *independent work*-(*Student-initiated*)-(*Task*^{1CW}) in the "Both Spaces" column indicates that during round 1, Victor *self-initiated independent work* on a given *task* both in class and in workshop. Since Victor enacted this form of participation in both spaces, the same code appears in the Class and Workshops spaces with superscripts that correspond to those spaces. These codes will appear purple in a superscript when the code was enacted in the current round of observation. As the diagram in Figure 2 represents the first round of observations, all superscripts are purple. In subsequent diagrams, gray superscripts indicate codes that were enacted in rounds previous to the one depicted in the diagram.

Figure 2

Victor's Round 1 Venn Diagram



Lastly, the mutual space of the Venn diagram includes participation codes for actions that were enacted in "Both Spaces" (i.e., class and workshop) in each round of data collection. This overlapping region is used to answer the "navigation" component of the research question. To ask how it is that students navigate the two instructional spaces is to ask whether one's participation in one space somehow impacts their participation in the other. If so, we refer to this phenomenon as an "interaction effect." We take up this question at the end of this section when navigation and interaction effects are explored. To better document and illustrate students' participatory trajectory in class and workshop throughout the entire semester, the participation codes they enacted in both instructional spaces will also be included in the individual instructional space in the Venn diagrams. With this setup, each set of Venn diagrams

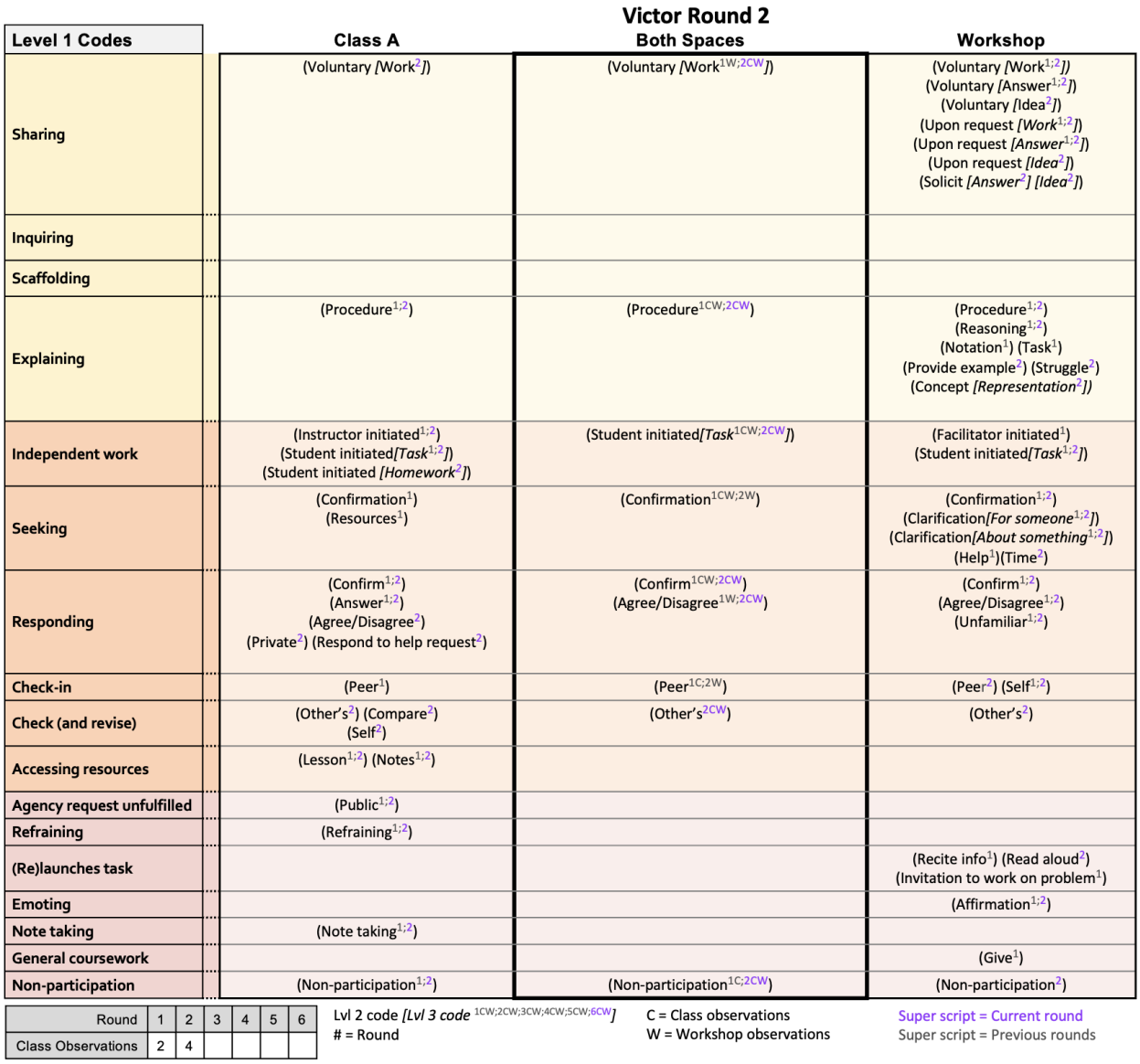
can clearly illustrate the student's participation trajectory as readers flip through each round's Venn diagram and attend to the participation actions enacted in each instructional space.

A review of Victor's set of Venn diagrams in Figures 3 to 7 shows a noticeable expansion period in rounds 2, 3, and 4. (We would encourage the reader to explore these Venn diagrams in the online Appendix [at tinyurl.com/kydissertationappendix] because the site enables the reader to scroll through a set of Venn Diagrams to get a sense of each student's trajectory of participation across a broad expanse of time.) Victor's growth in his participation in both spaces mainly occurs within the high (e.g., *explaining* and *sharing*) and moderate (e.g., *responding* and *seeking*) interactivity categories. Victor's expansion period entailed repetitions in the enactments of the same forms of Level 1 participation albeit in a greater variety of Levels 2 and 3 ways. For example, across the rounds of data collection, there is an observed increase in variation in the how and what (Levels 2 and 3) of Victor's *sharing* and *explaining* (Level 1).

Furthermore, in terms of the forms of participation that are encouraged in each instructional space, a closer analysis of the mutual area of Victor's set of Venn diagrams reveals a trajectory of participation codes enacted in both spaces. There are two conditions when a Level 2 and 3 participation code is added to the mutual area of a Venn diagram. First, the initial time an agentive participation action was enacted in both instructional spaces in the same round, not only will that participation code be added or noted in each instructional space, but it will also be added to the mutual space of the Venn diagram. For example, in Victor's round 1 Venn diagram (Figure 2), he enacted *explaining* (Level 1) *procedure* (Level 2) both in class and workshop. Therefore, the Level 2 code, *procedure*, is added to the Level 1 code, *explaining*, for both class and workshop. Furthermore, the code *procedure* is also added to the mutual space of this Venn diagram with its superscript highlighted in purple. The second condition for a Level 2 and 3 participation code to be added to the mutual space of a Venn diagram is when an agentive participation was first enacted in one instructional space and then later, during a different round of observation, it was enacted in the other instructional space. For example, in Victor's round 3 Venn diagram (Figure 4), he enacted the agentive participation of explaining task in one of the class observations in round 3. This *explaining* task agentive participation action was initially enacted in the first round of workshop. Since it was also observed to be enacted in class during round 3 of observations, the Level 2 participation code, *task*, was added to the Level 1 code, *explaining*, in the mutual space of the round 3 Venn diagram. However, since this agentive participation was not enacted in both instructional spaces within the same round of observation, its superscript in the mutual area is not highlighted in purple. This method enables the researcher to keep track of each act of agentive participation being added to the mutual space of each round's Venn diagram. A closer analysis of the new acts of agentive participation added to each Venn diagram shows that the participation actions (e.g., *sharing voluntary work*, *inquiring procedure*, and *seeking clarification*) that took place in both spaces are most often those that first took place in workshop and thereafter in both spaces. This phenomenon is used as evidence of an interaction effect. Furthermore, the Venn diagram's capacity to reveal this phenomenon is evidence of the analytic power of the representation.

Figure 3

Victor's Round 2 Venn Diagram



Victor Round 2

Level 1 Codes	Class A	Both Spaces	Workshop
Sharing	(Voluntary *[Work²]*)	(Voluntary *[Work^1W;2CW]*)	(Voluntary *[Work^1;2]*) (Voluntary *[Answer^1;2]*) (Voluntary *[Idea²]*) (Upon request *[Work^1;2]*) (Upon request *[Answer^1;2]*) (Upon request *[Idea²]*) (Solicit *[Answer²] [Idea²]*)
Inquiring			
Scaffolding			
Explaining	(Procedure^1;2)	(Procedure^1CW;2CW)	(Procedure^1;2) (Reasoning^1;2) (Notation¹) (Task¹) (Provide example²) (Struggle²) (Concept *[Representation²]*)
Independent work	(Instructor initiated^1;2) (Student initiated*[Task^1;2]*) (Student initiated *[Homework²]*)	(Student initiated*[Task^1CW;2CW]*)	(Facilitator initiated¹) (Student initiated*[Task^1;2]*)
Seeking	(Confirmation¹) (Resources¹)	(Confirmation^1CW;2W)	(Confirmation^1;2) (Clarification*[For someone^1;2]*) (Clarification*[About something^1;2]*) (Help¹)(Time²)
Responding	(Confirm^1;2) (Answer^1;2) (Agree/Disagree²) (Private²) (Respond to help request²)	(Confirm^1CW;2CW) (Agree/Disagree^1W;2CW)	(Confirm^1;2) (Agree/Disagree^1;2) (Unfamiliar^1;2)
Check-in	(Peer¹)	(Peer^1C;2W)	(Peer²) (Self^1;2)
Check (and revise)	(Other's²) (Compare²) (Self²)	(Other's^2CW)	(Other's²)
Accessing resources	(Lesson^1;2) (Notes^1;2)		
Agency request unfulfilled	(Public^1;2)		
Refraining	(Refraining^1;2)		
(Re)launches task			(Recite info¹) (Read aloud²) (Invitation to work on problem¹)
Emoting			(Affirmation^1;2)
Note taking	(Note taking^1;2)		
General coursework			(Give¹)
Non-participation	(Non-participation^1;2)	(Non-participation^1C;2CW)	(Non-participation²)

Round	1	2	3	4	5	6
Class Observations	2	4				

Lvl 2 code *[Lvl 3 code* ^1CW;2CW;3CW;4CW;5CW;6CW*]*
= Round

C = Class observations
W = Workshop observations

Super script = Current round
Super script = Previous rounds

Figure 4

Victor's Round 3 Venn Diagram

Victor Round 3

Level 1 Codes	Class A	Both Spaces	Workshop
Sharing	(Voluntary *[Work^{2}]*) (Upon request *[Work^{3}]*) (Upon request *[Idea^{3}]*)	(Voluntary *[Work$^{1W;2CW}$]*) (Upon request *[Work$^{1W;2W;3CW}$]*) (Upon request *[Idea$^{2W;3CW}$]*)	(Voluntary *[Work$^{1;2}$]*) (Voluntary *[Answer$^{1;2;3}$]*) (Voluntary *[Idea$^{2;3}$]*) (Upon request *[Work$^{1;2;3}$]*) (Upon request *[Answer$^{1;2;3}$]*) (Upon request *[Idea$^{2;3}$]*) (Solicit *[Answer$^{2;3}$] [Idea^{2}]*) (Solicit *[Work^{3}]*)
Inquiring	(Procedure^{3})		
Scaffolding			
Explaining	(Procedure$^{1;2;3}$) (Mistake*[Peer's^{3}]*) (Task^{3})	(Procedure$^{1CW;2CW;3CW}$) (Task$^{1W;3C}$)	(Procedure$^{1;2;3}$) (Reasoning$^{1;2;3}$*[Realistic^{3}]*) (Notation^{1}) (Task^{1}) (Provide example^{2}) (Struggle^{2}) (Concept *[Representation$^{2;3}$]*) (Concept *[Definition^{3}]*)
Independent work	(Instructor initiated$^{1;2;3}$) (Student initiated*[Task$^{1;2;3}$]*) (Student initiated *[Homework$^{2;3}$]*)	(Student initiated*[Task$^{1CW;2CW;3CW}$]*)	(Facilitator initiated^{1}) (Student initiated*[Task$^{1;2;3}$]*)
Seeking	(Confirmation$^{1;3}$) (Resources^{1}) (Clarification*[For someone^{3}]*) (Clarification *[About something^{3}]*)	(Confirmation$^{1CW;2W;3C}$) (Clarification*[For someone$^{1W;2W;3CW}$]*) (Clarification *[About something$^{1W;2W;3C}$]*)	(Confirmation$^{1;2}$) (Clarification*[For someone$^{1;2;3}$]*) (Clarification*[About something$^{1;2}$]*) (Help^{1})(Time$^{2;3}$)
Responding	(Confirm$^{1;2;3}$) (Answer$^{1;2;3}$) (Agree/Disagree^{2}) (Private^{2}) (Respond to help request^{2})	(Confirm$^{1CW;2CW;3CW}$) (Agree/Disagree$^{1W;2CW;3W}$)	(Confirm$^{1;2;3}$) (Agree/Disagree$^{1;2;3}$) (Unfamiliar$^{1;2}$)
Check-in	(Peer^{1}) (Self^{3})	(Peer$^{1C;2W;3W}$) (Self$^{1W;2W;3CW}$)	(Peer$^{2;3}$) (Self$^{1;2;3}$)
Check (and revise)	(Other's$^{2;3}$) (Compare^{2}) (Self$^{2;3}$)	(Other's$^{2CW;3C}$) (Compare$^{2C;3W}$)	(Other's^{2}) (Compare^{3})
Accessing resources	(Lesson$^{1;2}$) (Notes$^{1;2;3}$) (Online resources^{3})		
Agency request unfulfilled	(Public$^{1;2;3}$)		
Refraining	(Refraining$^{1;2;3}$)		
(Re)launches task			(Recite info$^{1;3}$) (Read aloud^{2}) (Invitation to work on problem^{1})
Emoting	(Confusion^{3})		(Affirmation$^{1;2}$)
Note taking	(Note taking$^{1;2;3}$)		
General coursework			(Give^{1})
Non-participation	(Non-participation$^{1;2;3}$)	(Non-participation$^{1C;2CW;3CW}$)	(Non-participation$^{2;3}$)

Round	1	2	3	4	5	6
Class Observations	2	4	4			

Lvl 2 code *[Lvl 3 code $^{1CW;2CW;3CW;4CW;5CW;6CW}$]*
= Round

C = Class observations
W = Workshop observations

Super script = Current round
Super script = Previous rounds

Figure 5

Victor's Round 4 Venn Diagram

Victor Round 4

Level 1 Codes	Class A	Both Spaces	Workshop
Sharing	(Voluntary *[*Work$^{2;4}$*]*) (Voluntary *[*Idea^{4}*]*) (Upon request *[Work^{3}]*) (Upon request *[Idea$^{3;4}$]*)	(Voluntary *[*Work$^{1W;2CW;4C}$*]*) (Voluntary *[*Idea$^{2W;3W;4CW}$*]*) (Upon request *[Work$^{1W;2W;3CW;4W}$]*) (Upon request *[Idea$^{2W;3CW;4CW}$]*)	(Voluntary *[*Work$^{1;2}$*]*) (Voluntary *[*Answer$^{1;2;3;4}$*]*) (Voluntary *[*Idea$^{2;3;4}$*]*) (Upon request *[Work$^{1;2;3;4}$]*) (Upon request *[Answer$^{1;2;3;4}$]*) (Upon request *[Idea$^{2;3;4}$]*) (Solicit *[Answer$^{2;3}$] [Idea^{2}]*) (Solicit *[Work^{3}]*)
Inquiring	(Procedure^{3})		
Scaffolding			
Explaining	(Procedure$^{1;2;3;4}$) (Mistake*[Peer's^{3}]*) (Task^{3})	(Procedure$^{1CW;2CW;3CW;4CW}$) (Task$^{1W;3C;4W}$)	(Procedure$^{1;2;3;4}$) (Reasoning$^{1;2;3;4}$*[Realistic^{3}]*) (Notation^{1}) (Task$^{1;4}$) (Provide example^{2}) (Struggle$^{2;4}$) (Concept *[Representation$^{2;3}$]*) (Concept *[Definition^{3}]*)
Independent work	(Instructor initiated$^{1;2;3;4}$) (Student initiated*[Task$^{1;2;3;4}$]*) (Student initiated *[Homework$^{2;3;4}$]*)	(Student initiated*[Task$^{1CW;2CW;3CW;4CW}$]*)	(Facilitator initiated^{1}) (Student initiated*[Task$^{1;2;3;4}$]*)
Seeking	(Confirmation$^{1;3;4}$) (Resources^{1}) (Clarification*[For someone$^{3;4}$]*) (Clarification *[About something$^{3;4}$]*)	(Confirmation$^{1CW;2W;3C;4CW}$) (Clarification*[For someone$^{1W;2W;3CW;4CW}$]*) (Clarification *[About something$^{1W;2W;3C;4C}$]*)	(Confirmation$^{1;2;4}$) (Clarification*[For someone$^{1;2;3;4}$]*) (Clarification*[About something$^{1;2}$]*) (Help^{1})(Time$^{2;3}$)
Responding	(Confirm$^{1;2;3;4}$) (Answer$^{1;2;3;4}$) (Agree/Disagree$^{2;4}$) (Private^{2}) (Respond to help request^{2})	(Confirm$^{1CW;2CW;3CW;4CW}$) (Agree/Disagree$^{1W;2CW;3W;4CW}$) (Respond to help request$^{2C;4W}$)	(Confirm$^{1;2;3;4}$) (Agree/Disagree$^{1;2;3;4}$) (Unfamiliar$^{1;2;4}$) (Respond to help request^{4})
Check-in	(Peer^{1}) (Self^{3})	(Peer$^{1C;2W;3W}$) (Self$^{1W;2W;3CW;4W}$)	(Peer$^{2;3}$) (Self$^{1;2;3;4}$)
Check (and revise)	(Other's$^{2;3}$) (Compare^{2}) (Self$^{2;3;4}$)	(Other's$^{2CW;3C}$) (Compare$^{2C;3W}$) (Self$^{2C;3C;4CW}$)	(Other's^{2}) (Compare^{3}) (Self^{4})
Accessing resources	(Lesson$^{1;2}$) (Notes$^{1;2;3;4}$) (Online resources$^{3;4}$)		
Agency request unfulfilled	(Public$^{1;2;3;4}$)		
Refraining	(Refraining$^{1;2;3;4}$)		
(Re)launches task			(Recite info$^{1;3}$) (Read aloud$^{2;4}$) (Invitation to work on problem$^{1;4}$)
Emoting	(Confusion^{3}) (Affirmation^{4})	(Affirmation$^{1W;2W;4CW}$)	(Affirmation$^{1;2;4}$)
Note taking	(Note taking$^{1;2;3;4}$)	(Note taking$^{1C;2C;3C;4CW}$)	(Note taking^{4})
General coursework			(Give^{1})
Non-participation	(Non-participation$^{1;2;3;4}$)	(Non-participation$^{1C;2CW;3CW;4CW}$)	(Non-participation$^{2;3;4}$)

Round	1	2	3	4	5	6
Class Observations	2	4	4	3		

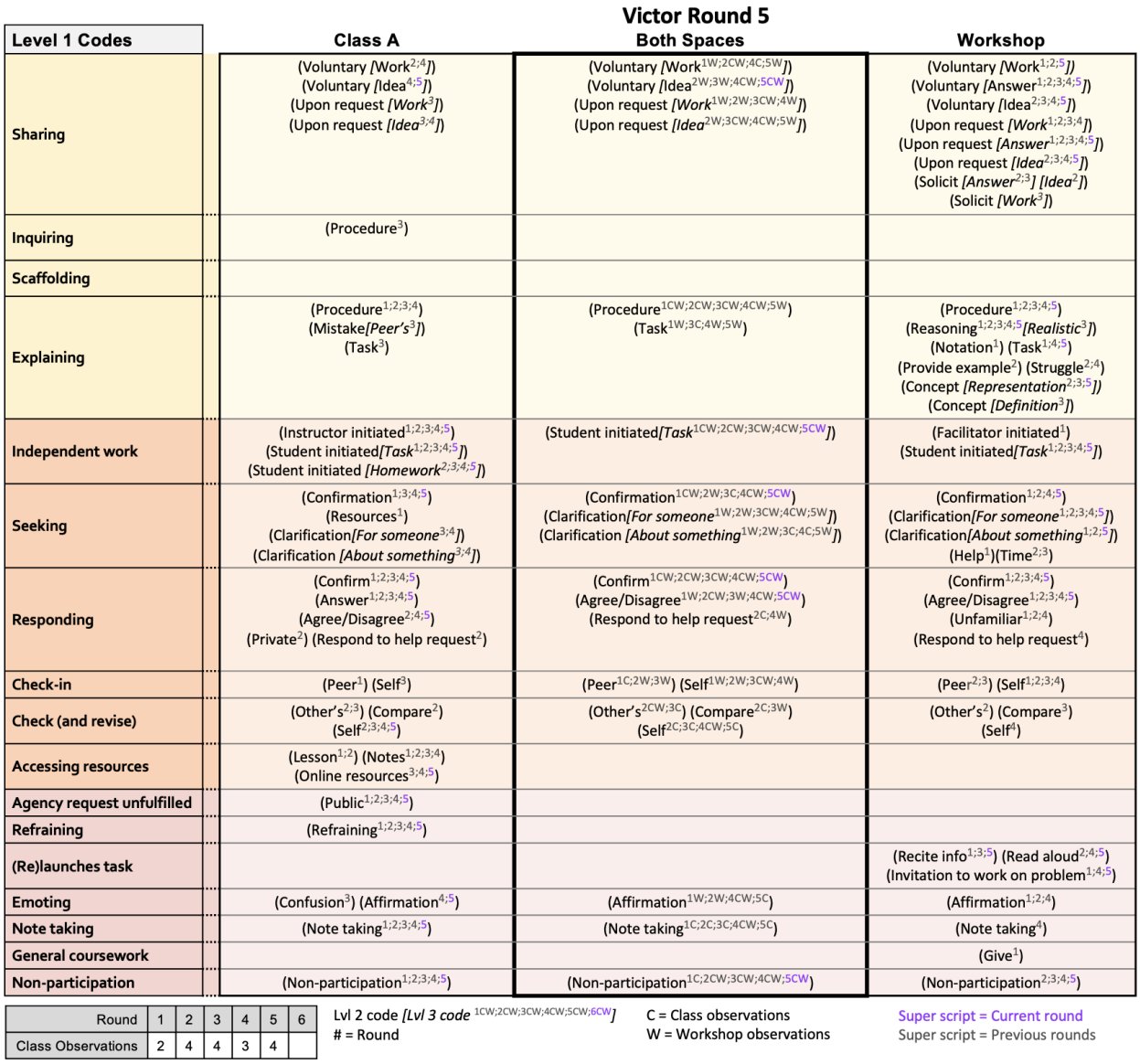
Lvl 2 code *[Lvl 3 code* $^{1CW;2CW;3CW;4CW;5CW;6CW}$*]*
= Round

C = Class observations
W = Workshop observations

Super script = Current round
Super script = Previous rounds

Figure 6

Victor's Round 5 Venn Diagram



Level 1 Codes	Class A	Victor Round 5 Both Spaces	Workshop
Sharing	(Voluntary *[Work*$^{2;4}$*]*) (Voluntary *[Idea*$^{4;5}$*]*) (Upon request *[Work*^{3}*]*) (Upon request *[Idea*$^{3;4}$*]*)	(Voluntary *[Work*$^{1W;2CW;4C;5W}$*]*) (Voluntary *[Idea*$^{2W;3W;4CW;5CW}$*]*) (Upon request *[Work*$^{1W;2W;3CW;4W}$*]*) (Upon request *[Idea*$^{2W;3CW;4CW;5W}$*]*)	(Voluntary *[Work*$^{1;2;5}$*]*) (Voluntary *[Answer*$^{1;2;3;4;5}$*]*) (Voluntary *[Idea*$^{2;3;4;5}$*]*) (Upon request *[Work*$^{1;2;3;4}$*]*) (Upon request *[Answer*$^{1;2;3;4;5}$*]*) (Upon request *[Idea*$^{2;3;4;5}$*]*) (Solicit *[Answer*$^{2;3}$*]* *[Idea*^{2}*]*) (Solicit *[Work*^{3}*]*)
Inquiring	(Procedure^{3})		
Scaffolding			
Explaining	(Procedure$^{1;2;3;4}$) (Mistake*[Peer's*^{3}*]*) (Task^{3})	(Procedure$^{1CW;2CW;3CW;4CW;5W}$) (Task$^{1W;3C;4W;5W}$)	(Procedure$^{1;2;3;4;5}$) (Reasoning$^{1;2;3;4;5}$*[Realistic*^{3}*]*) (Notation^{1}) (Task$^{1;4;5}$) (Provide example^{2}) (Struggle$^{2;4}$) (Concept *[Representation*$^{2;3;5}$*]*) (Concept *[Definition*^{3}*]*)
Independent work	(Instructor initiated$^{1;2;3;4;5}$) (Student initiated*[Task*$^{1;2;3;4;5}$*]*) (Student initiated *[Homework*$^{2;3;4;5}$*]*)	(Student initiated*[Task*$^{1CW;2CW;3CW;4CW;5CW}$*]*)	(Facilitator initiated^{1}) (Student initiated*[Task*$^{1;2;3;4;5}$*]*)
Seeking	(Confirmation$^{1;3;4;5}$) (Resources^{1}) (Clarification*[For someone*$^{3;4}$*]*) (Clarification *[About something*$^{3;4}$*]*)	(Confirmation$^{1CW;2W;3C;4CW;5CW}$) (Clarification*[For someone*$^{1W;2W;3CW;4CW;5W}$*]*) (Clarification *[About something*$^{1W;2W;3C;4C;5W}$*]*)	(Confirmation$^{1;2;4;5}$) (Clarification*[For someone*$^{1;2;3;4;5}$*]*) (Clarification*[About something*$^{1;2;5}$*]*) (Help^{1})(Time$^{2;3}$)
Responding	(Confirm$^{1;2;3;4;5}$) (Answer$^{1;2;3;4;5}$) (Agree/Disagree$^{2;4;5}$) (Private^{2}) (Respond to help request^{2})	(Confirm$^{1CW;2CW;3CW;4CW;5CW}$) (Agree/Disagree$^{1W;2CW;3W;4CW;5CW}$) (Respond to help request$^{2C;4W}$)	(Confirm$^{1;2;3;4;5}$) (Agree/Disagree$^{1;2;3;4;5}$) (Unfamiliar$^{1;2;4}$) (Respond to help request^{4})
Check-in	(Peer^{1}) (Self^{3})	(Peer$^{1C;2W;3W}$) (Self$^{1W;2W;3CW;4W}$)	(Peer$^{2;3}$) (Self$^{1;2;3;4}$)
Check (and revise)	(Other's$^{2;3}$) (Compare^{2}) (Self$^{2;3;4;5}$)	(Other's$^{2CW;3C}$) (Compare$^{2C;3W}$) (Self$^{2C;3C;4CW;5C}$)	(Other's^{2}) (Compare^{3}) (Self^{4})
Accessing resources	(Lesson$^{1;2}$) (Notes$^{1;2;3;4}$) (Online resources$^{3;4;5}$)		
Agency request unfulfilled	(Public$^{1;2;3;4;5}$)		
Refraining	(Refraining$^{1;2;3;4;5}$)		
(Re)launches task			(Recite info$^{1;3;5}$) (Read aloud$^{2;4;5}$) (Invitation to work on problem$^{1;4;5}$)
Emoting	(Confusion^{3}) (Affirmation$^{4;5}$)	(Affirmation$^{1W;2W;4CW;5C}$)	(Affirmation$^{1;2;4}$)
Note taking	(Note taking$^{1;2;3;4;5}$)	(Note taking$^{1C;2C;3C;4CW;5C}$)	(Note taking^{4})
General coursework			(Give^{1})
Non-participation	(Non-participation$^{1;2;3;4;5}$)	(Non-participation$^{1C;2CW;3CW;4CW;5CW}$)	(Non-participation$^{2;3;4;5}$)

Round	1	2	3	4	5	6
Class Observations	2	4	4	3	4	

Lvl 2 code *[Lvl 3 code* $^{1CW;2CW;3CW;4CW;5CW;6CW}$*]*
= Round

C = Class observations
W = Workshop observations

Super script = Current round
Super script = Previous rounds

Figure 7

Victor's Round 6 Venn Diagram

Victor Round 6

Level 1 Codes	Class A	Both Spaces	Workshop
Sharing	(Voluntary *[*Work$^{2;4}$*]*) (Voluntary *[*Idea$^{4;5}$*]*) (Upon request *[Work^{3}]*) (Upon request *[Idea$^{3;4}$]*)	(Voluntary *[*Work$^{1W;2CW;4C;5W;6W}$*]*) (Voluntary *[*Idea$^{2W;3W;4CW;5CW;6W}$*]*) (Upon request *[Work$^{1W;2W;3CW;4W;6W}$]*) (Upon request *[Idea$^{2W;3CW;4CW;5W;6W}$]*)	(Voluntary *[*Work$^{1;2;5;6}$*]*) (Voluntary *[*Answer$^{1;2;3;4;5;6}$*]*) (Voluntary *[*Idea$^{2;3;4;5;6}$*]*) (Upon request *[Work$^{1;2;3;4;6}$]*) (Upon request *[Answer$^{1;2;3;4;5;6}$]*) (Upon request *[Idea$^{2;3;4;5;6}$]*) (Solicit *[Answer$^{2;3;6}$] [Idea^{2}]*) (Solicit *[Work$^{3;6}$]*)
Inquiring	(Procedure^{3})		
Scaffolding			
Explaining	(Procedure$^{1;2;3;4;6}$) (Mistake*[Peer's^{3}]*) (Task^{3})	(Procedure$^{1CW;2CW;3CW;4CW;5W;6CW}$) (Task$^{1W;3C;4W;5W;6W}$)	(Procedure$^{1;2;3;4;5;6}$) (Reasoning$^{1;2;3;4;5;6}$*[Realistic^{3}]*) (Notation^{1}) (Task$^{1;4;5;6}$) (Provide example^{2}) (Struggle$^{2;4}$) (Concept *[Representation$^{2;3;5}$]*) (Concept *[Definition$^{3;6}$]*)
Independent work	(Instructor initiated$^{1;2;3;4;5;6}$) (Student initiated*[Task$^{1;2;3;4;5;6}$]*) (Student initiated *[Homework$^{2;3;4;5;6}$]*)	(Student initiated*[Task$^{1CW;2CW;3CW;4CW;5CW;6CW}$]*)	(Facilitator initiated^{1}) (Student initiated*[Task$^{1;2;3;4;5;6}$]*)
Seeking	(Confirmation$^{1;3;4;5;6}$) (Resources^{1}) (Clarification*[For someone$^{3;4}$]*) (Clarification *[About something$^{3;4}$]*) (Help^{6})	(Confirmation$^{1CW;2W;3C;4CW;5CW;6CW}$) (Clarification*[For someone$^{1W;2W;3CW;4CW;5W;6W}$]*) (Clarification *[About something$^{1W;2W;3C;4C;5W;6W}$]*) (Help$^{1W;6C}$)	(Confirmation$^{1;2;4;5;6}$) (Clarification*[For someone$^{1;2;3;4;5;6}$]*) (Clarification*[About something$^{1;2;5;6}$]*) (Help^{1})(Time$^{2;3}$)
Responding	(Confirm$^{1;2;3;4;5;6}$) (Answer$^{1;2;3;4;5;6}$) (Agree/Disagree$^{2;4;5}$) (Private^{2}) (Respond to help request^{2}) (Unfamiliar^{6})	(Confirm$^{1CW;2CW;3CW;4CW;5CW;6CW}$) (Agree/Disagree$^{1W;2CW;3W;4CW;5CW;6W}$) (Respond to help request$^{2C;4W}$) (Unfamiliar$^{1W;2W;4W;6C}$)	(Confirm$^{1;2;3;4;5;6}$) (Agree/Disagree$^{1;2;3;4;5;6}$) (Unfamiliar$^{1;2;4}$) (Respond to help request^{4})
Check-in	(Peer^{1}) (Self^{3})	(Peer$^{1C;2W;3W}$) (Self$^{1W;2W;3CW;4W;6W}$)	(Peer$^{2;3}$) (Self$^{1;2;3;4;6}$)
Check (and revise)	(Other's$^{2;3}$) (Compare^{2}) (Self$^{2;3;4;5;6}$)	(Other's$^{2CW;3C;6W}$) (Compare$^{2C;3W}$) (Self$^{2C;3C;4CW;5C;6CW}$)	(Other's$^{2;6}$) (Compare^{3}) (Self$^{4;6}$)
Accessing resources	(Lesson$^{1;2;6}$) (Notes$^{1;2;3;4;6}$) (Online resources$^{3;4;5}$)		
Agency request unfulfilled	(Public$^{1;2;3;4;5;6}$)		
Refraining	(Refraining$^{1;2;3;4;5;6}$)		
(Re)launches task			(Recite info$^{1;3;5}$) (Read aloud$^{2;4;5}$) (Invitation to work on problem$^{1;4;5}$)
Emoting	(Confusion^{3}) (Affirmation$^{4;5}$)	(Affirmation$^{1W;2W;4CW;5C}$)	(Affirmation$^{1;2;4}$)
Note taking	(Note taking$^{1;2;3;4;5;6}$)	(Note taking$^{1C;2C;3C;4CW;5C;6CW}$)	(Note taking$^{4;6}$)
General coursework			(Give$^{1;6}$)
Non-participation	(Non-participation$^{1;2;3;4;5;6}$)	(Non-participation$^{1C;2CW;3CW;4CW;5CW;6C}$)	(Non-participation$^{2;3;4;5}$)

Round	1	2	3	4	5	6
Class Observations	2	4	4	3	4	7

Lvl 2 code *[Lvl 3 code* $^{1CW;2CW;3CW;4CW;5CW;6CW}$*]*
= Round

C = Class observations
W = Workshop observations

Super script = Current round
Super script = Previous rounds

Word Clouds: Summative Representations of Student Participation

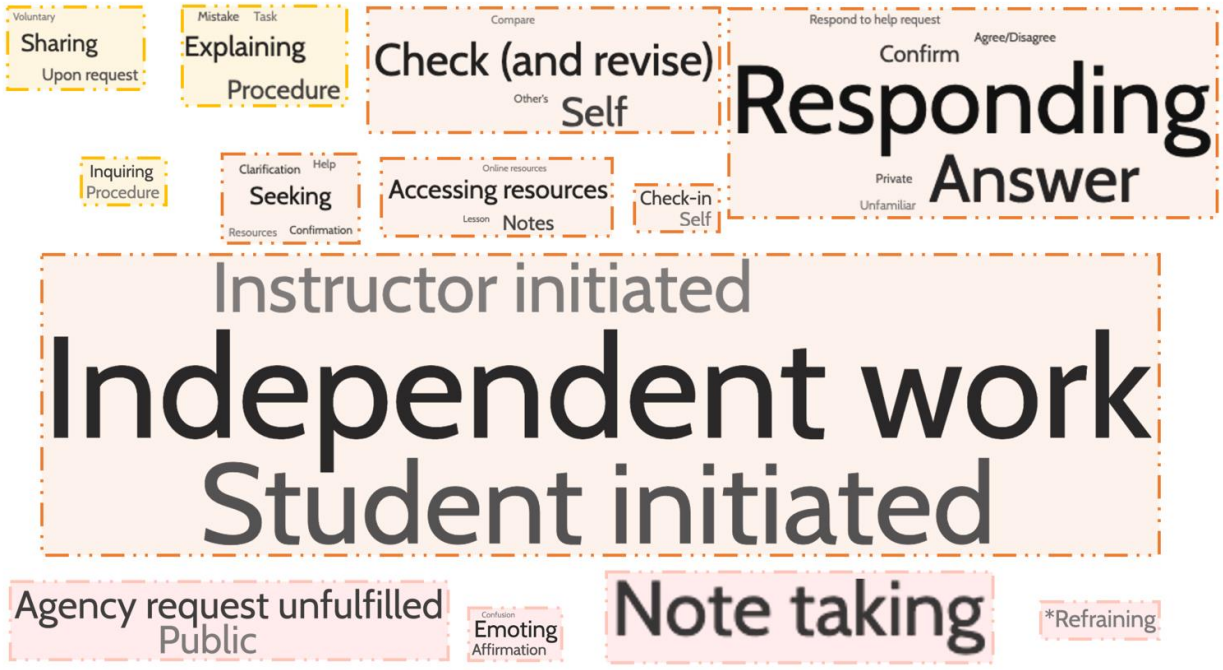
Whereas Venn diagrams were designed to provide a visual representation of each student's participation trajectory, participation world clouds were designed to provide a summary overview of each student's agentive participation in class and workshop. Figures 8 and 9 are Victor's class and workshop participation word clouds, respectively. These word clouds contain clusters of participation codes. The size of each cluster of participation codes is proportional to the frequency with which the participation codes were enacted across the semester. Minor scaling was applied to some clusters to ensure the legibility of all the codes. Hence, in the class word cloud, the *refraining* cluster was

reduced for this purpose. Although minor scaling was applied in the making of these word clouds, I[1] was committed to maintaining the overall sense of proportionality across all clusters. Each cluster consists of a Level 1 participation code with any corresponding Level 2 participation codes, such that the Level 1 code of *seeking* is grouped with its Level 2 codes *confirmation* and *clarification* in Figure 8. As illustrated in this example, the Level 1 code is always the largest code within a cluster. Level 2 codes are relatively smaller in size and appear within the same cluster. Lastly, it is important to note that in order to maintain the readability of these word clouds, Level 3 participation codes are not included.

I should point out that although these representations were used in a study that employed qualitative methods, there is precedent in the literature for the use of quantitative methods, provided that no new inferences are made based on the quantitative methods alone. In this case, frequency counts were used to construct word clouds that provided supplementary support for qualitative evidence (Maxwell, 2010) by yielding new insights into already found phenomena (Hannah & Lautsch, 2011).

Figure 8

Victor's Class Participation Word Cloud



[1] Aspects of this work and the preparation of this manuscript were collaborative between both authors. All instances of first-person pronouns refer to the first author only.

Figure 9

Victor's Workshop Participation Word Cloud

Regarding the opportunities that instructors provide for their students to participate, Victor refrained from participating 220 times across 23 in-person class observations, for an average of about 9 times in each class observation. Of all the students in this class, Victor was most likely the one willing to *respond* to the instructor's questions in class. Similarly, he was also the person most likely to respond to his peer's questions and help requests in class and in workshop by *explaining* or *sharing* his work, his *answers*, and his *ideas* with them. His willingness to *respond* to the instructor's questions is evident in two of the clusters in his class participation word cloud in Figure 8. First, the agentive participation move of *responding* is the second biggest cluster in Victor's class participation word cloud. Since Victor actively *responded* to the instructor's questions most of the time, the size of the cluster for the alienation code of *agency request unfulfilled* is also significantly bigger in his class participation word clouds than the other participants in this cohort.

A side-by-side comparison of Victor's class and workshop participation word clouds shows that the *independent work* characteristic of Victor's participation was magnified in class. His class word cloud further shows that Victor's *independent work* was more likely to be *initiated* by *himself* than by the *instructor*. This comparison is consistent with my observations, as Victor would typically try and work on practice problems independently, even when the instructor was working on examples and reviewing practice problems on the board.

As evident in Victor's workshop participation word cloud, he often took on the *sharer* and *explainer* roles in the group. As illustrated in his class participation word cloud, he also took on the *explainer* role in class to help his

peers. However, there were only limited opportunities for him to take on this role in class due to the norm of independent work. As evident in his class participation word cloud, while Victor offered other types of *explanations* (e.g., *mistakes* and *tasks*) to his peers in class, his *explanations* would most often focus on the *procedural* aspect. While in workshop, he also offered *explanations* about *procedures*, *tasks*, *notation*, and his *struggles* to his peers. This indicates that not only did Victor have more opportunities to enact the explainer role in workshop, there is an important distinction to be made between the kinds of explanations he offered in each instructional space. This is not to claim that students cannot enact these types of explanations in class. Rather, this contrast aims to show the opportunities made available to students to enact participation moves in the high-level interactivity category in each of the two spaces. All in all, the word cloud's capacity to reveal these findings is evidence of the analytic power of the representation.

Concluding Discussion

The analytic representations presented here come from a study that addressed the research question, *How do undergraduate students experience and navigate their calculus learning in the parallel spaces of coursework and inquiry-oriented complementary instruction?* A long list of codes and categories emerged from a grounded theory analysis of students' experiences through observations of the ways they participated with agency in each instructional space. That list could not adequately portray summative assessments of the ways each student participated. Such summative assessments were needed in order to construct participation profiles for each student so that their participation in each instructional space could be compared and contrasted. They were also needed for making inferences about the figured worlds in which these students participated. I developed participation word clouds to solve this analytic hurdle. In addition, efficient representations of students' participation were also needed to discern changes (if any) in the ways each student participated throughout the semester. Student-level Venn diagrams of each round of observation were created for this purpose. Analyses of each student's full set of six Venn diagrams made their participation trajectory throughout the semester apparent. In tandem, these novel representations proved useful for generating profiles of each student's emergent agentive participation and for making inferences about the norms and expectations for student participation in the two instructional spaces.

The power of these analytical representations of word clouds and Venn diagrams was demonstrated here in Victor's case and elsewhere in the case of six other students (Yu, accepted). In order to depict *how undergraduate students experience and navigate their calculus learning in the parallel spaces of coursework and inquiry-oriented complementary instruction*, both a summative view of their participation is needed, as is an analysis of how their participation changes over time. The ability to browse through a student's full set of Venn diagrams allowed for a more evident, visual representation of their participation trajectory as the semester unfolded. The summative analysis of the contrasting ways in which Victor participated in the two spaces was facilitated by side-by-side comparisons of his class and workshop word clouds. In addition, the analysis of Victor's set of Venn diagrams made evident the expansion periods he experienced both in class and in workshop; it also provided evidence of an interaction effect as he navigated those spaces.

We found that these novel representations gave clarity and efficiency to an analytic approach to understanding the nature of students' participation in two distinct instructional spaces for learning mathematics. Such is our rationale for sharing them with the STEM education research community. We propose that they would be of value to education researchers who seek to understand the nature of students' participation in any of the instructional spaces in which STEM education is undertaken.

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Corresponding Author Contact Information:

Author name: Karmen Yu

Department: Department of Mathematics

University, Country: Montclair State University, USA

Email: Yuk4@montclair.edu

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