UNDERGRADUATE STUDENTS' PARTICIPATION IN CALCULUS I COURSEWORK AND PEER-LED, COMPLEMENTARY INSTRUCTION WORKSHOP: CASE OF NEIL

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In postsecondary education, Calculus has been historically recognized as a "gateway course" for students to pursuit STEM fields. Responding to this issue, researchers at Montclair State University designed a model of complementary instruction to engage Calculus I students in collaborative problem solving on groupworthy tasks. This multiple-case study seeks to address the question, "How do undergraduate students experience their calculus learning in the parallel spaces of coursework and inquiry-oriented complementary instruction?" The findings of Neil's case study are presented here and include characterizations of the different forms of agentive participation afforded to students in the two spaces, as well as their complementary nature relative to learning calculus with understanding. Implications for dismantling the persistent barriers imposed by calculus on access to postsecondary STEM fields are also discussed.

Keywords: Calculus, Undergraduate Education, Problem Solving, Calculus Thinking

Calculus has the track record of serving as a "gateway course" that contributes to postsecondary students abandoning their pursuit of a STEM career (Hagman et al., 2017). The calculus reform effort in the 1990s emphasized to include fewer topics and incorporate an active learning and teaching approach aiming to transform calculus education to be "lean and lively" (Johnson et al., 2014). Twenty years later, the President's Council of Advisors on Science and Technology (2012) made a similar recommendation in order to provide students the time they need to develop robust understandings of mathematical concepts in order to succeed. Despite the continuing reform effort, the gate-keeping function of Calculus has hardly changed.

Drawing on the Mathematical Association of America's seven recommendations from the Insights and Recommendations (Bressoud et al., 2015), researchers at Montclair State University designed an inquiry-based *complementary* workshop, called Inquiry-Based Instructional Support (IBIS), facilitated by a peer leader (Roth et al., 2001) to run parallel to students' in class learning. During IBIS, students work collaboratively in small groups on groupworthy tasks (Buell et al., 2016) that are non-routine problems to promote conceptual understanding of calculus concepts.

The literature on peer-led cooperative learning models in postsecondary education confirms their effectiveness on students' academic achievement across different undergraduate mathematics courses (Altomare & Moreno-Gongora, 2018; Trenshaw et al., 2019). However, as the literature mainly focuses on evaluating the effectiveness using quantitative methods, there is a lack of insight into why, how, and what about peer-led cooperative learning models that contributes to these successful outcomes. Hence, this study seeks to address the question, *How do undergraduate students experience their calculus learning in the parallel spaces of coursework and inquiry-oriented complementary instruction?*

Perspectives and Methods

This exploratory (Yin, 2003) multiple-case study (Merriam, 1998) is grounded in a situated perspective (Lave & Wenger, 1991) leveraging the "learning as participation" aspect and utilized the concept of figured world (Holland et al., 1998) to examine the change in students' agentive participation and their identity formation (Vågan, 2011). To answer the research question, all of the observation video recordings were transcribed and analyzed using the grounded theory analytical approach (Corbin & Strauss, 2014). To depict a summary overview of each case study participant's enacted agentive participation in class and IBIS, a word cloud with agentive participation codes as clusters was created for each instructional space.

The participants of this study consist of two cohorts of Calculus I undergraduate students whose IBIS attendance is a part of their course requirement. Each cohort has four participants from the same class and attended the same IBIS sessions. Video recordings and field notes were taken for all 24 classes, six workshops, and three focus group interviews (Creswell, 2012).

Findings

The table in Figure 1 shows the various forms of participation enacted in class and IBIS by both cohorts' participants. These participation actions were organized into *high*, *moderate*, and *nominal* interactivity categories to describe students' participatory interactions with others, material resources, or tasks. Next, Neil's case (pseudonym) will illustrate how the participation codes and interactivity categories are used to characterize his participation in both spaces.

		Cohorts A & B		
Categories of Interactivity	Level 1 Codes	Class A	Class B	Cohorts A & B IBIS Workshop
High	Sharing	(Voluntary *[Answer] [Idea] [Resources] [Work]*) (Upon request *[Answer] [Idea] [Resources] [Work]*) (Solicit *[Answer] [Resources] [Work]*)		(Voluntary *[Answer^{A}] [Idea^{A}] [Resources^{A}] [Work^{A}]*) (Upon request *[Answer^{A}] [Idea^{A}] [Resources^{A}] [Work^{A}]*) (Solicit *[Answer^{A}] [Idea] [Work^{A}]*) (Offer *[Work] [Idea]*)
	Inquiring	(Conceptual) (Procedure)	(Procedure)	(Conceptual^{A}) (Other mathematical) (Procedure^{A})
	Scaffolding			(Scaffolding)
	Explaining	(Concept *[Representation]*) (Mistake *[Peer's]*) (Procedure) (Struggle) (Task) (Technicality)	(Mistake *[Instructor's]*) (Procedure) (Reasoning)	(Concept *[Definition] [Representation^{A}]*) (Mistake *[Facilitator's] [Peer's^{A}] [Self]*) (Notation) (Procedure^{AB}) (Provide Example) (Reasoning^{B} *[Realistic]*) (Struggle^{A}) (Task^{A}) (Technicality^{A})
Moderate	Independent work	(Student initiated *[Task] [Review] [Homework]*) (Instructor initiated)	(Student initiated *[Task] [Review] [Homework]*) (Instructor initiated)	(Student initiated *[Task^{AB}]*) (Facilitator initiated)
	Seeking	(Confirmation) (Help) (Resources) (Time) (Clarification *[About something] [For someone]*)	(Confirmation) (Help) (Clarification *[About something]*)	(Confirmation^{AB}) (Help^{AB}) (Resources^{A}) (Time^{A}) (Clarification *[About something^{AB}] [For someone^{A}]*)
	Responding	(Agree/Disagree) (Answer) (Confirm) (Respond to help request) (Private) (Uncertain) (Unfamiliar)	(Agree/Disagree) (Answer) (Confirm) (Private) (Unfamiliar)	(Agree/Disagree^{AB}) (Answer^{AB}) (Confirm^{AB}) (Respond to help request^{A}) (Uncertain^{A}) (Unfamiliar^{AB})
	Check-in	(Peer) (Self)	(Self)	(Peer^{AB}) (Self^{AB})
	Check (and revise)	(Compare) (Other's) (Self)	(Self)	(Compare^{A}) (Other's^{A}) (Self^{AB})
	Accessing resources	(Lesson) (Notes) (Online resources)	(Homework) (Notes) (Online resources) (Textbook)	(Homework^{B}) (Notes^{AB}) (Online resources^{AB})
Nominal	Agency request unfulfilled	(Public) (Private)	(Public)	(Private^{A})
	Refraining	(Refraining)	(Refraining)	
	(Re)launches task	(Read aloud)	(Read aloud)	(Read aloud^{AB}) (Recite info) (Invitation to work on problem)
	Emoting	(Affirmation) (Confusion) (Frustration) (Success)	(Affirmation) (Confusion) (Frustration) (Success)	(Affirmation^{AB}) (Confusion^{AB}) (Frustration^{AB}) (Relief) (Success^{AB})
	Note-taking	(Note-taking)	(Note-taking)	(Note-taking^{AB})
	General coursework	(Give) (Seeking)	(Seeking)	(Give^{AB}) (Seeking^{AB})
	Non-participation	(Non-participation)	(Non-participation)	(Non-participation^{AB})

(Lvl 2 code *[Lvl 3 code] [Lvl 3 code]*)
(Lvl 2 code *[Lvl 3 code^{ClassA}] [Lvl 3 code$^{Class B}$]*) = Occurred in both spaces

Figure 1: A table of participation actions in class and IBIS for Cohorts A and B.

Neil's Participation Profile

Neil was a private student both in class and IBIS. He spent most of his time in class *taking notes* and, in both spaces, *working independently* on the task at hand. Regarding the opportunities that the instructor provided for students to participate, Neil refrained from participating 323 times across 23 in-person class observations, for an average of about 14 times per class observation. The class and IBIS word clouds, in Figures 2A and 2B, provide a summary overview of Neil's participation in both spaces. A comparison of his class and IBIS word clouds shows that his independent participation characteristics tended to be magnified in class.

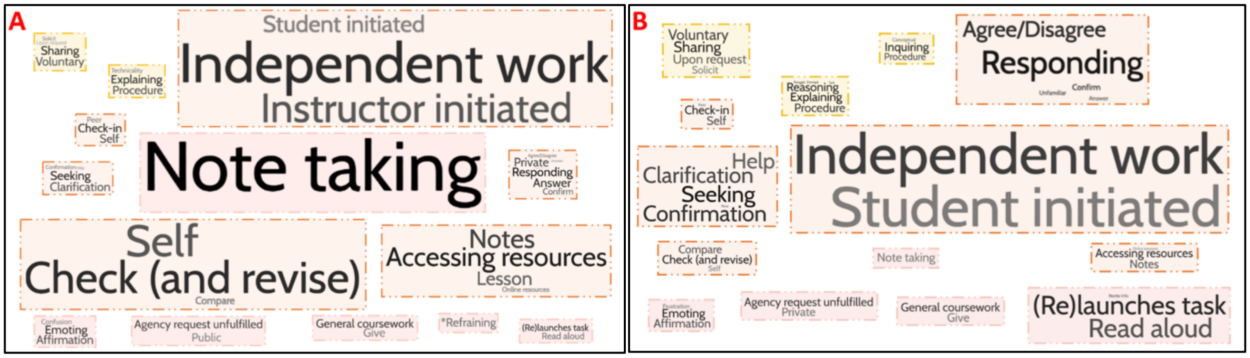

Figure 2: Neil's class (A) and IBIS (B) participation word clouds

Early in the course, Neil's participation consisted almost exclusively of *taking notes* and *working independently* on problems posed by the instructor, and then waiting for the instructor or another student to provide a solution. Every so often, as Neil worked on problems independently in both spaces, he would reference a variety of resources, such as his notes and online resources.

The size of the *independent work* cluster in Neil's IBIS word cloud suggests that though he also tended to work independently in IBIS, groupwork in IBIS offered opportunities and space for him to be a more active and interactive participant. The biggest clusters in his class word cloud are participation actions with moderate interactions with tasks and material resources (e.g., *note taking*, *independent work*, *accessing resources*, and *checking and revising*). In contrast, some of the biggest clusters in his IBIS word cloud are participation actions that have moderate interactions with his peers (e.g., *responding* and *seeking clarification*, *confirmation*, and *help)*. This suggests that more of his interactions in IBIS were with peers than with tasks and material resources. Moreover, as his IBIS word cloud also reveals, Neil was more likely to *respond* to his peers than to initiate interactions with them. He was also more likely to *seek confirmation*, *clarification*, and *help* from his peers than to enact the explainer role. The following excerpt illustrates some of these forms of participation from Neil during IBIS. In this excerpt, his group was discussing a composition function/chain rule problem, given the rates of change of profit per book sale, $p'(s)$, and book sales per month, $s'(t)$.

Table 1: An excerpt of a Chain rule discussion in the third IBIS session.

1	Amelia	Uhm. Well, for [#3] c, I just wrote using the chain rule. I kind of wrote the first part. And then for [#3] b, I just plugged in 4 to s. Like, to $s(t)$ I used 4. And then the answer I plugged it into $p(s)$. And then I just explained that in [#3] c. <...>
2	Neil	I thought you have to put one into the other.
3	Rachel	I did.
4	Amelia	Yea, that's kind of what I did.
5	Rachel	I thought you just have to plug it into the equation it goes with. Like for [#3] a, you just plug it into the $s(t)$. No? For $p(s)$…-cause like it gives you an example.
6	Neil	Yea, you put this portion here into s. <Points to his work on paper>
7	Rachel	So, for [#3] a, you got 16?
8	Neil	Huh? Yea.
9	Rachel	And then for [#3] b, you got 160, too?
10	Neil	Uh huh.
11	Rachel	And then what about [#3] c? What did you do?
12	Neil	You plug this portion into s and that becomes p and then you take the derivative and then you take the derivative becomes $p'(t)$.
13	Rachel	Oh! So, you do 3… <Continues to work on problem independently>
14	Neil	Yea, and then there is…we're done.
Note: <actions>; (unclear utterance); -interruption/cut off-; and [words inserted for clarity]		

In this excerpt, Neil responds to Amelia's (pseudonym) invitation by *sharing* his *ideas* about what to do for this problem (lines 2 and 6). Upon Rachel's (pseudonym) further request for him to share his work with her (line 11), Neil *explains* the *procedures* he took to determine $p'(t)$ (line 12). Even though Neil spent a lot of time in IBIS working independently, in contrast to his class participation, he was also a more active and interactive learner in that space by *sharing* with, *explaining* to, and *seeking* from his peers. As the semester progressed, there was some evolution in how Neil *shared*, *explained*, and what he *sought* from his peers in IBIS.

Overall, even though *note taking* continued to be the dominant form of Neil's participation throughout the semester, as the semester progressed, his participation in both spaces expanded from the predominantly nominal interactions of *note taking* and *working independently* to include both moderate and high interactions (e.g., *seeking*, *explaining,* and *sharing)*. The next excerpt illustrates his participatory expansion trajectory in class from mid-semester. In this excerpt, the class was working on finding the derivative of $f(x) = \sqrt[3]{2x^3 + \sin^2(5x)}$ posed by the instructor. Neil overheard Amelia expressing her confusion and took the initiative to check on her.

Table 2: An excerpt of Neil checking in on Amelia in class.

1	Amelia	I am so confused.
2	Neil	<Overhearing Amelia> Well, what are you confused about?
3	Instructor	<In the background of Neil and Amelia's interaction> Alright. Ah. So, let's quickly differentiate the whole equation.
4	Amelia	Where [does] cosine come from at the end?
5	Neil	<Looks back at his work> When I think about it- When I think about it in 2 pieces, [in terms of] 2 pieces. <Shows Amelia his work>
6	Amelia	Wait. Hold on. Let me finish [writing]. <Looks at Neil's work>
7	Neil	Well, she's talking about quotient (inaudible) first thing (inaudible) the number 2-
8	Amelia	Like, that I got.
9	Neil	Yeah, number 2, then (inaudible) it still has to be opened up (inaudible) so it will be (inaudible) plus (inaudible) sine. The opposite of sine is cosine, so cosine of 5 and then you just grab the last piece, (which is) 5.
10	Amelia	I kind of get it.

In this excerpt, Neil *seeks clarification* on what confused Amelia (line 2). Even when the instructor calls for the class's attention to go over the problem (line 3), Neil and Amelia continue to carry on with their conversation. After Amelia clarifies her confusion (lines 4 and 8), he offers his *explanation* to help her resolve it (lines 5, 6, and 9). This excerpt is one of the examples that illustrates the evolution in the interactivity of Neil's participation. As the semester progressed, Neil also enacted new kinds of *responding*, *sharing*, and *seeking* actions in both spaces.

Discussion and Conclusion

To summarize, this study found a range of agentive participation actions that were further categorized into *high*, *moderate*, and *nominal* interactivity categories based on the quality of their interactions with others, tasks, or material resources. These findings can inform and guide the design and implementation of parallel spaces of coursework and complementary instruction, particularly when the realities of coursework alone impose constraints that do not allow for adequate opportunities for high and moderately interactive participation. Specifically, these findings would be of value to postsecondary calculus educators and program directors who are committed to offering students the kinds of participatory experiences that are productive for their learning of calculus. That way, they can be more mindful in planning, structuring, and designing their calculus programs so as to dismantle the persistent barriers imposed by calculus on access to postsecondary STEM fields.

References

Altomare, T. K., & Moreno-Gongora, A. N. (2018). The role and impact of supplemental instruction in accelerated developmental math courses. *Journal of College Academic Support Programs*, *1*(1), 19-24.

Bressoud, D., Mesa, V., & Rasmussen, C. (Eds.). (2015). *Insights and recommendations from the MAA national study of college calculus*. The Mathematics Association of America.

Buell, C. A., Greenstein, S., & Wilstein, Z. (2016). Constructing an inquiry orientation from a learning theory perspective: Democratizing access through task design. *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies*, *27*(1), 75-95.

Corbin, J., & Strauss, A. (2014). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (4th ed.). Sage.

Creswell, J. W. (2012). *Educational research: Planning conducting and evaluating quantitative and qualitative research* (4th ed.). Pearson.

Hagman, J. E., Estrella, J., & Fosdick, B. K. (2017). Factors contributing to students and instructors experiencing a lack of time in college calculus. *International Journal of STEM Education*, *4*(1), 15.

Holland, D., Lachicotte Jr., W., Skinner, D., & Cain, C. (1998). *Identity and agency in cultural worlds*. Harvard University Press.

Johnson, E., Ellis, J., & Rasmussen, C. (2014). It's about time: How instructors and students experience time constraints in calculus I. the 38th Conference of the International Group for the Psychology of Mathematics Education and the 36th Conference of the North American Chapter of the Psychology of Mathematics Education, Vancouver, British Columbia: PME.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge University Press.

Merriam, S. B. (1998). *Qualitative research and case study applications in education: Revised and expanded from case study research in education*. Jossey-Bass Publishers.

President's Council of Advisors on Science and Technology (PCAST). (2012). *Report to the president—Engage to excel: Producing one million additional college graduates with degrees in science, technology, engineering, and mathematics*. Office of the President.

Roth, V., Goldstein, E., & Marcus, G. (2001). *Peer-led team learning: A handbook for team leaders*. Prentice Hall.

Trenshaw, K. F., Aish, N., Miskioglu, E. E., & Asare, P. (2019). *Leaders like me* The Collaborative Network for Engineering and Computing Diversity (CoNECD) Conference, Crystal City, VA.

Vågan, A. (2011). Towards a sociocultural perspective on identity formation in education. *Mind, Culture, and Activity*, *18*(1), 43-57.

Yin, R. K. (2003). *Case study research design and methods* (3rd ed.). Sage.