# A Novel Thermal Modeling Analysis for Liquid-Cooled High-Power EV Chargers

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Abstract—As transportation electrification keeps accelerating across a wide range of vehicle classes from light-duty cars to heavy-duty trucks, the need for high-power electric vehicle (EV) charging equipment continues to grow rapidly. Even though the advancements in power electronics are enabling higher efficiency for EV chargers, thermal management continues to be a significant challenge in high-power charger development. Liquid cooling with cold plates is commonly used for dissipating the heat generated by semiconductor devices in high-power chargers. To design an effective and optimized thermal management system, accurate thermal modeling and analysis are critical, especially in the preliminary design phases. Complex fluid dynamics (CFD) software such as Ansys has been widely used for thermal modeling and analysis in the literature; however, using CFD analysis tools can be expensive, time-consuming, and computationally intense. To address the technical needs for a rapid, accurate preliminary thermal analysis tool, this paper presents a novel and accurate thermal modeling and analysis approach for highpower EV chargers with liquid cooling and Silicon Carbide (SiC) MOSFETs mounted on cold plates. The proposed modeling and analysis approach utilizes a lumped element model for each of the many pieces within the system to mathematically represent the physical system and form thermal networks. The effectiveness, accuracy, and light computational load of the proposed approach have been validated through experimental results conducted on a 21 kW power converter module hardware from a 1 MW EV wireless charger developed by the team for Class 8 semi-trucks.

### I. INTRODUCTION

As the adoption of electric vehicles (EVs) continues increasing, the need for high-power charging infrastructure is growing. As the deployment of public high-power chargers keeps growing to meet this demand, EV users are also experiencing reliability issues in the current charging infrastructure [1]. To evaluate charging equipment reliability and advance their design to improve reliability, accurate and rapid thermal analysis and modeling methods for high-power EV chargers are necessary [2], [3].

Thermal modeling and analyses in the literature for highpower power converters often use finite element (FE) analysis tools [4], and often take hours to simulate. Thermal analysis tools often used in conjunction with reliability evaluation often neglect the thermal interactions between components in the same converter and may not consider the potential effects of cooling systems [5], [3]. Typical cooling system design tools often leverage complex fluid dynamic (CFD) tools [6], [7], [8]. These tools, while accurate, are not as practical when analyzing long mission profiles, performing life-cycle analysis, and quickly evaluating early design choices. In [9] a simplified FE model was developed to predict the thermal behavior of an ISOTOP diode module and TO-247 MOSFET module, while a simplified model may be quicker, FE software is still necessary and simulation time was not mentioned in the work. In [10], [4], and [11], lumped element (LE) models are used; however, the LE parameters were derived from impulse responses generated using FE analysis software.

The current literature is also sparse surrounding the effects of cooling on the thermal behavior of high-powered EV chargers. In [12], liquid cooling is modeled for a multi-cell battery but not for an EV charger. In [13], a high powered onboard charger is analyzed using CFD. In [14], a LE model of an onboard hybrid charger with heat sink cooling is presented, however, the complete system is simple, with passive cooling, and simulation data is only provided for 0.5 seconds of runtime.

CFD and FE analysis can provide very accurate, highresolution information about the thermal behavior of power converter components under load and their associated cooling hardware; however, due to computational complexity it is impractical to simulate large systems or create long-duration simulations. Due to the time required to perform these simulations, they are not as useful in evaluating the rapid design changes that occur early in system design.

An accurate and rapid thermal modeling analysis tool finds its place where thermal information about large systems needs to be gleaned rapidly or over a long period of simulation time, and where rapid thermal analysis can aid in design for reliability. Where this technology may fall short is in situations where high accuracy and resolution are critical.

This particular lumped element modeling and thermal analysis tool is best leveraged as a tool in power converter engineers toolbox, particularly in preliminary design. The quick simulation time and relative accuracy allow an engineer to quickly evaluate the effects of design changes on the thermal behavior of the power converter. From this thermal behavior converter reliability can also be quickly assessed. The ability to rapidly analyze converter thermal performance and reliability allow designers to consider these factors through the entire

This work is based in part upon work supported by the National Science Foundation (NSF) through the ASPIRE Engineering Research Center under Grant EEC-1941524 and CAREER Award under Grant 2239169.

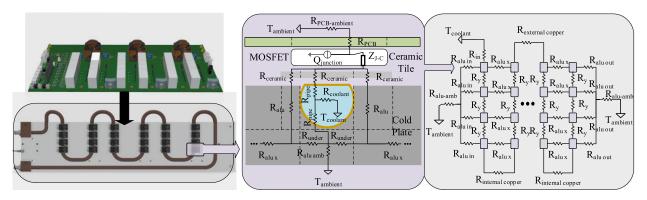


Fig. 1. Overview of the process of Lumped element formation on the 21 kw T-type converter circuit board which sits on its cold plate. Elements around the individual MOSFET are shown in pink and thermal interactions between each MOSFET and the cold plate are shown in grey. Thermal capacitance is omitted for simplicity.

design process.

To address the technical gaps in the literature and the need for accurate and rapid thermal modeling and analysis for achieving reliable high-power chargers, this paper proposes a novel and accurate thermal modeling and analysis approach for high-power EV chargers with liquid cooling without using FE or CFD software. The proposed approach utilizes the LE model to simply predict thermal flow in both the electrical and physical components of complex high-power chargers that require liquid cooling systems. The elements of the LE model are derived entirely from the system and component material properties and dimensions which are known by the designer or can be obtained from product data sheets. No additional thermal material testing or FE analysis is required such as in [15], [10] to obtain LE model parameters.

The major contributions of this work are the development of the novel thermal modeling and analysis approach based on LE theory for high-power converter systems with liquid cooling and a case study presenting experimental results conducted on a 21 kW power converter module hardware from a 1 MW EV wireless charger developed by the team for Class 8 semi-trucks [16]. The case study includes a long mission profile as well as a shorter, dynamic load profile to validate the model under multiple use cases. The thermal analysis model is explained in section II. The case study is presented in III.

## II. PROPOSED THERMAL ANALYSIS MODEL DEVELOPMENT

In an LE model, portions of a physical system are represented as lumped elements which can then be modeled as an equivalent circuit. Traditional circuit analysis techniques and circuit simulation software can then be utilized to solve and analyze the thermal behavior of the system. In this work, heat transfer between various parts of a system is represented using thermal equivalent circuits. The complexity of the resulting thermal equivalent model is dictated by the geometry and size of the physical system, and resulting number of lumped elements in the model.

Within the system, heat is generated due to power losses in the electrical components and flows from areas of high temperature to lower temperature areas. Electrical components such as MOSFETs, diodes, and resistors are modeled in the lumped element models as current sources where the current is equivalent to the power consumed by the device.

All of the physical materials in the system resist heat transfer and have the capacity to store heat energy. The physical materials in the system are modeled by thermal resistances  $R_{th}$  and thermal capacitances  $C_{th}$ . For many circuit components such as MOSFETs and diodes, the thermal resistance and capacitance of their cases can be obtained from data sheets. For other materials in the system the  $R_{th}$  and  $C_{th}$  are functions of their material properties. The thermal resistance of an element in a system can be determined from

$$R_{th} = \frac{l}{\lambda A}. (1)$$

Here,  $\lambda$  is material's thermal resistivity, A is the contact surface area between two elements, l is the length of the material in the direction of heat flow. An elements thermal capacitance to thermal ground can be determined from

$$C_{th} = \rho c_p V. (2)$$

Here,  $c_p$  is the material's specific heat,  $\rho$  is the density, and V is the element volume.

In a system with liquid cooling, both conductive (mentioned above) and convective heat flow must be modeled in the cooling fluid. The thermal resistance of coolant flowing through a tube with the tube wall surrounding it can be calculated using

$$R_{th-fluid} = \frac{1}{hA}. (3)$$

The convection heat transfer coefficient, h, for the coolant is calculated by  $h=\frac{Nu\lambda}{D_h}$ , where  $D_h$  is the hydraulic diameter and  $Nu=0.023Re^{0.8}Pr^{0.4}$  by the Dittus-Boelter approximation [17]. The Reynolds (Re) and Prandtl (Pr) numbers reflect the properties of the cooling system such as flow rate and coolant thermal resistivity. To determine Re and Pr for the

cooling system equations (4) and (5) are used, where  $u_{coolant}$ , L, and  $\mu_{coolant}$  are the coolant velocity, characteristic length, and coolant dynamic viscosity, respectively.

$$Re = \frac{\rho_{coolant} u_{coolant} L}{\mu_{coolant}},$$

$$Pr = \frac{c_{p_{coolant}} \mu_{coolant}}{\lambda_{coolant}}.$$
(5)

$$Pr = \frac{c_{p_{coolant}} \mu_{coolant}}{\lambda_{coolant}}.$$
 (5)

Temperature change along the coolant between inlet and outlet is modeled as a thermal resistance derived from the cooling system properties as in

$$R_{th} = \frac{1}{\dot{m}c_p\rho},\tag{6}$$

where  $\dot{m}$  is the coolant flow rate, and  $\rho$  is the density of the coolant.

The LE-based thermal model is constructed based on the specifications of the charger and cooling system. Elements should be of the same material and have similar proximity to heat-generating sources. Large portions of the system made up of a single material (such as a cold plate) may need to be divided into multiple elements especially if these pieces are connected to multiple head-generating sources. Relevant dimensions and thermal parameters are then determined for each of these elements. The thermal properties of the passive materials needed to determine  $R_{th}$  and  $C_{th}$  can be found in material data sheets or on materials reference sheets.

Contact surfaces between lumped elements are treated as nodes. Each thermal capacitance is connected between the relevant node closest to the heat generation source and ground. Thermal resistances are connected between relevant nodes across lumped elements. An element may have multiple elements across it representing heat flow in multiple directions across the element. If this is the case, all thermal resistances crossing the element are all split in half and joined in the center. The thermal capacitance of that element is connected at the central node.

A developed thermal equivalent circuit can be built in any circuit simulation software such as LTSpice, MAT-LAB/SIMULINK, and PLECS.

The proposed modeling and analysis process is illustrated for the 21 kW T-type converter of an unfolding-based, singlestage, ac-dc converter for MW-level EV wireless charging applications [16]. An overview of the lumped element model of the power converter board and associated liquid cooling system are shown in Fig. 1.

Lumping is performed based on material type, proximity to heat sources, and contact with other materials. Bulk regions of same materials are grouped, regions of similar material that make contact with multiple other elements are split into multiple elements to appropriately model heat transfer through each region. For example the area directly surrounding each MOSFET is divided as shown in Fig. 2.

The element regions are transformed into equivalent circuit components as shown in Fig. 3. For readability, element

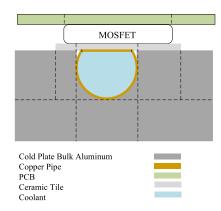


Fig. 2. Illustration of element separation in the areas directly surrounding each MOSFET where resolution is most important.

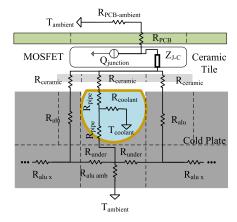


Fig. 3. Illustration of the resulting equivalent circuit model (ECM) in the areas directly surrounding each MOSFET overlaid on the image of element separation.

capacitances have been removed but in the full model, each element has an associated thermal capacitance representing its ability to store thermal energy. Some regions of the cold plate are ignored due to their distance from heat sources and the assumption that the majority of heat will flow through the path of least resistance.

## III. CASE STUDY

## A. Test Setup

To validate the proposed LE-based thermal modeling and analysis of high-power chargers with liquid cooling, a case study of a high-power charger introduced in Section II is presented. The t-type circuit is shown in Fig. 5 a t-type board featuring 3 t-type modules is shown in Fig. 6.

Two different scenarios were tested in hardware and simulated using the LE model. In the first, a single module of the 1 MW converter is tested at 21kW for about 40 minutes after the power was ramped up to full power and allowed to settle. This test serves to validate the models ability to quickly predict long term behavior of the power converter. The hardware test setup and parameters are similar to those described in [16].

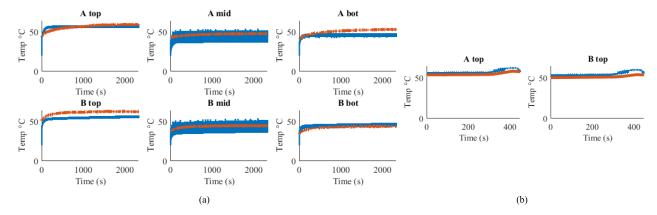


Fig. 4. (a) The predicted (blue) and measured (red) temperature of each all 6 MOSFETs monitored in the long mission profile test. (b) The predicted (blue) and measured (red) temperatures of the two top MOSFETs monitored in the changing load mission profile test.

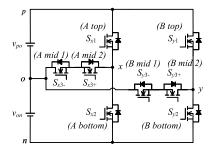


Fig. 5. The circuit diagram of the t-type converter topology. For more information on the circuit topology see [16]

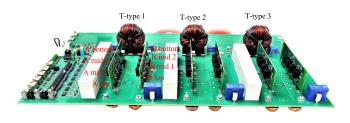


Fig. 6. One T-type module with each switch modeled. In the long mission profile verification 6 switches (Atop, Amid1, Abottom, Btop, Bmid1, and Bbottom) are monitored. In the dynamic load mission profile only the two top switches (Atop and Btop) are monitored.

The battery voltage is set to 715 V. A 50-50 mix of ethylene glycol and water is used as a coolant with a flow rate of 1.2 gpm. These conditions are accounted for in the calculation of the LEs as described previously. The case temperatures of each of the top and bottom MOSFETs and one of the center MOSFETs are recorded over a period of about 40 minutes. Refer to Fig. 5 and 6 for labeling of the MOSFETS.

In the second scenario, to verify the models ability to approximate changing load conditions the T-type power level was changed from 21 kW down to 12.4 kW over a period of about 2.5 minutes. The 12.4 kW power level is outside the ideal operating range and incurs greater losses in the T-type MOSFETS. Only the temperatures of the top two switches were measured in this test as these switches are under the greatest stress.

The model derived in Section II was simulated using MAT-LAB/SIMULINK and PLECS software.

The conduction and switching losses for each MOSFET are obtained analytically from equations presented in [16] over the entire mission profile. The temperature of each MOSFET in the simulation was recorded once each second.

#### B. Results

a) Prediction Accuracy: The results from each of the verification tests are presented in this section. In Fig. 4 (a) the predicted and actual temperature profiles of each of the MOSFETS in the longer mission profile are presented with the predicted temperature in blue and the measured temperature in red. In Fig. 4 (b) the predicted and actual temperature profiles of the MOSFETS under a changing load condition are presented with the predicted temperature in blue and the measured temperature in red. The temperature of each MOSFET was measured at its case.

$$\% Error = \frac{|T_{predicted} - T_{measured}|}{T_{measured}}$$
 (7)

Large differences in temperature at the beginning of the mission profile are present because the MOSFETS were allowed to heat up before data was recorded in the full power tests. The average prediction errors over the entire simulation are presented in table I. The maximum prediction error after the initial startup transient are presented in table II.

When a sliding window average is used to smooth the prediction results the maximum error is greatly improved. The average and maximum prediction errors with the filtered prediction temperatures are presented in tables IV, V, and VI. The filtered prediction data is shown overlaid on the plots of

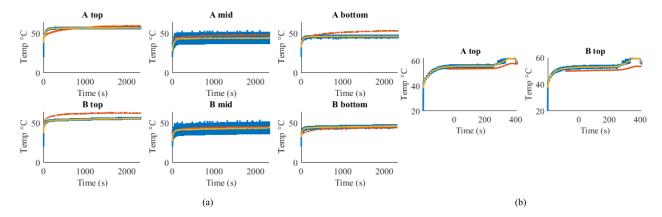


Fig. 7. Filtered temperature predictions (yellow) shown with the original prediction temperatures (blue) and measured temperatures (red) over (a) the long mission profile and (b) the dynamic load profile.

TABLE I
AVERAGE PREDICTION ERROR OF THE PREDICTION RESULTS FOR THE
LONG MISSION PROFILE

|                               | Average % Error |      |      |     |
|-------------------------------|-----------------|------|------|-----|
| Switch   Top   Middle   Botto |                 |      |      |     |
|                               | A               | 0.11 | 0.3  | 9.4 |
|                               | В               | 11.2 | 11.2 | 5.6 |

TABLE II

MAXIMUM PREDICTION ERROR OF THE PREDICTION RESULTS FOR THE LONG MISSION PROFILE

|        | Maximum % Error |        |        |  |
|--------|-----------------|--------|--------|--|
| Switch | Тор             | Middle | Bottom |  |
| A      | 6.9             | 23.6   | 19.3   |  |
| В      | 19.5            | 21.2   | 10.6   |  |

 $\begin{tabular}{ll} TABLE~III\\ Mean~and~Maximum~errors~for~the~changing~load~mission\\ PROFILE \end{tabular}$ 

| % Error |      |      |
|---------|------|------|
| Switch  | Mean | Max  |
| A       | 4.9  | 11.3 |
| В       | 6.1  | 14.1 |

unfiltered prediction data and measured data in yellow in Fig. 7.

b) Simulation Speed: The simulation was performed on a Windows 10 Enterprise PC with a 3.8GHz AMD Ryzen 5 3600X 6-Core Processor and 32 GB of RAM in MATLAB/SIMULINK R2023a and PLECS version 4.8.4. The long mission profile circuit simulation took 5.1 seconds to run and store all generated data for the 40 minutes of power converter hardware operation. The dynamic load profile circuit simulation took 1.7 seconds to run and store all thermal data.

TABLE IV AVERAGE PREDICTION ERROR OF THE FILTERED PREDICTION RESULTS FOR THE LONG MISSION PROFILE

| Average % Error |      |        |        |  |
|-----------------|------|--------|--------|--|
| Switch          | Тор  | Middle | Bottom |  |
| A               | 3.6  | 7.29   | 9.5    |  |
| В               | 11.2 | 3.42   | 5.2    |  |

TABLE V

MAXIMUM PREDICTION ERROR OF THE FILTERED PREDICTION RESULTS
FOR THE LONG MISSION PROFILE

|        | Maximum % Error |        |        |  |
|--------|-----------------|--------|--------|--|
| Switch | Тор             | Middle | Bottom |  |
| A      | 4.4             | 8.5    | 13.7   |  |
| В      | 12.2            | 4.3    | 5.8    |  |

TABLE VI
MEAN AND MAXIMUM FILTERED PREDICTION ERRORS OF THE CHANGING
LOAD MISSION PROFILE

| % Error |      |      |  |
|---------|------|------|--|
| Switch  | Mean | Max  |  |
| A       | 4.9  | 8.4  |  |
| В       | 6.1  | 12.0 |  |

## C. Discussion

The case study reveals the utility of the LETM in modeling the thermal behavior of high power converters. The models prediction error, in conjunction with its speed make it particularly useful in the design of high power converters and their associated cooling systems. For example, in [7] a liquid cooling system is designed for a power conversion module in a reliability-critical setting. The designers used ANSYS-fluent, a CFD software, and were limited to a isolated discrete operating points. Using a similar design process with a LETM similar to the one presented in this work eliminates the need for and CFD software and simplifies the simulation of dynamic

operating conditions.

The models accuracy is improved by the use of filtering. This likely due to the smoothing effects of the bulk of the cold plate that and the filtering effects of the sensing equipment used to measure the MOSFET temperatures. Increased accuracy could be obtained by modeling additional thermal mass of the charger and cooling system as well as any sensor effects; however this may make the model unnecessarily complicated. A balance must be struck between accuracy and simplicity to make the model useful in preliminary design.

#### IV. CONCLUSIONS AND FUTURE WORK

A novel LE-based thermal modeling and analysis method for rapid and accurate prediction of thermal behavior in high-power EV chargers is presented and shown to be an effective, rapid method of thermal evaluation. A case study with hardware results is presented to validate these claims. The model achieves realistic, and sufficiently accurate results to be utilized as a design tool. This model is especially useful in preliminary analysis where a rapid prediction of thermal behavior is critical for design and can result in substantial cost savings, and streamline charger design.

Future work will involve the application of LETMs to reliability evaluation and EV high-power charger failure prediction. A fast thermal behavior prediction tool allows for the simulation of long device mission profiles which can be used to determine the thermal cycling stress which a device is subject to. This information can be used in device lifetime models, such as those presented in [18], and [19] to predict device failure rates and lifetime. Additionally, in future work, the ECM thermal approximation can be simplified into matrix-based calculations to further simplify thermal simulation and eliminate the need for circuit simulation tools. Matrix-based calculations could also be extended to mesh-based tools similar to those in [20].

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