A Distributionally Robust Optimization Framework for Stochastic Assessment of Power System Flexibility in Economic Dispatch

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***Abstract*—Given the complexity of power systems, particularly the high-dimensional variability of net loads, accurately depicting the entire operational range of net loads poses a challenge. To address this, recent methodologies have sought to gauge the maximum range of net load uncertainty across all buses. In this paper, we consider the stochastic nature of the net load and introduce a distributionally robust optimization framework that assesses system flexibility stochastically, accommodating a minimal extent of system violations. We verify the proposed method by solving the flexibility of the economic dispatch problem on four distinct IEEE standard test systems. Compared to traditional deterministic flexibility evaluations, our approach consistently yields less conservative flexibility outcomes.**

***Index Terms*—Flexibility metric, net load uncertainty, distributionally robust optimization, economic dispatch**

I. Introduction

The national electricity sector has witnessed a significant rise in renewable energy integration. As this trend is projected to continue in the coming decades, net demand—calculated by subtracting electricity generation from the total load—has become more volatile and unpredictable. Confronted with these fluctuations, system operators must assess the grid's resilience across various scenarios. In this context, the term "flexibility" emerges in literature, describing a power system's capacity to manage the variability and uncertainty of net loads cost-effectively [1], [2]. Gaining insights into this metric is vital, equipping engineers to predict a system's endurance against unforeseen demand shifts and to guide reliability enhancements. Crucially, neglecting flexibility can lead to transient instabilities, cascading outages, and potential blackouts.

Numerous reviews [1], [3], [4] on power system flexibility categorize prior studies into two main groups based on the time scope of their target applications. First, from a short-term operational viewpoint, system frequency is a pivotal indicator of electrical power quality. Deviations from its nominal value require timely compensation from available resources. As such, the capacity for regulation, energy storage, power range, and ramping duration serves as their flexibility indices [2], [5]. However, these studies have inherent limitations: a flexibility index relevant in one context might be inapplicable in another. In contrast, defining flexibility based on system failure causes, rather than remedies, makes it more universally relevant across different operational systems, offering greater utility for power grid planners and operators.

Second, from a long-term planning perspective, scholars have introduced diverse technical and economic indices to gauge system flexibility from multiple facets. These include generation adequacy metrics such as the loss of load expectation [6]; ramping resource sufficiency indicated by the insufficient ramping resource expectation [7]; and flexibility endurance, i.e., periods of flexibility deficit [8]. These indices typically originate from simulations with preset net load probability assumptions. However, the task of computing the multi-dimensional joint probability distribution of net loads in real-world power systems, which often include numerous buses, is a formidable computational challenge. Furthermore, while these indices do well to capture system failures during certain variability patterns, such as Gaussian-distributed net loads, they don't fully represent the breadth of operational net load scenarios where systems function normally.

To address these challenges, state-of-the-art techniques concentrate on quantifying the utmost net load uncertainty that a system can accommodate [9]–[11]. These methodologies frequently employ a two-stage robust model [12], [13] to gauge the worst-case scenarios arising from renewable energy unpredictability. Instead of optimizing for cost-related objectives, some strategies [14], [15] apply a robust optimization model to deterministically identify the maximum net load deviation from its typical baseline. Nonetheless, such a conservative deviation range doesn't invariably ensure total operational safety, particularly given the rarity of worst-case scenarios. Normally, system operators exhibit a readiness to tolerate a slight degree of potential disruptions if it leads to augmented system adaptability. To cater to this perspective, we propose a novel stochastic assessment model grounded in distributionally robust optimization, offering twofold contributions:

- Identifying the maximum permissible net load variation ensures that the expected operational violations for any net load profile within this range stay within a predefined acceptable threshold.
- Adapting this model to the economic dispatch problem, wherein it evaluates system flexibility on an hourly basis through hyperbox metrics within a rolling optimization framework. Our case study emphasizes the superior performance of our stochastic assessment in comparison to traditional deterministic approaches.

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II. Assessment Methods

A. Flexibility Metric

The hyperbox metric evaluates the safe operating range, $\mathcal{U}$, of net loads [14], [15]. Defined by $\Delta d = [\Delta d_b]$ as the peak deviation of net load d_b on each bus, and $\bar{d}$ as the average or user-defined normal net load, these parameters can be empirically derived from historical observations. The hyperbox representation of the uncertainty set is thus given as:

$$U(\lambda) = \{\xi : \bar{d} - \lambda\Delta d \leq \xi \leq \bar{d} + \lambda\Delta d\}. \tag{1}$$

In (1), a higher generic value $\lambda \in [0, 1]$ indicates the system flexibility. This metric guarantees the minimum level of net load uncertainty tolerance. It is manifest that the flexibility set $U(\lambda) \subseteq \mathcal{U}$. A net load combination $\xi \notin U(\lambda)$ does not necessarily trigger a system failure.

B. Deterministic Assessment

The overall goal of the deterministic approach is to identify the largest possible λ based on the flexibility metric such that the system can accommodate all $\xi \in U(\lambda)$. Let $\mathbf{x}$ be a vector including all decision variables. We then propose the following general optimization framework:

$$\max \quad \lambda \tag{2a}$$
$$\text{s.t.} \quad \max_{\xi \in U(\lambda)} \phi(\xi) \leq 0, \tag{2b}$$

where

$$\phi(\xi) = \min_{\mathbf{x}, u} \quad \mathbf{1}^T u \tag{3a}$$
$$\text{s.t.} \quad A_1\mathbf{x} - u_1 \leq h_1 + H_1\xi, \tag{3b}$$
$$A_2\mathbf{x} + u_2^+ - u_2^- = h_2 + H_2\xi, \tag{3c}$$
$$u \geq 0. \tag{3d}$$

In the model, constraint (3b) represents all system inequalities, whereas (3c) captures all system equalities. The term u denotes system violations, and the objective is to determine the maximum deviation λ ensuring no system violations, even under the worst case ξ running within $U(\lambda)$, as indicated by constraint (2b). A detailed mathematical model of this concept in (2) for the power system's economic dispatch problem will be introduced in Section III.

Solution Approach: Several methods have been proposed to address problem (2). For example, [14] establishes that maximizing λ is equivalent to solving a mixed-integer program that reformulates the constraint $\phi(\xi) = 0$ using its first-order Karush-Kuhn-Tucker conditions. Furthermore, the cutting plane method in [15] also presents an alternative solution for addressing problem (2).

C. Stochastic Assessment

Building upon the deterministic approach, we extend our methodology [15] to develop a stochastic one, aimed at characterizing the uncertainty of the net load. We assume that ξ follows a probability distribution denoted as $P(\xi)$, which belongs to the following ambiguity set:

$$\mathcal{D}(\lambda) = \left\{ P(\xi) \middle| \int_{\xi \in U(\lambda)} dP(\xi) = 1, \ \int_{\xi \in U(\lambda)} \xi dP(\xi) = \bar{d} \right\}. \tag{4}$$

This ambiguity set indicates that we consider all distribution $P(\xi)$ if its support is on $U(\lambda)$ and the mean value is $\bar{d}$.

The goal of the stochastic approach is to identify the most extensive support set within the ambiguity set $\mathcal{D}(\lambda)$, ensuring that the expected constraint violation, considering the worst-case distribution within $\mathcal{D}(\lambda)$, remains below a predefined threshold β. The abstract formulation can be expressed as follows:

$$\max_{0 \leq \lambda \leq 1} \quad \lambda \tag{5a}$$
$$\text{s.t.} \quad \max_{P(\xi) \in \mathcal{D}(\lambda)} \quad \mathbb{E}_{P(\xi)}[\phi(\xi)] \leq \beta. \tag{5b}$$

The resulting formulation (5) is a distributionally robust optimization (DRO) model [16]. In this variant, the distribution of the random parameter is uncertain and can vary adversely within the decision-dependent (endogenous) ambiguity set $\mathcal{D}(\lambda)$, with the optimal solution determined by considering the worst-case distribution.

To tackle (5), we can treat the objective in (5a) as the master problem and redefine the internal maximization function within the constraints in (5b) as the subproblem. Employing the ambiguity set specified in (4), we reformulate the maximization function in (5b) as follows, where we represent $\mathbb{E}_{P(\xi)}[\phi(\xi)]$ as $\int_{\xi \in U(\lambda)} \phi(\xi) dP(\xi)$.

$$\max_{P(\xi)} \quad \left\{ \int_{\xi \in U(\lambda)} \phi(\xi) dP(\xi) : \int_{\xi \in U(\lambda)} dP(\xi) = 1, \right. \\ \left. \int_{\xi \in U(\lambda)} \xi dP(\xi) = \bar{d}. \right\} \tag{6}$$

Let α and γ serve as dual variables of two constraints in (6), its dual formulation can be expressed as follows:

$$\begin{aligned} \min_{\alpha, \gamma} \quad & \alpha + \bar{d}^T \gamma \\ \text{s.t.} \quad & \alpha + \xi^T \gamma \geq \phi(\xi), \quad \forall \xi \in U(\lambda) \\ & \alpha, \gamma \text{ free.} \end{aligned} \tag{7}$$

Using the minimax duality for the Lagrangian, (7) is equivalent to:

$$\min_{\gamma} \quad \left\{ \bar{d}^T \gamma \ + \max_{\xi \in U(\lambda)} \ (\phi(\xi) - \xi^T \gamma) \right\}. \tag{8}$$

To further express $\phi(\xi)$, we develop the dual formulation of the formulation (3):

$$\max_{\xi \in U(\lambda), \mu, \nu} \quad (h_1 + H_1\xi)^T \mu + (h_2 + H_2\xi)^T \nu \tag{9a}$$
$$\text{s.t.} \quad A_1^T \mu + A_2^T \nu \leq 0, \tag{9b}$$
$$-\mathbf{1} \leq \mu \leq \mathbf{0}, \ -\mathbf{1} \leq \nu \leq \mathbf{1}. \tag{9c}$$

Here, μ and ν are introduced as dual variables for constraints (3b) and (3c), respectively. Subsequently, we substitute $\phi(\xi)$ in (8) with the objective function derived in (9). This reformulation of the maximization function in (5b) is presented as follows:

$$\min_{\gamma} \quad \Big[\bar{d}^T\gamma \;+ \max_{\xi\in U(\lambda),\mu,\nu} \quad \Big\{(h_1+H_1\xi)^T\mu + (h_2+H_2\xi)^T\nu \\ -\xi^T\gamma : \text{Constraints (9b)} - \text{(9c)}\Big\}\Big] \leq \beta. \tag{10}$$

Considering that "min" in (10) indicates feasibility, it can be safely omitted. As a result, the minimax formulation in (10) can be alternatively represented by solving its inherent maximization problem.

It's worth noting that in this maximization problem, $U(\lambda)$ adopts a hyperbox-metric form, as described in (1). Therefore, ξ can be further expressed as $\xi = \bar{d} + \lambda\Delta dz^+ - \lambda\Delta dz^-$, with both z^+ and z^- being binary vectors indicating deviation direction. As outlined in [15], the optimal ξ must be achieved at the boundary of $U(\lambda)$.

For notation brevity, we suppose that $\xi \in \mathbb{R}^{N\times 1}$, $H_1 \in \mathbb{R}^{M_1\times N}$, and $H_2 \in \mathbb{R}^{M_2\times N}$. Given these, the expanded form of the maximization problem in (10) can be reformulated as:

$$\begin{aligned}\psi = \max_{z,\mu,\nu} \quad & h_1^T\mu + h_2^T\nu + \sum_{n=1}^{N}\sum_{m=1}^{M_1}(\bar{d}_n H_{1,m,n}\mu_m \\ & + \lambda\Delta d_n H_{1,m,n}\hat{\mu}^+_{n,m} - \lambda\Delta d_n H_{1,m,n}\hat{\mu}^-_{n,m}) \\ & + \sum_{n=1}^{N}\sum_{m=1}^{M_2}\big[\bar{d}_n H_{2,m,n}(\nu^a_m - \nu^b_m) \\ & + \lambda\Delta d_n H_{2,m,n}(\hat{\nu}^{a,+}_{n,m} - \hat{\nu}^{b,+}_{n,m}) \\ & - \lambda\Delta d_n H_{2,m,n}(\hat{\nu}^{a,-}_{n,m} - \hat{\nu}^{b,-}_{n,m})\big] \\ & - \sum_{n=1}^{N}(\bar{d}_n\gamma_n + \lambda\Delta d_n\gamma_n z^+_n - \lambda\Delta d_n\gamma_n z^-_n) \qquad (11a)\\ \text{s.t.} \quad & \text{Constraints (9b)} - \text{(9c)}, \qquad (11b)\\ & -z^+_n \leq \hat{\mu}^+_{n,m},\ \mu_m \leq \hat{\mu}^+_{n,m} \leq 1 - z^+_n + \mu_m, \\ & -z^-_n \leq \hat{\mu}^-_{n,m},\ \mu_m \leq \hat{\mu}^-_{n,m} \leq 1 - z^-_n + \mu_m, \\ & -1 \leq \hat{\mu}^+_{n,m} \leq 0,\ -1 \leq \hat{\mu}^-_{n,m} \leq 0, \\ & \forall n = 1\ldots N,\ \forall m = 1\ldots M_1. \qquad (11c)\\ & -z^+_n \leq \hat{\nu}^{\kappa,+}_{n,m},\ \nu^\kappa_m \leq \hat{\nu}^{\kappa,+}_{n,m} \leq 1 - z^+_n + \nu^\kappa_m, \\ & -z^-_n \leq \hat{\nu}^{\kappa,-}_{n,m},\ \nu^\kappa_m \leq \hat{\nu}^{\kappa,-}_{n,m} \leq 1 - z^-_n + \nu^\kappa_m, \\ & -1 \leq \hat{\nu}^{\kappa,+}_{n,m} \leq 0,\ -1 \leq \hat{\nu}^{\kappa,-}_{n,m} \leq 0, \\ & \forall \kappa \in \{a,b\},\ \forall n = 1\ldots N,\ \forall m = 1\ldots M_2. \qquad (11d)\\ & z^+_n + z^-_n = 1, \quad \forall z^+_n, z^-_n \in \{0,1\}. \qquad (11e)\end{aligned}$$

To tackle the bilinear term $\xi^T H_1^T\mu$ in the objective function, we introduce auxiliary variables $\hat{\mu}^+_{n,m}$ and $\hat{\mu}^-_{n,m}$ to denote the products $z^+_n\mu_m$ and $z^-_n\mu_m$, respectively. For the term $\xi^T H_2^T\nu$, we decompose ν into $\nu^a - \nu^b$. Both these components, ν^a and ν^b, are restricted to the range $[-1, 0]$. We then employ a method similar to (11c) to linearize the expressions $\xi^T H_2^T\nu^a$ and $\xi^T H_2^T\nu^b$. Consequently, (11) is transformed into a mixed-integer linear programming model.

Upon solving (11), the optimal solutions are denoted as (z^*, μ^*, ν^*) with the corresponding optimal value of ψ^*. In accordance with (10), we examine whether the following condition is satisfied:

$$\bar{d}^T\gamma \;+ \psi^* \leq \beta. \tag{12}$$

If (12) is met, the optimal solution to the master problem (5a), denoted as λ^*, becomes the final flexibility result.

Otherwise, we refine the master problem by incorporating a feasibility cut $\bar{d}^T\gamma + \psi(\lambda,\gamma) \leq \beta$. Here, $\psi(\lambda,\gamma)$ is derived by replacing with the optimal solution (z^*, μ^*, ν^*) from (11a). Subsequently, the reformed master problem is developed as:

$$\begin{aligned}\max_{0\leq\lambda\leq 1} \quad & \lambda \qquad (13a)\\ \text{s.t.} \quad & \sum_{n=1}^{N}\Big\{\sum_{m=1}^{M_1}(\Delta d_n H_{1,m,n}\hat{\mu}^{+,*}_{n,m} - \Delta d_n H_{1,m,n}\hat{\mu}^{-,*}_{n,m}) \\ & + \sum_{m=1}^{M_2}\big[\Delta d_n H_{2,m,n}(\hat{\nu}^{a+,*}_{n,m} - \hat{\nu}^{b+,*}_{n,m}) \\ & - \Delta d_n H_{2,m,n}(\hat{\nu}^{a-,*}_{n,m} - \hat{\nu}^{b-,*}_{n,m})\big] \\ & - (\Delta d_n z^{+,*}_n - \Delta d_n z^{-,*}_n)\gamma_n\Big\}\lambda \\ & + h_1^T\mu^* + h_2^T\nu^* + \sum_{n=1}^{N}\Big\{\sum_{m=1}^{M_1}\bar{d}_n H_{1,m,n}\mu^*_m \\ & + \sum_{m=1}^{M_2}\bar{d}_n H_{2,m,n}(\nu^{a*}_m - \nu^{b*}_m)\Big\} \leq \beta, \qquad (13b)\end{aligned}$$

where the bilinear term $\lambda\gamma_n$ from the feasibility cut (13b) is substituted with w_n, as depicted in (14a)-(14b) using McCormick Envelopes.

$$w_n \geq -\lambda K,\ w_n \geq \gamma_n + \lambda K - K,\ \forall n = 1\ldots N, \tag{14a}$$
$$w_n \leq \gamma_n - \lambda K + K,\ w_n \leq \lambda K,\ \forall n = 1\ldots N. \tag{14b}$$

Notably, K is a sufficiently large constant, and the constraint $-K \leq \gamma \leq K$ provides relaxation for the unrestricted γ.

Solution Approach: The subsequent steps outline the cutting plane algorithm used to resolve the DRO model (5):

1. Address the master problem (5a) to determine the optimal value, denoted as λ^*.
2. Assess the feasibility of the subproblem by resolving (11) using the derived λ^*.
3. Evaluate the validity of condition (12):
   - If it holds, conclude the process and yield both the optimal solution and the master problem's objective value, λ^*.
   - If not, refine the master problem by incorporating the feasibility cut from (13b) and revert to Step 1.

III. Economic Dispatch

In this section, we present the mathematical framework for a multi-period Economic Dispatch (ED) problem, accounting for flexible resources such as power generators and Energy Storage Systems (ESS). Our model optimizes the generation levels of each generator ($p^{\mathrm{G}}_{n,t}$) and the net power outputs from ESS ($p^{\mathrm{ESS}}_{i,t}$) per hour, indexed by t. Notably, a negative $p^{\mathrm{ESS}}_{i,t}$ indicates ESS charging, while a positive value signals discharging. With a predefined uncertainty space $\mathcal{U}$, the feasible domain for these decision variables can be expressed as:

$$X(d) = \Big\{ \mathrm{P}^{\min}_n \le p^{\mathrm{G}}_{n,t} \le \mathrm{P}^{\max}_n,\ \forall n \in \mathcal{G}, \forall t \in \mathcal{T}, \tag{15a}$$

$$-RD_n \le p^{\mathrm{G}}_{n,t} - p^{\mathrm{G}}_{n,t-1} \le RU_n,\ \forall n \in \mathcal{G}, \forall t \in \mathcal{T}, \tag{15b}$$

$$E^{\mathrm{ESS}}_{i,t} = E^{\mathrm{ESS}}_{i,t-1} - p^{\mathrm{ESS}}_{i,t},\ \forall i \in \mathcal{E}, \forall t \in \mathcal{T}, \tag{15c}$$

$$\mathrm{E}^{\min}_i \le E^{\mathrm{ESS}}_{i,t} \le \mathrm{E}^{\max}_i,\ \forall i \in \mathcal{E}, \forall t \in \mathcal{T}, \tag{15d}$$

$$-\mathrm{P}^{\max}_{c,i} \le p^{\mathrm{ESS}}_{i,t} \le \mathrm{P}^{\max}_{dc,i},\ \forall i \in \mathcal{E}, \forall t \in \mathcal{T}, \tag{15e}$$

$$-F_l \le \sum_{b\in\mathcal{B}} \mathrm{SF}_{b,l}\Big(\sum_{n\in\mathcal{G}^b} p^{\mathrm{G}}_{n,t} + \sum_{i\in\mathcal{E}^b} p^{\mathrm{ESS}}_{i,t} - d_{b,t}\Big) \le F_l, \forall l \in \mathcal{L}, \tag{15f}$$

$$\sum_{t\in\mathcal{T}} \Big\{ \sum_{n\in\mathcal{G}} C^{\mathrm{G}}_n p^{\mathrm{G}}_{n,t} + \sum_{i\in\mathcal{E}} C^{\mathrm{ESS}}_i p^{\mathrm{ESS}}_{i,t} \Big\} \le \tau,\ \forall n \in \mathcal{G}, \forall i \in \mathcal{E}, \tag{15g}$$

$$\sum_{n\in\mathcal{G}} p^{\mathrm{G}}_{n,t} + \sum_{i\in\mathcal{E}} p^{\mathrm{ESS}}_{i,t} - \sum_{b\in\mathcal{B}} d_{b,t} = 0,\ \forall d_{b,t} \in \mathcal{U}_{b,t}, \tag{15h}$$

$$p^{\mathrm{G}}_{n,t} \ge 0,\ \forall b \in \mathcal{B}, \forall n \in \mathcal{G}, \forall t \in \mathcal{T} \Big\}. \tag{15i}$$

We denote the sets of generators, ESSs, and transmission lines as $\mathcal{G}$, $\mathcal{E}$, and $\mathcal{L}$, respectively, with $\mathcal{G}^b$ and $\mathcal{E}^b$ indicating subsets of generators and ESSs at bus b. This model bypasses simultaneous ESS charging and discharging scenarios for arbitrage, given that its absence doesn't compromise system flexibility.

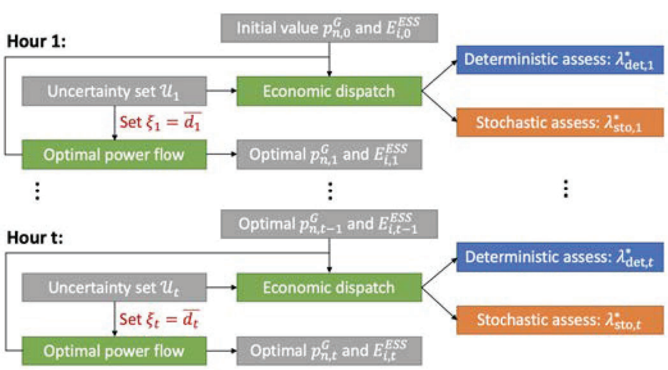

Fig. 1. Illustration of the flexibility assessment process for the ED model.

Fig. 1 depicts the process of assessing flexibility for the ED model on an hourly basis. Grounded in the uncertainty set for the net load $\mathcal{U}_t$, we optimize flexibility λ_t for each hour t using either the deterministic or stochastic assessment, subject to the constraints highlighted in (15) for the corresponding hour. In this context, while optimizing for hour t, we regard the preceding decision variables $p^{\mathrm{G}}_{n,t-1}$ and $E^{\mathrm{ESS}}_{i,t-1}$ from constraints (15b) and (15c) as pre-established constants, derived from the prior hour's optimal power flow results.

IV. Numerical Experiments

In this section, we evaluate the maximum extent of net load uncertainty across all buses in the ED model, as presented in Section III. We demonstrate both deterministic and stochastic methods using four IEEE standard systems. By integrating an hourly charging load demand into the ED model, the optimal power flow results vary hourly, leading to distinct flexibility outcomes optimized for each period.

A. Flexibility Metric

Fig. 2 displays the hourly-optimized flexibility metrics through both deterministic and stochastic assessments. We present results from both single-scenario assessments, based on the hourly normal net load, and those derived from 100-scenario assessments. Most of these 100 scenarios fall within a [0.99, 1.01] range relative to the normal net load per hour. Nevertheless, we incorporated an outlier—a scenario with 1.09 times the net load—to examine the responses of both deterministic and stochastic assessments to rare extreme cases in the power system. Notably, $\lambda_{\mathrm{sto},t}$ consistently outperforms $\lambda_{\mathrm{det},t}$. This difference arises from the DRO model's allowance in the stochastic assessment to accommodate minor ED constraint violations. Specifically, we set β in (5b) to 0.05, signifying an expected system operation breach below 5%, thus enhancing the system's adaptability. Moreover, as the number of scenarios expands, both the deterministic flexibility $\lambda_{\mathrm{det},t}$ and the stochastic flexibility $\lambda_{\mathrm{sto},t}$ diminish in the right subplot. Influenced by extreme cases, $\lambda_{\mathrm{det},t}$ plummets to zero in the day's final hour, suggesting the system lacks flexibility at that point. In contrast, $\lambda_{\mathrm{sto},t}$ retains a flexibility measure of 0.063, representing the flexibility exhibited in most scenarios, barring the extreme one.

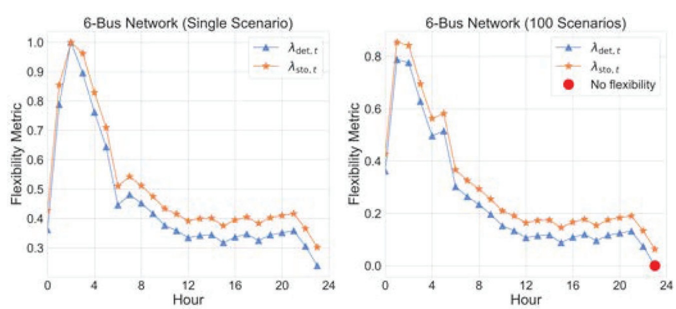

Fig. 2. Comparative flexibility outcomes: deterministic vs. stochastic assessments across single and multi net-load scenarios.

B. Sensitivity Analysis

We investigate the impact on the flexibility metric when there is a presence or absence of ESS in the ED model (15). By omitting ESS-related constraints, specifically (15c)-(15e), and optimizing the system's hourly flexibility, the resulting $\lambda_{\mathrm{sto},t}$ comparisons, both with and without ESSs under stochastic assessment, are depicted in Fig. 3.

In the early hours, power systems with ESSs demonstrate greater flexibility than those without. However, this advantage lessens over time, and sometimes, systems with ESSs can be less flexible. This is because as the ESS discharges, system generators curtail their power output in the initial hours. Given the ramping rate constraints, the power generation in subsequent hours can be inferior to systems devoid of ESSs, lowering overall flexibility as the ESS reaches its lowest SOC. This pattern is more evident in larger networks, i.e., the 24-bus and 30-bus systems, where the initial positive impact of ESSs on flexibility is diluted due to the ample generator resources present in these expansive networks.

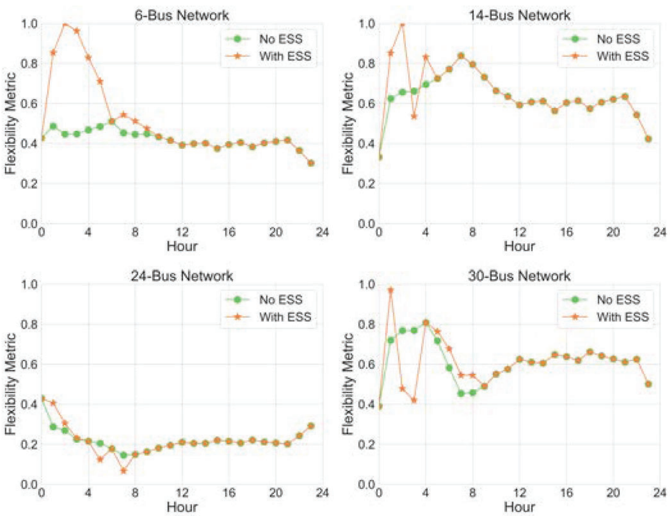

Fig. 3. ESS impact on system flexibility metric under stochastic assessment.

C. Computation Performance

To verify the efficiency of the McCormick relaxation technique in solving the stochastic flexibility assessment $\lambda_{\text{sto},t}$, we also address the master problem (13a) with the nonconvex feasibility cut directly in Gurobi, referred to as the Gurobi-NC method. We limit the stochastic cutting plane algorithm to 30 iterations per hour due to time constraints. Table I displays the computational times and the convergence performance for both approaches when handling the ED with ESSs. Specifically, the convergence metric represents the number of hours in a day both methods converge within those 30 iterations.

TABLE I
RUNNING TIME AND CONVERGENCE PERFORMANCE FOR THE GUROBI-NC AND MCCORMICK METHOD UNDER STOCHASTIC ASSESSMENT

Networks	Time (seconds)		Convergence Metric (hours)	
	Gurobi-NC	McCormick	Gurobi-NC	McCormick
6-bus	4.92	4.54	24	24
14-bus	28.72	18.31	23	24
24-bus	6853.59	1749.41	1	24
30-bus	19601.20	3007.45	8	24

As the system size increases, the efficiency of the McCormick method in optimizing the flexibility metric surpasses the Gurobi-NC method. While both methods converge optimally for the 6-bus system within the set iterations, the Gurobi-NC method struggles to do so for larger systems during certain hours. In contrast, the McCormick method consistently achieves convergence. Given its faster convergence and shorter computational time, the McCormick method stands out as the preferred choice for stochastic flexibility assessment in the DRO model.

V. Conclusion

Building on deterministic flexibility assessments of power system net load uncertainty, this paper introduces a stochastic assessment framework within the DRO model, tested through a multi-time interval economic dispatch model. Numerical results indicate that our stochastic approach yields less conservative flexibility metrics. Through sensitivity analysis, we observed that as system scales increase, abundant generators diminish the positive influence of ESS on system flexibility. Additionally, the efficiency of the McCormick envelope in solving the DRO model is confirmed against the direct nonconvex approach.

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