Extracting the dependence of directed flow differences on conserved charges

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Abstract. Recently, seven produced hadron species have been used to construct multiple hadron sets with given differences in the net electric charge (Δq) and strangeness (ΔS) between the two sides. A nonzero directed flow difference Δv_1 has been proposed as a consequence of the electromagnetic field produced in relativistic heavy ion collisions. Previously, we have shown with quark coalescence that Δv_1 and the slope difference $\Delta v_1'$ depend linearly on both Δq and ΔS with zero intercept. Here we emphasize that a two-dimensional function or fit is necessary for extracting the Δq- and ΔS-dependences of Δv_1. On the other hand, a one-dimensional fit gives a different value for the slope parameter of the Δq- or ΔS-dependence. Furthermore, a one-dimensional fit is incorrect because its slope parameter depends on the arbitrary scaling factor of a hadron set and is thus ill-defined. We use test data of Δv_1 to explicitly demonstrate these points.

1 Introduction

Direct flow analyses of produced hadrons that contain no u and d quarks were proposed [1, 2]. Seven hadron species, K^-, ϕ, $\bar{p}$, $\bar{\Lambda}$, $\bar{\Xi}^+$, Ω^- and $\bar{\Omega}^+$, were used [2] to construct multiple hadron sets with given differences in the net electric charge (Δq) and strangeness (ΔS) between the two sides. A nonzero directed flow difference Δv_1 (or the difference of the directed flow slopes at midrapidity $\Delta v_1'$) has been proposed as a consequence of the electromagnetic field produced in relativistic heavy ion collisions [2, 3], especially if Δv_1 or $\Delta v_1'$ increases with Δq.

In earlier studies [4, 5], we have examined the consequence of the coalescence sum rule (CSR) or quark coalescence on Δv_1 and $\Delta v_1'$ of the hadron sets. We found [4] that quark coalescence leads to $\Delta v_1' = c_q^* \Delta q + c_S^* \Delta S$, so in general $\Delta v_1' \neq 0$ for a hadron set with nonzero Δq and/or ΔS. Specifically, the CSR gives the following relation:

$$\Delta v_1' = \left(v_{1,\bar{d}}' - v_{1,\bar{u}}'\right)\Delta q + \left[\left(v_{1,\bar{s}}' - v_{1,s}'\right)/2 - \left(v_{1,\bar{d}}' - v_{1,\bar{u}}'\right)/3\right]\Delta S. \quad (1)$$

Therefore, the coefficients c_q^* and c_S^* reflect the v_1' difference of produced quarks with different electric charges. We then proposed two methods, the 5-set method and the 3-set method [4], to extract the coefficients for the Δq- and ΔS-dependences of Δv_1 or $\Delta v_1'$. Equivalently, we can write $\Delta v_1' = c_q^* \Delta q + c_B^* \Delta B$ that involves the difference in the net-baryon number (ΔB) between the two sides, where quark coalescence gives $c_B^* = -3c_S^*$ [5].

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Table 1 shows our choice of five hadron sets (Set 1-5) and the STAR Collaboration's choice (Index 1-5) [3]. For example, Δq_{ud} is the net electric charge in light quarks ($\bar{u}$ and $\bar{d}$ here) of the left side minus that of the right side including the weighting factor of each hadron (such as 1 or 1/2). For all the hadrons sets made of these seven produced hadron species, there are only five independent hadron sets [2, 4]. For example, each hadron index from the STAR Collaboration can be written as a linear combination of the hadron sets of our choice: Index1=-Set1, Index2=Set1+Set4+Set5, Index3=Set1+2*Set5, Index4=3*Set3+6*Set4, Index5=2*Set1-2*Set2+Set4+3*Set5.

Table 1. Set 1 to 5 are our choice of five independent hadron sets, while Index 1 to 5 are the STAR choice. Differences of electric charge, strangeness and baryon number between the two sides are given.

Set/Index #	Δq	Δq_{ud}	ΔS	ΔB	Left side	Right side
Set 1	0	0	0	0	$v_1[K^-(\bar{u}s)] + v_1[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})]$	$v_1[\bar{p}(\bar{u}\bar{u}\bar{d})] + v_1[\phi(s\bar{s})]$
Set 2	0	0	0	0	$v_1[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})]$	$\frac{1}{2}v_1[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s})] + \frac{1}{2}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})]$
Set 3	0	0	0	0	$\frac{1}{3}v_1[\Omega^-(sss)] + \frac{1}{3}v_1[\overline{\Omega}^+(\bar{s}\bar{s}\bar{s})]$	$v_1[\phi(s\bar{s})]$
Set 4	1/3	0	1	-1/3	$\frac{1}{2}v_1[\phi(s\bar{s})]$	$\frac{1}{3}v_1[\Omega^-(sss)]$
Set 5	2/3	1/3	1	-1/3	$\frac{1}{2}v_1[\phi(s\bar{s})] + \frac{1}{3}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})]$	$v_1[K^-(\bar{u}s)]$
Index 1	0	0	0	0	$v_1[\bar{p}(\bar{u}\bar{u}\bar{d})] + v_1[\phi(s\bar{s})]$	$v_1[K^-(\bar{u}s)] + v_1[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})]$
Index 2	1	1/3	2	-2/3	$v_1[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})]$	$\frac{1}{3}v_1[\Omega^-(sss)] + \frac{2}{3}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})]$
Index 3	4/3	2/3	2	-2/3	$v_1[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})]$	$v_1[K^-(\bar{u}s)] + \frac{1}{3}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})]$
Index 4	2	0	6	-2	$v_1[\overline{\Omega}^+(\bar{s}\bar{s}\bar{s})]$	$v_1[\Omega^-(sss)]$
Index 5	7/3	1	4	-4/3	$v_1[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s})]$	$v_1[K^-(\bar{u}s)] + \frac{1}{3}v_1[\Omega^-(sss)]$

2 Extracting the Δq or ΔS dependences

We have shown earlier [4] that Δv_1 and $\Delta v_1'$ of the hadron sets from quark coalescence depend linearly on both Δq and ΔS, as shown in Eq.(1). Therefore, we suggest to analyze the data with a two-dimensional (2-D) linear function: $\Delta v_1' = c_0^* + c_q^*\ \Delta q + c_S^*\ \Delta S$. Since the quark coalescence predicts $c_0^* = 0$, a non-zero intercept parameter c_0^* would mean the breaking of the coalescence sum rule. To extract the coefficients for the dependences on Δq and ΔS, one can use the above function to fit the five independent data points; we call this the 5-set method [4]. In the following, we use this method to demonstrate the proper way of extracting the Δq and ΔS dependences. We use the STAR 10-40% Au+Au data at 27A GeV as an example, where we only take the central values for demonstration (i.e., without considering the experimental error bars). These data are given in the STAR27 row in Table 2, where column 2 to 6 give the $\Delta v_1'$ values (multiplied by 10^4) of the hadron index 1 to 5, respectively.

Table 2. Values of $\Delta v_1' \times 10^4$ for Set 1-5 or Index 1-5 of different test data. Values of the coefficients c_0^*, c_q^* and c_S^* from 2-D fits and the coefficients $K_{\Delta q}$ and $K_{\Delta S}$ from 1-D fits are also given.

Test data	#1	2	3	4	5	Fit	c_0^*	c_q^*	c_S^*	$K_{\Delta q}$	$K_{\Delta S}$
STAR27	3	41	39	83	64	w/o c_0^*	0	12.8	9.67	33.4	15.2
						with c_0^*	6.66	8.66	9.93	30.5	13.0
Our27	-3	8	-55/3	23	21	w/o c_0^*	0	-6.00	25.0	-	-
						with c_0^*	-4.44	-6.00	29.4	-	-
Our27ideal	0	0	0	23	21	w/o c_0^*	0	-6.00	25.0	-	-
						with c_0^*	0.00	-6.00	25.0	-	-
STAR27ideal	0	44	42	138	86	w/o c_0^*	0	-6.00	25.0	47.2	22.4
						with c_0^*	0.00	-6.00	25.0	49.0	22.9
STAR27ideal2	0	44	42	69	86	w/o c_0^*	0	-6.00	25.0	40.1	21.9
(Index4 scaled by 1/2)						with c_0^*	0.00	-6.00	25.0	34.5	21.9

We first use the 2-D function to fit the five STAR27 data points. We obtain the fit function as $6.66 + 8.66\Delta q + 9.93\Delta S$ (or $12.8\Delta q + 9.67\Delta S$ if we do not allow the intercept c_0^*). Figure 1 shows the 2-D fit (w/o the intercept) of the STAR27 test data over the $\Delta q - \Delta S$ plane. We also follow the method used by the STAR Collaboration [3] and perform a one-dimensional (1-D) fit of the STAR27 test data as shown in Fig. 1. When fit with the function $K_{\Delta q}\Delta q$, we obtain $33.4\Delta q$ (or $5.33 + 30.5\Delta q$ with intercept). When using the function $K_{\Delta S}\Delta S$, we obtain $15.2\Delta S$ (or $9.65 + 13.0\Delta S$ with intercept). Note that the $K_{\Delta q}$ and $K_{\Delta S}$ values without intercept are rather close to those extracted by the STAR Collaboration (central value of 29 and 19, respectively) [3]. However, we see in Table 2 that the Δq and ΔS slope coefficients from the 1-D fits can be quite different from those from the 2-D fits. For example, the $K_{\Delta q}$ values from the 1-D fit are much larger than the c_q^* values from the 2-D fit.

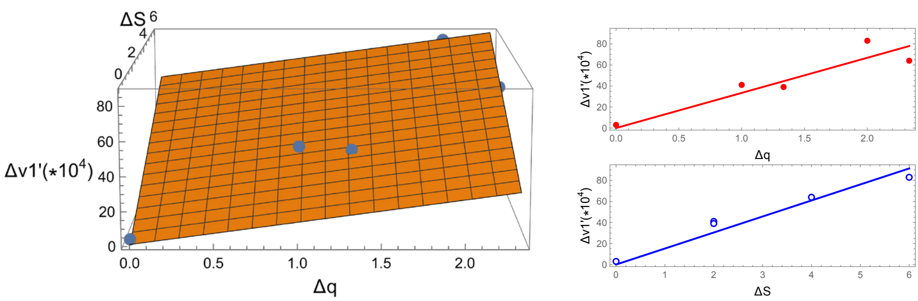

Figure 1. 2-D fit (left) and 1-D fits (right), all w/o intercept, of the STAR27 test data in Table 2.

We then convert the STAR27 test data into the five hadron sets of our choice: the Our27 row in Table 2. Fitting of Our27 test data with the 2-D function gives $-4.44-6.00\Delta q+29.4\Delta S$ (or $-6.00\Delta q+25.0\Delta S$ with no intercept). These c_q^* and c_S^* values are drastically different from those obtained from fitting the STAR27 test data. This difference may seem to be unexpected at first, since our hadron sets and the STAR hadron sets are related to each other linearly and thus equivalent. However, they are only equivalent when the Δv_1 data follow the coalescence sum rule. The left panel of Fig. 2 shows the 2-D fit (w/o intercept) of Our27 test data. We see that $\Delta v_1' \neq 0$ for Set 1, 2, 3 (all at $\Delta q = \Delta S = 0$); this clearly violates the coalescence sum rule.

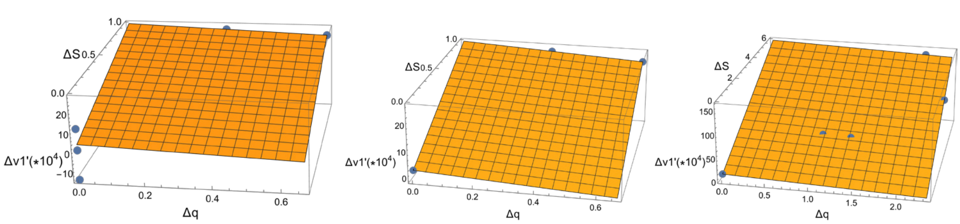

Figure 2. 2-D fit (w/o intercept) of Our27 test data (left). 2-D fits of Our27ideal test data (middle) and the STAR27ideal test data (right) give the same 2-D plane over the $\Delta q - \Delta S$ space.

To explicitly demonstrate this, we set the $\Delta v_1'$ values to zero for Set 1, 2, and 3 of Our27 test data to obtain the Our27ideal test data in Table 2. This test data thus satisfy the coalescence sum rule and serve as an ideal CSR case. A 2-D fit of Our27ideal test data gives $-6.00\Delta q + 25.0\Delta S$; this result is obtained with or without an intercept, which is expected for an ideal CSR case. We then convert Our27ideal test data into the five hadron indices chosen by STAR, labeled as STAR27ideal in Table 2. A 2-D fit of the STAR27ideal test data also gives $-6.00\Delta q + 25.0\Delta S$, as shown in Fig. 2. Therefore, the difference in the 2-D coefficients from fitting the STAR27 and Our27 test data is due to the breaking of CSR in the test data.

When we perform 1-D fits of the STAR27ideal test data, as shown in Fig. 3, we obtain $K_{\Delta q} = 47.2$ without intercept or 49.0 with intercept. They are drastically different from the Δq slope of -6.00 from the 2-D fits. Therefore, the coefficient for the Δq or ΔS dependence from 1-D fits can be quite different from that from 2-D fits even in the ideal CSR case and is thus incorrect. Note that the 2-D fit plane of the ideal CSR test data perfectly goes through each data point, as shown in Fig. 2. However, that is not the case for the 1-D fits (see Fig. 3).

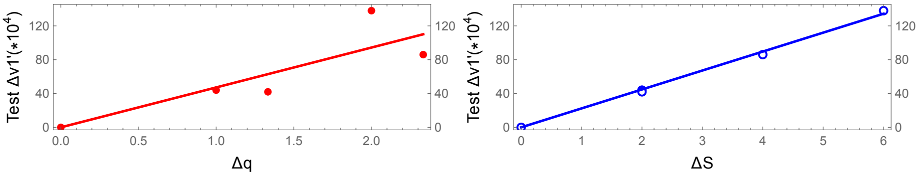

Figure 3. 1-D fits (w/o intercept) of the STAR27ideal test data that satisfy the coalescence sum rule.

In the ideal case where the Δv_1 data follow the CSR, the 2-D coefficients are unchanged when a given hadron set is scaled by a constant, e.g., changing the STAR Index 4 to $v_1[\overline{\Omega}^+(\bar{s}\bar{s}\bar{s})]/2$ (left side) and $v_1[\Omega^-(sss)]/2$ (right side). After all, the overall normalization of the hadron weighting factors for each hadron set is arbitrary. For demonstration, we scale Index4 of the STAR27ideal test data by 1/2 to get the STAR27ideal2 test data; the new Index4 now has $\Delta q = 1$ and $\Delta S = 3$ and STAR27ideal2 still represents the ideal CSR case. As expected, 2-D fits of the unscaled (STAR27ideal) and scaled (STAR27ideal2) test data give exactly the same coefficients. When we perform 1-D fits, however, the $K_{\Delta q}$ and $K_{\Delta S}$ values from fitting the unscaled and scaled test data are different, as shown in Table 2. This clearly shows that the coefficients from 1-D fits are ill-defined mathematically; therefore, 1-D fits should not be used for the analysis of the v_1 difference data.

3 Summary

On the direct flow difference of hadron sets consisting of produced hadrons, we show that 2-D fits give different Δq (or ΔS) slope values than 1-D fits, even for the ideal test data that follow the CSR relations perfectly. We also demonstrate that different choices of the five independent hadron sets are equivalent and give the same 2-D coefficient values when the v_1 difference data follow the CSR. On the other hand, if the extracted coefficients depend on the choice, it indicates the breaking of the CSR. Finally, we show that the coefficients from 1-D fits depend on the arbitrary scaling of a hadron set and are therefore ill-defined.

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