MATHEMATICS TEACHER
Learning & Teaching
PK 12

Mathematics Teacher: Learning and Teaching PK-12, is NCTM's newest journal that reflects the current practices of mathematics education, as well as maintains a knowledge base of practice and policy in looking at the future of the field. Content is aimed at preschool to 12th grade with peer-reviewed and invited articles. *MTLT* is published monthly.

ARTICLE TITLE:

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DIGITAL OBJECT IDENTIFIER: 10.5951/MTLT.2023.0366

VOLUME: 117

ISSUE NUMBER: 9

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Approved by the NCTM Board of Directors on July 15, 2017.

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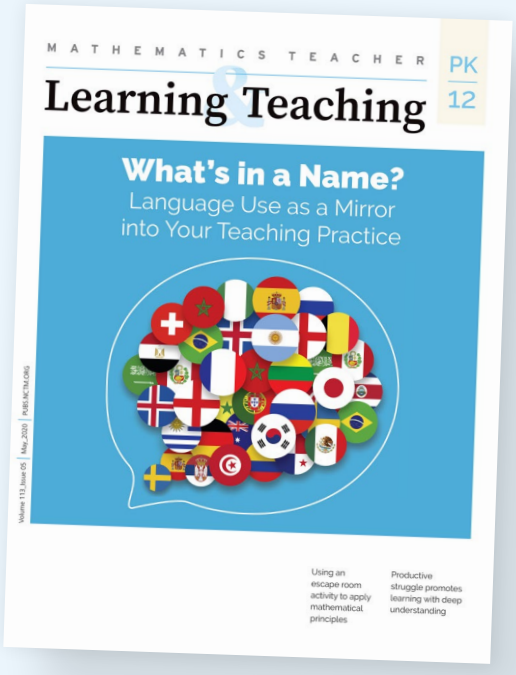

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What Do You Do With an Idea?

In a mathematics class where students are encouraged to freely share ideas, the room brims with diverse thoughts. How do students and teachers use them? Let's follow a group of multilingual students and teachers to see what they did with an idea!

Gladys H. Krause, Caroline Utne Finchum, and Maria-Emilia Borja

In a classroom environment where students are encouraged to talk and share their thinking about mathematics, it comes as no surprise to find an abundance of ideas flooding the room. Yet, what do we do with all these ideas? How do teachers and students use these ideas as they learn in the classroom? Moreover, how do teachers and students manage these conversations in a linguistically diverse classroom? These complex questions are major points of discussion for educators (Adams Corral & Sayer, 2023).

In this article, we (a teacher educator, a multilingual learner-teacher, and an artist) initiate a dialogue about the intersection of planning and spontaneity in classroom instruction. Often perceived as opposites, we assert that they can be complementary. We will begin by presenting an example from a linguistically diverse classroom to illustrate the origin of an idea, how the three of us navigated it with students, and how it enriched the learning experience of the classroom. We will then delve into two additional examples of how we included spontaneous student ideas within our planned mathematics lessons.

BACKGROUND

The classroom setting we describe is atypical because it involves students from second to eighth grade learning together. In this classroom, students spoke Spanish, Pashto, or Farsi as their first language. A cohort of eight students in this class received support daily in English and mathematics from the multilingual learner teacher. Additionally, at the end of the school year, these students participated in a summer camp. During this camp, the students, along with their teacher, a teacher educator, and a local artist, continued their mathematical learning while designing and painting a mural. The three of us came together to work as part of a collaborative research project focused on understanding how teacher and student knowledge intersect to create new knowledge that is relevant in racially, culturally, and linguistically diverse mathematics teaching contexts. As part of this endeavor, we view art as a crucial component in facilitating the expression and communication of this new knowledge. Our learning intentions for the summer camp included facilitating students' conceptual understanding of surface area, proportions, and estimation through the experience of painting a mural. We planned each stage of creating the mural to foster problem-solving scenarios for students to discover and apply the targeted mathematics concepts.

COLORS, MATHEMATICS, AND LANGUAGE

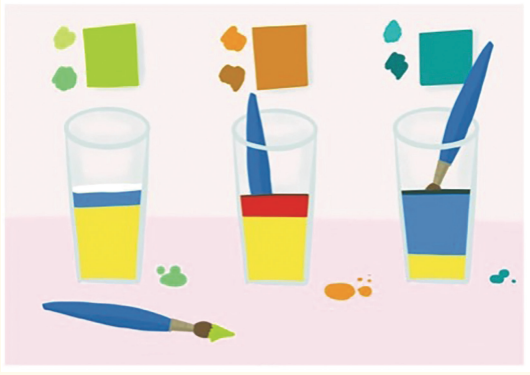
On one of the initial days of the summer camp, the students and the three of us collaborated in the process of mixing colors for the mural. Seated on the floor and engaged in a discussion about the amount of

paint required to achieve the ideal color shade for a section of our mural, each student received a small, transparent container, a brush, and the primary colors of paint to attempt to create the shade (Figure 1). As students actively mixed colors, a range of shades emerged. This facilitated a discussion on proportions, our planned learning goal.

While comparing colors, one student took a unique approach of inspecting the container from the bottom, as if seeking a different perspective to verify the accuracy of the color shade. The artist noticed this, and she too examined her color from the container's bottom, prompting other students to do the same. They found that it was easier to see the shade of the color from that angle. This sparked the artist's idea to flip the containers and arrange them from *darkest* to *lightest* to analyze the diversity of shades. The multilingual learner teacher used this as an opportunity to emphasize superlatives in English, one of the language goals. This brief conversation, lasting only a few minutes, was rich in meaning making. It introduced alternative ways of perceiving color, reviewed language features, and sparked a discussion about order, all while we pursued the intended shade for the mural (Figures 2a and 2b).

What made this student's idea turn into a rich learning experience? Perhaps it was the artist paying close attention to the students' actions and ideas. She was flexible and pivoted in the moment to create an interdisciplinary learning opportunity that was relevant to the students in front of her. Her flexibility in the delivery of the lesson allowed the teacher to emphasize other concepts inherent in the activity, illustrating the spontaneity of true learning. The rest of the lesson proceeded following our initial structured plan; however, we (students, artist, teacher, and teacher educator) continued to marvel at the power of one idea to spark deep and relevant learning experiences.

Figure 1 Materials for Creating a Shade for the Mural



FOSTERING STUDENT IDEAS

Using a soccer-game analogy, Krause et al. (2019) conceptualized the development of mathematical ideas within the classroom context. Through this analogy, they suggested that teachers can approach lesson planning as if they were preparing for a game. This approach suggests detailed planning (Stein et al., 2008) for various specific actions and other more flexible times that allow for space to adapt how teachers achieve their instructional goals. By planning for student input within the lesson, teachers can remain aware of the multiple paths through

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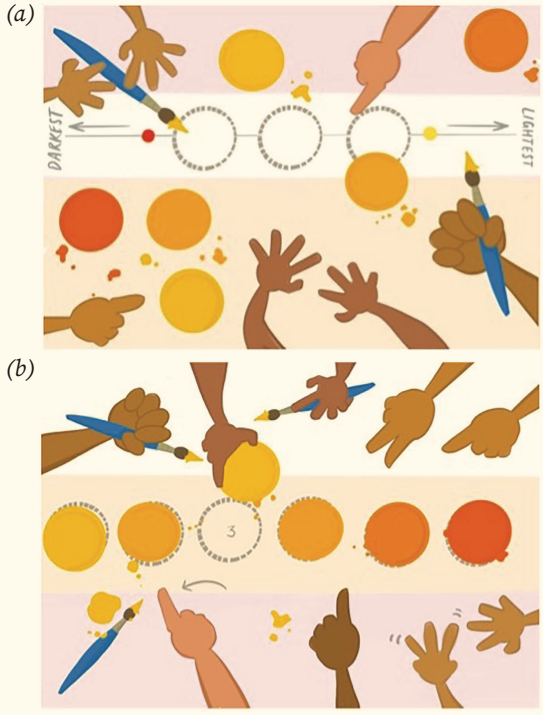
doi:10.5951/MTLT.2023.0366

which learning can be constructed at specific times within the lesson (Krause et al., 2019). While this may feel more challenging than delivering a fully scripted lesson plan, it aligns with the concept of co-creating knowledge, which makes deeper learning possible (Maldonado Rodríguez et al., 2022). We contend that teaching should encourage a dialogic process in which both students and teachers share their experiences, fostering an equitable exchange of ideas. In our example, the artist captured the moment when the student viewed the container from a different perspective. This spontaneous conversation aligned with our expectations for instruction. Creating room for such conversations can be integrated into an instructional plan. However, this space does not necessarily have to strictly adhere to the original plan. Teachers can have flexibility to deviate from it.

Introducing this duality (planning and flexibility) to educators can understandably seem overwhelming, as it challenges them to accept that the direction of the lesson is not fully controlled by them. However, it is important to emphasize that this kind of instruction does not occur without planning. Entering the classroom unprepared and merely observing how events unfold will not provide the space for intentional learning. Instructional planning goes beyond a teacher-centered scripted plan. As Stein et al. (2008) described in their five practices for facilitating mathematical discussions, instructional planning requires anticipating various ways in which students might think and includes meaningful activities to engage them in advancing their thinking. This is where the idea of planning and spontaneity as complementary elements in instruction, rather than opposing forces, comes into play. Below, we share two examples on different occasions from the same classroom. They provide some ideas on how both planning and spontaneity can work together.

Figure 2 (a) Representation of Ordering Shades; and (b) Order: Lightest to Darkest



Co-creating Problems

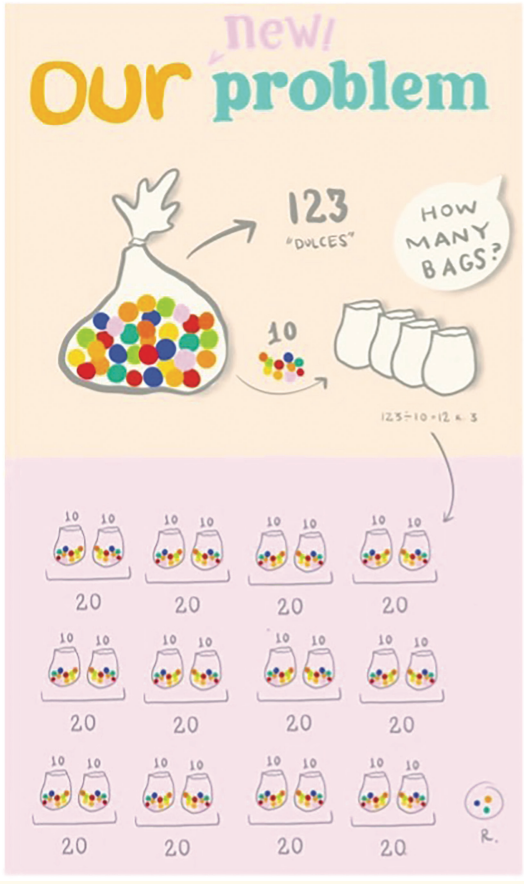
To practice division with two of the students from the summer camp, I (the teacher educator) collaborated with the multilingual learner teacher to plan and write a set of word problems. Our lesson objective was to reinforce the idea of 10 as a unit in alignment with the pacing of the general education curriculum. As part of our planning process, we decided to use measurement division problems because this problem type would allow for making groups to solve the problem, and the numbers facilitated the idea of 10 as a unit (Carpenter et al., 2014). The plan was to describe the context of the problem, unpack the problem with the students, and allow time for the students to solve the problem independently and then discuss the strategies as a group. As I posed the first problem to the students, they suggested that instead of solving my problem, they create their own problems. As I embraced this spontaneity, I compromised that we co-create them together. To keep our focus on division, measurement division in particular, I proposed that the problems involve arranging things into groups. One of the students immediately recognized this as a division problem. The students proposed the context and one set of numbers. I suggested grouping the items by 10, which allowed us to continue with the initial instructional goal of base 10 and grouping by tens, even though we did not use my problem. I found this moment of spontaneity allowed me to better engage the students in the learning process. Making the decision took me a split second, but it provided me with 45 minutes of rich mathematics instruction, where we

discussed division, grouping by tens and twenties (as we solved the same problem with different numbers), the meaning of the remainder, and writing equations that aligned with the strategies used by the students (Figure 3).

When Is Your Birthday?

I, the multilingual learner teacher, was also able to integrate students' ideas into my lesson plan. The plan for my lesson was to guide my two students into discovering and using multiple strategies for solving addition and subtraction relationships within familiar contexts. I created this plan because one of the students wanted to use the subtraction algorithm to solve subtraction relationships but was incorrectly following the procedure and was at a loss for how to determine if her answer was correct. I hoped that through contextualizing subtraction, introducing and co-creating new strategies for solving the problems, and using numbers within 30, the subtraction relationships would become clearer for her. Although I had several situations pre-planned, the social conversation we had leading up to the lesson created a rich context for solving the student's curiosity with addition or subtraction, so I pivoted and chose to start the lesson with her scenario. The student had shared that her twin brothers' birthday was coming up. I had asked her the specific date of their birthday, which prompted her to wonder how many days remained until their birthday. Thus, I proposed to the students that we figure it out. The students solved the problem through counting, and then I was able to help guide them into writing an equation that matched their strategy. Throughout the rest of the lesson, the students and I returned to this example as we solved new scenarios because the personal connection to the student made it a salient and a powerful connection to our learning goals.

Figure 3 Measurement Division Problem Co-Created With Students



WHERE DO IDEAS LEAD?

Perhaps the direction that mathematical ideas take depends on where you guide them. Maybe they transform into what you enable them to become. The choice ultimately lies with you. These examples demonstrate that by embracing the potential for diverse perspectives through planning for the sharing of students' mathematical thinking and ideas, we may uncover opportunities for spontaneous connections that deepen the learning experience for students.

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ACKNOWLEDGMENT

This work was supported by the National Science Foundation (DRL-2055419), but the opinions expressed do not necessarily reflect the position, policy, or endorsement of the funding agency.