Experimental exploration of quantitative optics-mechanics connection

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Abstract: We experimentally investigate polarization, entanglement, and complementary behavior of a light beam, and the center of mass and moment of inertia of a two-mass system, confirming an unexpected quantitative link between wave optics and mechanics.

While optics and mechanics are two distinct branches of physics, they are intrinsically connected. It is well known that the geometrical/ray treatment of light has direct analogies to mechanical descriptions of particle motion. More interestingly, an unexpected quantitative connection between wave optics and classical mechanics is reported recently [1]. A generic complementary identity relation between polarization and entanglement is demonstrated for arbitrary light fields and shown to be quantitatively associated with the mechanical concepts of center of mass and moment of inertia via the Huygens-Steiner theorem for rigid body rotation. Here, we propose an experimental investigation of the optical complementary identity relation, as well as the unexpected optics-mechanics quantitative link. Specific measurement approaches are designed to register both optical and mechanical quantities.

To derive the complementary identity relation between polarization and entanglement for a 2-dimensional (2D) light field, we start with the normalized form of the planar-transverse light field, which can be written with Dirac notations as [2]

$$|e\rangle = \alpha|x\rangle\,|e_x\rangle + \beta\,|y\rangle\,|e_y\rangle\,, \tag{1}$$

where $|e\rangle$ is the light field normalized by its intensity, $|x\rangle$, $|y\rangle$ are polarization components and $|e_x\rangle$, $|e_y\rangle$ are corresponding amplitude components, and α, β are real normalized coefficients such that $\alpha^2 + \beta^2 = 1$. The degree of polarization is defined as [3]

$$\mathcal{P}_2 = \sqrt{2\left(\mathrm{Tr}\,\mathcal{W}_{2\mathrm{D}}^2 - \frac{1}{2}\right)}, \tag{2}$$

where

$$\mathcal{W}_{2\mathrm{D}} = \begin{bmatrix} \alpha^2 & \alpha\beta\delta \\ \alpha\beta\delta^* & \beta^2 \end{bmatrix} \tag{3}$$

is the normalized 2D polarization coherence matrix with the cross correlation parameter $\delta = \langle e_x|e_y\rangle$. Here, $\mathcal{P}_2$ is normalized between 0 (complete unpolarization) and 1 (complete unpolarization). Entanglement of the 2D field can be quantitatively measured by the Schmidt weight parameter as

$$\mathcal{K}_2 = \sqrt{2\left(1 - \frac{1}{K_2}\right)}, \tag{4}$$

where $K_2 = 1/(m_1^2 + m_2^2)$ is the Schmidt number, with m_1 and m_2 being the eigenvalues of the 2×2 coherence matrix $\mathcal{W}_{2\mathrm{D}}$. Here, $\mathcal{K}_2$ is bounded between 0 (zero entanglement) and 1 (maximal entanglement). Combining the degree of polarization and entanglement for the 2D light field, one is then led to the complementary identity

$$\mathcal{P}_2^2 + \mathcal{K}_2^2 = 1. \tag{5}$$

The complementary relation can be visually illustrated by a geometric mapping which links to mechanical concepts. We let the polarization coherence matrix eigenvalues m_1 and m_2 to represent the weights of two point masses and place the two point masses symmetrically about the center point O, see illustration in Fig. 1 (a). Then the value of the degree of polarization $\mathcal{P}_2$ is exactly the distance from O to the center-of-mass point M, i.e., $\mathcal{P}_2 = \overline{OM}$, and the value of degree of entanglement $\mathcal{K}_2 = \overline{MB}$ where $\overline{MB} \perp \overline{OM}$ and B is the cross point with the unit circle O [2]. As a result, the 2D complementary relation $\mathcal{P}_2^2 + \mathcal{K}_2^2 = 1$ is represented by $\overline{OM}^2 + \overline{MB}^2 = \overline{OB}^2$ of the right triangle $\triangle OMB$ as shown in Fig. 1 (a).

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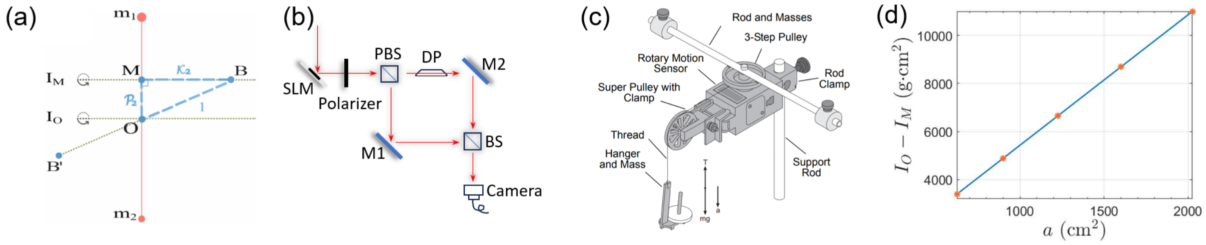

Fig. 1. (a) Geometric illustrations of mapping optical polarization coherence and entanglement to mechanical concepts for 2D light field. (b) Proposed optical setup to measure polarization and entanglement of the beam. (c) Mechanical setup to measure moment of inertia of a two mass system. (d) Relation between moment of inertia difference and the separation of the two masses in setup (c).

The optical experiment to verify the complementary identity relation is composed of a polarizer, a spatial light modulator (SLM), a Dove prism, and a Mach-Zehnder interferometer as shown in Fig. 1 (b). The SLM generates a first order Hermite-Gaussian mode, which enters a polarizer and a polarized beam splitter (PBS), before splits into two beams with continuously varying intensities. In one path of the interferometer, a Dove prism (DP) is used to rotate the spatial mode by $\pi/2$. The output beam from the beam splitter is exactly 2D light field in (1).

The moment of inertia (MOI) of a system is related to center of mass. In 2D case, when the rotation axis passes through the mass center M, the MOI is $I_M = m_1(1-\overline{OM})^2 + m_2(1-\overline{OM})^2$. The MOI with respect to a parallel axis that passes through the geometric center O is then $I_O = m_1 + m_2$ (unit length from O to masses). With the relation $\mathcal{P}_2 = \overline{OM}$ and $\mathcal{P}_2^2 + \mathcal{K}_2^2 = 1$, the degrees of polarization and entanglement can be expressed as

$$\mathcal{P}_3 = \sqrt{I_O - I_M}, \quad \text{and} \quad \mathcal{K}_3 = \sqrt{1 - I_O + I_M}. \tag{6}$$

These expressions establish direct quantitative connections between optical coherence quantities and mechanical properties. In classical mechanics, the MOI about paralleled axis is described by the Huygens-Steiner theorem. We can use the optical setup in Fig. 1 (b) to verify this theorem. Furthermore, to illustrate the similarity between optical and classical mechanical property, the Huygens-Steiner theorem can be also tested by the mechanics experiment: Two weights with masses m_1 and m_2 are mounted on the rod with a separation of a. The MOI about the geometric center I_O and mass center I_M are measured using the setup shown in Fig. 1 (c) [4]. The relation between the moment of inertia difference $I_O - I_M$ and distance a can be obtained as

$$I_O - I_M = (m_1 + m_2)\left[\frac{m_1 - m_2}{2(m_1 + m_2)} a\right]^2 \tag{7}$$

For weights with $m_1 = 148.25$ g and $m_2 = 78.13$ g, the experimental data is expected to be aligned just as shown in Fig. 1 (d).

In conclusion, we propose a systematic experimental procedure to test the theoretical results of quantitative links (6) between optics and mechanics. Using meticulously designed optical and mechanical setups, one can not only verify the optical coherence properties but also draw parallel conclusions in classical mechanics. The experiments leveraged the structured relationships derived from the Huygens-Steiner theorem, applying them to complex light fields and mechanical analogs. Our preliminary findings already support the notion that optical coherence properties such as polarization and entanglement can be described using classical mechanical principles, thus confirming the theoretical predictions. Furthermore, the successful execution of these experiments provide a new platform through which the interplay between optics and mechanics can be tested and comprehended. This will pave the way for innovative applications in fields such as quantum computing, optical engineering, and materials science. The validation of these concepts not only enriches our understanding of light and motion but also underscores the unified nature of physical laws across seemingly disparate domains.

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