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Superresolution of three point-sources assisted with machine learning

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Abstract: We demonstrate super-resolved localization of three point sources with the assistance of a machine learning model that is based on the decomposition of the source signal into Hermite Gaussian modes. High fidelity of over 80% is achieved.

Superresolution of two passive point sources has been extensively studied recently by proposing techniques that are capable of maintaining high accuracy in estimating the source separation far beyond the Abbe-Rayleigh diffraction limit [1–5]. This promises the possibility of superimaging which relies on the superesolution of neighboring point sources of an object. A crucial step toward the realization of superimaging is the capability of superresolving multiple point sources. In this work, we explore the superresolution of three-point sources with the assistance of a specially trained machine learning model to precisely estimate the locations of three partially coherent point sources with arbitrary intensity unbalancedness.

Our methodology employs the power of a machine learning model to extract essential separation information of the three sources through the decomposition of complex intensity patterns into the spatial basis of Hermite Gaussian (HG) modes. Remarkably, this AI-assisted technique achieves superresolution beyond the Abbe-Rayleigh diffraction limit, enabling high accuracy localization.

We consider the point spread function of the point sources to be Gaussian. Then the overall three sources in the image plane can be represented as

$$\Phi(x,y) = \sum_{i=1}^{3} A_i e^{-\frac{(x-x_i)^2+(y-y_i)^2}{\sigma^2}} \tag{1}$$

where (x_i, y_i) are the coordinates of the i-the source and A_i are the corresponding amplitudes. See an illustration of the three sources by the schematic plot in Fig. 1. By decomposing $\Phi(x,y)$ into Hermite Gaussian modes, onr can re-express it as:

$$\Phi(x,y) = \sum_{n,m=0}^{\infty} C_{nm} H_n(x) H_m(y) e^{-\frac{x^2+y^2}{\sigma^2}} \tag{2}$$

where C_{nm} are the coefficients for the corresponding HG_{nm} modes. Here we have absorbed the propagating z dependent terms into C_{nm} as we will analyze the signal at a fixed plane. The coefficients C_{nm} can be analytically obtained through

$$C_{nm} = \int\int H_n^*(x) H_m^*(y) e^{-\frac{x^2+y^2}{\sigma^2}} \Phi(x,y) dxdy. \tag{3}$$

For example, the first three coefficients (C_{00}, C_{01}, and C_{10}) can be achieved as

$$C_{00} = \left(e^{-\frac{x_1^2+y_1^2}{2\sigma^2}} + e^{-\frac{x_2^2+y_2^2}{2\sigma^2}} + e^{-\frac{x_3^2+y_3^2}{2\sigma^2}} \right), \tag{4}$$

$$C_{10} = \frac{1}{2}\left(e^{-\frac{x_1^2+y_1^2}{2\sigma^2}} x_1 + e^{-\frac{x_2^2+y_2^2}{2\sigma^2}} x_2 + e^{-\frac{x_3^2+y_3^2}{2\sigma^2}} x_3 \right), \tag{5}$$

$$C_{01} = \frac{1}{2}\left(e^{-\frac{x_1^2+y_1^2}{2\sigma^2}} y_1 + e^{-\frac{x_2^2+y_2^2}{2\sigma^2}} y_2 + e^{-\frac{x_3^2+y_3^2}{2\sigma^2}} y_3 \right). \tag{6}$$

As illustrated in Fig. 1, by choosing the first point source to be at origin ($x_1 = 0, y_1 = 0$) of the coordinate system, the second to be on the x-axis ($x_2, y_2 = 0$), and the third to be arbitrary (x_3, y_3), one can in principle invert the equations above and solve the position parameters x_2, x_3, y_3. Therefore the locations of all three sources can be determined.

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To obtain the coefficients C_{nm}, we train a machine learning program with a dataset of 50,000 images (38x38 pixels) containing three-point sources with known positions but with unknown random partial coherence and unbalancedness. Each image is decomposed into a series of Hermite Gaussian modes to achieve the corresponding coefficients [6].

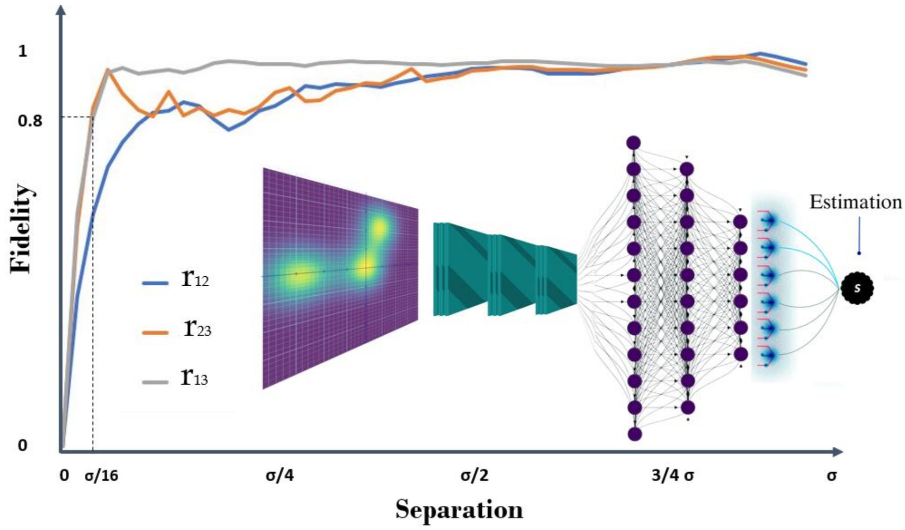

Fig. 1. The fidelity of the predicted positions of the three-point sources.

We designed a convolutional neural network (CNN) architecture with three convolutional layers followed by fully connected layers. The final output layer consisted of three nodes, each representing one of the coefficients, see schematic illustration in Fig. 1. During training, the model parameters were optimized using mean squared error loss and stochastic gradient descent to minimize the discrepancy between the predicted coefficients and the ground truth values. The trained model demonstrated high accuracy in predicting the coefficients on a separate validation dataset. This allows us to leverage the model's predictive capabilities for any new input image containing point sources, enabling the subsequent locations (x_i, y_i) of the three sources.

To quantify the accuracy of predicting the separation of the three sources, we define a quantity called fidelity F given as:

$$F_{ij} = 1 - \frac{|s_{ij} - r_{ij}|}{s_{ij} + r_{ij}}, \tag{7}$$

where s_{ij} is the separation between i-th and j-the source and r_{ij} is the actual distance.

With the trained CNN model we achieve separation fidelity exceeding 80% for point-source separations $\sigma/16 \leq s \leq \sigma$, well beyond the traditional Rayleigh resolution limit, see fidelity F results presented in Fig. 1.

In conclusion, we have demonstrated high accuracy superresolution of three-point sources assisted by a machine-learning model. This technique will be beneficial across different fields such as astronomy, biological microscopy, precise sensing, etc., particularly when the source is non-controllable. Future work involves refining the methodology, exploring alternative AI architectures, and handling more complex scenarios with additional point sources. Our work is supported by NSF Grant No. PHY-2316878 and the U.S. Army under Contact No. W15QKN-18-D-0040.

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