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AI-assisted superresolution of two practical point sources

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Abstract: We investigate super-resolution of two spatially separated practical point sources using machine learning. High fidelity of over 90% is achieved for separations that are 16 times smaller than the conventional resolution limit. © 2024 The Author(s)

The Abbe-Rayleigh diffraction criterion has long been an obstacle for fine resolution of particularly non-controllable sources. Recent studies have reported overcome of this long standing limit via way of spatial mode decomposition for two balanced incoherent sources [1]. Extensions to more practical situations of partial coherent [2] and unbalanced sources [3, 4] have been explored, however, with various limitations. In reality there are more realistic factors, such as unknown center of the two sources, detection inefficiency, background noise, propagation noise, etc. A comprehensive investigation of the simultaneous effects of all these factors is still missing, largely due the complexity of including all these practical elements into a solvable model. Here we propose a machine learning approach incorporated with the spatial mode decomposition technique. We develop a Convolutional Neural Network (CNN) that effectively extracts the first six coefficients of the Hermite-Gaussian (HG) modes, enabling it to estimate the separation between two point sources with a high accuracy of 0.9 fidelity with the consideration of various practical challenges such as background noise, photon loss, detection inefficiency, unknown two-source center, partial coherence, arbitrary unbalancedness, random phase, etc. Our findings represent a promising step toward achieving practical superresolution applications.

We consider two arbitrarily unbalanced partially coherent point sources separated with a distance s in the x-axis. At the imaging plane, the spatial domains of the two sources can be described as $h_{\pm}(x) = h(x \pm s/2)$ representing positive and negative $s/2$ displacements of the point-spread function $h(x)$, which is taken to be Gaussian $h^2(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp[-\frac{x^2}{2\sigma^2}]$ with σ being the width. The general optical field of the two sources is described as [3, 5]:

$$|\Psi\rangle = a\,|h_+\rangle\,|\phi_1\rangle + be^{i\varphi}\,|h_-\rangle\,|\phi_2\rangle\,, \tag{1}$$

where φ is an arbitrary phase, a and b are arbitrary amplitudes representing general unbalancedness of the two sources. Here $|\phi_1\rangle$ and $|\phi_2\rangle$ are the non-spatial (can be statistical) degree of freedom of each source where the overlap $\langle\phi_1|\phi_2\rangle = \gamma$ represents arbitrary coherence of the two sources. The total intensity of the two sources is given as $I(s,a,b,\gamma,\varphi) = \langle\Psi|\Psi\rangle$.

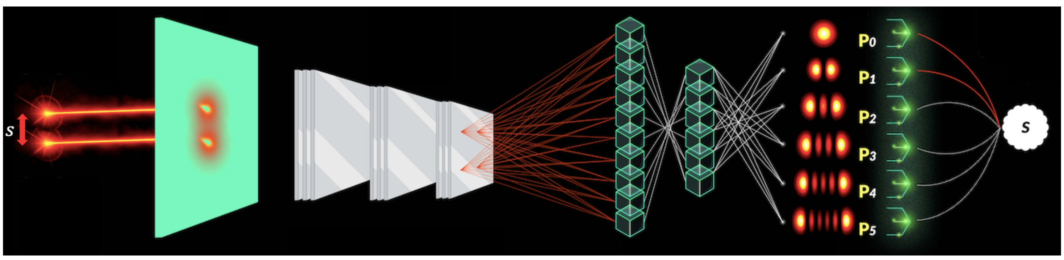

Fig. 1. Schematic illustration of CNN approach to achieve two-source superresolution.

According to the approach [1], to achieve the two-source separation s, measurements should be performed projecting onto HG modes $\{|\Phi_q\rangle\}$. The two-sources state can always be expressed in terms of the HG mode representation, i.e., $\langle x|\Psi\rangle = \Psi(x) = \sum_{q=0}^{\infty} C_q \Phi_q(x)$, where $C_q = \langle\Phi_q|\Psi\rangle$ is the corresponding coefficient of each HG mode. Then the probability of detecting photons in the q^{th} mode can be obtained as $p_q = |C_q|^2 = \eta_q(s)[1 + 2ab\Re(e^{i\varphi}\gamma)]$, where the coefficient

$$\eta_q(s) = e^{\frac{-s^2}{16\sigma^2}} \frac{(\frac{s^2}{16\sigma^2})^q}{q!} \tag{2}$$

contains the information of two-source separation s. To achieve this unknown parameter s, conventionally one needs to develop an estimation theory taking into account all practically unknown parameters. In reality, this is an extremely complex, if not impossible, task as too many realistically unknown parameters such as background noise, photon loss, two-source center, random phase, unbalancedness, partial coherence, need to be included.

Here we employ the CNN, see Fig. 1, to obtain the probability p_q of an arbitrary input signal on each of the HG modes. This approach avoids the trouble of building a complex estimation theory, while is still capable to obtain the two-source separation with high accuracy. In training the CNN model, a dataset consisting of 30,000 simulated images with 27x27 pixels are generated. Each of these images randomly chooses a separation, phase, unbalancedness, partial coherence, of the two sources. In its architecture, the CNN employed three convolutional layers equipped with large 9x9-sized kernels dedicated for detailed source image features.

With such a model we are able to achieve the separation s based on the ratio of coefficients. For example the ratio of the first two modes yields the separation information through $s = s_{01} = 4\sigma\sqrt{p_1/p_0}$. To assess the performance of the trained CNN, we use 100 simulated testing samples that were not part of the training dataset. We adopt a quantity called Fidelity defined as:

$$F = 1 - \frac{|s_{01} - s_R|}{s_{01} + s_R}, \tag{3}$$

to quantify the accuracy of the machine learning predicted separation (s_{01}) comparing to the actual separation (s_R).

When the parameters a, b, γ and φ are taken as known parameters, Fig. 2 shows that our trained CNN displays a very high fidelity over 0.95 (red curve) when the two-source separation $s > \frac{\sigma}{16}$, where σ is the conventional resolution limit. When all these parameters are taken as unknown, the fidelity maintains to be a high value of 0.9 (blue curve). It is important to note that our CNN model is very robust against real-world noisy conditions. That is, it can predict separation distances with high fidelity $F > 0.85$ even when there is significant background noise or phase distortions introduced during propagation or photon losses due to source properties.

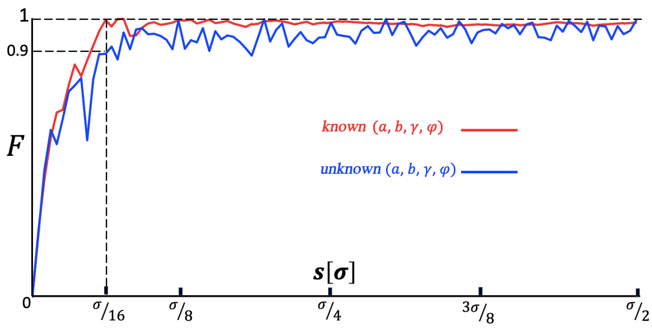

Fig. 2. Fidelity of the CNN predicted separation when compared with the actual separation.

As a conclusion, we have demonstrated superresolution of two practical point sources with high accuracy. Our method not only deals with the realistic factors of arbitrary partial coherence, unbalancedness, and random phase, it is also robust against background noise, photon loss, unknown two-source center. Our work is supported by NSF Grant No. PHY-2316878 and the U.S. Army under Contact No. W15QKN-18-D-0040.

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