Frontiers in Optics + Laser Science 2024 (FiO/LS) © Optica Publishing Group 2024

Simulating Quantum State Revival with Paraxial Beam Propagation

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Abstract: We explore the equivalence between paraxial optical beam propagation and 2D harmonic oscillator evolution. The phenomenon of quantum state revival of harmonic oscillator is shown to be simulated with the propagation of a focusing beam.

Quantum harmonic oscillator (QHO), as one of the few exactly solvable models in quantum mechanics, has significant impacts on both foundational and practical quantum issues. Its state time evolution is crucial for various quantum dynamical processes. For example, quantum revival, is a typical phenomenon in QHO dynamics that exhibits the periodic recurrence of a quantum wave packet [1]. On the other hand, it has been long known that the form of Schrödinger's equation for a two dimensional harmonic oscillator is isomorphic to that of the paraxial Helmholtz equation of light [2–4]. Therefore, the solutions (or eigen wave functions) of the two wave equations are also analogous to each other. Here we investigate the simulation of the quantum revival phenomenon for two dimensional QHO with the propagating light beam.

The wave equation of a linearly polarized transverse electromagnetic beam under the paraxial approximation is described by

$$\frac{\partial^2\Phi(r)}{\partial x^2}+\frac{\partial^2\Phi(r)}{\partial y^2}+2ik\frac{\partial\Phi(r)}{\partial z}=0. \tag{1}$$

With some simple coordinate transformations $(x=\xi\omega(z)/\sqrt{2}, y=\eta\omega(z)/\sqrt{2}$ and $z=b\tan\tau)$ where b denotes the Rayleigh range and $\omega(z)$ is the width of the beam at z given by $\omega(z)=\sqrt{\omega_0^2(1+z^2/b^2)}$, the above light wave equation (1) can be shown to be equivalent to the Schrödinger's equation for the two dimensional QHO [4], i.e.,

$$\left[-\frac{\partial^2}{\partial\xi^2}-\frac{\partial^2}{\partial\eta^2}+\xi^2+\eta^2\right]\psi(\xi,\eta,\tau)=2i\frac{\partial}{\partial\tau}\psi(\xi,\eta,\tau). \tag{2}$$

Here $\hbar$ and mass(m) are both set to 1 for convenience of analysis. The transverse electromagnetic (TEM) mode solutions of the paraxial light equation is equivalent to the Hermite-Gaussian solution to the harmonic oscillator Schrödinger's equation, and can be obtained as:

$$\Phi_{mn}(r)=\frac{\omega_0}{\omega(z)}\phi_m\Big(\frac{\sqrt{2}x}{\omega(z)}\Big)\phi_n\Big(\frac{\sqrt{2}y}{\omega(z)}\Big)e^{\frac{ik(x^2+y^2)}{2R(z)}}e^{-i(m+n+1)\varphi(z)},\ \text{where}\ \phi_n(\xi)=\frac{1}{\sqrt{2^n n!\sqrt{\pi}}}H_n(\xi)e^{-\xi^2/2}. \tag{3}$$

Here ω_0 is the waist of the beam given by $\omega_0=\sqrt{2b/k}$, k is the wave number, $\varphi(z)$ denotes the Gouy phase and $R(z)$ is the radius of curvature of the wavefront of the beam. The function ϕ_n is exactly the solution to a one dimensional HO given by the product of a constant $\frac{1}{\sqrt{2^n n!\sqrt{\pi}}}$, Hermite polynomial $H_n(\xi)$, and a Gaussian function $e^{-\xi^2/2}$. In other words, the solution to the paraxial wave equation in Eq. (1) is the same as the solution to a two dimensional HO up to a constant and a phase difference.

Fig. 1a and Fig. 1b illustrates the similar behavior of two corresponding eigen wave function solutions as it evolves in the time (or propagates in the z direction). Fig. 1a represents the probability distribution of the QHO eigenstate $\phi_3(\xi)$ and Fig. 1b represents the intensity distribution of the light beam for the TEM(30) mode. The two diagrams show similar evolution (propagation) behaviors except for the intensities focusing for the paraxial beam at $z=0$ Fig. 1b. This can be shown to be the same as QHO states after performing the aforementioned coordinate transformation.

Similarly, Fig. 1c (or Fig. 1d) illustrates the evolution (or propagation) of a wave packet, composed of a superposition of five eigen functions of ξ (or x) for the quantum harmonic oscillator (or paraxial beam). The phenomenon of quantum revival [1], which signifies the periodic return of the initial quantum wave packet after some finite

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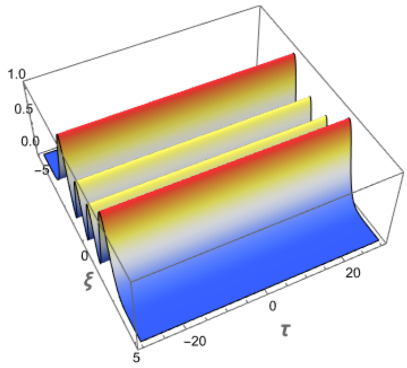
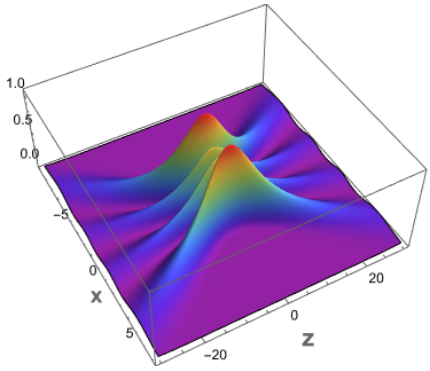
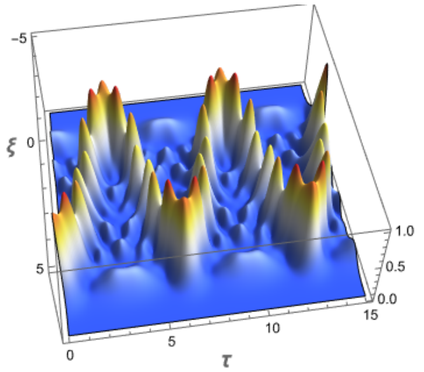


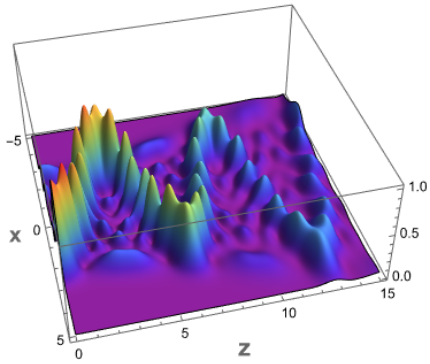
(a) Single QHO eigenstate: $\psi_3(\xi,\tau) = \phi_3(\xi)e^{3i\tau}$



(b) Single TEM mode: $\Phi_3(x,z) = \omega_0/\omega(z)\phi_3\left(\sqrt{2}x/\omega(z)\right)e^{ik(x^2)/2R(z)}e^{-(3+1))i\varphi(z)}$



(c) Superposition of 5 QHO modes: $\Psi(\xi,\tau) = \sum_n \psi_n(\xi,\tau)$ for n= {2,3,4,5,7}



(d) Combination of 5 TEM modes: $\Phi(x,z) = \sum_n \Phi_n(x,z)$ for n= {2,3,4,5,7}

Fig. 1. Time evolution of the solutions of the QHO (left) and the z-propagation of the solutions of the paraxial beam (right) plotted in 1D for simplicity.

time interval, can be observed in both figures at about τ(or z)=6. This further suggests that time evolution of QHO states can be mapped onto the propagation of the focusing paraxial beams. To experimentally verify the revival phenomenon of QHO with a light beam, one can employ a Spatial Light Modulator (SLM) to generate the initial wave packet with corresponding amplitude and phase information. Then by observing the light propagation in the z, one can record the time evolution of the QHO.

In conclusion, we have shown the wave equation and wave function equivalence of a two dimensional quantum harmonic oscillator with these of a paraxial light beam. Therefore, the propagating light beam provides a classical robust platform for simulations of the quantum revival phenomenon of a two dimensional harmonic oscillator. Our results suggest an alternative efficient way of quantum simulation. Future work will involve the experimental realization of the simulation. This work is supported by NSF Grant No. PHY-2316878 and Stevens Institute of Technology.

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