Countercyclical Unemployment Benefits: A General Equilibrium Analysis of Transition Dynamics

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Abstract

We analyze the general equilibrium effects of countercyclical unemployment benefit policies. Our heterogenous-agent model features costly job search with imperfect insurance of unemployment risk and individual savings. Our model predicts: (1) the additional unemployment under a countercyclical policy relative to that under an acyclical policy to be a superlinear function of the aggregate shock?s size, (2) a higher unemployment rate sensitivity to UI policy changes when individual savings are relatively low. Our estimates of the effects of UI policy changes are based on transition dynamics following a large, unanticipated increase in the unemployment rate.

Keywords: Unemployment Benefit policy, General equilibrium, Mean field game, Transition dynamics

JEL Classification: E24 , E32 , E62 , J64 , J65

1 Introduction

Unemployment insurance (UI) in the United States increases in recessions.[1] Such increases in the generosity of benefits involve an enormous expansion of total UI expenditures, especially in severe recessions, and are therefore accompanied by a debate on whether the benefits of such an expansion outweigh their costs. This debate is guided by existing work that has quantified the cost and benefit of UI benefit increases; in particular, the tradeoff between preventing large consumption declines of the unemployed on one hand, and reducing the incentives of job search on the other (see e.g., the review by [1]). This literature has pointed out that in analyzing the effect of benefit increases, it is important to model the ability of individuals to self-insure through accumulated savings (see e.g., [2]).

Existing general equilibrium analyses which allow for individual savings have been restricted to an analysis of steady-state equilibria (see e.g., [3] and [4]). Since such analyses compare steady-state equilibria with different parameter values characterizing UI policies, they provide estimates of a *permanent* change in UI payments. In reality, however, changes in UI payments are short-lived. Therefore, there is a need for a general equilibrium analysis of the transition dynamics following a change in the UI policy.

Our paper fills this gap. Our heterogenous agent model features costly job search. Individuals are unable to perfectly hedge their unemployment risk, but self-insure through savings. The UI policy in this economy is countercyclical. In particular, we assume that the increase in the UI payment from its steady-state value is directly proportional to the increase in the unemployment rate from its steady-state value. While we assume the form of the UI policy, the actual level of benefits and the unemployment rate are endogenous and jointly determined in the general equilibrium.

We analyze the transition dynamics following an unanticipated aggregate shock which results in an increase in the unemployment rate. While analyzing the transitional dynamics is harder, since it features a time-dependent cross-sectional distribution of agent savings as a state variable, we believe this analysis provides better estimates of the effect of UI extensions than those inferred from steady-state analyses, since expansions in UI are short-lived in practice. We use numerical methods from mean field game theory (see e.g., [5]) to solve for our model's equilibrium.

Our model provides two insights. First, we find that the additional unemployment under a countercyclical UI policy relative to an acyclical policy, which we call the "excess unemployment rate", is a superlinear function of the size of the aggregate shock. For instance, in our quantitative exercise, we find that the excess unemployment rate following a shock that raises the unemployment rate to the level seen during the Great Recession to be 1% when we fully solve the model, while a linear extrapolation from the steady-state underpredicts the excess unemployment rate to be only 0.6%. Our result therefore implies that gauging the effect of UI benefit increases on the

[1] The CARES Act of 2020, for instance, added $600 per week to the pre-existing level of UI benefits (the maximum pre-existing benefits varied from $200 per week to $600 per week across states) and extended the duration of benefits, initially to a maximum of 39 weeks, and later extending this to 53 weeks (the pre-existing duration for most states was 26 weeks). Similarly, UI benefits following the Great Recession were extended by 99 weeks from the usual 26 weeks.

unemployment rate in disaster states by a linear extrapolation of existing empirical estimates during typical recessions would significantly understate the size of the effect.

The intuition for the rapid increase in the excess unemployment rate with the size of the aggregate shock is the following. The excess unemployment rate at a small time Δt following the realization of the aggregate shock is approximately proportional to the product of the shock's size and the difference between the aggregate job finding rates under the countercyclical and acyclical policies. The difference between the job finding rates under these two policies is an increasing function of the size of the aggregate shock. This is because a larger aggregate shock is associated with more generous UI payments, and hence lower equilibrium job search, under the countercyclical policy. Therefore, the product of the size of the aggregate shock and the difference between the aggregate job finding rates is a superlinear function of the aggregate shock's size. This superlinearity is further enhanced with the passage of time as fewer workers find employment under the countercyclical policy relative to that under the acyclical policy (due to lower job search effort). This in turn leaves UI payments at a relatively high level for a longer duration, further discouraging job search.[2]

Second, we find that the sensitivity of the equilibrium unemployment rate to UI policy changes is larger when individual savings is relatively low. This implies that the same increase in UI payments in 2020 results in higher excess unemployment than it would have in 1980, since there has been a large secular decline in savings by the lower 90% of the U.S. income distribution over the past four decades (see [6]).

We illustrate our result of the effect of the savings distribution through a comparative static exercise in which we consider two otherwise identical economies that are populated by individuals who differ in the value of their time preference parameter (i.e., degree of patience). The economy populated by less patient individuals saves less than the economy populated by more patient individuals. We compare the effect of the same UI policy change in response to the same aggregate shock in these two economies. We find that the economy with less patient individuals (and hence lower equilibrium savings) has a much higher unemployment rate sensitivity to policy changes. This result arises due to a combination of two effects that we explain below.

First, the reduction in the equilibrium job search effort in response to a given increase in UI payments is inversely related to individual savings. Intuitively, individuals with lower savings face a stronger pressure to find a job in order to prevent a sharp consumption decline. As a result, a given increase in UI payments implies a larger reduction in the aggregate flow out of unemployment in the economy populated by individuals with lower patience. Second, for the same level of savings, an individual in the economy with lower patience reduces their job search intensity by a greater amount. This is because impatient individuals attach a lower value to a future job. Taken together, these imply that the economy populated by individuals with lower patience, and hence lower equilibrium savings, has a higher sensitivity of the unemployment rate to changes in UI payments.

[2] This intuition holds over a short, finite period following the aggregate shock. The excess unemployment rate approaches zero as $t \to \infty$ when the equilibria under both policies revert back to the same steady-state.

Related Literature. We contribute to the literature that uses dynamic stochastic general equilibrium models to quantify the effect of UI policies on welfare and aggregate dynamics. Within this class of quantitative models, our paper relates two strands of the literature. The first allows agents to self-insure through savings. However, the analyses in these papers are in steady-state without any aggregate shock. Examples include [3] and [4]. The second analyzes the effect of UI policies in the presence of aggregate shocks. However, these papers do not allow for self-insurance by individuals through savings. Examples include [7–10].[3] The closest paper to ours is [12] who compare the effects of two countercyclical UI policies, both of which increase the generosity of UI payments during a recession: one raises the level of UI payments while the other increases the duration of benefits during a recession. In contrast to that paper, we obtain results which compare the effect of a countercyclical policy (which features a combination of an increase in duration and level of benefits in a recession) versus an *acyclical* policy in which the UI policy remains unchanged in a recession.

While we consider a restricted class of UI policies and analyze its implications, there is an important strand of the literature that addresses the optimal design of UI policies by taking a contract theory approach. Examples include [13–15]. The analyses in this literature are in partial equilibrium and are qualitative.

Our general equilibrium model builds on the partial equilibrium models that analyze the trade-off between consumption insurance and the provision of sufficient incentives for the unemployed to search for jobs. Recent examples include [2, 16–18]. In contrast to the partial equilibrium analyses in these papers, our general equilibrium analysis differs along two dimensions. First, our model captures the externality of changes in job search intensity of one individual on another. Second, the UI policy and the unemployment rate in our environment are endogenous and jointly determined, while the UI policy in the partial equilibrium analyses are exogenous.

2 The Model

We construct a general equilibrium model with costly job search to analyze the effects of a class of countercyclical UI policies in an economy with imperfect risk-sharing. The focus of our analysis is the transitional dynamics following an unanticipated, aggregate shock at $t = 0$ which increases the unemployment rate.

The Environment

The economy is populated by a continuum of infinitely-lived individuals of measure one with identical preferences:

$$\mathbb{E}_t \int_t^{\infty} e^{-\rho(\tau - t)} u(c_\tau) d\tau \, . \tag{1}$$

where t is current time, $u(c)$, $\{c_\tau\}_{\tau \geq t}$, and ρ are the individual's running utility function, the consumption trajectory, and the time preference parameter, respectively. At

[3] [11] analyzes implications of UI policy over the business cycle in a model featuring individual savings but do not consider the disincentives to job search from higher UI payments since the model does not feature costly job search.

any point in time, an individual is in one of two possible employment states $\epsilon \in \{\epsilon_1, \epsilon_2\}$, where ϵ_1 and ϵ_2 are the unemployment and employment states, respectively.

Each individual owns capital and is endowed with a single, indivisible unit of labor which they supply when employed. Individuals rent capital to a representative firm and earn rental income at a rate $r_t k_t$, where r_t is the market-wide rental rate for capital, and k_t is the individual's time-t capital stock. They also earn labor income $y_t \in \{y_{1t}, y_{2t}\}$, that is either equal to UI benefit y_{1t} for unemployed individuals, or after-tax wages $y_{2t} = (1-\theta_t)w_t$ for employed individuals, where w_t is the market-wide wage and θ_t is the income tax rate. We assume that y_{1t} depends on the individual's capital k, and we describe its specific form in equation (8) below.

Each individual uses total income partly for consumption and invests the remainder $i_t = r_t k_t + y_t - c_t$, where i_t is the investment rate. The law of motion for an individual's capital k_t is:

$$dk_t = -\delta k_t dt + i_t dt \,, \tag{2}$$

where δ is the depreciation rate. Individuals face a liquidity limit on capital

$$k_t \geq \underline{k}, \tag{3}$$

for some constant $\underline{k} \geq 0$.

We assume that the transition intensity from employment to unemployment λ_1 is an exogenously specified constant. The transition intensity from unemployment to employment $\lambda_2(s)$, on the other hand, depends on the job search intensity s of an unemployed individual. We model the tradeoffs faced by an unemployed individual using an off-the-shelf costly job search model (see e.g., [2, 16, 19–21]). Increasing search intensity s results in a linear increase in the transition intensity from unemployment to employment:

$$\lambda_2(s) = s \,, \tag{4}$$

where we have normalized the proportionality constant between λ_2 and s to one. Searching is costly, incurring a flow cost

$$\psi(s) = \frac{\phi s^{1+\kappa}}{1+\kappa} \,, \tag{5}$$

where $\phi > 0$ and $\kappa > 0$ are constant.[4]

We will denote the distributions of capital of unemployed and employed individuals by $g_1(k,t)$ and $g_2(k,t)$, respectively. These densities satisfy $\int_{\underline{k}}^{\infty} g_1(k,t)dk + \int_{\underline{k}}^{\infty} g_2(k,t)dk = 1$ for all t. The first term on the left-hand side of this equation corresponds to the aggregate unemployment rate, that is: $U_t = \int_{\underline{k}}^{\infty} g_1(k,t)dk$.

The production side of the economy consists of a representative firm which produces output with a Cobb-Douglas technology

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}, \tag{6}$$

[4]Our choice of this search cost is standard in the literature (see e.g., [2], and [21]). Recently [22], [23], and [24] provide direct evidence that higher UI benefits lower job search activity, thus providing a justification for costly job search.

where K_t and L_t are capital and labor inputs, respectively, and $0 < \alpha < 1$ is the capital share parameter. For simplicity, we assume that total factor productivity stays constant, and we normalize this to one. The firm rents capital and labor in competitive spot markets, taking the rental rate for capital r_t and the wage w_t as given. There are no adjustment costs for factor inputs and the firm chooses K_t and L_t to maximize the profit flow $\Pi_t = Y_t - r_t K_t - w_t L_t$. The first order optimality conditions imply that the rental rate $r_t = \alpha\big(K_t/L_t\big)^{\alpha-1}$ and the wage $w_t = (1-\alpha)\big(K_t/L_t\big)^{\alpha}$.

Finally, the income tax rate θ_t is determined by requiring total UI expenditure equal total tax collected from employed individuals: $\int_{\underline{k}}^{\infty} g_1(k,t) y_{1t}(k) dk = \theta_t \int_{\underline{k}}^{\infty} w_t g_2(k,t) dk$.

The Shock

Over the period $t < 0$, the economy is in the steady-state with time-invariant density functions $g_1^\star(k)$ and $g_2^\star(k)$ of unemployed and employed individuals, respectively, and a constant unemployment rate $U^\star = \int_{\underline{k}}^{\infty} g_1^\star(k) dk$. We call this the precrisis period. At $t = 0$, an unanticipated negative shock is realized. It results in a fraction of employed individuals becoming unemployed, that is, the distributions $g_1(k,t)$ and $g_2(k,t)$ change discontinuously at $t = 0$. The size of the aggregate shock is characterized by the change in the unemployment rate $U_0 - U^\star$, where $U_0 = \int_{\underline{k}}^{\infty} g_1(k,0) dk$ is the time-0 unemployment rate immediately after the shock is realized. For simplicity we assume that the probability of job loss is independent of the individual's capital holding k. This implies that for all k

$$g_1(k,0) = \frac{U_0}{U^\star} g_1^\star(k). \tag{7}$$

UI policy

We focus on UI payments y_{1t} of the form:

$$y_{1t} = \min\left(a + b(k_t - \underline{k}), \overline{y}\right) + \eta\left(U_t - U^\star\right), \tag{8}$$

where a, b, $\overline{y}$, and $\eta \geq 0$ are four constants which represent the minimum level of UI benefits, the sensitivity of UI benefits to k, the maximum level of UI benefits, and the sensitivity of changes in UI benefits to changes in the unemployment rate, respectively. In choosing the functional form (8), we incorporate a feature of UI payments in the U.S., namely, that such payments have a floor, increase with recent earnings, and have a ceiling. In particular, the dependence of y_{1t} on k_t implies that UI payments depend on recent earnings since the agent's savings k_t endogenously depend on past labor income.[5]

In the precrisis period $t < 0$, $U_t = U^\star$, and therefore the last term on the right-hand side of equation (8) drops out. During this period, the UI payments are a piece-wise

[5]Instead of modeling the dependence of UI payments on recent earnings through accumulated savings, an alternate choice would be to make UI payments depend explicitly on the agent's past year's labor income. The latter modeling choice would require us to introduce cross-sectional heterogeneity in wages in order to capture differences among high and low income agents in their incentives to search for a job. Because heterogeneous wages would significantly deviate from standard general equilibrium models in the UI literature, we leave its analysis for future research.

linear function of k, increasing linearly with slope b for individuals with $k \leq k_H = \underline{k} + \frac{\overline{y}-a}{b}$. All unemployed individuals with $k \geq k_H$, receive the same amount $\overline{y}$.

When the shock is realized at $t = 0$, UI payments increase by $\eta(U_0 - U^\star)$ for all unemployed individuals. We see that the increase depends both on the size of the aggregate shock $U_0 - U^\star$ and also on the value of the policy parameter η. The policy with $\eta = 0$ is an acyclical policy since changes in the aggregate unemployment rate U_t are not accompanied by changes in UI benefits (see equation (8)). In contrast, a policy with $\eta > 0$ is a countercyclical policy since an increase in U_t is accompanied by an increase in UI payments under this policy.

The individual's problem

Taking prices r_t and w_t, the tax rate θ_t, an individual chooses consumption $c(k,t)$ and, if unemployed, job search intensity $s(k,t)$ to solve the following optimal control problem

$$\max_{c,s} \quad \int_0^\infty e^{-\rho t} u(c(k_t,t)) - \psi(s(k_t,t)) \mathbb{I}_{\{\epsilon_t=\epsilon_1\}}(t) dt$$

$$\text{s.t.} \quad \begin{cases} k'(t) = (r_t - \delta)k_t + y_{1t}\mathbb{I}_{\{\epsilon_t=\epsilon_1\}} + y_{2t}\mathbb{I}_{\{\epsilon_t=\epsilon_2\}} - c(k_t,t) \\ k_t \geq \underline{k} \\ \mathbb{P}(\epsilon_{t+\Delta t} = 1 | \epsilon_t = 2) = \lambda_1 \Delta t + o(\Delta t) \\ \mathbb{P}(\epsilon_{t+\Delta t} = 2 | \epsilon_t = 1) = s(k,t)\Delta t + o(\Delta t) \end{cases} , \forall t \geq 0.$$

By a standard derivation, we obtain an unemployed individual's Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned} \rho v_1(k,t) \; &= \; \max_{c_1,s} u\big(c_1\big) + \big(y_{1t} + (r_t - \delta)k - c_1\big)\partial_k v_1(k,t) \\ &+ \; \big(v_2(k,t) - v_1(k,t)\big)\lambda_2(s) - \psi(s) + \partial_t v_1(k,t)\,, \end{aligned} \tag{9}$$

where c_1 is the consumption of an unemployed individual, $v_1(k,t)$ and $v_2(k,t)$ are the individual value functions in the unemployed and employed states, respectively. The first order condition for job search effort $s = s(k,t)$ is

$$\psi'(s(k,t)) = \lambda_2'(s(k,t))\big(v_2(k,t) - v_1(k,t)\big)\,. \tag{10}$$

Note that the optimal s depends on an individuals' current savings and on the aggregate conditions through $v_1(k,t)$ and $v_2(k,t)$. The envelope condition is $u'\big(c_1(k,t)\big) = \frac{d}{dk}v_1(k,t)$.

An employed individual's HJB equation is

$$\begin{aligned} \rho v_2(k,t) \; &= \; \max_{c_2} u\big(c_2\big) + \big(y_{2t} + (r_t - \delta)k - c_2\big)\partial_k v_2(k,t) \\ &+ \; \big(v_1(k,t) - v_2(k,t)\big)\lambda_1 + \partial_t v_2(k,t)\,. \end{aligned} \tag{11}$$

where c_2 is the consumption of an employed individual. The envelope condition is $u'\big(c_2(k,t)\big) = \frac{d}{dk}v_2(k,t)$.

The Kolmogorov forward (KF) equation for the evolution of the distributions g_1 and g_2 are:

$$\begin{aligned}\partial_t g_1(k,t) &= -\frac{d}{dk}\big(g_1(k,t)(y_{1t} + (r_t-\delta)k - c_1(k,t))\big) - \lambda_2(s(k,t))g_1(k,t) + \lambda_1 g_2(k,t)\\ \partial_t g_2(k,t) &= -\frac{d}{dk}\big(g_2(k,t)(y_{2t} + (r_t-\delta)k - c_2(k,t))\big) - \lambda_1 g_2(k,t) + \lambda_2(s(k,t))g_1(k,t)\,.\end{aligned} \tag{12}$$

Equilibrium

The competitive equilibrium consists of consumption and job search policies of unemployed individuals $c_1(k,t)$ and $s(k,t)$, respectively, the consumption policy of employed individuals $c_2(k,t)$, the densities of capital for the unemployed and employed $g_1(k,t)$ and $g_2(k,t)$, aggregate capital K_t and labor L_t, the rental rate r_t and the wage w_t, the tax rate θ_t, and the UI policy y_{1t}, given the initial distributions $g_1(k,0)$ and $g_2(k,0)$ immediately after realization of the shock, such that: (i) unemployed individuals choose consumption and job search according to (9), (ii) employed individuals choose consumption to according to (11), (iii) the densities $g_1(k,t)$ and $g_2(k,t)$ satisfy (12), (iv) the firm chooses capital K_t and labor L_t to maximize firm profit Π_t, (v) the capital market clears: $K_t = \int_{\underline{k}}^{\infty} k\big(g_1(k,t)+g_2(k,t)\big)dk$, (vi) the labor market clears: $L_t = \int_{\underline{k}}^{\infty} g_2(k,t)dk$, (vii) the goods market clears: $C_t = Y_t - I_t$, where the aggregate investment $I_t = \int_{\underline{k}}^{\infty} (i_1(k,t)g_1(k,t) + i_2(k,t)g_2(k,t))\,dk$ and the total consumption $C_t = \int_{\underline{k}}^{\infty} (c_1(k,t)g_1(k,t) + c_2(k,t)g_2(k,t))\,dk$, (viii) total UI expenditure equals total tax collected $\int_{\underline{k}}^{\infty} g_1(k,t)y_{1t}(k)dk = \theta_t \int_{\underline{k}}^{\infty} w_t g_2(k,t)dk$, and (ix) the UI payments satisfy equation (8).

Remark 1. *We numerically solve the HJB-KF equation system (9, 11,12) employing algorithms detailed in Appendix A. The intricate coupling between the HJB and KF systems, along with the presence of hybrid controls and state constraints, makes establishing the existence and uniqueness of the equilibrium an open question. For a more in-depth exploration and discussion of the theoretical analysis and challenges involved, we refer to [25] and [5].*

3 Quantitative Results

3.1 Calibration

We use the parameters in Table 1. All values are annual. We choose commonly-used values for the preference and technology parameters. We choose the capital share parameter $\alpha = 0.33$, the depreciation rate $\delta = 0.10$, and the liquidity limit $\underline{k} = 0$. We choose the time preference parameter $\rho = 0.05$, and we assume that individuals have a constant relative risk aversion $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, with relative risk aversion $\gamma = 2$.

We choose the 3 parameters of the UI benefit function (8) which are, the minimum and maximum benefits $a = 0.018$ and $\overline{y} = 0.09$, respectively, and the threshold

Table 1 Parameter values

Parameter	Symbol	Model
Capital share	α	0.33
Depreciation rate	δ	0.10
Time-preference parameter	ρ	0.05
Risk aversion	γ	2
Job separation intensity	λ_1	0.4
UI benefit parameters	a	0.018
	k_H	1.039
	$\overline{y}$	0.09
Job search cost parameters	ϕ	0.01
	κ	0.55

All values are annual.

level of capital at which the UI benefit reaches its maximum value $k_H = 1.039$, to approximately match: (i) the slope of the UI policy with respect to income (over the increasing part of the UI policy), (ii) the location of the threshold income in the income distribution at which the UI policy reaches its maximum value, and (iii) the ratio of the maximum to minimum UI payments. We rely on recent estimates of UI insurance payments documented by Ganong et al. [26] (GNV) for two of these data values. In particular, GNV find that the slope of the UI policy with respect to income is 0.5. In comparison, our model-implied estimate is 0.47. GNV find that UI benefits reach the maximum value for individuals whose income is close to the mean of the income distribution.[6] In our calibration, UI benefits reach their maximum for individuals with total income (rent from capital plus wages) equal to 1.15, while the mean income is equal to 1.49. Finally, Krueger and Meyer [1] find the average value of the ratio of the maximum to minimum UI benefits across states to be 0.2. Our model-implied value for $a/\overline{y}$ is also 0.2.

In order to compare the effects of an acyclical and a countercyclical UI policy, we choose $\eta = 10$ for the countercyclical policy to approximately match the increase in total UI spending as a fraction of output in the data. We obtain data estimates for this ratio from https://data.oecd.org/socialexp/public-unemployment-spending.htm which reports the ratio of total UI expenditures as a fraction of GDP at an annual frequency. The value of this ratio increased to 1.1% during the Great Recession from its average value of 0.4% over the period 1980-2019. Thus the relative increase in this ratio in the data was $(1.1 - 0.4)/0.4 = 1.75$. The corresponding value in our model is 1.9 with our choice of $\eta = 10$ in (8). The acyclical policy has $\eta = 0$.

We choose the job loss intensity $\lambda_1 = 0.4$ to match the annualized job separation rate of non-farm payroll workers between 2000M1 - 2019M12 as estimated from the monthly FRED series JTUTSR (not seasonally adjusted). We choose the values of the job search cost parameters $\kappa = 0.55$ and $\phi = 0.1$ to approximately match the unemployment rate and the elasticity of unemployment duration with respect to the benefit level for the median k individual. Our model-implied unemployment rate in the precrisis state is 5.7%, which matches the U.S. unemployment rate of 5.7% over the

[6] For instance, Ganong et al. [26] report that UI benefits in Nevada reach their maximum for individuals with weekly income above \$902. This is close to the mean income of \$886 per week in that state. Both of these are values for 2019.

Table 2 **Aggregate Quantities and Prices in precrisis state**

	$L^\star$	$K^\star$	$Y^\star$	$C^\star$	$w^\star$	$r^\star$
Precrisis values	0.943	3.138	1.408	1.094	0.995	0.150

Steady state values for $t < 0$. We use the parameter values shown in Table 1.

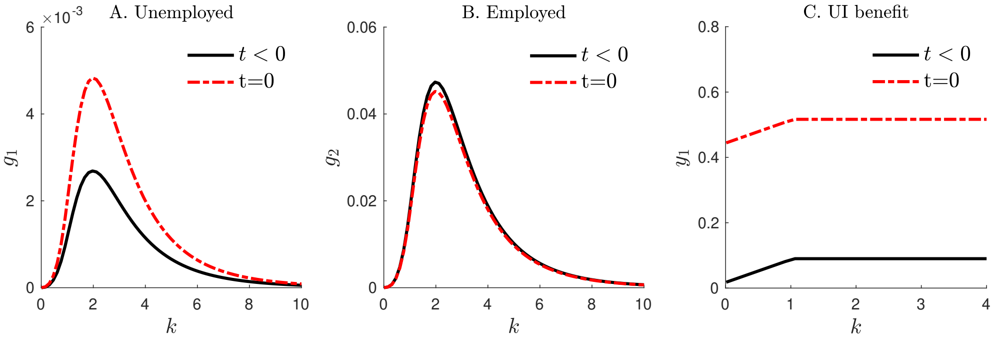

Fig. 1 **The Shock and UI policy.** Panels A and B show the distributions of the unemployed and the employed, respectively, before the shock $t < 0$, and immediately after the shock is realized at $t = 0$. Panel C shows the UI payments y_1 as a function of individual capital k. The dot-dash line in panel C shows UI payments in equation (8) with $\eta = 10$.

period 1948M1—2019M12, where the latter is computed from the seasonally adjusted monthly rate series. Our model-implied average elasticity of unemployment duration with respect to unemployment benefits is 0.45, while [2] estimates this value to be 0.5 in the data.

Table 2 shows the values of aggregate quantities and prices in the precrisis state. The solid lines in panels A and B of Figure 1 show the precrisis state stationary distributions $g_1^\star(k)$ and $g_2^\star(k)$ of the unemployed and employed, respectively.

3.2 Baseline Analysis

In this subsection we consider a shock which increases the unemployment rate from its steady-state value $U^\star = 5.7\%$ to $U_0 = 10\%$ at $t = 0$. Panels A and B of Figure 1 show the distributions of the unemployed g_1 and the employed g_2, respectively, before the shock $t < 0$, and immediately after the shock is realized at $t = 0$.

We compare the effect of the acyclical UI benefit policy with that of the countercyclical policy with $\eta = 10$. UI payments remain unchanged after the shock under the acyclical policy, whereas they increase by 0.44 at $t = 0$ for all unemployed individuals under the countercyclical policy (see panel C of Figure 1). Thereafter, the future evolution of UI payments y_{1t} under the countercyclical policy is determined by the equilibrium path of U_t though equation (8), reverting back to its steady-state value as $t \to \infty$ when $U_t \to U^\star$.

Table 3 Effects of the two UI policies in the cross-section

Wealth Distribution	Unemployment duration		$\Delta c/c$ percent		Value function	
Percentiles	AC (1)	CC (2)	AC (3)	CC (4)	AC (5)	CC (6)
Bottom 1	5.6	7.6	-6.1	-4.2	-22.4	-22.4
Bottom 10	6.8	9.0	-2.1	-1.9	-21.7	-21.7
Bottom 25	7.3	9.7	-1.3	-1.2	-21.1	-21.1
Median	7.7	10.2	-1.0	-0.9	-20.4	-20.4
Top 25	8.4	11.0	-0.9	-0.9	-19.4	-19.4
Top 10	9.2	12.0	-0.9	-0.8	-18.2	-18.1
Top 1	11.9	15.1	-0.9	-0.8	-15.3	-15.3

This table compares the effect of the acyclical policy (AC) with the countercyclical policy (CC). Unemployment duration is the expected duration of unemployment immediately after realization of the negative shock at $t = 0$. The values are in weeks. The fractional decline in consumption, denoted as $\Delta c/c$, is calculated as the ratio of Δc, which represents the discontinuous change in consumption experienced by individuals who suffer job loss at $t = 0$, to c, which is their consumption immediately prior to job-loss at $t = 0^-$. The value functions correspond to unemployed individuals.

3.2.1 Individual utilities

In this section we quantify the cost and benefit of the UI policies as measured by their effects on the job search intensity and consumption smoothing, respectively. We also quantify cross-sectional differences in the value function of the unemployed under the two policies.

Table 3 shows our results for different levels of the agent's capital k. First, comparing columns (1) and (2), we see that the expected duration of unemployment is shorter under the acyclical policy than under the countercyclical policy. For instance, individuals at the bottom 10 percentile of the capital distribution have unemployment durations that are lower by 2.2 weeks under the acyclical policy compared to that under the countercyclical policy. This difference in unemployment duration under the two policies increases with the individual's capital stock holding; for individuals in the top 1 percentile of the capital distribution, unemployment durations under the acyclical policy are 3.2 weeks shorter than under the countercyclical policy. The countercyclical policy results in higher equilibrium unemployment durations compared to the acyclical policy because higher UI benefits under the countercyclical policy reduces the incentive to search for a job.

Next, we compare the consumption smoothing benefits of the two UI policies. From columns (3) and (4) of Table 3, we see that individuals who undergo job-loss following the negative shock at $t = 0$, experience a larger drop in initial consumption under the acyclical policy compared to that under the countercyclical policy. This is a consequence of higher UI benefits following the negative shock at $t = 0$ under the countercyclical policy. We measure the consumption decline as the fraction $\Delta c/c$, where Δc is the discontinuous change in consumption experienced by individuals who suffer job loss at $t = 0$ and c is their consumption immediately prior to job-loss at $t = 0^-$. From columns (3) and (4), we see that the consumption drop $|\Delta c/c|$ is higher

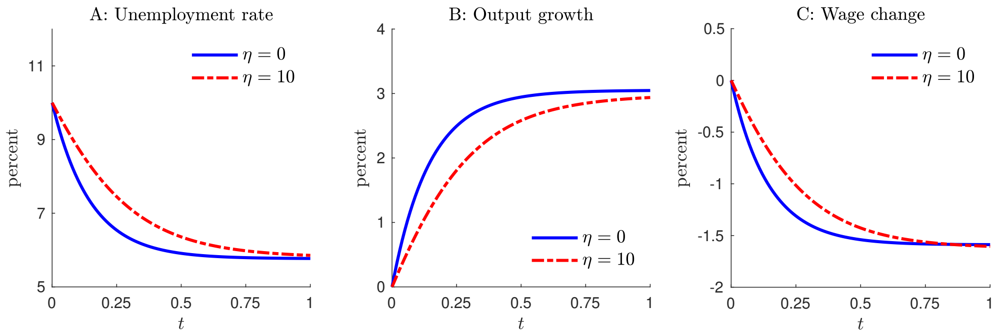

Fig. 2 Aggregate results. Panel A shows the path of the aggregate unemployment rate under the acyclical ($\eta = 0$) and the countercyclical ($\eta = 10$) policies. Panels B and C show the growth in output and the change in wage, respectively, from their values immediately after realization of the shock at $t = 0$ under each policy. Time is measured in years in these figures.

under the acyclical policy by about 2% (0.1%) for individuals at the bottom (top) 1 percentile of capital distribution.

Comparing columns (5) and (6) of Table 3, we see that both policies deliver approximately the same ex-ante expected utilities to all unemployed individuals. This approximate equality arises because the larger initial consumption drop under the acyclical policy compared to the countercyclical policy is offset by individuals finding jobs faster.

3.2.2 Response of Aggregate Quantities

In Panel A of Figure 2, we compare the paths of the equilibrium unemployment rate under the two UI policies. We see that the unemployment rate is slower to revert to its precrisis level under the countercyclical policy (dot-dash line) than under the acyclical policy (solid line): the difference in the equilibrium unemployment rates under the two policies is large, with the maximum difference being 1% and occurring at $t = 0.17$ years (i.e., 2 months following the shock).

Labor markets recover more slowly under the countercyclical policy because higher UI benefits reduce the incentives for the unemployed to search for a job. This reduction in incentives is partly due to current higher benefits y_{1t} under the countercyclical policy (see equation (8)). In addition, there is a general equilibrium feedback effect. Lower job search effort from all unemployed individuals under the countercyclical policy (relative to the acyclical policy) results in a higher equilibrium unemployment rate (both current and future). This, in turn, leads to higher benefits according to equation (8), increasing the value of unemployment. This further reduces equilibrium job search effort (see equation (10)).

The lower employment level in response to the countercylical benefit policy results in slower recovery of output. The time-0 shock results in an immediate output drop of 3% from its precrisis level. Panel B of Figure 2 shows its subsequent evolution. We see that under the acyclical policy, output increases by 2.1% from its value at the trough (at $t = 0$) in two months ($t = 0.18$). In comparison, the corresponding growth is only 1.36% under the countercyclical policy.

Panel C of Figure 2 show that wages are higher and decline more slowly under the countercyclical policy than under the acyclical policy. This is simply because there are fewer workers under the countercyclical policy than under the acyclical policy along the transition path (note that we assume that total factor productivity is constant and normalized to one in both economies, see equation (6)).

3.3 Countercyclical policy and shock size

In this section we show that the severity of the effect of countercyclical UI policies on labor supply is a rapidly increasing, non-linear function of the size of the aggregate shock U_0. We establish this result through a comparative static exercise in which we vary U_0 while holding all other model parameters fixed. Once again, we fix $\eta = 10$ for countercyclical policies.

Panel A of Figure 3 shows the time-0 total UI expenditure normalized by output as a function of U_0. The dashed line corresponds to the acyclical policy $\eta = 0$. This line is linear because while individual UI payments do not change with U_0, the measure of recipients increases linearly with U_0. The solid line corresponds to the countercyclical policy $\eta = 10$. Total UI expenses increase faster under this policy than under the acyclical policy because, in addition to the increase in the measure of recipients, individual payments also increase linearly with U_0 under the countercyclical policy. The difference is quantitatively large when unemployment U_0 is high; for $U_0 = 10\%$, total UI expenditure is 0.4% of GDP under the acyclical policy, while it is 1.07% of GDP under the countercyclical policy.

Panel B of Figure 3 shows the excess unemployment rate at $t = 2$ months as a function of U_0. We define the excess unemployment rate as the difference between the unemployment rates under the countercyclical and the acyclical policies. We see that the excess unemployment rate increases non-linearly with the shock size U_0. For instance, for $U_0 = 6\%$ (the maximum unemployment rate following the Great Recession), the excess unemployment rate is only 0.024%, but for $U_0 = 10\%$, it is forty times higher at 0.98%. Recent findings in the empirical literature suggest that the adoption of extended unemployment benefits following the Great Recession likely had a small effect on the unemployment rate (e.g., [27] estimate a 0.3% increase in the unemployment rate). Our result of a superlinear increase in the excess unemployment rate as a function of U_0 implies that naively extrapolating these estimates to large disasters would significantly understate the effect.

Panel C of Figure 3 shows aggregate output two years following the shock relative to its precrisis level. The solid and dash lines refer to countercyclical and acyclical policies, respectively. This figure also highlights the non-linear effect of countercyclical UI policies on the recovery of output. For instance, for $U_0 = 6\%$, the path of output is quite similar under the countercyclical and acyclical policies—at $t = 2$ months, output is 0.11% and 0.09% below precrisis levels under these policies, respectively. However, the difference in output growth at $t = 2$ is much larger under the two policies for $U_0 = 10\%$: while output is 1.01% below the precrisis level under the acyclical policy, it is 1.71% below the precrisis level under the countercyclical policy.

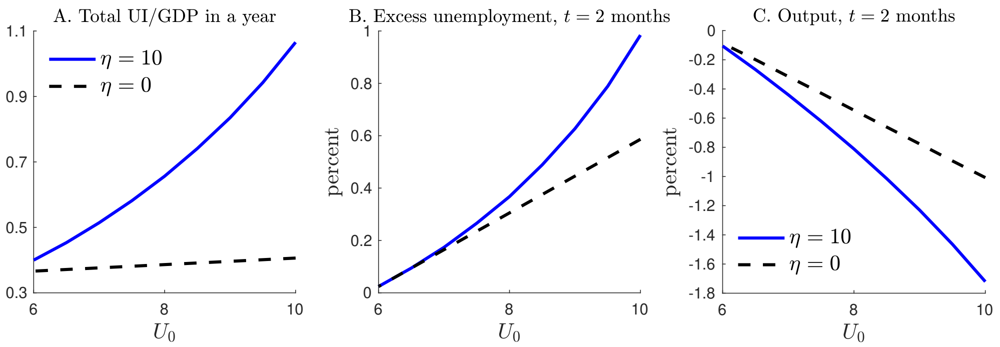

Fig. 3 Countercyclical policies and shock size. Panels A through C compare total UI expense normalized by output, the excess unemployment rate at $t = 2$ months, and the output relative to its precrisis level also at $t = 2$ months as a function of the size of the shock U_0. The dashed line in panel B shows the estimate of the excess unemployment rate obtained by a linear extrapolation.

3.4 Savings distribution and effect of UI policies

The main result of this section is that the sensitivity of the equilibrium unemployment rate to UI policy changes is larger when individual savings is relatively low (i.e., when the cross-sectional distribution of savings is shifted to the left). Mian et al. [6] document a secular decline in savings by the lower 90% of the U.S. income distribution since 1980 (see their Figure 6). Our model therefore implies that increases in UI benefits will result in much higher equilibrium unemployment rates now than they did in 1980.

We illustrate how changes in the distribution of savings affect the costs and benefits of countercyclical UI policies through a comparative static exercise. In particular, we analyze the same environment as in 3.2 (including the aggregate shock at $t = 0$) for two different values of the time-preference parameter—the baseline value of $\rho = 0.05$ that we used in 3.2 and a lower value of $\rho = 0.015$. 4 shows the results.

Columns (1) and (2) of Table 4 compare the steady-state distribution of capital. As expected, individuals in the less-patient economy with $\rho = 0.05$ save less compared to individuals in the more-patient economy with $\rho = 0.015$. The median savings in the less-patient economy is 2.57 compared to 4.08 in the more-patient economy. Since the reduction in job search intensity is inversely related to savings[7] (see column (1) and (2) of Table 4), we expect that the same increase in UI payment will lead to a greater overall reduction in job search and hence a lower flow rate of individuals out of unemployment, that is, longer unemployment durations, under the countercyclical policy.

There is a second effect which reinforces the effect discussed above. Namely, controlling for the level of savings k, individuals in the economy with a higher value of ρ optimally reduce their job search intensity by a larger amount and therefore have

[7] In our model, the cost of being unemployed is inversely related to the agent's savings k because a higher capital stock provides: (i) a higher capital rental income and (ii) a larger buffer to weather an unemployment spell for a longer period of time.

Table 4 Effects of the two UI policies in the cross-section for two economies with different initial savings distribution

Wealth Distribution	Capital level		Excess U duration	
Percentiles	$\rho = 0.05$ (1)	$\rho = 0.015$ (2)	$\rho = 0.05$ (3)	$\rho = 0.015$ (4)
Bottom 1	0.61	0.91	2.07	1.70
Bottom 10	1.22	1.97	2.26	1.80
Bottom 25	1.82	2.72	2.36	1.83
Median	2.57	4.08	2.45	1.87
Top 25	3.93	5.47	2.60	1.91
Top 10	5.47	7.85	2.79	1.95
Top 1	11.31	13.42	3.35	2.06

Columns (1) and (2) compare the level of savings (i.e., individual capital holding k) at different percentiles of the savings distribution. Columns (3) and (4) compare the excess unemployment duration between the two economies. Excess unemployment duration is the expected duration of unemployment immediately after realization of the negative shock at $t = 0$ under the countercyclical policy (CC) minus that under the acyclical policy (AC). The values are in weeks.

longer unemployment durations. Indeed, impatient individuals attach a lower value of finding a job in the future. Columns (3) and (4) of Table 4 provide a quantitative comparison of the excess unemployment duration for the two economies, where the "excess unemployment duration" is the expected duration of unemployment immediately after realization of the negative shock at $t = 0$ under the countercyclical policy (CC) minus that under the acyclical policy (AC) in the corresponding economy.

The combination of these two effects is non-neglibigle. While the maximum excess unemployment rate along the transition path is 1% in the $\rho = 0.05$ economy, it is about 27% smaller in the $\rho = 0.015$ savings-rich economy, where the maximum excess unemployment rate is 0.73%. This result shows that it is important to determine the distribution of savings among the unemployed in order to assess the potential effect of changes in UI payments.

4 Conclusion

In this paper we analyze the general equilibrium effects of countercyclical UI policies. The key innovation of our paper is to quantify such effects by analyzing the transitional dynamics following an aggregate shock, as opposed to steady-state analyses in the existing literature. Our approach is therefore able to capture the effect of short-lived increases in UI payments that are observed in practice. Solving for the equilibrium along the transition path involves tracking a time-dependent cross-sectional distribution of individual savings; we solve our model using numerical methods from mean field game theory.

Our model provides the following insights. First, we find that the additional unemployment under a countercyclical UI policy relative to an acyclical policy is a superlinear function of the size of the aggregate shock. Second, we find that the sensitivity of the equilibrium unemployment rate to UI policy changes is larger when individual savings is relatively low.

We hope that our approach of using mean field games to analyze the general equilibrium effects of countercyclical UI policies can be used in more realistic models. An important extension of our current model is to feature heterogeneity in labor productivity and investigate how countercyclical UI policies influence job matching dynamics. Existing empirical evidence such as [28] suggests that while an increase in UI payments may initially reduce job search intensity, it could lead to enhanced job matches over the long term, thereby improving the overall wages. Our current model assumes a single common productivity for the worker-firm match and therefore cannot speak about the effect of UI policies on worker-firm match quality. In other words, our model does not feature the choice between a job with a better match and a job with a poor match. Rather, the choice is between a job and no job. By adding heterogeneity in labor productivity and calibrating such a model to the data, our framework is well-suited to analyze the equilibrium effects of countercyclical UI policies in both short term and long term scenarios, offering explicit insights into transitional dynamics. Another promising avenue of research is to prove the existence and uniqueness of the equilibrium of our heterogenous agent model.

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Declarations

Financial interests The authors have no competing interests to declare that are relevant to the content of this article.

Appendix A Algorithm

A.1 Algorithm for stationary equilibrium

In the precrisis period $t < 0$, the UI payments are a a piece-wise linear function of k, $y_1^\star(k) = \min\left(a + b(k - \underline{k}, \bar{y})\right)$, which increases linearly with slope b for individuals with $k \leq k_H = \underline{k} + \frac{\bar{y}-a}{b}$. Given these parameters $a, b, \bar{y}$, we consider a stationary equilibrium which can be summarized by a coupled HJB-KF equation system. For $k \in [\underline{k}, \infty)$,

$$\rho v_1(k) = \max_{c_1 s} u(c_1) + \left(y_1^\star(k) + (r^\star - \delta)k - c_1\right)\dot{v}_1(k) + \left(v_2(k) - v_1(k)\right)\lambda_2(s) - \psi(s) \tag{A1}$$

$$\rho v_2(k) = \max_{c_2} u(c_2) + \left((1-\theta^\star)w^\star + (r^\star - \delta)k - c_2\right)\dot{v}_2(k) + \left(v_1(k) - v_2(k)\right)\lambda_1 \tag{A2}$$

$$0 = -\frac{d}{dk}\left(g_1(k)\left(y_1^\star(k) + (r^\star - \delta)k - c_1^\star\right)\right) - g_1(k)\lambda_2(s^\star) + g_2(k)\lambda_1 \tag{A3}$$

$$0 = -\frac{d}{dk}\left(g_2(k)\left((1-\theta^\star)w^\star + (r^\star - \delta)k - c_2^\star\right)\right) - g_2(k)\lambda_1 + g_1(k)\lambda_2(s^\star) \tag{A4}$$

where $c_1^\star, s^\star$ are such that the right hand side of (A1) attains maximum, $c_2^\star$ is such that the right hand side of (A2) attains maximum, more specifically,

$$c_1^\star(k) = (\dot{v}_1(k))^{-1/\gamma}$$
$$c_2^\star(k) = (\dot{v}_2(k))^{-1/\gamma}$$
$$s^\star(k) = \left(\frac{v_2(k) - v_1(k)}{\phi}\right)^{1/\kappa}$$

and $r^\star, w^\star, \theta^\star$ satisfy market clearing conditions

$$r^\star = \alpha\left(K^\star/L^\star\right)^{\alpha-1} \tag{A5}$$
$$w^\star = (1-\alpha)\left(K^\star/L^\star\right)^{\alpha} \tag{A6}$$
$$\theta^\star w^\star L^\star = \int_{\underline{k}}^{\infty} g_1(k) y_1^\star dk \tag{A7}$$

with $K^\star = \int_{\underline{k}}^{\infty} k(g_1(k) + g_2(k))dk$ and $L^\star = \int_{\underline{k}}^{\infty} g_2(k)dk$.

Notice that $w^\star = w(r^\star) = (1-\alpha)(\frac{r^\star}{\alpha})^{\alpha/(\alpha-1)}$ while $\theta^\star$ depends on both the interest rate and the distribution functions.

Our computational approach is adopted from [5] (hereafter referred to as "AHLLM"). In AHLLM, individuals make decisions solely on consumption and their employment state is determined by an exogenous two-state continuous-time Markov chain. Consequently, the employment rate in the stationary equilibrium is a constant $L = \frac{\lambda_{1,2}}{\lambda_{1,2}+\lambda_{2,1}}$ where $\lambda_{1,2}(\lambda_{2,1})$ is the constant intensity rate of transition from the unemployment(employment) state to the employment(unemployment) state. For a each given interest rate r, the total capital of demand $K_d(r) = L(\frac{r}{\alpha})^{1/(\alpha-1)}$ can be computed directly and a total capital of supply $K_s(r) = \int_{\underline{k}}^{\infty} k(g_1^r(k) + g_2^r(k))dk$ can be obtained by solving a system of HJB-KF equations. Here, g_1^r and g_2^r are solutions of KF equations whose coefficients depend on the solution of the HJB equation given the interest rate r and the corresponding $w = w(r)$.

In AHLLM, a fixed-point algorithm is employed to find the interest rate r such that $K_s(r) = K_d(r)$, indicating a stationary equilibrium. In each iteration of the fixed point algorithm, AHLLM uses a variate of finite difference method, "upwind-scheme", to solve HJB equations. The solution of HJB equations are then used to generate coefficients in KF equations, whose solution is obtained by solving a linear equation system. The value of r will be adjusted after each iteration based on whether there is an excess demand or an excess supply of total capital.

In our case, we aim to find a pair $(r^\star, \theta^\star)$ such that not only does $K_s(r^\star) = K_d(r^\star)$ hold but also (A7) holds. We decompose this task into a two-layer fixed-point algorithm. The algorithmic details are outlined in Algorithm 1, where, in the second loop of the fixed point algorithm, we obtain L and θ such that (A7) holds for each $r^{(n)}$ in the first loop. In the first loop, we iterate over $r^{(n)}$ until $K_s(r^\star) = K_d(r^\star)$ holds.

Remark 2. *In the algorithm, all value functions $v_i^\star$ and density functions $g_i^\star$ are represented by $2 \times I$ matrices, where I is the number of mesh points on interval $[\underline{k}, \bar{k}]$ for a large $\bar{k}$. The $\|\cdot\|$ norm is $\|\cdot\|_{\sup}$ on $2 \times I$ matrices.*

Remark 3. *For fixed $r^{(n)}, \theta^{(n)}$, the solution of (A2,A1) is obtained using "upwind-scheme", as in AHLLM. For detailed information, please refer to the Online appendix of AHLLM.*

A.2 Algorithm for time dependent equilibrium

Although we have infinite horizon in our problem, we do not start from the steady state due to the unanticipated shock at time $t = 0$. Moreover, with a time-dependent UI payments y_{1t}, the equilibrium solution for our model is in form of transition dynamics. As in AHLLM, for the convenience of numerical computation, we assign a large time T as the terminal time and assume that with this long enough time T, the economy has converged to the same steady state as in the precrisis period. We use the time-dependent equilibrium of this finite horizon problem to approximate the transition dynamics of the original infinite horizon problem, which can be summarized by a coupled HJB-KF equation system. For $k \in [\underline{k}, \infty), t \in [0, T]$,

$$\begin{aligned}
\rho v_1(k,t) &= \max_{c_1,s} u(c_1) + \left(y_{1t}(k) + (r_t - \delta)k - c_1\right) \partial_k v_1(k,t) \\
&\quad + \left(v_2(k,t) - v_1(k,t)\right) \lambda_2(s) - \psi(s) + \partial_t v_1(k,t) \qquad \text{(A8)} \\
\rho v_2(k,t) &= \max_{c_2} u(c_2) + \left((1-\theta_t)w_t + (r_t - \delta)k - c_2\right) \partial_k v_2(k,t) \\
&\quad + \left(v_1(k,t) - v_2(k,t)\right) \lambda_1 + \partial_t v_2(k,t) \qquad \text{(A9)} \\
\partial_t g_1(k,t) &= -\partial_k \left(g_1(k,t) \left(y_{1t}(k) + (r_t - \delta)k - c_1(k,t)\right)\right) \\
&\quad - g_1(k,t)\lambda_2(s(k,t)) + g_2(k,t)\lambda_1 \qquad \text{(A10)} \\
\partial_t g_2(k,t) &= -\partial_k \left(g_2(k,t) \left((1-\theta_t)w_t + (r_t - \delta)k - c_2(k,t)\right)\right) \\
&\quad - g_2(k,t)\lambda_1 + g_1(k,t)\lambda_2(s(k,t)) \qquad \text{(A11)}
\end{aligned}$$

where in (A10, A11)

$$\begin{aligned}
c_1(k,t) &= \left(\partial_k v_1(k,t)\right)^{-1/\gamma} \\
c_2(k,t) &= \left(\partial_k v_2(k,t)\right)^{-1/\gamma} \\
s(k,t) &= \left(\frac{v_2(k,t) - v_1(k,t)}{\phi}\right)^{1/\kappa}
\end{aligned}$$

and $r_t, w_t, y_{1t}, \theta_t$ satisfy

$$\begin{aligned}
r_t &= \alpha \left(K_t/L_t\right)^{\alpha-1} \\
w_t &= (1-\alpha)\left(K_t/L_t\right)^{\alpha} \\
y_{1t}(k) &= \min\left(a + b(k - \underline{k}), \bar{y}\right) + \eta(U_t - U^\star) = \min\left(a + b(k - \underline{k}), \bar{y}\right) + \eta(L^\star - L_t)
\end{aligned}$$

Algorithm 1 Algorithm for stationary equilibrium

Require: $\alpha, \gamma, a, b, \bar{y}, \phi, \lambda_1, \kappa, \underline{k}, \epsilon, N, r_{\max}, r_{\min}$

$y_1^\star(k) \leftarrow \min\left(a + b(k - \underline{k}, \bar{y})\right)$

Take an initial guess of $r^\star := r^{(0)} \in [r_{\min}, r_{\max}]$.

$\Delta_K^{(0)} \leftarrow \infty$; $n \leftarrow 0$;

while $n < N$ **do**

$w^{(n)} \leftarrow (1-\alpha)\left(\frac{r^{(n)}}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}$

Take an initial guess of $L^\star := L^{(0)}$ and $\theta^\star := \theta^{(0)}$.

$\Delta_\theta^{(0)} \leftarrow \infty$; $m \leftarrow 0$

while $\Delta_\theta^{(m)} > \epsilon$ **do**

Use $r^{(n)}, w^{(n)}, \theta^{(m)}$ as given coefficients in (A1, A2)

Take an initial guess of $v_2^{(0)}, v_1^{(0)}$.

$\Delta_v^{(0)} \leftarrow \infty$; $l \leftarrow 0$

while $\Delta_v^{(l)} > \epsilon$ **do**

Compute the optimal $c_2^\star, c_1^\star, s^\star$ using $v_2^{(l)}, v_1^{(l)}$

Solve for $v_2^{(l+1)}, v_1^{(l+1)}$ in (A1, A2) given the above $c_2^\star, c_1^\star, s^\star$

$\Delta_v^{(l+1)} \leftarrow \|(v_2^{(l)}, v_1^{(l)}) - (v_2^{(l+1)}, v_1^{(l+1)})\|$

$v_2^{(l)} \leftarrow v_2^{(l+1)}$; $v_1^{(l)} \leftarrow v_1^{(l+1)}$

$l \leftarrow l + 1$

end while

With the above solution $v_2^\star, v_1^\star$, solve for $g_2^\star, g_1^\star$ in (A3, A4)

$L^{(m+1)} \leftarrow \int_{\underline{k}}^{\infty} g_2^\star(k)dk$

$\theta^{(m+1)} \leftarrow \left(\int_{\underline{k}}^{\infty} g_1^\star(k) y_1^\star(k) dk\right) / \left(w(r^{(n)}) L^{(m+1)}\right)$

$\Delta_\theta^{(m+1)} \leftarrow \|\theta^{(m+1)} - \theta^{(m)}\|$

$m \leftarrow m + 1$

end while

$K^{demand} \leftarrow L^{(m)} \left(\alpha / r^{(n)}\right)^{1/(1-\alpha)}$

$K^{supply} \leftarrow \int_{\underline{k}}^{\infty} k\left(g_2^\star(k) + g_1^\star(k)\right) dk$

if $K^{supply} - K^{demand} > \epsilon$ **then**

$r_{\max} \leftarrow r^{(n)}$; $r^{(n+1)} \leftarrow \frac{1}{2}(r_{\min} + r^{(n)})$

else if $K^{supply} - K^{demand} < -\epsilon$ **then**

$r_{\min} \leftarrow r^{(n)}$; $r^{(n+1)} \leftarrow \frac{1}{2}(r_{\max} + r^{(n)})$

else

print: "Stationary equilibrium founded"

return $r^{(n)}, w^{(n)}, L^{(m)}, \theta^{(m)}, v_u^\star, v_2^\star, g_1^\star, g_2^\star, c_2^\star, c_1^\star, s^\star, K^{demand}$

end if

$n \leftarrow n + 1$

end while

print: "No equilibrium founded"

return

$$\theta_t w_t L_t = \int_{\underline{k}}^{\infty} g_1(k,t) y_{1t}(k) dk$$

with given employment rate $L^\star$ in steady state, $K_t = \int_{\underline{k}}^{\infty} k(g_1(k,t) + g_2(k,t))dk$ and $L_t = \int_{\underline{k}}^{\infty} g_2(k,t)dk$.

The terminal conditions are given by $v_2(k,T) = v_2^\star(k), v_1(k,T) = v_1^\star(k)$ where $v_2^\star(k), v_1^\star(k)$ are the solutions of the stationary equilibrium (A1,A2,A3,A4). The initial distributions $g_1(k,0), g_2(k,0)$ are given by (7) and $g_2(\cdot,0) = g_1^\star(\cdot,0) + g_2^\star(\cdot,0) - g_1(\cdot,0)$. This indicates a uniform job loss for all k and guarantees a drop of employment rate from $L^\star$ to an exogenous constant L_0.

With the above initial and terminal conditions, Algorithm 2 outlines the procedure for solving (A8,A9, A10, A11). In each iteration the value functions $v_i(\cdot,t)$ is derived backward in time given the terminal condition and the density function $g_i(\cdot,t)$ is derived forward in time given the initial condition. A detailed description is also available in the Online appendix of AHLLM. Paths of (K_t, L_t, θ_t) are updated simultaneously in each iteration based on the density functions $g_i(\cdot,t)$. The algorithm converges to the equilibrium when the maximum change in paths of (K_t, L_t, θ_t) between two consecutive iterations is sufficiently small.

Algorithm 2 Algorithm for time-dependent equilibrium

Require: $\alpha, \gamma, \phi, \lambda_1, \kappa, a, b, \bar{y}, \eta, L^\star, \epsilon, N, T, g_1(\cdot, 0), g_2(\cdot, 0), v_1^\star(\cdot), v_2^\star(\cdot)$

Take an initial guess of $K_t^{(0)}, L_t^{(0)}$ for $t \in [0, T]$

Compute the corresponding $r_t^{(0)}, w_t^{(0)}, y_{1t}^{(0)}$ given $K_t^{(0)}, L_t^{(0)}$ for $t \in [0, T]$

Take an initial guess of $\theta_t^{(0)}$ for $t \in [0, T]$

Let $n = 0$

while $n < N$ **do**

  Given coefficients $r_t^{(n)}, w_t^{(n)}, y_{1t}^{(n)}, \theta_t^{(n)}$ and the terminal condition $v_1^\star(\cdot), v_2^\star(\cdot)$, apply finite difference method to compute $(v_1^{(n)}, v_2^{(n)})$ of (A8, A9) backward in time. $\left\{c_1^{(n)}(\cdot, t), c_2^{(n)}(\cdot, t), s^{(n)}(\cdot, t)\right\}_{t\in[0,T]}$ are obtained simultaneously.

  Given $g_1(\cdot, 0), g_2(\cdot, 0)$ and $\left\{c_1^{(n)}(\cdot, t), c_2^{(n)}(\cdot, t), s^{(n)}(\cdot, t)\right\}_{t\in[0,T]}$, solve for $(g_1^{(n)}, g_2^{(n)})$ in (A10, A11) forward in time.

  $K_t^{(n+1)} \leftarrow \int_{\underline{k}}^\infty k \left(g_1^{(n)}(k, t) + g_2^{(n)}(k, t)\right) dk$

  $L_t^{(n+1)} \leftarrow \int_{\underline{k}}^\infty g_2^{(n)}(k, t) dk$ for $t \in [0, T]$

  Compute $r_t^{(n+1)}, w_t^{(n+1)}, y_{1t}^{(n+1)}$ given $K_t^{(n+1)}, L_t^{(n+1)}$ for $t \in [0, T]$

  $\theta_t^{(n+1)} \leftarrow \left(\int_{\underline{k}}^\infty g_1^{(n)}(k, t) y_{1t}^{(n+1)}(k) dk\right) / \left(w^{(n+1)} L^{(n+1)}\right)$ for $t \in [0, T]$

  if $\max\left\{\|K^{(n+1)} - K^{(n)}\|, \|L^{(n+1)} - L^{(n)}\|, \|\theta^{(n+1)} - \theta^{(n)}\|\right\} < \epsilon$ **then**

    print: "Equilibrium founded"

    return $\left\{v_i^{(n)}(\cdot, t), g_i^{(n)}(\cdot, t), c_i^{(n)}(\cdot, t)\right\}_{t\in[0,T]}^{i\in\{1,2\}}, \left\{s^{(n)}(\cdot, t), r_t^{(n)}, w_t^{(n)}, K_t^{(n)}, L_t^{(n)}, \theta_t^{(n)}\right\}_{t\in[0,T]}$

  else

    $n \leftarrow n + 1$

  end if

end while

print: "No equilibrium founded"

return

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