Polyplane:

Art, Visualization, and Outreach

Alex Kontorovich and Glen Whitney

The *Polyplane* exhibit we produced (shown here) began with a seemingly simple connection that Alex realized during a brainstorming meeting at the National Museum of Mathematics in 2019: Euler's polyhedron formula,

$$V - E + F = 2,$$

can be interpreted as the equation of a plane if the numbers of vertices (V), edges (E), and faces (F) of a simply connected polyhedron represent coordinates in three-space $(x, y, z) = (F, E, V)$.

So why not illustrate that relationship by placing a whole slew of polyhedra in a physical space, each centered at their corresponding coordinates! A visitor with no prior mathematical experience might notice the arrangement and wonder why the shapes align in this way.

Alex tested the idea by posting a digital rendering on social media (*bit.ly/3E71jH3*), and the post received an encouraging response. Meanwhile, Glen had led courses on hands-on mathematical construction at several institutions. We connected at MoMath years earlier because we both love finding ways to physically embody deep, modern, mathematical ideas for a broad audience. At another meeting in 2019, we decided to jointly offer a version of Glen's course at Rutgers, Alex's home institution. COVID intervened, but the course finally took place in the spring of 2023.

The course "finale" consisted of a large-scale crowd-sourced construction of *Polyplane*. Through a public "call for polyhedra," we enlisted the help of roughly 30 artists and mathematicians (each contributor being some combination thereof) who generously provided the bulk of the models in the exhibit. We hope the visual richness and variety afforded by the diversity of media in the exhibition draws people in and leads them to notice the truly striking and surprising arrangement of shapes. Alex's contributed shape, one higher genus polyhedron, creates a sort of exception that proves the rule. Visitors will notice that this object seems out of place, thereby emphasizing the planar arrangement of all of the remaining polyhedra.

We believe that more polyhedra, filling in a greater portion of the "Euler plane," would make the relationship even more salient. So the exhibition is still accepting additional contributions of polyhedra—please visit *polyplane.org* for more information, including how your institution can host a showing of *Polyplane*.

Alex Kontorovich *is a Distinguished Professor of Mathematics at Rutgers University, and* ***Glen Whitney*** *currently serves as the Problem Warden for the Prison Math Project.*

No potential conflict of interest was reported by the author.

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