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A Statistical Method Assessing the Influence of Cobots' Technical Parameters on Performance Indices

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***Abstract*— Coexisting-cooperative-cognitive (Tri-Co) robots are advanced systems designed for interaction with environments, humans, and other robots. Collaborative robots (cobots), a subset of Tri-Co robots, specifically work safely alongside humans. When it comes to improving cobot performance, understanding the relationship between their technical parameters and performance indices becomes crucial. This paper proposes a method that combines the idea of experimental science and statistics, using Pearson correlation analysis to find this relationship. We define local Pearson correlation coefficients and influential indicators to measure each technical parameter's impact on performance indices. A case study on Rethink Sawyer (Sawyer) cobot validates our theoretical framework and underscores the practical applicability of our method.**

I. INTRODUCTION

Tri-Co robots represent an advanced category of robotics designed to interact seamlessly with their environment, humans, and other robots. They independently adjust to complex and changing surroundings and collaborate effectively [1]. Tri-Co Capabilities (TCCs) encompass a set of advanced functionalities enabling these robots to operate as designed.

Collaborative robots (cobots) [2], as a representative type of Tri-Co robots, are specifically designed to work alongside human workers in a shared workspace. They are focused primarily on collaboration and safety. Cobots are a high-value research object due to the versatility in their applications such as assembly, polishing, machine tending, quality inspection [3] and stacking within industrial scenes.

Recent literature has focused on various aspects of cobot performance and interaction. For example, Ajoudani [4] explored human-robot collaboration dynamics, emphasizing the importance of intuitive control interfaces and adaptive algorithms. Despite these advancements, the systematic evaluation of TCCs still remains under-explored. Current methodologies primarily address individual aspects of cobot performance, such as intuitive user interfaces [5] and safety [6], but do not provide a comprehensive assessment of TCCs.

The technical parameters significantly impact the kinematic and dynamic performance of the cobots. Identifying which parameters have the greatest influence on performance is crucial. The sensitivity analysis quantifies the specific impact of each technical parameter on the performance indices [7] [8]. However, this method usually involves constructing complex mathematical formulae to describe these relationships, which makes it challenging to model accurately in engineering practice. Moreover, when dealing with a large number of technical parameters and performance indices in a robotic system, the practicality and efficiency of sensitivity analysis methods are substantially limited.

This leads to a need for more accessible methodologies to optimize cobot performance effectively. To fulfill the need, this work proposes a simple yet novel statistical method to explore the relationship between cobots' technical parameters and performance indices using Pearson correlation analysis. This method combines the idea of experimental science and statistics. Preferred for its simplicity, interpretability, and minimal assumptions, Pearson correlation analysis is accessible to individuals with limited statistical expertise and can be complemented with various visualizations. Gu [9] discussed the use of heatmaps to visualize Pearson correlations among various genomic features. Other statistical graphs like scatter plots and residual plots could also be used. This work uses line charts and bar charts for the presentation of our results.

We chose Pearson correlation analysis over other statistical methods due to its robustness and simplicity in practical applications, as highlighted in Bishara's study [10] comparing various correlation measures. There are numerous existing studies that involve the application of Pearson correlation analysis. Hao's study [11] used such method to examine the relationship between internet addiction and interpersonal relationships among teenagers and college students.

Compared to existing studies, this work aims to make the following distinctive contributions:

- Proposing a method that determines technical parameters' impact on performance indices. This method combines the idea of experimental science and statistics, serving as a toolkit in the field of robotics.
- Defining local Pearson correlation coefficients and influential indicators. Since we essentially need to assess the relationship between a scalar and a column vector, which cannot be achieved by traditional Pearson correlation analysis. By removing one entry from the matrix at a time and analyzing the remainder, we manipulate the column vectors that fall under the restriction of the Pearson correlation analysis, allowing us to iteratively repeat the procedure. Ultimately, we can gain insights on the relationship of these entries in an indirect fashion.

The structure of this paper is organized as follows. Section II introduces technical parameters and performance indices, and gives a framework of the proposed method. Section III describes TCCs testing. Section IV demonstrates the correlation analysis method from a theoretical perspective.

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Section V is a case study on Sawyer to actualize the proposed method. Finally, section VI concludes and summarizes the work done in this research and suggests future work.

II. Background

A. Technical Parameters

Technical parameters are the specific characteristics and capabilities that define a cobot's performance, functionality, and suitability for various tasks. They can be used for cobots' design and evaluation. In this work, the cobot has 19 specific technical parameters. We label them from $Y1$ to $Y19$, as shown in Table I.

TABLE I: Technical Parameters and Notations

Notation	Technical Parameter
Y1	Degree of freedom
Y2	Mass
Y3	Conventional power consumption
Y4	Peak power consumption
Y5	Real load
Y6	IP level
Y7	Maximum speed of each joint
Y8	Range of each joint motion
Y9	Maximum torque of each joint
Y10	Repetitive positioning accuracy
Y11	The arm span of robot
Y12	Bed area
Y13	Vision sensor
Y14	Tactile sensor
Y15	Force and torque sensor
Y16	Speed sensor
Y17	Position sensor
Y18	Auditory sensor
Y19	Acceleration sensor

B. Performance Indices

Performance indices are the metrics and criteria used to assess the efficiency, effectiveness, and overall performance of the cobot. In this work, the performance indices of our cobot were obtained using a text clustering method called K-means algorithm [13]. First, we constructed a text set that consists of sufficient amount of abstracts. Then the text set would go through a series of steps including text preprocessing, text representation, feature selection, cluster analysis using K-means, high-correlation feature word extraction, experts closed-loop feedback correction, and clustering effect check. During clustering, based on the elbow rule [12], we set the number of clusters k to 4 since it was when the degree of distortion significantly improved. By the end of the process, we obtained four clusters: cobots-human, cobots-cobots, cobots-environment, and safety protection & flexible intelligent switching control, with each cluster having its own corresponding indices. We label them from $X1$ to $X47$, as shown in Table II. Note that some indices belong to multiple clusters. Although they share the same name, they represent different measures in different clusters. For example, "Adaptive impedance control" can be found in "TCCs of cobots-human" cluster, "TCCs of cobots-cobots static task allocation" cluster, and "TCCs of cobots-environment" cluster. We label them separately as $X8$, $X21$, and $X31$.

C. Framework of the Proposed Method

The framework of our proposed statistical method is shown in Fig 1. Stage 1 calculates and collects the technical parameters of our cobot. Performance indices are constructed using the K-Means algorithm via clustering. The TCCs Test Tasks and Sub-Tasks are designed based on the characteristics of the performance indices of the cobot. Then TCCs testing is performed. This is where the original data is generated. Stage 2 mathematically normalizes all the original data to make them computation ready. Stage 3 calculates the influential indicator for our cobot. Stage 4 analyzes and determines the impact of technical parameters on performance indices. Note: Our experiment consists specifically of 19 technical parameters and 47 performance indices. The scope of these parameters and indicators can be further extended to cover more factors that can affect the performance of cobots in future work.

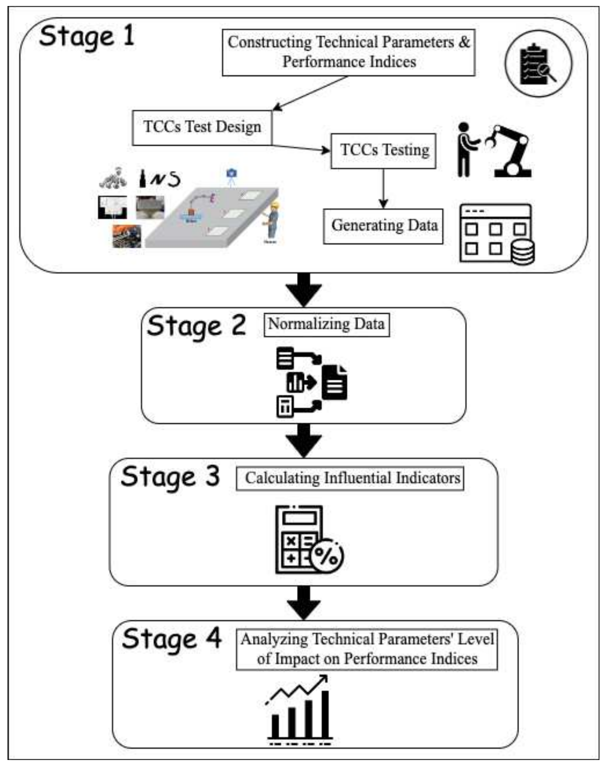


Fig. 1: Framework of the Proposed Method

III. TCCs Testing

A. Test Design

In line with the Tri-Co concept, test tasks were categorized into three types: cobots-human, cobots-cobots, and cobots-environment, with their quantity determined by covering all corresponding performance indices. Therefore, five tasks were designed accordingly: two cobots-human, two cobots-cobots, and one cobots-environment task. This work takes one cobots-human test task as an example. This particular test task has four sub-tasks which will be given in sub-section B. Our test setup is shown in Fig 2.

TABLE II: Performance Indices and Notations

TCCs of cobots-human			
Notation	Index	Notation	Index
X1	Human feature recognition	X7	Hybrid force/position control
X2	Active collision avoidance	X8	Adaptive impedance control
X3	Task planning and coordination	X9	Motion planning
X4	Autonomous decision-making	X10	Human-robot collaboration
X5	Imitation learning	X11	Human intentions understanding
X6	Contact based human-robot interaction	X12	Human-robot contactless interaction

TCCs of cobots-cobots static task allocation			
Notation	Index	Notation	Index
X13	Static task allocation	X20	Loose coordinated motion control
X14	Real-time communication	X21	Adaptive impedance control
X15	Dynamic task allocation	X22	Collision detection
X16	Adaptive learning	X23	Hybrid force/position control
X17	Motion planning	X24	Tight coordination motion control
X18	Real-time obstacle avoidance	X25	Dynamic collaboration
X19	Static collaboration		

TCCs of cobots-environment			
Notation	Index	Notation	Index
X26	Intelligent control	X32	Real-time obstacle avoidance
X27	Collision detection	X33	Motion planning in an unstructured environment
X28	Adaptive complementary filtering algorithm	X34	Hybrid force/position control
X29	Kalman filtering algorithm	X35	Multimodal information fusion and processing
X30	Collision avoidance	X36	Modeling of work environment
X31	Adaptive impedance control		

Safety protection rigid & flexible intelligent switching control			
Notation	Index	Notation	Index
X37	Rigid and flexible intelligent switching control	X43	Power and force limitations
X38	Fixed point safety monitoring stop	X44	Active compliance control
X39	Traction teaching	X45	Safe skin
X40	Flexible joint	X46	Security decision-making mechanism
X41	Speed and distance monitoring	X47	Passive compliant mechanism
X42	Active and passive compliance control		

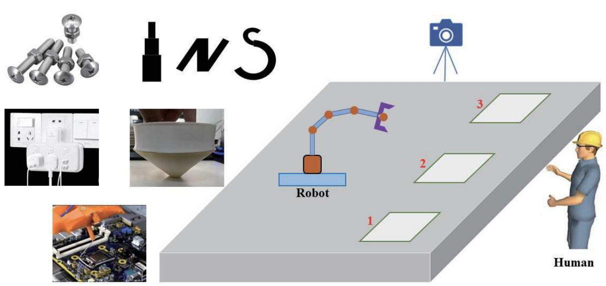


Fig. 2: TCCs Test Setup

B. Test Tasks

Sub-Task 1: As shown in Fig 3, bolts and nuts are randomly placed in area 1 and area 3. The tester selects a nut from area 1 and holds it in the air right above area 2. At this point, the cobot uses visual sensors to identify and match, grabbing the corresponding bolt from area 3 and cooperating with the tester in area 2 to assemble the bolt and nut.

Sub-Task 2: A phone charger and a three-pin plug are randomly placed in area 1, a power strip is placed in area 2, and three USB cables are randomly placed in area 3. The tester sequentially installs the phone charger and the three-pin plug on the power strip, then installs the three USB cables in their respective positions. Finally, the USB cables, three-pin plug, and phone charger are unplugged and returned to their original areas. At this point, the cobot should capture the tester's actions, then imitate the actions to replicate the plugging-unplugging process.

Sub-Task 3: Area 1 has a memory module, area 2 has a computer motherboard, and area 3 has a CPU. The tester picks up the memory module and CPU and hands them to the cobot, which then grips and installs the memory module and CPU in their respective positions on the motherboard.

Sub-Task 4: Breaking down area 2 into three zones, three different types of tracks are set in each zone. The cobot uses visual sensors to recognize the tracks to determine the magnitude of the vertically applied force F, then grabs a cone-shaped sponge brush dipped in paint and vertically draws out three different tracks on the working plane.

C. Test Explanation

In order to ensure quality and completeness of the test, the following requirements need to be met:

- The cobot is aware of the test item information and the layout information of the working area.
- The cobot is placed in a position where it can complete the test tasks.

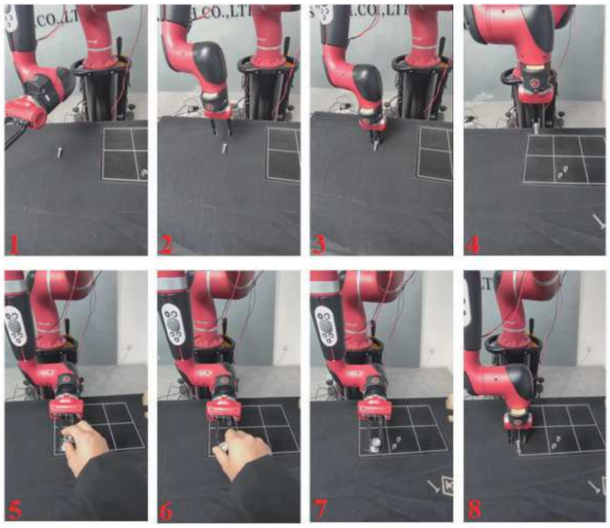


Fig. 3: TCCs Testing with Sawyer

- The cobot is equipped with visual sensors and force-control functions.
- The fitting curve of the line width D_0 generated by the cone-shaped sponge brush and the vertically applied force F have been determined.

D. Test Scoring Criteria

- In sub-task 1, for each successfully assembled pair of bolts and nuts, add 5 points.
- In sub-task 2, add 4 points for each successful insertion or removal of a mobile phone charger or three-pin plug, and add 4 points for each successful insertion or removal of a USB cable.
- In sub-task 3, add 12 points for successfully installing the CPU on the computer motherboard and add 8 points for successfully installing a memory module.
- In sub-task 4, add points based on the difference between the actual track drawn and the preset track, formulated as $15\frac{D_0}{D_{\text{read}}}$. Up to 45 points can be added for three different tracks.
- Add 10 points for total completion time within (0, 30] minutes, add 6 points within (30, 35] minutes, add 2 points within (35, 40] minutes. No points are added for total completion time exceeding 40 minutes.
- With three complete operations, add the job score and bonus score together, then take the arithmetic average as the total score. If the total score exceeds 100, it should be counted as a full score of 100.

IV. Correlation Analysis Method

We have previously established the technical parameters and performance indices for the cobot. In this section, we are conducting a quantitative analysis on how technical parameters impact performance indices utilizing the Pearson correlation analysis [14]. In order to proceed, we need to build two column vectors with same dimension. First, we put together the technical parameter data as a column vector.

Then, we have the cobot go through the TCCs testing procedure introduced in section III. Note that we need to make sure the number of tests is at least equivalent to the number of technical parameters. By the end of the test, we can obtain a column vector of K performance indices, whose dimensions are consistent with those of the technical parameters. To analyze the relationship between the parameters with the indices, we use the Pearson correlation coefficient. The mathematical formula is listed below:
Denote each parameter as $P_1, \ldots, P_M$ and each index as $I_1, \ldots, I_K$, where M is the number of parameters and K is the number of indices, For the data matrix D, $D_{i,j}$ means the i, j-th entry, and $\bar{D}_{\cdot,l}$ is the average of the l-th column of D.

Define the two correlation coefficients:
For each I_k for $k \in \{1, 2, \ldots, K\}$, the global correlation coefficients is

$$r_{I_k} = \frac{\sum_{j=1}^{M} \left(D_{j,1} - \bar{D}_{\cdot,1}\right)\left(D_{j,k} - \bar{D}_{\cdot,k}\right)}{\sqrt{\sum_{j=1}^{M} \left(D_{j,1} - \bar{D}_{\cdot,1}\right)^2 \sum_{j=1}^{M} \left(D_{j,k} - \bar{D}_{\cdot,k}\right)^2}} \tag{1}$$

For each I_k, the local correlation coefficients with parameter P_m removed for $m \in \{1, 2, \ldots, M\}$ is

$$r_{I_k,-m} = \frac{\sum_{j=1, j\neq m}^{M} \left(D_{j,1} - \bar{D}_{\cdot,1}\right)\left(D_{j,k} - \bar{D}_{\cdot,k}\right)}{\sqrt{\sum_{j=1, j\neq m}^{M} \left(D_{j,1} - \bar{D}_{\cdot,1}\right)^2 \sum_{j=1, j\neq m}^{M} \left(D_{j,k} - \bar{D}_{\cdot,k}\right)^2}} \tag{2}$$

Define an influential indicator: For each I_k, the influential indicator for parameter P_m is

$$r_{I_k,P_m} = r_{I_k} - r_{I_k,-m} \tag{3}$$

The global correlation coefficients and the local correlation coefficients have values between -1 and 1, and the influential indicator have values between -2 and 2. The greater the absolute value, the stronger the correlation between two variables. From above formulae, it can be observed that there are $M \times K$ indicators, reflecting each technical parameter's level of impact on performance indices.

V. Case Study

A. Data Generation

We had Sawyer cobot go through the TCCs testing procedure introduced in section III for exactly 19 times to match the number of technical parameters. Recall that we have 47 performance indices. This would give us a 19 $\times$ 47 matrix of original data from Sawyer. Then we proceeded to normalize all the data to make them computation ready. Part of the normalized data is shown in Table III .

TABLE III: TCCs Data

Y \ X		X1	X2	X3	X4	···	X25	···	X47
Y1	0.7	0.8	0.8	0.5	0.8	···	0.5	···	0.3
Y2	0.3	0.5	0.3	0.7	0.2	···	0.6	···	0.5
Y3	0.4	0.8	0.3	0.3	0.3	···	0.4	···	0.4
Y4	0.4	0.6	0.7	0.5	0.4	···	0.5	···	0.6
Y5	0.4	0.3	0.6	0.3	0.4	···	0.3	···	0.3
Y6	0.5	0.3	0.8	0.7	0.4	···	0.3	···	0.8
Y7	0.9	0.8	0.5	0.5	0.8	···	0.3	···	0.6
Y8	0.2	0.6	0.4	0.7	0.4	···	0.2	···	0.5
Y9	0.2	0.2	0.4	0.4	0.6	···	0.7	···	0.2
Y10	0.2	0.2	0.5	0.8	0.5	···	0.3	···	0.5
Y11	0.8	0.3	0.4	0.4	0.3	···	0.7	···	0.6
Y12	0.2	0.7	0.8	0.7	0.5	···	0.3	···	0.4
Y13	0.3	0.8	0.7	0.6	0.3	···	0.7	···	0.5
Y14	0.2	0.3	0.3	0.6	0.2	···	0.2	···	0.3
Y15	0.1	0.3	0.4	0.4	0.6	···	0.4	···	0.3
Y16	0.8	0.4	0.4	0.5	0.7	···	0.3	···	0.3
Y17	0.1	0.8	0.3	0.6	0.7	···	0.5	···	0.4
Y18	0.8	0.3	0.4	0.5	0.6	···	0.7	···	0.3
Y19	0.5	0.3	0.2	0.7	0.3	···	0.5	···	0.7

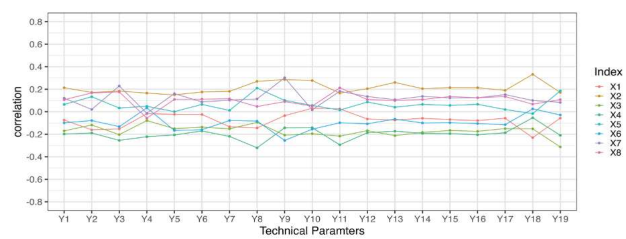
B. Computation and Results

Plug the normalized data into equation (1) for global Pearson correlation coefficients, equation (2) for local Pearson correlation coefficients, where M and K are 19 and 47, respectively. Use equation (3) for the influential indicators. The calculation results are in the form of 19×47 correlation matrices. We have visualized them in Fig. 4. and Fig 5. where they are the local Pearson correlation coefficients and the influential indicators, respectively. There are some noticeable observations:

- All six line charts present mild fluctuation in the plot lines. This means that the 19 technical parameters have consistent impacts on the 47 performance indices. Namely, there are very few parameters that have a huge impact on one index but have a small impact on others.
- Fig 4(b) shows that almost all 19 technical parameters have a large negative impact on index $X11$, which represents ”Human intentions understanding”.
- Fig 4(d) shows that almost all 19 technical parameters have a large negative impact on index $X32$, which represents ”Real-time obstacle avoidance”
- Fig 4(f) shows that almost all 19 technical parameters have a large positive impact on index $X41$ and index $X44$ which represent ”Speed and distance monitoring” and ”Active compliance control”, respectively.

Recall that an influential indicator is defined as each technical parameter's level of impact on each performance index. By visualizing our influential indicators, one could easily tell which technical parameter has the highest impact on the performance indices, therefore potentially improve the cobot's performance in a more time-efficient fashion for future studies. Taking our case as an example, Fig. 5 (b) shows that $X37$ is outstanding. This observation can be interpreted as, ”the arm span of robot” has a relatively high impact on ”rigid and flexible intelligent switching control”.

Due to space limitations, please refer to [13] for more detailed descriptions of specific test tasks, scoring criteria, and calculation results.



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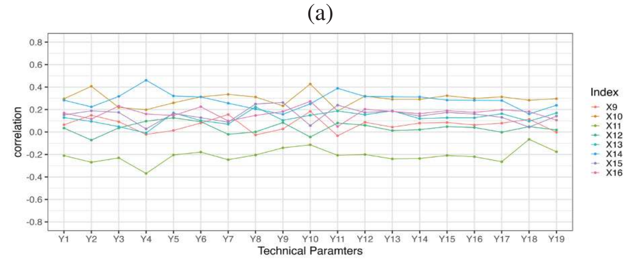

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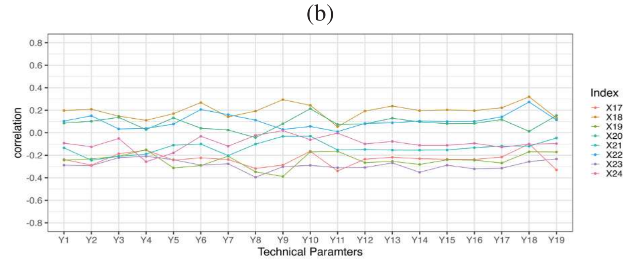

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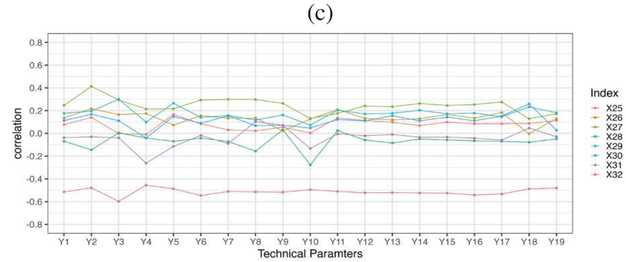

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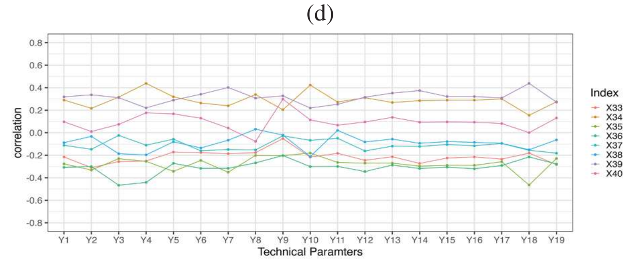

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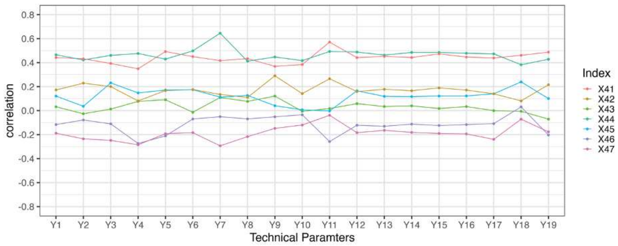
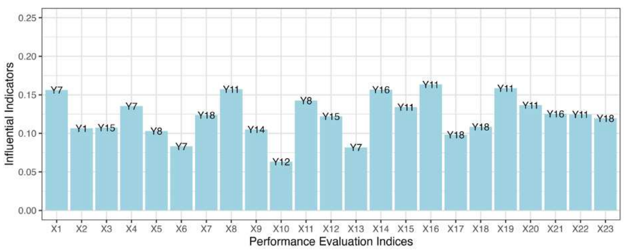
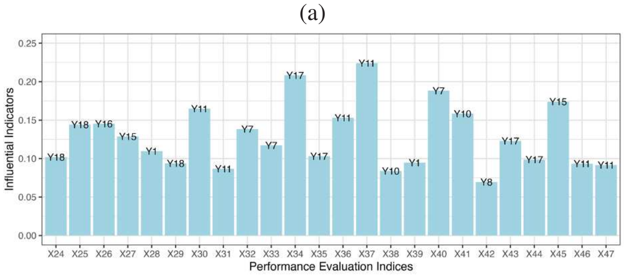


(f)

Fig. 4: Local Pearson Correlation Coefficients



(a)



(b)

Fig. 5: Influential Indicators

VI. CONCLUSION

We proposed a statistical method to optimize cobot performance by analyzing the relationship between technical parameters and performance indices using Pearson's correlation analysis. Our method is accessible to those with limited statistical expertise due to its simplicity, interpretability, and practicality. We defined local Pearson correlation coefficients and influential indicators to evaluate the impact of technical parameters on performance indices. Our results confirmed the feasibility and effectiveness of this method in providing a comprehensive assessment of cobot capabilities, addressing a significant gap in the literature that often focuses on individual performance aspects. The case study on Sawyer in Section V validates our theoretical framework and demonstrates the practical applicability of our method.

Future research should extend this methodology to complex and non-linear relationships between technical parameters and performance indices by applying non-linear transformations like polynomial transformations or exponential transformations to the data. Furthermore, integrating this method with advanced machine learning algorithms could enhance predictive power and provide deeper insights into the optimization of cobot performance. Future work should also include applying this methodology in various industrial scenes and with different types of robotic systems.

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