



Hot hand thinking in children

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ABSTRACT

A tendency to perceive illusory streaks or clumps in random sequences of data—the hot hand phenomenon—has been identified as a human universal tied to our evolutionary history of foraging for clumpy resources. We explored how this misperception of randomness and, more generally, ecologically relevant statistical thinking develops ontogenetically. Based on previous work with adults, we developed three tablet-based decision-making tasks that assessed how 3- to 10-year-old children in the U.S. and Germany decide whether sequential events will continue in a streak or not, their understanding of randomness, and their ability to reason about randomness in spatially dependent terms. Our analyses suggest that children, like adults, hold strong expectations of clumpy resources when they search through and reason about 1- and 2-dimensional statistical distributions. This evolved psychological default to clumped resources decreases somewhat with age. Future research should explore possible early interventions to improve statistical literacy and minimize the detrimental effects that (mis)perceptions of streaks and patterns can have on everyday life.

1. Introduction

1.1. Humans as intuitive statisticians

The world is full of statistical patterns. Distributions of objects and events in space and time are generated by underlying processes that are partly ordered, partly stochastic. For example, sequences of coin tosses show elements of order as well as randomness—no sequence is exactly predictable, but certain aggregate properties are, such as the expected proportion of heads and tails. The same holds for natural processes in space and time including weather, the distributions of plants and animals, and human social behavior. A large and growing literature in psychology has examined how humans and other animals grasp this statistical nature of the world and use it in their judgments and decisions (e.g., [Anderson & Schoeler, 1991](#); [Brase et al., 1998](#); [Cosmides & Tooby, 1996](#); [Gigerenzer & Hoffrage, 1995](#); [Gigerenzer & Murray, 1987](#); [Pacheco-Cobos et al., 2019](#); [Petersen & Beach, 1967](#); [Simon, 1956](#)). This

literature considers humans as “intuitive statisticians,” revealing ways in which our intuitions conform to, and sometimes depart from, principles of sound statistical inference and decision making (e.g., [Brase, 2002](#); [Fawcett et al., 2014](#); [Haselton et al., 2009](#); [Hertwig et al., 2019](#); [Wilke & Todd, 2012](#)). Importantly, this literature includes comparative dimensions, examining facets of intuitive statistics across species (e.g., [Rakoczy et al., 2014](#); [Stephens, 2008](#)), across human societies (e.g., [Pica et al., 2004](#); [Uskul et al., 2008](#); [Wilke & Barrett, 2009](#)), and across the human lifespan, including, crucially, studies of intuitive statistics in infants and children (e.g., [Gopnik & Schulz, 2004](#); [Johnson, 2020](#); [Xu & Garcia, 2008](#)). However, these developmental and comparative studies are orders of magnitude sparser than the vast literature on college-educated human adults (c.f. [Nielsen et al., 2017](#)). In particular, the growing literature on the development of intuitive statistics, while empirically impressive, has only begun to explore how the diverse and many-faceted aspects of adult statistical intuitions originate in childhood (e.g., [Gopnik, 2012](#); [Kushnir et al., 2010](#); [Schulz et al., 2019](#)). We

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propose to expand our understanding of this aspect of development.

Intuitive statistics in human adults has been described as comprising a “toolbox” of different mechanisms and processes that we bring to bear in understanding and making use of the statistical nature of the world (Gigerenzer et al., 1999). What this means is that statistical thinking in adults is not a consistent, uniform entity generated by a single, underlying process (Kahneman, 2013). Instead, human adults bring to bear both data-driven processes, such as statistical learning and Bayesian inference (e.g., Gopnik & Tenenbaum, 2007), as well as a variety of heuristics that embody assumptions about the statistical structure of the world (e.g., Gigerenzer & Goldstein, 1996). Importantly, many of these heuristics appear to be “ecologically rational”: they embody assumptions about statistical distributions that allow for rapid and efficient decision making in environments where those assumptions hold true (e.g., Pleskac & Hertwig, 2014; Todd et al., 2000; Wilke & Todd, 2012). There is an evolutionary component here, because these heuristics are likely to have been selected and tuned over the course of human evolution to take advantage of robust statistical patterns that are widespread over space and time. They are thus not only *ecologically* but also *evolutionary* rational, operating adaptively when used in appropriate environments (e.g., Fawcett et al., 2014). A downside of this, of course, is that when these statistical assumptions are not met in particular environments, using heuristics that rely on them may produce judgments and behavior that appear “irrational,” maladaptive, or incorrect.

A case that we and others have explored at length is judgments about *randomness*, and randomness of a particular kind. A key distinction in formal statistics is the distinction between *independent* and *non-independent* events. Many statistical techniques assume the statistical independence of events being sampled, meaning that any given event has no connection to and hence no predictive information regarding any other event; this is a basic staple of introductory statistics courses, with coin tosses often used as a canonical example. One can expect sets of coin tosses, for example, to conform in the aggregate to binomial (Poisson) distributions precisely because of the independence of each toss, which does not affect or predict the outcome of the next. A large literature, however—to which we have contributed, and which we review briefly below—suggests that adult humans and animals rarely assume this independence is the case. Instead, we much more frequently assume that events in the world exhibit some mutual dependence, e.g., *autocorrelation*, either positive or negative (e.g., Falk & Konold, 1997; Scheibehenne et al., 2011). For example, if we catch a fish in a pond we might cast our line again, assuming there will be more, and if the first person we encounter in a village speaks our language we might assume that others will too. From an ecological rationality perspective, this makes perfect sense if we evolved in a world where events tend not to be completely independent (at least the events that have impacted our fitness).

However, in settings where events are largely independent, these intuitions will lead to mistakes. The intuitions we have just described reflect an assumption of *positive* autocorrelation of events. This assumption will fail—indeed, it is “irrational”—in some situations, including where events are statistically independent. In such cases, our hunches should instead rely only on the overall base rates of events and should not be biased by any prior observation. For example, if the occurrence of water in a particular location in the desert is an independent event, then observing a pool should not cause us to increase our estimate of the probability of another pool nearby above the base rate of pools (c.f. Wang, 1996).

1.2. The misperception of randomness

Human adults are often very poor at making judgments about randomness—that is, about independent events (terms we use interchangeably here). As we and others have found, human adults robustly assume that events tend to be autocorrelated, usually positively. We expect that events come in “streaks,” or “clumps” (see Wilke & Barrett,

2009) and often see patterns that simply are not present in the data that is inspected (e.g., Bar-Hillel & Wagenaar, 1991; Scheibehenne et al., 2011).

This is revealed in a phenomenon that has been well described in the judgment and decision-making literature, known as the *hot hand* effect (see Gilovich et al., 1985). Empirically, the hot hand effect occurs when humans expect sequentially occurring (binary) events to be positively autocorrelated (e.g., a basketball player’s chance of hitting a shot is perceived to be greater following a successful shot than a miss), even when in fact these sequential events are independent (see Reifman, 2011). This finding can be considered a form of “illusory pattern detection”, or “misperception of randomness” as it reveals an implicit assumption of streaks, clumps or aggregation in these sequential events. The hot hand expectation has been found to be very robust in adults, across many contexts and across cultures (cf. Wilke & Barrett, 2009). In recent years, psychologists have further explored the *proximate mechanisms* of the hot-hand phenomenon (e.g., Oskarsson et al., 2009; Yu et al., 2018) and looked at the role of the hot hand bias in other sports (e.g., Morgulev & Avugos, 2023; Raab et al., 2012), its role in gambling behavior and finance (e.g., Croson & Sundali, 2005; Wilke et al., 2014), its occurrence in other age groups such as older adults (Castel et al., 2012), and the statistical limitations of Gilovich’s original findings (e.g., Miller & Sanjurjo, 2017).

A second well-known confusion about randomness has been reported by Falk and Konold (1997). Their results inform us that people not only perceive spatial clumps in random 1-dimensional binary sequences—as in the hot hand phenomenon—but also in random 2-dimensional resource patterns. Falk and Konold (1997) elegantly showed this with a set of 10×10 grids in which half of the 100 squares were empty and half were filled. Each pattern was generated according to an alternation rate $p(A)$ that specified the probability that the next square would differ from the previous one. Whereas grids with an alternation rate $p(A)$ near 0.5 are least predictable (and most random), lower alternation rates create clusters or clumps of empty or full squares and higher alternation rates lead to more dispersion. But when asked to rate the randomness of the visual grid arrangements, participants did not give the highest ratings to grids with alternation rate near 0.5—they chose grids which were more dispersed (with a $p(A)$ around 0.60–0.65) as they thought they saw non-random patterns in grids with $p(A)$ values near 0.5. Thus, as in one-dimensional sequences, the least predictable two-dimensional random grid arrangements were nonetheless perceived as having clusters of resources (see Falk & Konold, 1997; cf. Falk et al., 2009).

We have argued that this apparent irrationality regarding random distributions results from an assumption that is ecologically rational in many natural contexts—that events come in clumps—but that leads to errors in experimental or real-world contexts where events truly are random, as in computer-generated random sequences used in psychology experiments, in sequences of coin tosses, or in casino gambling (Gaismaier et al., 2016; Yu et al., 2018).

1.3. An evolutionary perspective to the problem

Research on the *ultimate function* of the hot hand phenomenon and the misperception of random data sets suggests that these behaviors reflect an underlying adaptive human universal, tied to an evolutionary history of foraging for clumpy resources, rather than an erroneous cognitive fallacy that only occurs in sports or financial settings.

Wilke and Barrett (2009) found that the hot hand phenomenon occurs in both Western and traditional foraging cultures (i.e., the Shuar of Ecuador) when participants predict hits and misses foraging for natural and man-made artificial resources, and that it seems to be a psychological default which is only partly erased by experience with true randomizing mechanisms like coin tosses. Blanchard et al. (2014) showed that rhesus macaques, a non-human primate species with a similar foraging history shares our proclivity for seeing positive recency (clumps) in independent sequential events too—either by *homology* (i.e.,

a common ancestor of primates showed the hot hand phenomenon and now modern descendants do too) or by *convergence* (i.e., because similar patchy foraging environments selected for a misperception of randomness in disparate species). Thus, such possible ancient homology increases the likelihood that the misperception of randomness is a genuine human universal and could predict a robust and canalized developmental trajectory, possibly appearing in early childhood (see Barrett, 2015).

Aggregation in space and time, rather than randomness, is likely to have been common for most of the natural resources humans encountered over evolutionary time. Resources that primates forage for (e.g., plants and animals) rarely distribute themselves in a random manner in their natural environment, because individual organisms are not independent from another (Taylor, 1961; Taylor et al., 1978; cf. Hutchinson et al., 2008). While deviations from randomness could go either in the direction of aggregation (forming clusters) or in the direction of greater dispersal (being spread out), more often these deviations are toward aggregation, because aggregation offers considerable benefits to individual organisms coming together for mating, parenting, habitat exploration, and group foraging (see Bell, 1991; Krause & Ruxton, 2002). We propose that our species' long history as hunters and gatherers pushed our evolved psychology to take such aggregated resource distributions as the default (Tooby & DeVore, 1987). Wilke et al. (2018) found empirical support for the claim that many resources are clumps by investigating the exact ecological spatial patterns of different classes of resources in the environment. After observing and coding 15 different resources from both developed and natural domains—such as seats taken at a café and in a restaurant, occupied parking spots, groupings of geese and cows, and patterns of wilderness, wild forest, and water—the results showed that natural resource domains (e.g., animal distributions, habitat structures) and many human-developed resource domains contain aggregation. Random distributions occurred much less frequently than aggregated ones and dispersed distributions were very rare. Agent-based simulation models and behavioral multi-person foraging tasks support this argument by demonstrating that patchy structures in environments may have coevolved with the emergence of cognition for searching and exploiting such patches (Legge et al., 2012; Luthra et al., 2020; Talbot et al., 2009; Wilke & Barrett, 2009).

Thus, assuming clumpiness is adaptive in contexts where clumps exist but dispersal is rare, as in natural foraging settings. It is beneficial, because when trying to predict the best foraging site, using a strong prior expectation for clumped resources is likely to provide better predictions than a random prior (c.f. error management theory; see Haselton & Nettle, 2006; Haselton et al., 2009). At the same time, it is not more costly than other assumptions in random environments: When faced with sequences of independent and equiprobable events, specific "cognitive biases" like hot hand do not decrease accuracy, because all strategies produce chance-level performance (see Scheibehenne et al., 2011). This means that what has been seen as a systematic error in our decision-making apparatus may actually be a design feature of our cognitive system to help us find the locations of forageable resources in physical environments. This explanation also highlights the role of *ecological* (and *evolutionary*) *rationality* as introduced above—the principle that there is a match between the statistical structure of objects and information of current (and past) environments and the judgment and decision-making strategies of humans and other organisms (e.g., Brase et al., 1998; Griffiths et al., 2018; Rosati, 2017; Todd et al., 2012; Wang, 1996).

1.4. A developmental perspective to the problem

Little is known about the perception of randomness in human infants or children. Some early work tried linking children's reasoning about randomness producing physical devices to distinct stages of cognitive development (see Piaget & Inhelder, 1975), but these studies neither involved sequential foraging tasks nor reflected on the statistical

distributions of the *natural* environment that our minds evolved to respond to (cf. Hoemann & Ross, 1971; Kuzmak & Gelman, 1986; Metz, 1998). What has been shown in recent years, however, using cleverly designed empirical studies, is that human babies and children are impressive intuitive statisticians in multiple ways. The statistical intuitions of infants can be examined by, for example, showing them a sample stimulus (e.g., balls being drawn from an urn containing a larger sample of balls), and then measuring their reactions when, for instance, the rest of the urn is revealed (e.g., are they surprised or not at the features of the larger distribution?). Using methods of this kind, Xu and Garcia (2008) showed that infants are able to make inferences about the statistical properties of a larger population from a small sample of that population, and even base their expectations on whether or not the sample was drawn randomly (cf. Xu & Denison, 2009). Using similar methods, Téglás et al. (2007) showed that 12-month old infants can reason about probabilities without observing outcomes, forming intuitions about the probabilities of future events from small samples of events. Furthermore, computational models using infant data suggest that babies conform to principles of Bayesian statistical inference (e.g., Téglás et al., 2011). Work by Gopnik and colleagues shows that babies and children are able to draw sound causal inferences from data, and even that they operate as "little scientists," conducting interventions targeted at revealing statistical causal relationships (e.g., Bonawitz et al., 2014; Gopnik, 2012; Gopnik et al., 2001; Gopnik & Schulz, 2004). And work by Johnson and colleagues shows that human babies and children are good statistical learners, using the statistical properties of sequences of information, such as speech patterns, to learn the statistical properties of that information (see Marcus et al., 2007; for an overview, Johnson, 2020). What is lacking in this literature, to our knowledge, is any systematic investigation of possible departures from an ecological statistical rationality in young children, to complement the large literature in adults and, more importantly, to begin a developmental map of how and when the deployment of various statistical heuristics used by adults begins in childhood. We are, therefore, proposing an initial investigation of this kind.

Prior studies of statistical reasoning in young children did not explore the hot hand intuitions commonly seen in adults. Would young children show the same expectations of clumps in sequences as adults do? If not—if young children do not expect clumps but adults do—then this would raise questions regarding how and why these intuitions appear in adulthood: It could be that hot hand beliefs are acquired through experience, via a relatively slow developmental process, or alternatively that while hot hand thinking is an evolved bias it only appears later—for reasons to be further investigated—via maturation. If hot hand thinking is present in children, however, particularly at very young ages, then this suggests that hot hand intuitions either develop very early and do not require a lifetime of experience, or that hot hand thinking is an evolved default setting, present at birth. Of course, it is also possible that children may exhibit a hot hand effect that is weaker than that seen in adults, again arguing for a role of learning. Thus, by examining a relatively wide age range, we should be able to examine whether there is any evidence for developmental change during this time. Children aged 3 to 6 years, for instance, are of particular interest as, ethnographically-speaking, this age window is when children first begin to become autonomously active beyond the household and, sometimes, beyond adult supervision. In many traditional foraging societies, for example, children begin to forage by age 5 (often in multi-aged groups of children; see Bird & Bird, 2002; Hawkes et al., 1995).

With the present study we aim to explore answers to the following open questions: 1) Do young children exhibit the hot hand effect when presented with random resource distributions? 2) How does their perception of randomness develop ontogenetically? 3) At what age do they start to grasp spatial dependencies such as clustering and dispersion?

2. Methods

2.1. Participants

We tested children in two locations to obtain a large total sample as well as adequately-sized subsamples for inspection of developmental trends by age group across our experimental paradigms. The data collection setting for one location was geographically small and rural (Potsdam, NY, USA) while the other was large and urban (Berlin, Germany). For the present study, we focused on testing children 3–10 years of age. The Potsdam sample consisted of $n = 160$ children (girls: 73, boys: 87, average age = 5 years and 9 months) and the Berlin sample of $n = 186$ (girls: 93, boys: 93, average age = 6 years and 9 months). Since there were no meaningful statistical differences between the samples in preliminary analyses (and the present study did not aim to be cross-cultural), we merged the data from both locations into one combined sample of $N = 346$ children (girls: 166, boys: 180, average age: 6 years and 5 months). **Table 1** shows subsample sizes across each age group.

Children were tested at daycare facilities (e.g., SUNY Potsdam's Child Care Center, Potsdam, NY; Kinder Academy Learning Center, Potsdam, NY; Canton Daycare Center, Canton, NY), at science museums (i.e., the North Country Children's Museum, Potsdam, NY; Natural History Museum Berlin, Germany), at a zoo (i.e., the Berlin Zoological Garden, Berlin, Germany), and in psychological laboratories (i.e., the Evolution and Cognition Lab at Clarkson University, Potsdam, NY; the Max Planck Institute for Human Development, Berlin, Germany). While parents had to provide written consent for their children's participation ahead of time, children provided their consent orally to the research assistant at the beginning of the study. All children received a completion certificate and a stuffed toy animal for their participation.

We also collected data from a group of adults $N = 85$ that were made up of a convenience sample of Introductory Psychology students at Clarkson University (women: 43, men: 42, average age: 19 years and 8 months). Students provided written consent and received course credit for their study completion.

Institutional Review Board approval for the study was granted by both Clarkson University (#20–03) and by the Ethics Committee at the Max Planck Institute for Human Development (#i2019–05).

Table 1

Means and standard deviations of subjective alternation probability scores (Task 1).

Sample	<i>n</i>	Full sequence	<i>p</i> vs. 0.5	1st half	2nd half	<i>p</i> difference
3-year olds	49	0.47 (0.10)	.034	0.48 (0.15)	0.46 (0.11)	.525
		0.44 (0.15)		0.44 (0.16)	0.43 (0.19)	
4-year olds	56	0.40 (0.13)	.002	0.40 (0.17)	0.38 (0.17)	.666
		0.39 (0.13)		0.42 (0.17)	0.36 (0.17)	
5-year olds	54	0.40 (0.12)	.001	0.42 (0.16)	0.36 (0.16)	.487
		0.39 (0.12)		0.43 (0.16)	0.35 (0.16)	
6-year olds	47	0.49 (0.11)	.001	0.53 (0.14)	0.44 (0.15)	.047
		0.49 (0.11)		0.43 (0.14)	0.35 (0.15)	
7-year olds	46	0.49 (0.12)	.001	0.53 (0.13)	0.44 (0.15)	.008
		0.49 (0.12)		0.54 (0.13)	0.43 (0.15)	
8-year olds	47	0.51 (0.10)	.235	0.56 (0.14)	0.45 (0.16)	.005
		0.51 (0.10)		0.56 (0.14)	0.45 (0.16)	
9-year old	22	0.52 (0.11)	.540	0.56 (0.12)	0.49 (0.14)	.016
		0.51 (0.11)		0.56 (0.12)	0.45 (0.14)	
10-year olds	25	0.52 (0.11)	.630	0.56 (0.12)	0.49 (0.14)	.003
		0.52 (0.11)		0.47 (0.16)	0.41 (0.16)	
Combined kids	346	0.44 (0.13)	.001	0.47 (0.16)	0.41 (0.16)	< .001
Adults	85	0.52 (0.12)	.973	0.56 (0.13)	0.49 (0.14)	.003

Note. Standard deviations are presented in parentheses; *p*-values testing the subjective alternation probability for the full sequence against the objective alternation probability of 0.5 are left-tailed tests; *p*-values testing difference between first and second half of sequences are two-tailed.

2.2. Tasks and testing procedures

Children were tested individually on all three tasks in one session. A research assistant read out the instructions for each of the tasks to the children and explained, if needed, on how to navigate the touchscreen interfaces. All tasks were presented on iPad Pro tablets. Children's behavioral choice data was saved via local Wi-Fi networks into a secure database along with their age, gender and participant number information. Each testing session took about 20 min. Full video walkthroughs of each task can be found among the materials that were uploaded along with the raw data to the Open Science Framework (OSF; see link in Supplementary Materials section).

2.2.1. Task 1 (sequential search): The animal foraging task

Through helping an animal rabbit friend named Maxi to find food, children had the opportunity to either predict the presence of a plant resource (a hit) or the absence of it (a miss) at a sequence of locations, one location at a time. Choices were made by tapping one of two possible outcome buttons (i.e., carrot hit vs. no carrot/miss). The touchpad software recorded the binary choice, provided the feedback animation (correct prediction or not) and showed the rabbit moving to the next location. Children were told that Maxi knows that half of the spots have carrots in them and half are empty, but that Maxi does not know where those carrots are. Children were also told that their task was to help Maxi guess if there is something to eat or nothing to eat under each spot. Each child saw one training location at the beginning of the task that contained a resource hit. Based on pilot data collection that focused on attention and fatigue effects—specifically for the young children—we chose 40 predictable locations as an adequate length of the resource foraging path. **Fig. 1** (top) shows two task screenshots.

2.2.2. Task 2 (randomness perception): The raindrop task

Children were told that a group of friends were playing basketball on a playground. When it suddenly started to get cloudy outside and then began to rain, the friends left to head home. Children were instructed that their task is to show us where they think each of the raindrops were falling. To do so, children tapped with a finger each specific location on a grey basketball court area that they thought will be hit by a raindrop. The task ended after the child placed 50 raindrops inside the dedicated area. The task software recorded the spatial location of each raindrop as well as their respective time stamps. Placed raindrops only remained visible on the screen for a very short period of time (1 s) before they disappeared. Pilot testing revealed that the basketball court area needed to be displayed without any line markings or hoops to not introduce spatial demand effects. **Fig. 1** (middle) shows task screenshots along with a child's data from the pilot study.

2.2.3. Task 3 (spatial dependency): The tree task

Children were asked to place 10 individual resources on a tree that had 24 empty zones. The resources were initially located at the bottom of the screen where they could be picked up one at a time, with a finger, and moved to a tree location. The same resource could be picked up and dropped in a location repeatedly, but only until a new resource was chosen from the resource stack. All participants performed two different task conditions, presented counterbalanced, in which they either indicated where apples will grow on a tree or where birds will build their nests. Children were instructed that the apples like to grow close to the sun to get lots of sunlight (positive spatial dependency) and that the birds like to build their nests away from each other (negative spatial dependency). Counterbalanced within each condition was the additional placement of a sun or a cloud in the sky. **Fig. 1** (bottom) shows task screenshots along with a child's data from the pilot study.

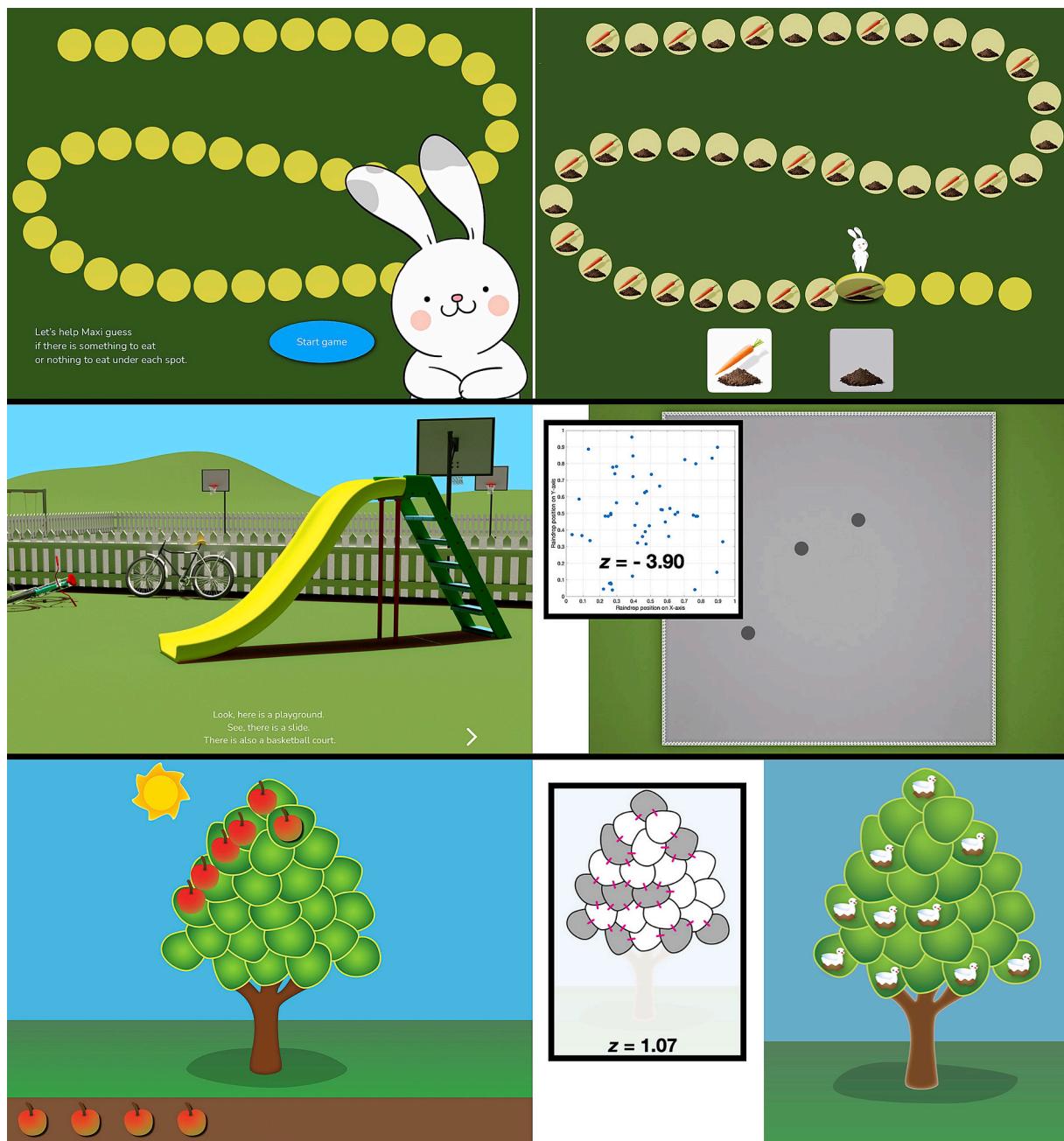


Fig. 1. Screenshots and select pilot data for three iPad decision-making tasks: The animal foraging task (top row images), the raindrop task (middle row images) and the tree task (bottom row images). Shown z -scores indicate spatial pattern type of pilot data (see main text for details).

3. Results

3.1. Task 1 (sequential search): The animal foraging task

To answer whether the children exhibited the hot hand effect in our sequential search task, we need to assess whether or not they subjectively expected to find resources in clumps, akin to a hot hand streak. The touchpad software showed each child a different random sequence with an *objective* alternation probability $p(A) = 0.5$ that had a length of 40 locations (with 20 of these locations containing carrots/hits). Based on the choices that the children made at each location (after receiving feedback about the previous location), we can compute their *subjective* alternation probability $p(A)$ and compare it to the random 0.5 alternation probability that they actually encountered (see Wilke & Barrett, 2009 for methodological details on computing alternation

probabilities). As in the 2-dimensional grid case described above, here, in a 1-dimensional sequential path case, lower subjective alternation probabilities [e.g., $p(A) = 0.4$] indicate an assessment of clumpiness or streakiness that is not present in the data, and higher subjective alternation probabilities [e.g., $p(A) = 0.6$] indicate an assessment of dispersion and more frequent alternations than expected by chance.

Fig. 2 shows histograms for our sample of young children (blue bars) and adult comparison sample (red bars). As can be seen, the distribution of subjective alternation probabilities for the children is shifted somewhat to the left of the vertical dashed red line indicating the objective randomness threshold of $p(A) = 0.5$. The distributions of child and adult data are also significantly different from each other such that the adults, at least for the sequences in the present study, were able to more or less accurately perceive, on average, that the sequences were random. Overall, children's subjective alternation probabilities had an average p

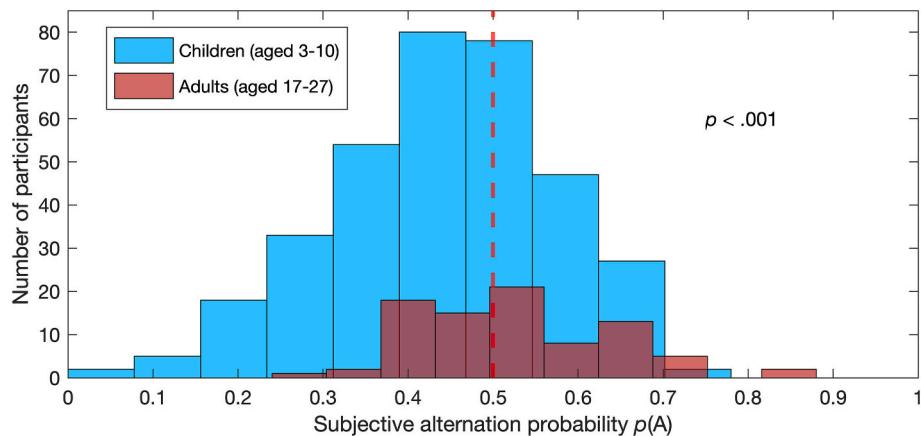


Fig. 2. Histograms showing individual subjective alternation probabilities $p(A)$ computed for a foraging path of 40 locations for children ($N = 346$, blue bars) and adults ($N = 85$, red bars) in Task 1. Randomness threshold with objective alternation probability of $p(A) = 0.5$ is plotted as dashed vertical line. Shown p -value tests for sample difference. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

(A) of 0.44, showing that many children perceived the random sequences to be more clumpy/streaky than they truly were.

There are important differences, however, when we break down this distribution into distinct age groups. As seen in Table 1, the shift toward the left side of the threshold was mostly driven by the subgroups aged 3–7 years. Children aged 8, 9 or 10 years, on average at least, did not misperceive these sequences of 40 resource locations to be streaky. Based on earlier results in adults (see Scheibehenne et al., 2011; Wilke & Barrett, 2009), we also split the full length of the foraging path into two subsets of the first 20 and the last 20 locations to check if the subjective perception of the encountered (random) pattern changes over time. Here, older children aged 6–10 years have lower subjective alternation probabilities in the second half of the sequence indicating that their perception of streakiness increases with longer presentations of random data. More specifically, for the 6- and 7-year olds, existing perceptions of clumpiness become more extreme over time, whereas for the 8–10 year olds, a fairly accurate understanding of randomness turns into seeing clumps and streaks when the full sequence is explored further. We found no differences across gender.

3.2. Task 2 (randomness perception): The raindrop task

Past psychological research on investigating why humans are so prone to misunderstand random data has typically focused on empirical resource distributions of *equal base rates* as described and analyzed for Task 1. In 2-dimensional cases of this problem, researchers typically use *square (or rectangular) arrangements*—such as a 10×10 grid of resource locations with 50 resources/tokens in it—and again compute alternation probabilities that indicate the degree of spatial aggregation, randomness, or dispersion (see above). In this case, alternation probabilities have to be computed for each row and column of data separately and then get averaged into one alternation probability for the entire grid (see Falk & Konold, 1997; cf. Wilke et al., 2015). Most natural spatial resource distributions, however, do not occur with such regular features. If our hunter-gatherer past shaped our evolved psychology toward a default assumption of aggregation—and not randomness—then we must be able to also look at stimuli and resource distributions of the natural world in which binary patterns have *unequal base rates* and *irregular arrangements*. To overcome these methodological limitations, we incorporate a more robust statistical methodology from the spatial ecology literature (see Wilke et al., 2018).

Spatial autocorrelation measures are widely used in ecological data analysis. One of these methods—the Join Count Statistic (JCS; see Fortin et al., 2002)—was developed for describing spatial patterns of binary data (e.g., the presence and absence of a particular resource, voting for

one of two parties across different counties, etc.). This statistic uses counts of which neighboring regions belong to the same binary category (e.g., presence-presence or absence-absence) and which adjacent regions are not in the same category (e.g., presence-absence or absence-presence). The JCS makes it possible then to deduce the degree and direction of positive or negative spatial dependency by describing the underlying nature of the distribution according to three types that are interpreted via a common z -score distribution: aggregation (z -scores < -1.96), randomness (z -scores from -1.96 to 1.96) or dispersion (z -scores > 1.96). While Wilke et al. (2018) applied this statistical technique to analyze real-world resource distributions such as people, plants and animals to determine how commonly clumpiness and aggregation occur in nature, the same technique allows us here to explore children's perception of randomness in more realistic (and less constrained) spatial configurations. With the JCS, the placement of 50 raindrops in our 2-dimensional randomness perception task can be turned into a single spatial coefficient irrespective of how close or far apart certain clusters of raindrops appear within the grid that is placed on top of all rain locations. Fig. 1 (middle right) shows the coding of a pilot data plot obtained from the first-author's older daughter when testing protocols for children were still limited by the restrictions imposed by COVID-19. In the plot, each small dot represents a raindrop she placed that gets translated into a distinct zone that is either in presence or absence to another nearby raindrop. If more than one raindrop fell into the same zone, neighboring zones equal to the surplus got translated into another zone with a presence (see Wilke et al., 2018 for a detailed visualization and explanation of the coding process). Taking as a whole, her raindrop pattern can then be analyzed with regard to what distribution type it indicates (here, a clumpy raindrop pattern as the computed z -score is less than -1.96).

The upper part of Fig. 3 shows histograms for the two calculated JCS score distributions—the children sample (blue bars) and the adult comparison sample (red bars). As explained above, all scores fall into one of three distinct spatial distribution categories—aggregation (with patterns where raindrops were placed into clumps and clusters), randomness (with patterns of raindrops that are non-systematic), and dispersion (with patterns of raindrops that spread out more evenly than expected by chance). The two vertical dashed red lines indicate the lower and upper bound of the raindrop distributions that get categorized as random based on their z -score. As can be seen, a very large majority of the raindrop patterns that children created fell left of this lower bound indicating that their understanding of falling rain is such that raindrops cluster into clumps (and in many instances very strongly so). While some raindrop patterns fell into the random range, not a single dispersed pattern was created by any child. The distribution of children and adult

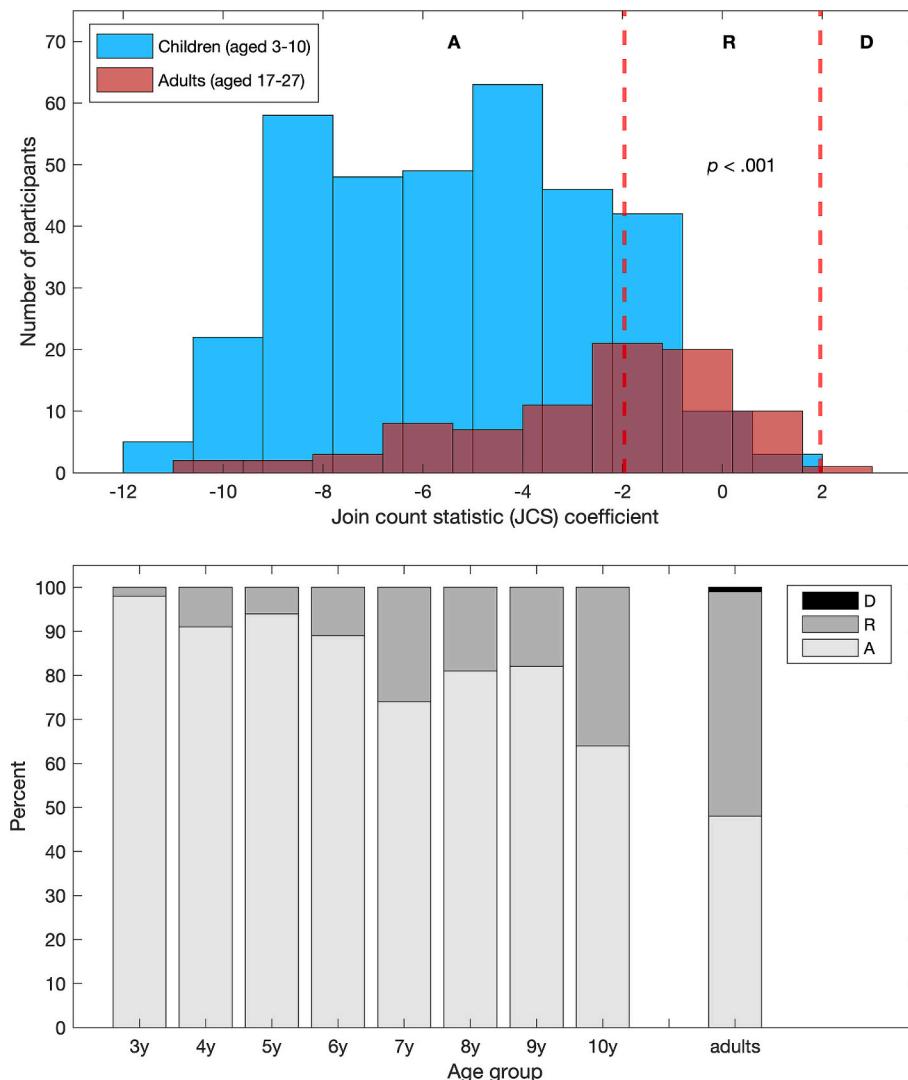


Fig. 3. Histograms (top subplot) showing individual join count statistic (JCS) coefficients computed for each distribution of raindrops placed by either children ($N = 346$, blue bars) or adults ($N = 85$, red bars) in Task 2. Lower and upper randomness threshold at $z = \pm 1.96$ shown in dashed vertical lines. JCS scores can indicate aggregation (A), randomness (R) or dispersion (D). Shown p -value tests for sample difference. Stacked bar plots (bottom subplot) showing join count statistic (JCS) coefficients in Task 2. Each age group is shown by a separate bar showing the percentage of JCS scores that fall into the dispersed (D), random (R) or aggregated (A) range. Sample sizes for each age group are shown in Table 2. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

data is again significantly different such that adults more frequently produce raindrop patterns that are indeed random. Nevertheless, many adults still produce clumpy raindrop patterns and dispersion remains essentially absent.

Table 2 shows the average JCS scores and standard deviations for both samples overall, and also for the subgroups of children by age. While every age subgroup distribution is significantly below the lower randomness threshold, the youngest age groups show the very lowest JCS scores (and therefore the most strongly clumped raindrop distributions). With increasing age, the strength of this effect weakens with JCS scores shifting higher even though most individual scores still remain falling largely below the lower threshold.

The lower part of Fig. 3 shows the percentage of JCS scores within each age group falling into each of three spatial distribution categories. Complementary to the data in Table 2, young children essentially produce only clumps and clusters of raindrops when asked to decide how (random) rain would fall onto a basketball court area. Older children seem to acquire a better understanding of these 2-dimensional distributions and can represent randomness somewhat more accurately. In

Table 2
Means and standard deviations of join count statistic coefficients (Task 2).

Sample	<i>n</i>	JCS	<i>p</i> lower	<i>p</i> upper
3-year olds	49	-6.47 (2.28)	< .001	1
4-year olds	56	-6.32 (2.69)	< .001	1
5-year olds	54	-5.73 (2.71)	< .001	1
6-year olds	47	-5.51 (2.39)	< .001	1
7-year olds	46	-4.46 (2.95)	< .001	1
8-year olds	47	-4.45 (2.49)	< .001	1
9-year old	22	-3.97 (3.13)	.003	1
10-year olds	25	-3.98 (3.11)	.002	1
Combined kids	346	-5.33 (2.97)	< .001	1
Adults	85	-2.64 (2.78)	.014	1

Note. Standard deviations are presented in parentheses; p -values are testing JCS values against aggregation (below -1.96 , lower bound) and dispersion (above 1.96 , upper bound) thresholds.

the oldest age group we tested, 10-year-old children, the spatial distribution percentages start to approximate the data we see in adults. Differences in the motor capacities of younger versus older children may partially contribute to these improvements. We again found no differences for gender. Additional analyses on time effects for subsequently placed raindrops or time taken to complete the task did not produce meaningful insights. Other spatial distance measures—such as Euclidean distance—also did not reveal systematic patterns regarding how children place raindrops early on and then systematically shift to different distribution types later.

3.3. Task 3 (spatial dependency): The tree task

In addition to *absolute* distributional assessments (e.g., is the obtained z -score low enough to indicate an aggregated, clumpy distribution?), the JCS can also allow us to make *relative* distributional comparisons when responses from multiple empirical distributions are collected from the same child (e.g., is the z -score obtained from one

condition larger than the score from another condition?). Fig. 1 (bottom), for instance, shows again the coding of a pilot data subplot under COVID-19 restrictions. Here, a young girl of 3 years was asked to place birds on a tree in such a way that the bird nests spread out (i.e., negative spatial dependency condition). While the resulting z -score did not fall above the upper threshold for randomness—therefore not indicating a truly dispersed distribution—the score can be compared to the same child's z -score from a condition in which apples were supposed to be placed close together (i.e., positive spatial dependency condition) to see if the former arrangement of nests was more dispersed than the latter arrangement of apples.

The upper part of Fig. 4 visualizes these comparisons following the same logic for our data from Task 2. As before, histograms for our children sample (blue bars) are plotted against the data from the adult comparison sample (red bars). Each subplot shows one of two conditions, the positive spatial dependency condition (i.e., apples, upper plot) in which JCS scores were expected to fall left of the lower bound and the negative spatial dependency condition (i.e., bird nests, lower plot) in

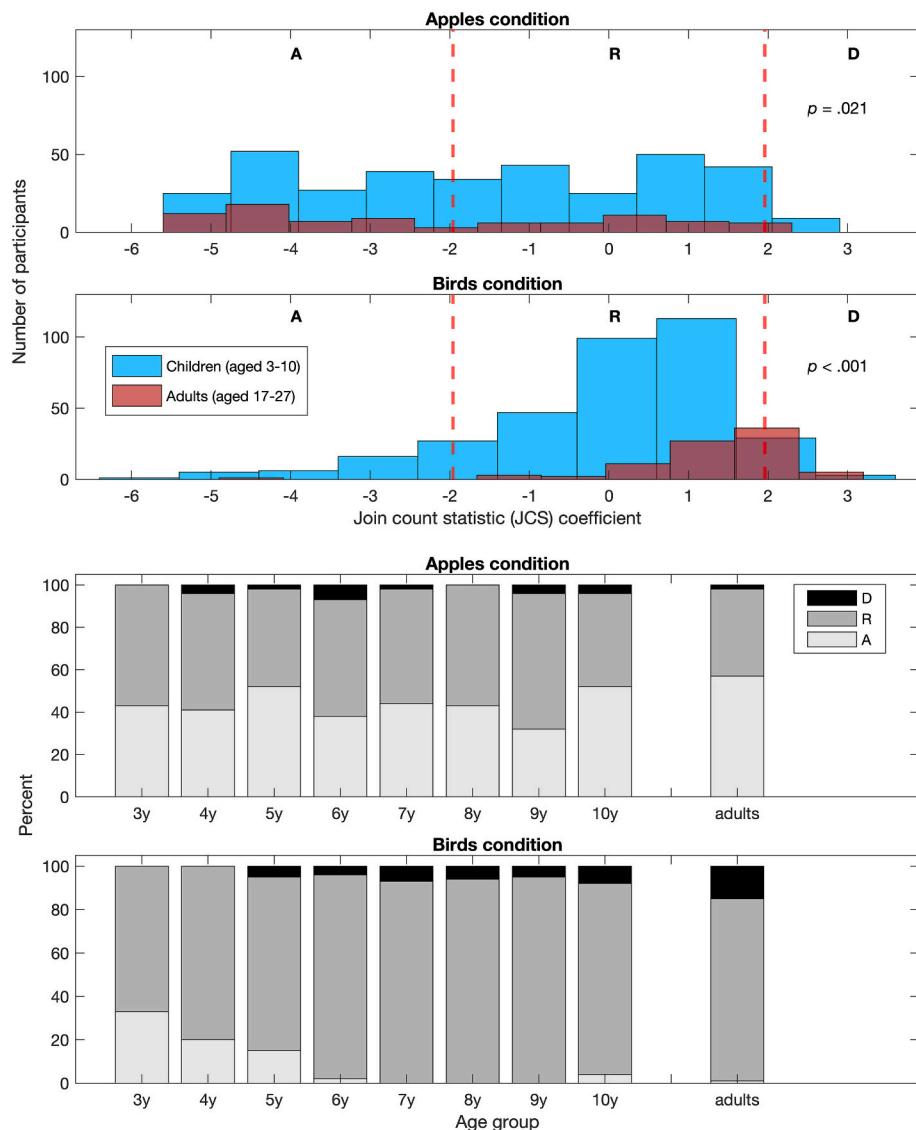


Fig. 4. Histograms (top two subplots) showing individual join count statistic (JCS) coefficients computed for each of two conditions (apples vs. birds) placed by either children ($N = 346$, blue bars) or adults ($N = 85$, red bars) in Task 3. Lower and upper randomness threshold at $z = \pm 1.96$ shown in dashed vertical lines. JCS scores can indicate aggregation (A), randomness (R) or dispersion (D). Shown p -values test for sample differences. Stacked bar plots (bottom two subplots) showing join count statistic (JCS) coefficients for each of two conditions (apples vs. birds) in Task 3. Each age group is shown by a separate bar showing the percentage of JCS scores that fall into the dispersed (D), random (R) or aggregated (A) range. Sample sizes for each age group are shown in Table 3. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

which scores were expected to be right of the upper bound. Lower and upper bounds for the range of random distributions are indicated again by vertical red dashed lines. As seen, children had an easier time correctly moving resources closer together on the tree (upper plot) than they had with spreading them out across the tree (lower plot): Close to half of the JCS scores in the apple condition fell into the aggregated category range, but only very few JCS scores in the bird condition fell into the dispersed distribution range. While many scores in the positive dependency condition fell into the random range, many more—almost all of them—fell into the random range in the negative dependency condition. JCS score classifications for dispersed distributions are indeed more frequent in the birds condition, but nevertheless quite rare in either condition. Thus, while overall the spatial distribution classifications in both conditions did not fall by majority into the predicted ranges in absolute terms, there are some clear relative shifts between the conditions that indicate that participants treated the conditions differently and adjusted their placement behavior across conditions correctly.

For both of the overall children and adult samples as well as the subgroups by age, we can compute the average obtained JCS scores for each condition. **Table 3** shows these averages and their standard deviations along with statistical tests checking for a difference in conditions. In both samples, children and adults had significantly lower JCS scores in the positive dependency condition. This finding also holds when inspecting individual children's age groups. There was no systematic trend in the JCS score averages across age subgroups in either condition, but what appears noteworthy is that the percentage of individually correct JSC score pairs differs (i.e., when an individual's JCS score from the apple condition is lower, and therefore more clumpy, than their corresponding JCS score from the bird condition). This percentage of correct relative score pairs goes up with age as shown in the right-most column.

Breaking down the JCS scores in each condition by distributional type and age in the lower part of **Fig. 4** indicates that while the understanding of resources clumping together in space does not systematically change across age subgroups (upper plot), there may be more learning with age of what spreading out means (lower plot) and an improved concept of negative spatial dependency, which could be what drives the increasingly correct relative pair proportions with age. There were no differences across gender. Additional analyses looking for the existence of left- or right-side biases with regard to how and where resources were placed upwards from the resource stack onto the tree did not produce meaningful insights. There were also no meaningful differences in the underlying distribution types when analyzing and comparing the first 5 placed resources against the second 5 placed ones. There were no order effects for condition.

4. Discussion

An important part of foraging cognition for some species is a learning

Table 3
Means and standard deviations of join count statistic coefficients (Task 3).

Sample	n	JCS apples	JCS birds	p difference	% A < B
3-year olds	49	-1.64 (1.69)	-1.04 (1.75)	< .001	53.1
4-year olds	56	-1.73 (2.01)	-0.49 (1.63)	< .001	58.9
5-year olds	54	-2.10 (2.13)	-0.17 (1.85)	< .001	64.8
6-year olds	47	-1.22 (2.65)	0.55 (0.97)	< .001	59.6
7-year olds	46	-1.16 (2.32)	0.64 (0.84)	< .001	67.4
8-year olds	47	-1.38 (2.51)	0.46 (1.01)	< .001	66.0
9-year old	22	-1.08 (2.36)	0.81 (1.02)	< .001	72.7
10-year olds	25	-1.73 (2.30)	0.08 (1.69)	.003	76.0
Combined kids	346	-1.54 (2.24)	0.03 (1.53)	< .001	63.3
Adults	85	-2.18 (2.40)	1.27 (1.00)	< .001	87.1

Note. Standard deviations are presented in parentheses; p-values testing for group condition are two-tailed; percentages show proportion of correct JCS score pairs with values for apples (A) being lower than birds (B).

mechanism that adjusts decision-making parameters based on experience (e.g., how clumpy a resource is or not). Such a mechanism would likely be adaptive for omnivores (like humans) and other primates that face multiple and variable statistical foraging environments. Furthermore, a learning system that started with broadly ecologically applicable priors would likely be selected over one with no defaults. In terms of the statistical regularities of environments (see [Wilke et al., 2018](#)), our previous research has shown that human adults indeed possess such default expectations of clumpiness, but that these expectations can be somewhat altered by experience with independent (non-clumpy) resource distributions (see [Wilke & Barrett, 2009](#)). Further, this default expectation of clumps appears very robust: it is present in adults across different cultures, it is present in another species of primate, and is difficult to experimentally reverse by manipulating the type of resource (see [Blanchard et al., 2014](#); [Scheibehenne et al., 2011](#); [Wilke & Barrett, 2009](#)). Moreover, its impacts in daily life are important: The expectation of clumps can influence peoples' behavior in other high-stakes situations such as gambling ([Wilke et al., 2014](#)), financial markets (e.g., [Kahneman & Riepe, 1998](#)), meteorological predictions and insurance market purchases ([Dodge et al., 2019](#)), responses to social media posts (e.g., [Garimella & West, 2019](#)), and general worldviews that people hold ([van Prooijen et al., 2018](#)). Thus, hot hand thinking is both relevant and pervasive in adult everyday decision making.

What has not been previously examined, however, is how this prior expectation develops ontogenetically and if the assumption of clumpiness—hot hand thinking—could be part of a developmental program that manifests later in life or if it is already present in early life (c.f. [Panchanathan & Frankenhuys, 2016](#)). Our findings here from children in the age window of 3 to 10 years across three different tasks indicate that these expectations of clumpiness are indeed operating at very young ages and are typically stronger than those found in adults. Children subjectively perceive more streaky patterns in 1-dimensional sequences (as in our animal foraging task), produce more clumps and patterns when asked to generate non-systematic 2-dimensional distributions (as in our raindrop task), and show—ontogenetically speaking—an earlier readiness for an understanding of a positive autocorrelation than they do for the concept of a negative autocorrelation (as in our tree task). Across the three tasks, we find evidence that an understanding of randomness and dispersion improves with age. However, notions of clumpiness and streakiness are very common, appear harder to learn the limits of, and remain widespread at each age group we studied in this project.

Our findings point to multiple possibilities for future research. First, it would be informative to look at the presence of hot hand thinking even earlier, including during infancy, to investigate what expectations of clumpiness look like with relatively minimal experience and exposure to socio-cultural context. Second, investigating these expectations of clumpiness also later in childhood, toward adolescence, could inform the limited empirical research on youth gambling and youth gambling addictions (see [Felsher et al., 2004](#); [Griffiths, 1989](#)). Jeff [Derevensky et al. \(1996\)](#), for example, found children as young as 9 years begin to gamble for money. Some adolescents may be more prone to experience illusory patterns in random data than others, and this could in turn be related to gambling risk for them too (as we have found to be true for adults before; see [Wilke et al., 2014](#); [Gaismaier et al., 2016](#)). Third, our findings could inform better methods of science education for helping students to more accurately recognize what are likely to be reliable patterns and what is random. A sound understanding of randomness is central to teaching statistics, informs our decision-making processes, and provides guidance when facing judgments under risk and uncertainty. Targeted statistical literacy interventions could systematically address the detrimental effects that misperceptions of randomness have and provide young adults with an additional skillset to make better life choices. Lastly, ongoing research by our team currently looks at the adaptivity that adults and children show when they have to differentiate among different objective alternation probabilities of various kinds, specifically when events are non-random and do indeed contain

statistical regularities that can be exploited (Wilke et al.). Such research will complement prior studies on exploitation-exploration trade-offs and how children direct their search behavior (e.g., Meder et al., 2021; Rosetti et al., 2017) and shed further light on why human participants—children and adults alike—persist in seeing patterns where none exist.

CRediT authorship contribution statement

Andreas Wilke: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Gracie DeLaBruere:** Project administration, Investigation, Formal analysis, Conceptualization. **Steven Pedersen:** Visualization, Software, Methodology, Conceptualization. **Bang-Geul Han:** Software, Methodology, Data curation, Conceptualization. **Hannah Spilman:** Investigation. **Yadhira Garcia:** Investigation. **H. Clark Barrett:** Writing – review & editing, Writing – original draft, Funding acquisition, Conceptualization. **Peter M. Todd:** Writing – review & editing, Writing – original draft, Funding acquisition, Conceptualization. **Annie E. Wertz:** Writing – review & editing, Writing – original draft, Project administration, Methodology, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Supplementary materials

All participant data (in MatLab format) plus task videos are available on the Open Science Framework (<https://osf.io/5g6w3>).

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