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Financing green entrepreneurs under limited commitment ☆

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ABSTRACT

Risk-averse entrepreneurs interact with financiers to fund their projects. Projects can be operated under green or dirty technologies. We explore the role of limited commitment in determining the adoption of green technologies when governments enact carbon taxes and/or directed investment subsidies. We show that entrepreneurial (respectively, financier) limited commitment makes it more (less) costly for governments to encourage green technology adoption. Because green technologies are still at an early stage, the cash flows they generate are back-loaded. Entrepreneurial limited commitment forces consumption to increase over time, thereby undermining risk-sharing and making dirty technologies more attractive. By contrast, under financier limited commitment, the possibility that front-loaded dirty technologies become obsolete forces consumption to decrease over time, thereby impairing risk-sharing and making green technologies more attractive. We also show that carbon taxes (directed technology subsidies) are more cost-effective when entrepreneurs (financiers) display limited commitment.

1. Introduction

Since Nordhaus's (1994) seminal work, economists have coalesced around the need to implement carbon taxes to address the negative externalities of greenhouse gas emissions on temperatures.[1] Under this paradigm, properly designed carbon taxes induce firms to internalize the social cost of their emissions. More recently, however, Acemoglu et al. (2012) argue that path dependencies and positive spillovers for green technologies imply that a directed investment subsidy represents an ideal complement to carbon taxes in delivering the optimal transition toward environmental sustainability. As such, the relative merits of *both* these policy instruments need to be evaluated by policymakers when designing a comprehensive environmental tax policy.

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[1] The letter titled "Economists' Statement on Carbon Dividends" (Akerlof et al., 2019), signed by over 3,000 economists including 28 Nobel Laureates in economics and four former Federal Reserve chairs, provides a detailed rationale for carbon taxes. Moreover, the monograph by Gollier (2019) contains a thorough discussion about the nuances involved in computing the appropriate carbon tax.

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By and large, those discussions have assumed away financial and agency frictions and focused on situations in which firms do not display any kind of commitment, moral hazard, or adverse selection frictions.[2] While theoretically important, these frictionless (first-best) benchmarks are not sufficiently informative for policymakers given the broad empirical evidence documenting the importance of financial frictions in driving firm behavior (e.g., Hennessy et al., 2007; Gilchrist et al., 2014; Buera et al., 2015). It is thus natural to ask: How are the relative merits of carbon taxes and directed investment subsidies affected when firms face financial frictions? Do frictions tilt the scale in favor of either of these policy instruments relative to the frictionless benchmark?

This paper contributes to partially addressing these questions by focusing on a particular type of financial friction: limited commitment, i.e., the possibility for either party in a contract to renege on its obligation in case a better outside option arises. To that end, we study the design of a contract between an entrepreneur and a financier when either party can display limited commitment.[3] The *optimal* contract entails a joint optimization over (a) the choice of technology (green vs dirty), (b) the entrepreneur's compensation scheme, and (c) the firm's investment policy. This optimization allows us to characterize the type and magnitude of government intervention required to incentivize green technologies.

We model firm cash flows as given by the product of the firm's size and profitability. The size dynamics depend on the type of technology the firm operates, its investment rate, and an idiosyncratic firm-specific shock. A firm operating the dirty technology ("a dirty firm") is assumed to be more profitable than one operating the green technology ("a green firm"). This assumption reflects the head-start advantage enjoyed by dirty firms over the past century. However, green firms grow faster than dirty ones, reflecting growing environmental concerns and the consequent consumer demand for environmentally friendly products. We study a situation in which governments can enact carbon taxes and/or directed investment subsidies to encourage firms to adopt the green technology in lieu of the dirty one.[4]

Our model adopts the following timeline: a risk-averse entrepreneur contracts with a risk-neutral financier to fund the firm he operates. The entrepreneur takes as given the tax/subsidy policies enacted by the government and chooses the operating technology (green vs dirty). The agreed-upon contract specifies the firm's investment plan and the entrepreneur's compensation (as a function of cash flows generated by the venture). Notably, contracts are *endogenously incomplete* due to the fact that entrepreneurs and/or financiers can display limited commitment.[5] That is, we allow for either party to renege on the initial contract and quit when the continuation contract (after a given history) is worse than his/her respective outside option. As a result, feasible contracts are required to satisfy a limited commitment constraint. The optimal contract has to balance the trade-off between insuring the risk-averse entrepreneur and the need to adjust his consumption to satisfy the limited commitment constraint. In particular, under entrepreneurial (financier) limited commitment, the optimal contract features a constant consumption stream for the entrepreneur whenever the constraint does not bind, and an increment (reduction) in consumption after good (bad) performance in order to keep the entrepreneur (financier) in the venture.

Our model delivers three important insights. First, under limited commitment, it is not sufficient for policymakers to determine the difference in net present value (NPV) between the green vs the dirty technology in order to deduce the tax incentive required to incentivize the adoption of green technologies. Thus, additional considerations regarding the term-structure of cash flows and the extent of either party's commitment problems are needed to determine the required incentive.

Second, the required carbon tax on the dirty technology should be higher (lower) than the difference in NPVs between the dirty and green technologies when the entrepreneur (financier) features limited commitment. Because the green technology commands higher growth potential, the entrepreneur's (financier's) limited-commitment constraint is more (less) likely to bind when the firm adopts the green technology, thereby undermining (enhancing) risk-sharing. As a result, entrepreneurial (financier) limited commitment makes green projects relatively less (more) desirable than their dirty counterparts. Therefore, the required tax incentive to close this wedge is higher (lower) than that under the first-best benchmark without commitment considerations (i.e., the NPV criterion).

Third, the allocation between directed investment subsidies and carbon taxes depends on which type of commitment friction prevails in the economic environment. When entrepreneurs display limited commitment, direct investment subsidies constitute a prohibitive approach to incentivizing green technology adoption. Because investment subsidies encourage firms to grow, the entrepreneurial limited-commitment constraint is more likely to bind, inducing an inferior risk-sharing arrangement for green firms. By contrast, when financiers exhibit limited commitment, investment subsidies have a double benefit for green firms: they not only render investment cheaper (a standard effect) but also make the financier's limited-commitment constraint less likely to bind (a novel effect). The latter effect arises because stronger growth increases the firm's future cash flows, making the financier less prone to renege on the contract. This additional benefit renders investment subsidies more cost-effective than carbon taxes when the financier has limited commitment. Table 1 summarizes our results schematically.

Our findings uncover novel forces governing the interaction between limited-commitment frictions and the back-loaded (front-loaded) structure of the cash flows generated by green (dirty) firms. As a result, we can provide guidance to policymakers on the type

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[2] Section 3.6 of the special Climate Finance issue by Hong et al. (2020) highlights this inconvenient void: "Given that corporations and insiders (CEOs) ultimately need to make investments to address climate change, two traditional corporate finance issues loom large: agency problems associated with corporate short-termism and financing frictions.".

[3] For the purpose of our model, "financiers" can be interpreted in a broad sense as any type of investor willing to provide contingent funding to the project (e.g., private equity, venture capital, or a bank line of credit).

[4] Carbon taxes are borne by firms operating dirty technologies and are proportional to their greenhouse gas emissions. Investment subsidies are enjoyed by green firms and reduce their investment costs.

[5] By contrast, *exogenously* incomplete contracts directly restrict the contract space by imposing, for instance, an exogenous borrowing limit (e.g., Aiyagari, 1994).

Table 1
Schematic summary of our results.

Policy Intervention	Entrepreneurial Limited Commitment	Financier Limited Commitment
Carbon Taxes	*Higher* carbon taxes required relative to the first-best benchmark. (see Section 4.1)	*Lower* carbon taxes required relative to the first-best benchmark. (see Section 4.2)
Investment Subsidies	*Significantly higher* investment subsidies needed relative to the first-best benchmark. (see Section 5.1)	*Significantly lower* investment subsidies needed relative to the first-best benchmark. (see Section 5.2)

and magnitude of the intervention required to encourage the adoption of green technologies in settings with limited commitment. Because our analysis delivers delicate asymmetric implications depending on whether entrepreneurs or financiers are more likely to feature commitment problems, it is essential that policymakers tailor their policies to the specific economic environment in which they are intervening. In practice, empirical work and institutional knowledge helping to assess which party is more likely to feature limited commitment is a required input for our analysis to be fruitful in policy-making circles.

1.1. Literature review

Our paper closely relates to the two taxation paradigms regarding climate externalities. The first, pioneered by the Dynamic Integrated Climate Change (DICE) models starting with Nordhaus (1994), developed realistic scenarios integrating insights from geophysics and climate science with models of economic growth. These models focus on the negative externalities associated with greenhouse emissions and on the optimal carbon tax. Recent contributors include Acemoglu et al. (2016), who endogenize the growth rate by modeling a firm's innovation-decision, and Golosov et al. (2014), who model taxes on fossil fuels as a finite resource.

The second paradigm, pioneered by Acemoglu et al. (2012), emphasizes positive externalities associated with path dependencies for innovations in green technologies. In their recent work, Aghion et al. (2016) show empirical evidence of path dependency and aggregate spillovers in the automotive industry for dirty vs clean technologies. This paradigm argues that directed investment subsidies are required for firms to effectively internalize this externality, which carbon taxes cannot achieve on their own. Most of this vast body of work assumes away commitment issues. We contribute to this debate by providing a new perspective on the relative merits of these two policy instruments when limited commitment is a friction of first-order importance in the economic environment.[6]

Our contribution is based on the large microeconomic literature on contracting under limited commitment. Harris and Holmstrom (1982) study a model of optimal insurance for a risk-averse worker unable to commit to a long-term contract. Hart and Moore (1994) develop a theory of endogenous debt capacity arising from the inalienability of human capital, a form of limited commitment. Building on these contributions, Alvarez and Jermann (2000) extend the welfare theorems to a macroeconomic setting with limited commitment. Albuquerque and Hopenhayn (2004) extend previous analyses to highlight the impact of limited commitment on firms' growth and dynamics.[7] We contribute by providing the first exploration of the role played by limited commitment in the environmental transition and its associated tax/subsidy policies.

Our methods borrow from the continuous-time contracting literature under limited commitment. Grochulski and Zhang (2011) study a one-sided limited-commitment problem in which the agent features Constant Relative Risk Aversion (CRRA) preferences. The authors lever on the tractability of continuous-time modeling to provide a closed-form solution for the optimal contract, and they directly relate their findings to the solvency constraints in Alvarez and Jermann (2000). Zhang (2014) sheds light on the dynamics of intrafirm risk-sharing under the labor-side limited commitment and its power to explain the cross-sectional cash flow volatility. Miao and Zhang (2015) use duality techniques, in the spirit of Marcet and Marimon (2019), to obtain a tractable linear partial differential equation for both types of limited commitment. More recently, Ai and Li (2015) explore the impact of limited commitment on firms' investments and CEO compensation, while Bolton et al. (2019) focus on the joint determination of investment and risk-management policies. These papers provide computationally fast and reliable algorithms, allowing us to compute the optimal contracts and value functions under various government tax/subsidy policies.

[6] Notably, a large, growing literature in economics and finance studies climate-change externalities from a variety of angles beyond carbon taxes and investment subsidies. These contributions include the study of capital reallocation to ESG funds (Halbritter and Dorfleitner, 2015; Goldstein et al., 2021), the implications of financing constraint and socially responsible capital on firms' technology and production choices (Oehmke and Opp, 2024), the use of environmentally friendly investing mandates among college endowments (Bessembinder, 2016) and sovereign wealth funds (Bolton et al., 2012), the potential benefit of investment income taxes (Nguyen et al., forthcoming), the impact of active mandates on firms' policies (Broccardo et al., 2022; Oehmke and Opp, 2024), the impact of green investing on firms' carbon emissions (De Angelis et al., 2022), and the role of corporate social responsibility in helping firms commit to lower product prices and harness network effects (Xiong and Yang, 2023).

[7] Limited commitment has also been successfully applied to other areas in economics, including labor dynamics (e.g., Rudanko, 2009), development economics (e.g., Ligon et al., 2002), international finance (e.g., Kehoe and Perri, 2002), and firm dynamics (e.g., Ai et al., 2021). Golosov et al. (2016) and Ljungqvist and Sargent (2018, Ch. 17) provide excellent overviews of this vast literature.

2. Model

2.1. Technology and preferences

We consider a continuous-time setting with an infinite horizon in which there are two types of players: financiers and entrepreneurs. Following Ai and Li (2015) and Bolton et al. (2019), we assume that financiers are risk-neutral while entrepreneurs are risk-averse. Everyone discounts the future[8] at a constant rate $r > 0$.

The firm can use either of two technologies: the ecofriendly ("green") or the non-ecofriendly ("dirty") technology. We denote the first type with the index g and the second type with the index d. At the outset, the entrepreneur chooses the technology.[9] Because financiers cannot operate the project, they must hire an entrepreneur to operate it, as described below. We denote by Y_t the cumulative cash flows generated by a project until time t. Cash flows are proportional to the project's capital stock as in Hayashi (1982), i.e.,

$$dY_t = b_n K_t dt, \tag{1}$$

where $b_n > 0$ captures the profitability of technology $n \in \{g, d\}$ per unit of firm size K_t. The law of motion of the firm size follows standard neoclassical dynamics

$$dK_t = (\mu_n K_t + I_t)dt + \sigma_n K_t dB_t \quad \text{and} \quad K(0) = K_0, \tag{2}$$

in which B_t is a standard Brownian motion capturing the idiosyncratic risk of the project and μ_n the baseline growth rate of the firm driven by exogenous factors (e.g., demand for clean energy, appetite for electric vehicles). We denote by $\mathcal{B}^t$ the σ-algebra generated by the Brownian motion, $\mathcal{B}^t = \sigma(B_s, s \leq t)$. The firm's investment policy I_t is adapted to the filtration $(\mathcal{B}^t)_t$.

Let C_t be the compensation offered to the entrepreneur for operating the project, which is a control, also adapted to the filtration $(\mathcal{B}^t)_t$. The entrepreneur's preference is represented by the CRRA utility function with a coefficient of risk-aversion $\gamma \in \mathbb{R}_+$. For tractability, it is easier to work with the entrepreneur's certainty-equivalent flow payment

$$X_t = \left[E_t \left(\int_t^\infty r e^{-r(s-t)} C_s^{1-\gamma} ds \right) \right]^{\frac{1}{1-\gamma}}, \tag{3}$$

with E_t denoting the mathematical expectation conditional on the time t-information $\mathcal{B}^t$.

A contract $(I_t, C_t)_{t\geq 0}$ specifies the compensation to the agent and the investment rate. Admissible contracts are subject to standard integrability conditions specified in Appendix 8.2. The payoff to the financier from a given contract $J(I(.), C(.))$ is equal to the NPV of the project cash flows (net of the entrepreneur's compensation C_t and the investment cost $h(i_t)K_t$)

$$J(I(.), C(.)) = E_0 \left[\int_0^\infty e^{-rt} \left(b_n K_t - h(i_t)K_t - C_t \right) dt \right]. \tag{4}$$

In equation (4), $i_t := I_t / K_t$ denotes the investment rate, and we assume that the investment cost is homogeneous of degree one in capital, i.e., given by $h\left(i_t\right) K_t$.

2.2. Entrepreneurial limited commitment

We first focus on the case in which the entrepreneur has limited commitment. Following Albuquerque and Hopenhayn (2004), the entrepreneur can always take away a fraction of the firm's capital and default on the contract. Upon default, the entrepreneur can use this capital to produce consumption goods but is no longer allowed to contract with the financier. The entrepreneur's outside option is thus proportional to the firm's current capital, K_t. Entrepreneurial limited commitment implies that to keep the entrepreneur in this venture, the certainty equivalent of the entrepreneur must be larger than his outside option at all times, i.e.,

$$X_t \geq \bar{x} K_t, \tag{5}$$

where the parameter $\bar{x}$ captures in reduced form the outside option for the entrepreneur per unit of capital.[10]

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[8] The rationale for our modeling choices is the following: In practice, the financier represents an infinitely lived, fully diversified set of investors, hence her risk-neutrality. By contrast, the entrepreneur is under-diversified and, therefore, risk-averse. We assume an identical discount rate to highlight the risk-sharing motive and abstract away from the potential gains from trade arising under heterogeneous time-discount rates.

[9] In practice, many ongoing firms can invest in R&D technologies to decrease or mitigate pollution and thereby transition from brown to green. Studying the optimal transition is beyond the scope of this paper. Thus, our model, in which technology is chosen at the outset of the project, is most relevant for firm formation.

[10] For tractability purposes, we model the outside option in reduced form. However, a more realistic model would endogenously determine $\bar{x}$ from the discounted utility obtained by the entrepreneur if he was to leave the firm, as modeled in, e.g., Eisfeldt and Papanikolaou (2013); Dou et al. (2021). Our results would remain unchanged as long as the outside option does not depend on whether the entrepreneur is currently operating a green or a dirty firm.

2.3. Financier limited commitment

We also consider the benchmark in which financiers face limited commitment and can renege on their contractual obligations when the value they obtain from the *continuation* contract is negative. That is, we consider the case in which, after a history of poor performance, the financier may decide not to pay the entrepreneur, which implies that at all times the following constraint must be satisfied to keep her in this venture

$$E_t\left[\int_t^{\infty} e^{-r(s-t)}\left(b_n K_s - h(i_s)K_s - C_s\right)ds\right] \geq 0. \tag{6}$$

2.4. Externalities and taxes

We incorporate two types of environmental externalities and associated instruments in our setting:

Social Cost of Carbon (SCC). We introduce an externality associated with CO2 emissions as in the neoclassical DICE model pioneered by Nordhaus (1994). Dirty technologies emit more CO2, a greenhouse gas that contributes to increased temperatures and, therefore, to climate disasters. Under the assumption that firms operate in competitive markets, cash flows and output are linearly related. Therefore, to incorporate the negative externality associated with CO2 emissions, governments may introduce a tax capturing the social cost of carbon (SCC), denoted by τ. In equation (4), this means that the profitability of the dirty technology becomes

$$b_d \longrightarrow (1-\tau)b_d, \quad \text{following the tax implementation.} \tag{7}$$

Directed Technological Change (DTC). Profitability is not exogenous, but instead affected by positive spillovers from other firms' investments, as suggested by Acemoglu et al. (2012). The more firms invest in the green (dirty) technology, the more efficient this technology becomes and the larger the market share of green (dirty) projects.[11] To internalize this externality, a DTC investment subsidy for green firms $s \in [0,1]$ must be introduced.[12] In equation (4), this implies that the investment cost becomes

$$h(i_t) \longrightarrow (1-s)h(i_t), \quad \text{if the government puts such a scheme in place.} \tag{8}$$

3. Solution to the optimal contract

We now determine the optimal contract offered to the entrepreneur by the financier. We first provide the first-best neoclassical benchmark without limited commitment. Then we identify the recursive structure for the problem under limited commitment using the entrepreneur's certainty equivalent and the firm size as state variables. Finally, we exploit the problem's homogeneity to reduce it to a single-state stochastic control problem for which there is a complete characterization (see Ai and Li, 2015; Bensoussan et al., 2022).

3.1. First-best benchmark

In the first-best benchmark, the financier's value function V^{FB} is given by

$$V^{FB}(K_0, X_0) = K_0 \underbrace{\frac{b - h(i^{FB})}{r - (\mu + i^{FB})}}_{v^{FB}} - \frac{X_0}{r}, \tag{9}$$

where i^{FB} is the neoclassical first-best investment rate (see proof in Appendix 8.1). In this case, the financier perfectly insures the entrepreneur by providing him with a constant stream of consumptions that has an NPV of $\frac{X_0}{r}$ and collects all the project cash flows (under the optimal investment policy), a claim with an NPV of $K_0\frac{b-h(i^{FB})}{r-(\mu+i^{FB})}$. As shown later, under limited commitment, there are two sources of welfare losses beyond climate-related externalities: (i) perfect consumption insurance for the entrepreneur will not be feasible and (ii) the first-best investment rate will be distorted. Both forces reduce the financier's value (relative to the first-best benchmark) and indirectly determine the (equilibrium) entrepreneur's choice of technology.

[11] See Aghion et al. (2016) for direct evidence of this mechanism in the auto industry.

[12] One way to think about the DTC externality is to allow the profitability of firms to depend on the average investment rate in the industry. That is, $b_g \longrightarrow b_g + \lambda\bar{i}$, where $\bar{i}$ is the average investment rate by firms operating green technologies and $\lambda > 0$ is a parameter capturing the positive spillovers from aggregate investment in green technology on individual firms.

3.2. Recursive formulation

In Appendix 8.2, we show that if a stochastic process solves the SDE given by

$$dX_t = X_t\left[\tfrac{r}{1-\gamma}\left(1-\left(\tfrac{C_t}{X_t}\right)^{1-\gamma}\right)+\tfrac{1}{2}\gamma\sigma^2\phi_t^2\right]dt+\sigma X_t\phi_t dB_t, \tag{10}$$

where ϕ_t is measurable with respect to $\mathcal{B}^t$, then it corresponds to the certainty equivalent in eq. (3). The contract features C_t, I_t, and ϕ_t as control variables, while X_t and K_t are state variables. Importantly, the choice of ϕ_t corresponds to choosing the agent's certainty equivalent sensitivity to the shocks, which can be interpreted as a measure of the pay-performance sensitivity (PPS) of the manager's contract. This insight played a critical role in the development of modern dynamic contracting, following the seminal contribution of Sannikov (2008).

3.3. Dimensional reduction

Next, we use the firm size K_t as a scaling factor and consider the stochastic processes

$$x_t = \frac{X_t}{K_t}, \quad i_t = \frac{I_t}{K_t}, \quad \text{and} \quad c_t = \frac{C_t}{K_t}. \tag{11}$$

We can now reformulate equations (2), (4), and (10). In particular, the scaled certainty equivalent of the entrepreneur x_t solves

$$dx_t = x_t\left[\tfrac{r}{1-\gamma}\left(1-\left(\tfrac{c_t}{x_t}\right)^{1-\gamma}\right)-i_t-\mu+\sigma^2\left(1-\phi_t+\tfrac{\gamma}{2}\phi_t^2\right)\right]dt+x_t(\phi_t-1)\sigma dB_t \tag{12}$$

for given initial value $x(0)=x_0$ to be determined in equilibrium, while the dynamics of capital K_t are given by equation (2). The payoff to the financier in equation (4) is now given by

$$J_{K,x}\big(i(\cdot),c(\cdot),\phi(\cdot)\big) = E_0\left[\int_0^\infty e^{-rt}K_t\big(b_n-h(i_t)-c_t\big)dt\right]. \tag{13}$$

Furthermore, the limited-commitment constraint (5) becomes

$$x_t \geq \bar{x}, \tag{14}$$

while the limited-commitment constraint (6) is equivalent to

$$x_t \leq x^*, \tag{15}$$

where the free boundary x^* is determined from boundary conditions specified later.

Finally, the value function for the financier V maximizes her payoff within the set of admissible contracts, which satisfy—among other conditions—the limited-commitment constraints in eq. (14) or (15). That is,

$$V(K,x) = \sup_{i(\cdot),c(\cdot),\phi(\cdot)} J_{K,x}\big(i(\cdot),c(\cdot),\phi(\cdot)\big). \tag{16}$$

We will show in the sequel that $V(K,x)=Kv(x)$, where $v(\cdot)$ is the solution to a Bellman equation, for which we provide a complete characterization.

3.4. Bellman equation solution

The Bellman equation for the stochastic control problem (16) obtains as

$$\begin{aligned}0 = \sup_{c,i,\phi}\Bigg\{ & b-h(i)-c+v(x)(i+\mu-r)+\left[\tfrac{r}{1-\gamma}\left(1-\left(\tfrac{c}{x}\right)^{1-\gamma}\right)-i-\mu+\tfrac{1}{2}\gamma\sigma^2\phi^2\right]xv'(x) \\ & +\tfrac{1}{2}\sigma^2(\phi-1)^2x^2v''(x)\Bigg\},\end{aligned} \tag{17}$$

where the optimal controls can be computed from the scaled value function $v(x)$ in feedback form by taking first-order conditions of equation (17). The treatment of the boundary conditions for this equation is delicate; we discuss it below for each case.

Entrepreneurial limited commitment: We required the following boundary conditions at $x=\bar{x}$ and at $x\to\infty$:

$$v''(\bar{x}) = -\infty, \quad \text{and} \quad v(x)-(v^{FB}-\tfrac{x}{r})\to 0, \text{as } x\to+\infty. \tag{18}$$

The first boundary condition is designed to ensure the limited commitment constraint is satisfied. That is, to ensure that the state variable $x_t \in [\bar{x},\infty)$, $\forall t$. Intuitively, to ensure that x_t does not go below $\bar{x}$, the optimal controls need to "kill" the volatility of x_t at $\bar{x}$; otherwise, the process will cross through that boundary, violating the limited-commitment constraint. From equation (12), we notice that this objective is achieved if $\phi(\bar{x}) = 1$, which, as shown in Appendix 8.3, obtains when we impose $v''(\bar{x}) = -\infty$. The second boundary condition, which is more straightforward, states that the first-best value characterized in equation (9) realizes when the limited constraint is infinitely far away from binding (i.e., when $x \to +\infty$).

Financier limited commitment: We require the following boundary conditions at $x = 0$ and at $x = x^*$:

$$v(x^*) = 0, \quad v''(x^*) = -\infty, \quad \text{and} \quad v(x) \to v^{FB}, \text{as } x \to 0. \tag{19}$$

The first boundary condition identifies x^* as the maximum value x_t can take such that the financier does not want to abandon the venture, since for $x_t > x^*$ the financier's continuation value is negative. The second boundary condition ensures that the limited-commitment constraint is satisfied (i.e., that the state variable $x_t \in [0, x^*]$, $\forall t$). Hence, following a similar intuition as in the previous case, $v''(x^*) = -\infty$ "kills" the volatility of x_t at x^* and prevents it from violating the financier's limited-commitment constraint. Finally, the third boundary condition states that the first-best value is achieved when x goes to 0 (i.e., when the constraint is infinitely far away from binding).

Theorem 1 *(Existence and uniqueness). Under technical assumptions specified in Appendix 8.2, Bellman equation (17), subject to the boundary conditions specified in Appendix 8.3, has one and only one solution $v(\cdot)$.*

Proof. See Bensoussan et al. (2022). □

With a slight abuse of notation, we denote subsequently by $v_n(\cdot)$ the value function obtained under a given choice of technology $n \in \{g, d\}$.

4. Carbon taxes and technology adoption

This section studies the carbon tax implications on technology adoption under *limited commitment*. In particular, we are interested in the forces driving the choice between green and dirty technologies. To that end, we first make a standard zero-profit condition on the financier side.

Zero-Profit Condition: We assume the financial sector is competitive, so that the rents earned by the financiers are driven down to zero.[13] Therefore, the initial promised value to an entrepreneur operating the green technology g (dirty technology d), denoted by x_0^g (x_0^d), must be such that

$$v_g(x_0^g) = 0 \quad \text{and} \quad v_d(x_0^d) = 0. \tag{20}$$

We discuss next the calibration of our model's parameters distinguishing between green and dirty technologies.

Calibration: The calibration should address two considerations. First, the dirty technology is more productive because of a head-start advantage. Second, the green technology is more likely to experience growth in the future as the demand for its output grows. That is, the cash flows generated by the green technology are more back-loaded, which implies a higher growth rate for green capital. Taken together, these two key features distinguishing the two technologies imply that $b_d > b_g$ and $\mu_d < \mu_g$, which we impose below. The parameters for the dirty firm are calibrated in line with Ai and Li's (2015) limited-commitment model: $b_d = 0.21$, $\sigma_d = 0.36$, $\gamma = 2$, $r = 0.08$, and $\bar{x} = 0.0164$. Following Nikolov and Whited (2014), we set a decay rate for the dirty technology of $\mu_d = -0.09$. For the green technology, we keep the same calibration except for the parameters b_g and μ_g. Acemoglu et al. (2016) estimate the innovation step size and the distribution of the initial productivity gap between dirty and green technologies. We adhere to their study, setting a lower profitability for the green technology (per unit of capital), with $b_g = \frac{b_d}{1.063^4}$ to match the innovation step size and the long tail of the productivity gap distribution. In addition, to match the average difference in estimations for the dirty and green sectors documented in Li et al. (2016), we set $\mu_g = -0.06$.[14]

4.1. Carbon tax under entrepreneurial limited commitment

This section illustrates the implications of the carbon tax rate τ on the adoption of green technologies when the entrepreneur has limited commitment. Because the financial sector is competitive, to determine whether the green technology is adopted at the outset, it suffices to compute the entrepreneurial value under the green (x_0^g) and dirty (x_0^d) technologies. Thus, our quantity of interest is the difference in the (initial) scaled entrepreneurial values

$$\Delta x := x_0^g - x_0^d, \tag{21}$$

where x_0^n is the initial value of the state process in equation (12). Fig. 1 plots this quantity $\Delta x(\tau)$ as a function of the carbon tax rate τ. We also plot the first-best benchmark $\Delta x^{FB}(\tau)$, in which we compute (21) under full commitment as described in Section 3.1.

13 Our results do not depend on the degree of competition in the financial sector; we make this assumption only to deal with the most tractable case.

14 Table 2 of the Appendix considers alternative calibrations in the neighborhood of our baseline calibration. It shows that our subsequent results are robust to alternative calibrations as long as the defining features of both sectors (i.e., $b_d > b_g$ and $\mu_d < \mu_g$) hold.

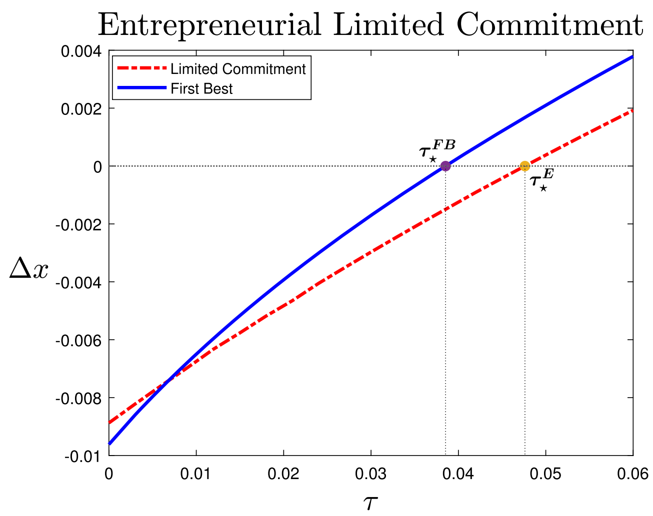

Fig. 1. Difference between certainty equivalents in Equation (21) as a function of the carbon tax rate τ under entrepreneurial limited commitment. The dashed and solid vertical lines depict the minimal carbon tax rate required to incentivize entrepreneurs to adopt green technologies under entrepreneurial limited commitment (τ_*^E) and in the first-best benchmark (τ_*^{FB}). Observe that the required carbon tax rate under entrepreneurial limited commitment is greater than that in the first-best benchmark (i.e., $\tau_*^E > \tau_*^{FB}$). Parameter values are $b_g = 0.1645$, $b_d = 0.21$, $\mu_g = -0.06$, $\mu_d = -0.09$, $\sigma_g = 0.36$, $\sigma_d = 0.36$, $\gamma = 2$, $r = 0.08$, and $\theta = 3$.

We obtain two important insights. First, and a direct consequence of our calibration, is the fact that $\Delta x(0) < 0$. That is, in the absence of carbon taxes $\tau = 0$, the entrepreneur prefers to operate a dirty technology. This is because the dirty technology is intrinsically more productive, and the firm does not internalize the damage this technology causes to the environment. Hence, the acknowledged need for government intervention, e.g., via a carbon tax. As this tax increases, adopting the green technology becomes increasingly lucrative for the entrepreneur. Above a cut-off value for that tax, denoted by τ_*^E (τ_*^{FB}), an entrepreneur featuring limited (full) commitment will favor the green technology. Second, and more telling,

$$\tau_*^E > \tau_*^{FB}, \tag{22}$$

which means that, when entrepreneurial limited commitment matters in the economic environment, the government needs to raise the carbon tax rate to a higher value to encourage the adoption of green technologies (compared to the full-commitment benchmark).

Intuitively, because the cash flows generated by green technologies are more back-loaded in time (since a green firm grows faster than a dirty one), it is more difficult for the optimal contract to provide insurance for the entrepreneur since a green entrepreneur will be tempted to quit in the future as the firm's capital grows. As time goes by, two sources of welfare loss result from the fact that the entrepreneur's limited-commitment constraint binds more often under the green technology than under the dirty technology: (i) the entrepreneur's consumption needs to increase over time, limiting risk-sharing, and (ii) to prevent the entrepreneur from leaving the firm, the optimal contract features underinvestment relative to the first-best investment rate (see Fig. 2).

Because the financial sector is competitive, these welfare losses imply that entrepreneurial limited commitment makes green technologies even less attractive for entrepreneurs. A larger carbon tax on dirty technologies is thus required to encourage the adoption of green technologies. As we show in Section 4.2, this result is reversed under financier limited commitment.

To conclude this section, we stress for policymakers the need to carefully design incentives that encourage the adoption of green technologies. They must beware that, in the presence of limited-commitment frictions, the difference in NPV between green and dirty technologies is *not* a sufficient statistic to determine the necessary tax incentives. One must look not only at the NPV of each technology but also at how the cash flows of each technology are distributed over time. This insight plays a critical role in all of our subsequent findings.

4.2. Carbon tax under financier limited commitment

This section explores the implications of the carbon tax rate τ on the adoption of green technologies when the financier (instead of the entrepreneur) has limited commitment. To that end, Fig. 4 again plots $\Delta x(\tau)$ in equation (21) as a function of the carbon tax rate τ. The order between the carbon tax needed in the first-best vs financier limited commitment case is now reversed compared to the one in equation (22) (i.e., under entrepreneurial limited commitment). That is, we observe that

$$\tau_*^E > \tau_*^{FB} > \tau_*^F,$$

where τ_*^F here denotes the minimum carbon tax needed for the entrepreneur to adopt green technology when the financier has limited commitment and can walk away from her contractual obligations.

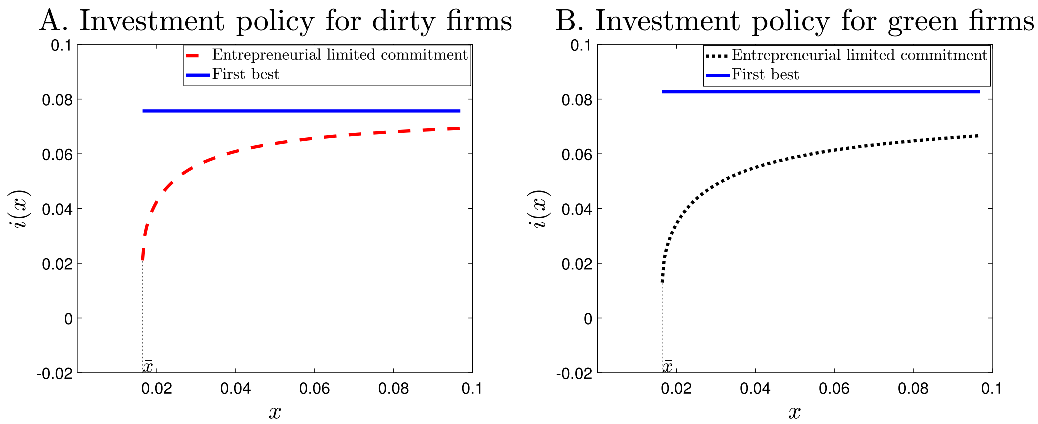

Fig. 2. Green and dirty firms' normalized investment rates featuring entrepreneurial limited commitment under the minimum tax rate required for green technology adoption. Entrepreneurial limited commitment leads to underinvestment relative to the first-best investment policy. Comparing Panels A and B, we observe that this distortion is greater for green firms than for dirty firms under entrepreneurial limited commitment. Parameter values are $b_g = 0.1645$, $b_d = 0.21$, $\mu_g = -0.06$, $\mu_d = -0.09$, $\sigma_g = 0.36$, $\sigma_d = 0.36$, $\gamma = 2$, $r = 0.08$, and $\theta = 3$.

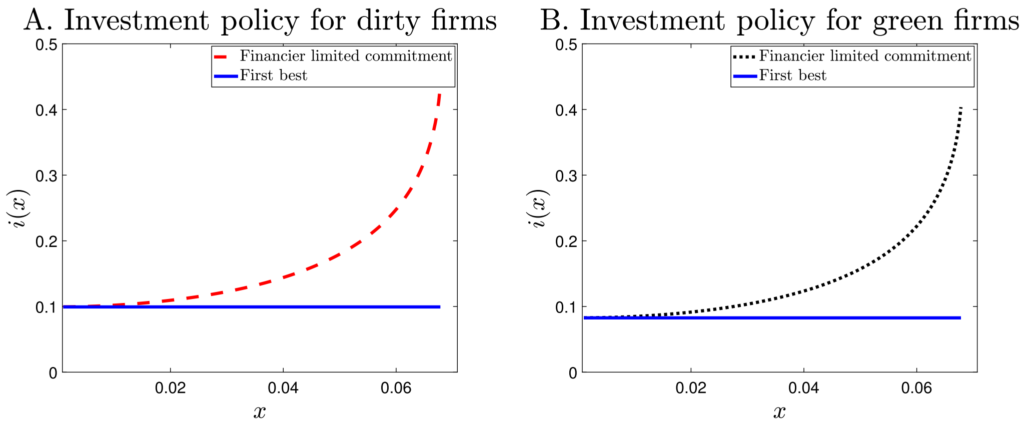

Fig. 3. Green and dirty firms' normalized investment rates featuring financier limited commitment under the minimum tax rate required for green technology adoption. Financier limited commitment leads to overinvestment relative to the first-best investment policy. Comparing Panels A and B, we observe that this distortion is greater for dirty firms than for green firms under financier limited commitment. Parameter values are $b_g = 0.1645$, $b_d = 0.21$, $\mu_g = -0.06$, $\mu_d = -0.09$, $\sigma_g = 0.36$, $\sigma_d = 0.36$, $\gamma = 2$, $r = 0.08$, and $\theta = 3$.

Intuitively, because the dirty technology generates front-loaded cash flows, the financier's limited-commitment constraint is more likely to bind in the future. That is, as the dirty technology becomes obsolete, the financier will have an incentive to walk away from her commitment to finance such a project. To prevent her from doing so, it is necessary to (i) reduce the consumption of the entrepreneur over time, thereby undermining risk-sharing, and (ii) distort, above the first-best benchmark, the investment rate of the project, as depicted in Fig. 3. Both of these mechanisms reduce the value of projects operated under dirty technologies, and hence make green technologies relatively more attractive. Interestingly, financier limited commitment can make it "easier" for governments to incentivize green technologies than in the absence of this friction.

We conclude Section 4 by highlighting the asymmetric impact of limited-commitment constraints on the carbon tax required to incentivize green technologies. Entrepreneurial (financier) limited commitment increases (decreases) the required carbon tax needed to embrace green technologies relative to the frictionless first-best benchmark. In practice, it is an empirical question which of these two types of commitment issues is more relevant in a given context. Our goal herein is to inform policymakers regarding the expected direction carbon taxes need to be adjusted, depending on which friction is more relevant in their economic context.

5. Directed subsidies and technology adoption

We now consider directed technology subsidies aimed at fostering the adoption of green technologies. Our goal in this section is to contrast the merits of the traditional carbon approach (Nordhaus, 1994) vs the directed technological change approach (Aghion et al., 2016) when limited commitment is a critical feature in the economic environment. As in Section 4, we proceed in two steps, first focusing on entrepreneurial limited commitment, then on financier limited commitment.

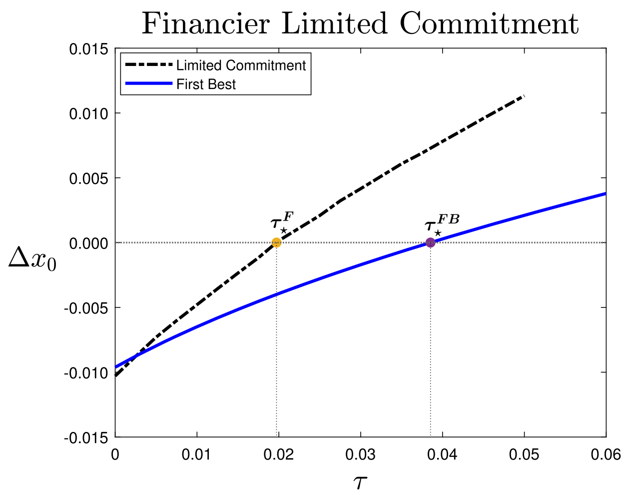

Fig. 4. Difference between certainty equivalents in Equation (21) as a function of the carbon tax rate τ under financier limited commitment. The dashed and solid vertical lines depict the minimal carbon tax rate required to incentivize entrepreneurs to adopt green technologies under financier limited commitment (τ_*^F) and in the first-best benchmark (τ_*^{FB}). Observe that the required carbon tax rate under financier limited commitment is lower than that in the first-best benchmark (i.e., $\tau_*^F < \tau_*^{FB}$). Parameter values are $b_g = 0.1645$, $b_d = 0.21$, $\mu_g = -0.06$, $\mu_d = -0.09$, $\sigma_g = 0.36$, $\sigma_d = 0.36$, $\gamma = 2$, $r = 0.08$, and $\theta = 3$.

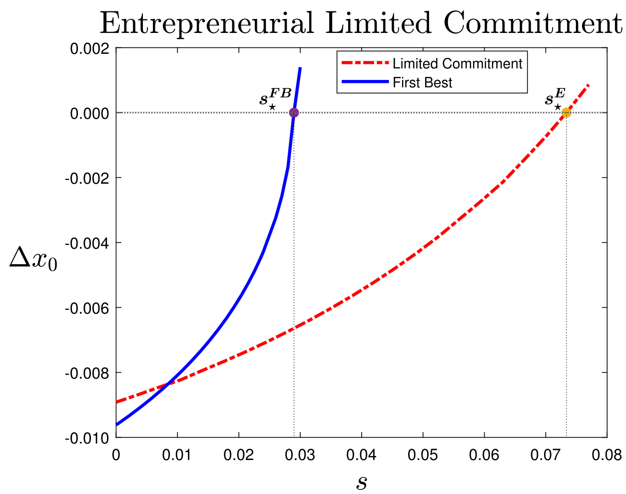

Fig. 5. Difference between certainty equivalents in Equation (21) as a function of the directed subsidy s under entrepreneurial limited commitment. The dashed and solid vertical lines depict the minimal investment subsidy required to incentivize entrepreneurs to adopt the green technologies under entrepreneurial limited commitment (s_*^E) and in the first-best benchmark (s_*^{FB}). Observe that the required investment subsidy under entrepreneurial limited commitment is greater than that in the first-best benchmark (i.e., $s_*^E > s_*^{FB}$). Parameter values are $b_g = 0.1645$, $b_d = 0.21$, $\mu_g = -0.06$, $\mu_d = -0.09$, $\sigma_g = 0.36$, $\sigma_d = 0.36$, $\gamma = 2$, $r = 0.08$, and $\theta = 3$.

5.1. *Directed subsidies under entrepreneurial limited commitment*

Recall that we model directed subsidies as a transfer of $\$s$ from the government to green firms for every $1 of capital expenditure, as seen in equation (8). Fig. 5 depicts $\Delta x(s)$ as a function of the directed technological subsidy s. We observe two things: First, entrepreneurial limited commitment necessitates a larger subsidy from the government than under the first-best setting (i.e., $s_*^E > s_*^{FB}$), in a fashion similar to our finding in Section 4.1 for the carbon tax case. Second, the investment subsidy needed under entrepreneurial limited commitment is significantly larger than in the first-best setting.

Intuitively, investment subsidies are beneficial for green firms because, by definition, they make investment cheaper. However, entrepreneurial limited commitment makes investment less desirable, because high growth makes it more difficult to retain the entrepreneur in the contract, making the limited-commitment constraint more likely to bind and hence reducing firm value. As a result, this unintended negative effect of investment subsidies reduces firm value. Therefore, the subsidy needed to incentivize green technology adoption is significantly larger than in the first-best case.

To summarize, this section uncovers a novel asymmetry between carbon taxes and directed tax subsidies under limited commitment. This asymmetry stems from our recurring theme regarding the delicate role played by any government intervention beyond its impact not only on the NPV of the firms but also on the term-structure of cash flows. Put differently, because investment, by

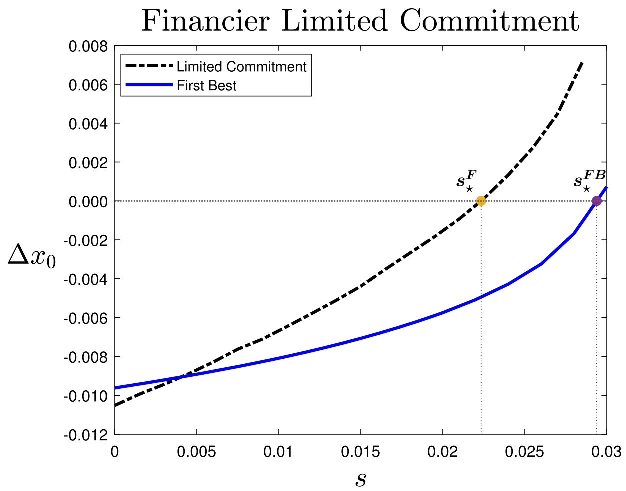

Fig. 6. Difference between certainty equivalents in Equation (21) as a function of directed subsidy s under financier limited commitment. The dashed and solid vertical lines depict the minimal investment subsidy required to incentivize entrepreneurs to adopt green technologies under financier limited commitment (s_*^F) and in the first-best benchmark (s_*^{FB}). Observe that the required investment subsidy under financier limited commitment is lower than in the first-best benchmark (i.e., $s_*^F < s_*^{FB}$). Parameter values are $b_g = 0.1645$, $b_d = 0.21$, $\mu_g = -0.06$, $\mu_d = -0.09$, $\sigma_g = 0.36$, $\sigma_d = 0.36$, $\gamma = 2$, $r = 0.08$, and $\theta = 3$.

nature, delivers back-loaded cash flows, and entrepreneurial limited commitment is more problematic in such a situation, directed investment subsidies constitute a very expensive approach to inducing green technology adoption. By contrast, and as we will see in the next section, these results are reversed when financiers (instead of entrepreneurs) feature limited commitment.

5.2. *Directed subsidies under financier limited commitment*

We complete our analysis by studying the magnitude of the directed subsidy required to incentivize green technologies when the financier faces limited commitment. Fig. 6 depicts $\Delta x(s)$ defined in equation (21), as a function of the directed technological subsidy s. We note that limited commitment on the financier's side renders a smaller subsidy sufficient to incentivize green technologies relative to the subsidy needed in first-best. This finding mirrors our findings for carbon taxes in Section 4.2.

Intuitively, financier limited commitment is more of a problem when the cash flows of the firm are front-loaded. In this case, as time goes by, the remaining cash flows may result in negative NPV, and the financier may want to renege on her contractual obligations. To prevent that, the contract stipulates reductions in the entrepreneur's compensation (undermining risk-sharing) and an excessively high investment rate. Therefore, an investment subsidy to the green firm is highly beneficial because it not only increases the firm's NPV but also helps relax the financier's limited-commitment constraint by allowing the firm to back-load cash flows via a higher investment rate.

6. Policy impact on climate change

This section builds on our previous insights to characterize the impact of each type of policy intervention (carbon taxes vs directed investment subsidies) on alleviating the climate change externality under entrepreneurial vs financier limited commitment. To that end, we extend our baseline model along two dimensions: First, we model the heterogeneity in the cross-section of entrepreneurs concerning their social preferences for choosing green technologies. Second, we model the negative climate change externalities imposed on other economic agents resulting from dirty firms' greenhouse emissions.[15]

6.1. *Heterogeneity*

We consider heterogeneity in the fraction of entrepreneurs choosing a dirty vs a green technology. Specifically, we assume that there is a mass 1 of potential entrepreneurs who differ in their altruistic preferences for operating green firms, where the additional utility u they derive from operating a green firm is uniformly distributed between 0 and $\bar{u} > 0$ (i.e., $u \sim U[0, \bar{u}]$). That is, an entrepreneur with preference parameter u will choose to operate the green technology if and only if

$$x_0^g + u \geq x_0^d. \tag{23}$$

[15] For concreteness, we abstract away from positive investment externalities, such as those discussed in Section 2.4.

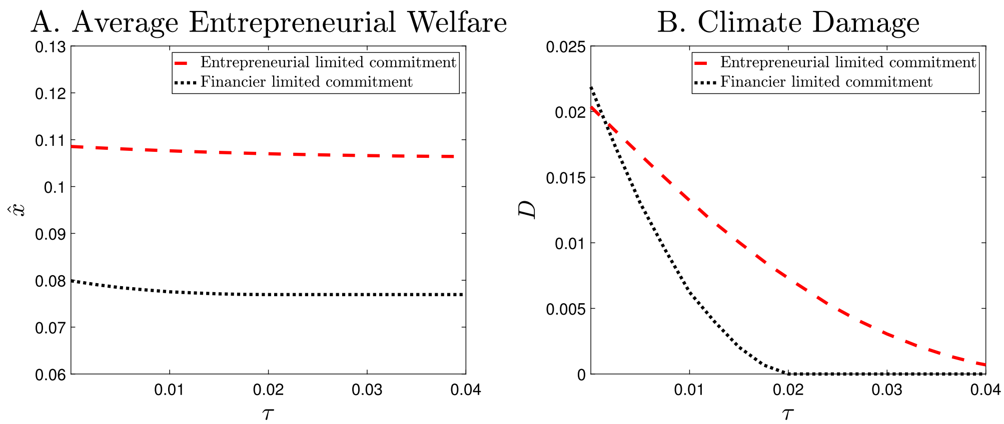

Fig. 7. Average welfare and climate damage under entrepreneurial and financier limited commitment with different tax rates. Panel A shows that average entrepreneurial welfare is decreasing in the carbon tax rate τ under entrepreneurial limited commitment and financier limited commitment. Panel B compares the effectiveness of carbon taxes in mitigating climate change damages under entrepreneurial and financier limited commitment. Parameter values are $b_g = 0.1645$, $b_d = 0.21$, $\mu_g = -0.06$, $\mu_d = -0.09$, $\sigma_g = 0.36$, $\sigma_d = 0.36$, $\gamma = 2$, $r = 0.08$, $\theta = 3$, $\bar{D} = 0.04$, $\bar{u} = 0.018$, and $\alpha = 1.5$. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

6.2. Climate change externalities

Our baseline model considered the tax required to induce entrepreneurs to operate green technologies, without explicitly modeling the social benefits of embracing such technologies. Here, we proceed to endogenize the benefit of green technologies in a parsimonious manner. We suppose the economy is subject to a damage function $D(\cdot)$ that captures the monetary value of the losses to society due to adverse climate events (e.g., forest fires, droughts, hurricanes) arising from firms operating dirty technologies. The value $\tilde{u}$ denotes the entrepreneur with the lowest u who is willing to operate the green technology (i.e., $x_0^g + \tilde{u} = x_0^d$). Therefore, a fraction $\tilde{u}/\bar{u}$ of entrepreneurs run dirty firms. Moreover, we denote by $I(d)$ the total investment expenditure by dirty firms ($I(g)$ for green firms), i.e.,

$$I(d) = E_0\left[\int_0^\infty e^{-rt} K_{d,t} h(i_{d,t}) dt\right], \qquad I(g) = E_0\left[\int_0^\infty e^{-rt} K_{g,t} h(i_{g,t}) dt\right]. \tag{24}$$

We parametrize the damage function as follows

$$\bar{D}\left(\frac{\tilde{u} I(d)}{(\bar{u}-\tilde{u}) I(g) + \tilde{u} I(d)}\right)^{\alpha}, \text{where } \alpha > 1. \tag{25}$$

That is, we assume that damages are increasing in the fraction of investment in the economy made by dirty firms, where the parameter $\bar{D} > 0$ captures the maximum value of the damage function, attained when the entire economy is dirty. Especially, when each entrepreneur decides whether to operate the green or dirty technology, he fails to internalize that choosing to operate the dirty technology will increase the fraction of dirty firms in the economy and thereby impose higher climate-related damages to society.

6.3. Carbon taxes and climate damages

We now explore the role of carbon taxes in mitigating climate change externalities. We start by studying the effectiveness of carbon taxes on reducing the equilibrium share of investment by dirty firms in the economy. For each carbon tax rate τ, we compute (i) the fraction of firms that operate the dirty technology $\frac{\tilde{u}}{\bar{u}}$, and (ii) the average entrepreneurial welfare in the economy

$$\hat{x}(\tau) = \int_0^{\tilde{u}} x_d(\tau) du + \int_{\tilde{u}}^{\bar{u}} (x_g + u) du, \tag{26}$$

where we make explicit the dependence of x_d on the carbon tax rate τ through equation (7). We then proceed to compute (iii) the climate-associated damages for each tax rate τ.

Panel A of Fig. 7 depicts average entrepreneurial welfare as computed in equation (26). In line with intuition, a higher carbon tax reduces entrepreneurial welfare because entrepreneurs choosing to operate the dirty technology obtain a lower payoff due to the more onerous taxation. This result is true under both entrepreneurial (red dashed line) and financier limited commitment (black dotted line).

Next, panel B depicts the climate damage computed in equation (25). We observe that carbon taxes discourage entrepreneurs from operating the dirty technology; as a result, a larger fraction of investment in the economy takes place inside green firms. Thus,

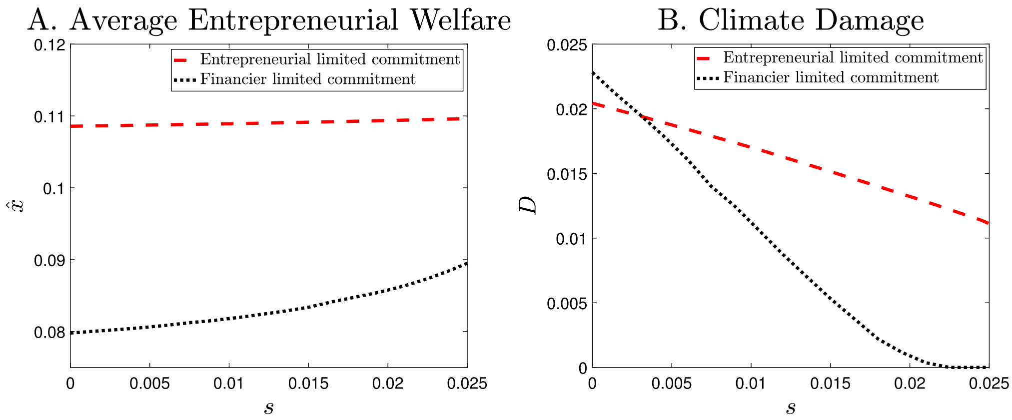

Fig. 8. Average welfare and climate damage under entrepreneurial and financier limited commitment with different subsidy rates. Panel A shows that average entrepreneurial welfare is increasing in the investment subsidy s under entrepreneurial limited commitment and financier limited commitment. Panel B compares the effectiveness of investment subsidies in mitigating climate change damages under entrepreneurial and financier limited commitment. Parameter values are $b_g = 0.1645$, $b_d = 0.21$, $\mu_g = -0.06$, $\mu_d = -0.09$, $\sigma_g = 0.36$, $\sigma_d = 0.36$, $\gamma = 2$, $r = 0.08$, $\theta = 3$, $\bar{D} = 0.04$, $\bar{u} = 0.018$, and $\alpha = 1.5$.

the climate-associated damages are mitigated through carbon taxes. What is more, the ability of carbon taxes to reduce damages is higher under financier limited commitment. This result matches our analysis from the baseline model because, under financier limited commitment, firms with back-loaded cash flows (i.e., green firms) command higher values. Thus, a lower carbon tax suffices to incentivize green technologies and reduce climate damages under financier limited commitment. The opposite holds under entrepreneurial limited commitment.

6.4. Investment subsidies and climate damages

We now explore the role of investment subsidies in mitigating climate change externalities. Panel A of Fig. 8 depicts the average entrepreneurial welfare given by

$$\hat{x}(s) = \int_0^{\tilde{u}} x_d \, du + \int_{\tilde{u}}^{\bar{u}} (x_g(s) + u) du. \tag{27}$$

A higher subsidy increases the average entrepreneurial welfare, since entrepreneurs choosing to operate the green technology obtain a higher payoff from the more generous subsidy. This result is true under both entrepreneurial (red dashed line) and financier limited commitment (black dotted line).

Furthermore, Panel B depicts the climate damage function from equation (25). We observe that investment subsidies encourage entrepreneurs to operate green technologies; therefore, a larger fraction of investment in the economy takes place inside green firms. Thus, the climate-associated damages are alleviated through investment subsidies. Notably, investment subsidies are less effective at reducing damages under entrepreneurial limited commitment. In line with our baseline analysis, entrepreneurial limited commitment is more problematic for firms with higher growth rates (i.e., green firms). Therefore, subsidizing investment induces green firms to grow even faster, thereby compounding the negative effect of entrepreneurial limited commitment. This necessitates a much higher subsidy to significantly reduce climate damages, as depicted by the flat slope of the red dashed line.

7. Conclusion

This paper compares the merits of carbon taxes and directed technological subsidies in encouraging the adoption of green technologies when limited commitment is the key friction in the financial contract between the entrepreneur and the financier. We show that, contrary to the results of the frictionless first-best benchmark studied in the prior literature, policymakers need to not only look beyond the impact of their policies on the NPV of each technology but also assess their impact on the term-structure of cash flows. Such is the case because green (dirty) firms' cash flows are more back-loaded (front-loaded), which makes the entrepreneurial (financier) limited commitment more likely to bind.

Our analyses yield two important insights for policymakers. First, when entrepreneurial (financier) limited commitment is the main friction in the environment, the carbon taxes required to encourage green technologies should be higher (lower) than what a simple NPV rule would imply. Second, carbon taxes (directed technology subsidies) are the more cost-effective interventions when entrepreneurial (financier) limited commitment is the main friction in the economic environment. As a consequence, our analysis does not prescribe a simple rule for choosing between these policy instruments. However, our goal is to inform policymakers of the forces that can be at play in the industry and country in which they operate. In reality, entrepreneurial and financier limited commitment are both likely to be at play, and policymakers can use our insights as an additional item in their checklist of economic considerations.

Our results raise various important questions for future research. For instance, which policy intervention is most desirable when other frictions such as moral hazard (e.g., DeMarzo and Sannikov, 2006; Wong and Yu, 2023; Oehmke and Opp, 2024[16]), or adverse selection (e.g., Daley and Green, 2012), also matter? What is the optimal intervention when firms can choose to "reform" themselves and transition from being dirty to green (as in Heinkel et al., 2001)? What are the quantitative implications of a structurally estimated model with two-sided limited commitment?

8. Appendix

8.1. Appendix for Section 3.1

Lemma 2 *(First-best benchmark). In the first-best benchmark, the financier's value function V^{FB} is given by*

$$V^{FB}(K_0, X_0) = K_0 \frac{b - h(i^{FB})}{r - (\mu + i^{FB})} - \frac{X_0}{r}, \tag{28}$$

where i^{FB} is the neoclassical first-best investment rate that satisfies

$$h'(i^{FB}) = \frac{b - h(i^{FB})}{r - (\mu + i^{FB})}. \tag{29}$$

Proof. Under full commitment, the optimization problem is to choose optimal investment and compensation policies to maximize

$$E\left[\int_0^\infty e^{-rt}\left(bK_t - h(i_t)K_t - C_t\right)dt\right], \tag{30}$$

subject to the participation constraint

$$\left[E\left(\int_0^\infty r e^{-rt} C_t^{1-\gamma} dt\right)\right]^{\frac{1}{1-\gamma}} \geq X_0, \tag{31}$$

with the dynamic of the capital following equation (2).

The problem can be broken into two parts: First, choose the investment policy to maximize $E[\int_0^\infty e^{-rt}(bK_t - h(i_t)K_t)dt]$ with the dynamics of capital given by equation (2). Second, choose the compensation policy in order to minimize $E[\int_0^\infty e^{-rt}C_t dt]$ subject to the participation constraint (31).

Denote $V_1(K) = \sup_i E[\int_0^\infty e^{-rt}(bK_t - h(i_t)K_t)dt]$. The HJB equation associated with the optimal investment problem is

$$rV_1(K) = \sup_i \left\{ bK - h(i)K + V_1'(K)\, K\, (\mu + i) + \tfrac{1}{2} V_1''(K)\, K^2\, \sigma^2 \right\}. \tag{32}$$

We obtain the optimal feedback controls:

$$h'(i^{FB}) = V_1'(K). \tag{33}$$

The solution for (32) is given by $V_1(K) = v^{FB} K$, where

$$rv^{FB} = \left[b - h(i^{FB}) + v^{FB}\,(\mu + i^{FB})\right]. \tag{34}$$

This implies

$$v^{FB} = \frac{b - h(i^{FB})}{r - (\mu + i^{FB})}. \tag{35}$$

From (33) and (35), we obtain (29). The value function for the maximization problem is

$$V_1(K) = K\, \frac{b - h(i^{FB})}{r - (\mu + i^{FB})}. \tag{36}$$

For the minimization problem, denote $V_2(X) = \sup_{C,\phi} E[\int_0^\infty -e^{-rt} C_t dt]$, where the dynamic of X_t is given by (10). The HJB equation for the minimization problem is

$$rV_2(X) = \sup_{C,\phi} \left\{ -C + V_2'(X)\, X\, \left[\tfrac{r}{1-\gamma}\left(1 - \left(\tfrac{C}{X}\right)^{1-\gamma}\right) + \tfrac{1}{2}\gamma\sigma^2\phi^2\right] + \tfrac{1}{2} V_2''(X)\sigma^2 X^2 \phi^2 \right\}. \tag{37}$$

[16] Oehmke and Opp (2024) explore a setting in which moral hazard limits the cash flows that can be pledged by entrepreneurs. This consideration pushes green firms significantly below their optimal size and affects the magnitude of the optimal intervention needed relative to the frictionless benchmark.

The optimal feedback controls are

$$C = X\left[-rV_2'(X)\right]^{\frac{1}{\gamma}} \quad \text{and} \quad \phi = 0. \tag{38}$$

The solution for (37) is given by

$$V_2(X) = \frac{-X}{r}, \tag{39}$$

and the optimal compensation policy is $C = X$.

Finally, combining (36) and (39), we obtain (28). □

8.2. Appendix for Sections 3.2 and 3.3

The following integrability conditions are needed for the admissibility of a contract:

$$c_t, i_t > 0, \quad E\left(\int_0^\infty e^{-rt} K_t c_t dt\right) < \infty, \tag{40}$$

$$E\left(\int_0^{+\infty} e^{-rt} K_t dt\right) < \infty, \quad E\left(\int_0^{+\infty} e^{-rt} K_t h(i_t) dt\right) < \infty, \tag{41}$$

$$E\left(\int_0^{+\infty} e^{-2rt} X_t^{2(1-\gamma)} \phi_t^2 dt\right) < \infty. \tag{42}$$

Lemma 3 *(Certainty equivalent). Assume that*

$$E\left[\left(\int_0^{+\infty} e^{-rt} C_t^{1-\gamma} dt\right)^2\right] < \infty, \tag{43}$$

then X_t has the Ito differential

$$dX_t = X_t\left\{\frac{r}{1-\gamma}\left[1 - \left(\frac{C_t}{X_t}\right)^{1-\gamma}\right] + \frac{1}{2}\gamma\sigma^2\phi_t^2\right\}dt + \sigma X_t \phi_t dB_t, \tag{44}$$

where the process $(\phi_t)_t$ is adapted to the filtration $(\mathcal{B}^t)_t$ and satisfies

$$E\left(\int_0^{+\infty} e^{-2rt} X_t^{2(1-\gamma)} \phi_t^2 dt\right) < \infty. \tag{45}$$

Introducing the stochastic processes x_t, i_t, and c_t (see (11)), then

$$dx_t = x_t\left\{\frac{r}{1-\gamma}\left[1 - \left(\frac{c_t}{x_t}\right)^{1-\gamma}\right] - i_t - \mu + \sigma^2\left(1 - \phi_t + \frac{\gamma}{2}\phi_t^2\right)\right\}dt + \sigma x_t(\phi_t - 1)dB_t. \tag{46}$$

Proof. From the definition of X_t (see equation (3)), we have

$$e^{-rt} X_t^{1-\gamma} = -r\int_0^t e^{-rs} C_s^{1-\gamma} ds + rE\left(\int_0^{+\infty} e^{-rs} C_s^{1-\gamma} ds \middle| \mathcal{B}^t\right).$$

From the martingale representation theorem, we can state that

$$E\left(\int_0^{+\infty} e^{-rs} C_s^{1-\gamma} ds \middle| \mathcal{B}^t\right) = E\left(\int_0^{+\infty} e^{-rs} C_s^{1-\gamma} ds\right) + \int_0^t \zeta_s dB_s,$$

with ζ_t adapted and

$$E\left(\int_0^{+\infty} \zeta_s^2 ds\right) \le E\left[\left(\int_0^{+\infty} e^{-rt} C_t^{1-\gamma} dt\right)^2\right]. \tag{47}$$

Table 2
Robustness check. Baseline parameter values are $b_g = 0.1645$, $b_d = 0.21$, $\mu_g = -0.06$, $\mu_d = -0.09$, $\sigma_g = 0.36$, $\sigma_d = 0.36$, $\gamma = 2$, $r = 0.08$, $\theta = 3$, $\bar{D} = 0.04$, $\bar{u} = 0.018$, and $\alpha = 1.5$.

Panel A: Carbon Taxes			
Parameters	τ_*^F	τ_*^{FB}	τ_*^E
$b_g = 0.160$	0.041	0.069	0.075
$b_g = 0.169$	0.000	0.006	0.023
$\mu_g = -0.057$	0.000	0.005	0.025
$\mu_g = -0.066$	0.059	0.085	0.088
Panel B: Investment Subsidies			
Parameters	s_*^F	s_*^{FB}	s_*^E
$b_g = 0.160$	0.051	0.055	0.111
$b_g = 0.169$	0.000	0.003	0.036
$\mu_g = -0.057$	0.000	0.003	0.040
$\mu_g = -0.066$	0.073	0.076	0.131

Denote $V_t = X_t^{1-\gamma}$. We get immediately that the Ito differential of V_t is

$$dV_t = (rV_t - r\,C_t^{1-\gamma})dt + r\,e^{rt}\,\zeta_t dB_t.$$

Since $X_t = V_t^{\frac{1}{1-\gamma}}$, by applying Ito's calculus and defining ϕ_t as

$$\zeta_t = \frac{\phi_t\,(1-\gamma)\,\sigma\,e^{-rt}\,X_t^{1-\gamma}}{r},$$

we obtain the results (44) and (45). Finally, applying Ito's lemma to the process x_t, we achieve the dynamics of the scaled certainty equivalent of the entrepreneur as stated in equation (46). □

8.3. Appendix for Section 3.4

Using the optimality principle of Dynamic Programming, we get equation (17), to which we must add boundary conditions at $x = \bar{x}$ and at $x \to +\infty$ in the case of entrepreneurial limited commitment, since the domain of interest is $x \in [\bar{x}, \infty)$. Similarly, we require boundary conditions at $x = 0$ and at $x = x^*$ in the case of financier limited commitment, since the domain of interest in this case is $x \in [0, x^*]$. We can optimize with respect to c and ϕ, which leads to the following feedback:

$$c(x) = x\,[-r\,v'(x)]^{\frac{1}{\gamma}} \quad \text{and} \quad \phi(x) = \frac{x\,v''(x)}{\gamma\,v'(x) + x\,v''(x)}. \tag{48}$$

Note that admissible controls must satisfy conditions (40), (41), and (42).

Thus, equation (17) reduces to

$$0 = b - (-\mu + r)\,v(x) + \sup_i\{-h(i) + i\,[v(x) - x\,v'(x)]\} \tag{49}$$

$$+ \left(\tfrac{r}{1-\gamma} - \mu\right) x\,v'(x) + \tfrac{\gamma}{1-\gamma}\,x\,[-r\,v'(x)]^{\frac{1}{\gamma}} + \tfrac{1}{2}\sigma^2\gamma\,\tfrac{x^2\,v'(x)\,v''(x)}{\gamma\,v'(x) + x\,v''(x)}. \tag{50}$$

When the entrepreneur displays limited commitment, $v(x)$ is the solution to (49) on $[\bar{x}, \infty)$ subject to the following boundary conditions:

$$v''(\bar{x}) = -\infty;\; v(x) - (v^{FB} - \tfrac{x}{r}) \to 0, \text{as } x \to +\infty. \tag{51}$$

When the financier displays limited commitment, $v(x)$ is the solution to (49) on $(0, x^*]$ subject to the following boundary conditions:

$$v(x^*) = 0;\, v''(x^*) = -\infty;\; v(x) \to v^{FB}, \text{as } x \to 0. \tag{52}$$

Appendix. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jedc.2024.104930.

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