

# Spatial Correlation-Aware Opportunistic Beamforming in IRS-Aided Multi-User Systems

L. Yashvanth *Student Member, IEEE*, Chandra R. Murthy, *Fellow, IEEE*, and Bhaskar D. Rao, *Life Fellow, IEEE*

**Abstract**—This paper addresses the high overheads associated with intelligent reflecting surface (IRS) aided wireless systems. By exploiting the inherent spatial correlation among the IRS elements, we propose a novel approach that randomly samples the IRS phase configurations from a carefully designed distribution and opportunistically schedules the user equipments (UEs) for data transmission. The key idea is that when IRS configuration is randomly chosen from a channel statistics-aware distribution, it will be near-optimal for at least one UE, and upon opportunistically scheduling that UE, we can obtain nearly all the benefits from the IRS without explicitly optimizing it. We formulate and solve a variational functional problem to derive the optimal phase sampling distribution. We show that, when the IRS phase configuration is drawn from the optimized distribution, it is sufficient for the number of UEs to scale exponentially with the *rank* of the channel covariance matrix, not with the number of IRS elements, to achieve a given target SNR with high probability. Our numerical studies reveal that even with a moderate number of UEs, the opportunistic scheme achieves near-optimal performance without incurring the conventional IRS-related signaling overheads and complexities.

**Index Terms**—Intelligent reflecting surfaces (IRS), spatial correlation, opportunistic scheduling, multi-user diversity.

## I. INTRODUCTION

A n intelligent reflecting surface (IRS) comprises multiple passive elements that can be independently configured to reflect signals in required directions, thereby controlling the overall channel and improving the spectral efficiency (SE) of next-generation wireless systems [1]. However, optimally configuring the IRS entails *three-fold* control overheads: 1) acquisition of channel state information (CSI), 2) optimization of IRS phase angles, and 3) phase transportation from the base station (BS) to the IRS via control links. These overheads can easily undermine the professed benefits of an IRS when the number of IRS elements is large. This paper overcomes this bottleneck by leveraging a spatial correlation-aware opportunistic beamforming framework that reaps optimal IRS gains without optimization/three-fold overheads, as described above.

In the pursuit of reducing the complexity while maximizing the IRS-aided performance, [2] leverages correlation among different user equipments (UEs) to minimize the pilot overheads, and [3] proposed to use only the partial CSI of the channel. In [4], a blind BF approach without CSI estimation is proposed; however, it suffers from high time complexity. In this view, [5] and [6] utilize opportunistic scheduling techniques to mitigate both time and computational complexity. However, they consider independent fading channels and need a very large number of UEs to achieve optimal gains.

L. Yashvanth and C. R. Murthy are with the Dept. of ECE, Indian Institute of Science, Bangalore, India 560 012 (e-mail: {yashvanthl, cmurthy}@iisc.ac.in). B. D. Rao is with the Dept. of ECE, University of California, San Diego, USA. (e-mail: brao@ucsd.edu). The work of LY was financially supported by the PMRF, Govt. of India, that of CRM was supported by the Qualcomm 6G UR India Grant, and that of BDR was supported by the National Science Foundation (NSF) Grant CCF-2225617.

In this paper, we progress upon this problem by exploiting the inherent spatial correlation at the IRS and show that the performance of the opportunistic scheme can be significantly improved even with a small number of UEs at very low time and computational complexities. Our key contributions are:

- 1) We pose and solve a variational functional problem to obtain the optimal sampling distribution for the random IRS phases, as a function of the channel statistics. (Sec. IV-C.)
- 2) We show that, when the derived spatial-correlation-aware distribution is used, it is sufficient for the number of UEs to scale exponentially in the *rank* of the channel covariance matrix, to obtain near-optimal SNR in every slot. (Sec. V.)
- 3) In the process, we derive the tail probability of the Rayleigh quotient of a heteroscedastic complex Gaussian random vector, which may be of independent interest. (Lemma 3.)

We empirically illustrate the efficacy of the opportunistic scheme when the IRS phase is randomly sampled from the optimal distribution. For example, with  $N = 32$  IRS elements, the sum-SE is only 0.7 bps/Hz (5%) away from that achieved via IRS optimization with just  $K = 25$  UEs (see Fig. 2a).

## II. SYSTEM MODEL AND PROBLEM DESCRIPTION

We consider an  $N$ -element IRS-aided downlink scenario where a BS serves  $K$  UEs in a time-division multiple-access fashion. Let the small-scale channel from the BS to IRS be  $\mathbf{h}_1 \in \mathbb{C}^N$ , and from IRS to UE- $k$  be  $\mathbf{h}_{2,k} \in \mathbb{C}^N$ . Since the BS and IRS are envisioned to be deployed at fixed positions, we model  $\mathbf{h}_1$  as a deterministic vector, with entries [7]

$$[\mathbf{h}_1]_n = \exp(j2\pi d_n/\lambda), \quad n = 1, 2, \dots, N, \quad (1)$$

where  $d_n$  is the distance between the BS antenna and  $n$ th IRS element. The channel from IRS to UE can be random; so, we model  $\mathbf{h}_{2,k} \sim \mathcal{CN}(\mathbf{0}, \Sigma_k)$ , where  $\Sigma_k$  is the spatial correlation matrix at the IRS for UE- $k$ . The overall channel at UE- $k$  is

$$h_k = \sqrt{\beta_{d,k}} h_{d,k} + \sqrt{\beta_{r,k}} \mathbf{h}_{2,k}^T \tilde{\Theta} \mathbf{h}_1,$$

where  $h_{d,k} \sim \mathcal{CN}(0, 1)$  is the direct channel from BS to UE- $k$ ,  $\tilde{\Theta} \in \mathbb{C}^{N \times N}$  is a diagonal matrix containing the IRS phase shifts, and  $\beta_{d,k}$ ,  $\beta_{r,k}$  denote the path loss of the direct path and cascaded path via the IRS, respectively. We now write

$$h_k = \sqrt{\beta_{d,k}} h_{d,k} + \sqrt{\beta_{r,k}} \tilde{\Theta}^T (\mathbf{h}_{2,k} \odot \mathbf{h}_1) \triangleq \boldsymbol{\theta}^H \mathbf{h}_{f,k}, \quad (2)$$

where  $\odot$  is the hadamard product,  $\boldsymbol{\theta} \triangleq [1, \tilde{\Theta}^{*T}]^T \in \mathbb{C}^{N+1}$  is the effective IRS vector and  $\tilde{\Theta} \in \mathbb{C}^N$  has the diagonal elements of  $\tilde{\Theta}$ , and  $\mathbf{h}_{f,k} \triangleq [\sqrt{\beta_{d,k}} h_{d,k}, \sqrt{\beta_{r,k}} \mathbf{h}_{2,k}^T]^T$  is the fading vector with  $\mathbf{h}_{r,k} \triangleq \mathbf{h}_{2,k} \odot \mathbf{h}_1$ . The system is illustrated in Fig. 1.

Let  $P$ ,  $\sigma^2$  be the transmit and noise power, respectively. Configuring the IRS to UE- $k$  with the SE-optimal phase:

$$\boldsymbol{\theta}^{\text{opt}} = \arg \max_{\boldsymbol{\theta}} \log_2 \left( 1 + |\boldsymbol{\theta}^H \mathbf{h}_{f,k}|^2 P / \sigma^2 \right), \quad (\text{P1})$$

$$\text{s.t. } [\boldsymbol{\theta}]_1 = 1, \quad |[\boldsymbol{\theta}]_n| = 1, \quad n = 2, \dots, N+1, \quad (\text{C1-1})$$

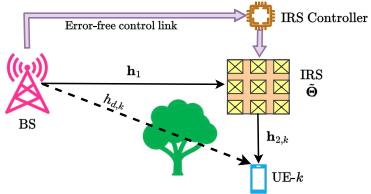


Fig. 1: System model for one UE.

incurs inordinate overheads (see Sec. III.) So, we answer:

- 1) Can we leverage the spatial correlation to randomly configure  $\theta$  (without determining  $\theta^{\text{opt}}$ ), opportunistically schedule the best UE, and thereby obtain benefits from the IRS?
- 2) For the above scheme, what is the probability that a random IRS phase procures a target near-optimal SE as in (P1)?

To this end, we first analyze the benchmark SE obtained under IRS optimization with round-robin (RR) scheduling of UEs.

### III. THE BENCHMARK: SUM-SE VIA IRS OPTIMIZATION

Under RR scheduling, the BS sequentially schedules UEs using a pre-defined ordering. Considering that the IRS is configured as per (P1) in every time slot, the achievable beamforming (BF) sum-SE is characterized in the following [6].

**Lemma 1.** *With  $K$ -UEs, under RR scheduling, the BF sum-SE obtained when the BS optimizes the IRS to the channel of the scheduled UE in every time slot is given by  $R_K^{\text{opt}} =$*

$$\frac{1}{K} \sum_{k=1}^K \log_2 \left( 1 + \left| \sqrt{\beta_{d,k}} |h_{d,k}| + \sqrt{\beta_{r,k}} \sum_{n=1}^N |\mathbf{h}_{r,k}|_n \right|^2 \frac{P}{\sigma^2} \right),$$

and is achieved with the optimal IRS configurations given by  $[\theta^{\text{opt}}]_n = \exp \left\{ j \left( \angle [\mathbf{h}_{r,k}]_{n-1} - \angle h_{d,k} \right) \right\}, n = 2, \dots, N+1$ . (3)

**Remark 1.** Achieving  $R_K^{\text{opt}}$  as in Lemma 1 incurs computationally expensive three-fold overheads in every time slot:

- 1) *Channel estimation: The BS acquires the CSI of all the links; this potentially requires  $\mathcal{O}(N)$  pilot transmissions.*
- 2) *Phase optimization: The BS must optimize the IRS to achieve the best SE during data transmission.*
- 3) *Phase transportation: The BS transports the optimal phase of each IRS element to the IRS controller via an error-free control link, and its overhead scales as  $\mathcal{O}(N)$ .*

### IV. SPATIAL CORRELATION-AWARE OPPORTUNISTIC BF

#### A. The Proportional-fair Scheduler

In each slot, the PF scheduler selects a UE with the highest instantaneous-to-average SE ratio [8], thereby opportunistically enhancing throughput via the multi-user diversity effect while ensuring fairness in UE scheduling. Let  $R_k(t) \triangleq \log_2(1 + |h_k(t)|^2 P / \sigma^2)$  be the achievable SE of UE- $k$  at time  $t$ . The PF scheduler selects the  $k^*(t)$ th UE, where

$$k^*(t) = \arg \max_{k \in \{1, \dots, K\}} R_k(t) / T_k(t),$$

where  $T_k(t)$  is the exponentially weighted moving average (EWMA) SE seen by UE- $k$  till time  $t$ , which is parameterized by the EWMA factor  $\tau$  [8]. Smaller (larger) values of  $\tau$  favor short (long)-term fairness in UE scheduling. We will refer to  $R_k(t) / T_k(t)$  as the *PF metric* of UE- $k$  at time  $t$ .

#### B. Opportunistic Communication using an IRS

The proposed opportunistic communication (OC) scheme has two steps per slot: 1) the IRS configuration is randomly chosen from an *appropriate* sampling distribution, and 2) the BS opportunistically selects a UE using the PF scheduler. In this view, we next state a lemma, proved similar to [8].

**Lemma 2.** *In a  $K$ -UE system, using a PF scheduler with  $\tau \rightarrow \infty$ , when the IRS configurations are randomly sampled from a spatial correlation-aware distribution, the sum-SE of the IRS-aided OC scheme, denoted by  $R_K^{\text{opp}}$  obeys*

$$\lim_{K \rightarrow \infty} (R_K^{\text{opp}} - R_K^{\text{opt}}) = 0,$$

where  $R_K^{\text{opt}}$  is the optimal sum-SE as given in Lemma 1.

From Lemma 2, we deduce that with a large number of UEs, the PF scheduler selects the UE for which the random IRS phase is close to its BF configuration and procures the BF benefits without explicitly optimizing the IRS [8]. This is called *opportunistic beamforming*. We next characterize the IRS phase sampling distribution that satisfies Lemma 2 from a variational perspective, which is one of our key contributions.

#### C. Optimal Distribution for Sampling the Random IRS Phases

We observe that the optimal IRS vector in (3) is obtained as the deterministic map  $\mathcal{F} : \mathbb{C}^{N+1} \rightarrow \{1\} \times \mathbb{U}^N$ , given by

$$\mathcal{F} : \mathbf{h}_{f,k} \mapsto \left[ 1, \left( \exp \left\{ j \left( \angle \mathbf{h}_{r,k} - \angle h_{d,k} \right) \right\} \right)^T \right]^T, \quad (4)$$

where  $\mathbb{U}^N \triangleq \{ \mathbf{z} \in \mathbb{C}^N \mid |\mathbf{z}_i| = 1, i = 1, \dots, N \}$ . As a consequence, the design of the random distribution is coupled with the statistics of the channels to the UEs. We can write the small-scale channel between the IRS and UE- $k$  as

$$\mathbf{h}_{2,k} = \Sigma_k^{1/2} \widetilde{\mathbf{h}_{2,k}} \stackrel{(a)}{\approx} \Sigma^{1/2} \widetilde{\mathbf{h}_{2,k}}, \quad (5)$$

where  $\widetilde{\mathbf{h}_{2,k}} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$ ,  $M = \text{rank}(\Sigma)$ ,  $\Sigma_k^{1/2}$  contains the first  $M$  columns of a square root of  $\Sigma_k$ , and in (a), we used  $\Sigma_k = \Sigma, \forall k$ . This corresponds to a Kronecker channel model where the correlation is induced by local spatial scattering at the IRS elements or a scenario where many UEs are located in a hotspot area [5], [9].<sup>1</sup> Since  $\mathcal{R}(\Sigma^{1/2}) = \mathcal{R}(\Sigma)$ , where  $\mathcal{R}(\mathbf{A})$  is the range space of  $\mathbf{A}$ , from (5), we get  $\mathbf{h}_{2,k} \in \mathcal{R}(\Sigma)$ . Thus,  $\mathbf{h}_{2,k}$  lies in an  $M$ -dimensional subspace of  $\mathbb{C}^N$ . Let  $\Sigma = \mathbf{U} \Lambda \mathbf{U}^H$  be the spectral decomposition of  $\Sigma$ ;  $\Lambda$  contains the non-zero eigenvalues of  $\Sigma$ . Then, for every  $\mathbf{h}_{2,k} \in \mathcal{R}(\Sigma)$ , by the Karhunen–Loëve Theorem, there exists  $\{\alpha_{k,i}\}_{i=1}^M$  s.t.

$$\mathbf{h}_{2,k} = \sum_{i=1}^M \alpha_{k,i} \mathbf{u}_i, \text{ and } \alpha_{k,i} = \langle \mathbf{h}_{2,k}, \mathbf{u}_i \rangle, \quad (6)$$

with  $\mathbf{u}_i$  being the  $i$ th orthonormal eigenvector of  $\Sigma$ . Hence, the channel at each UE is uniquely determined by the UE-specific coefficients  $\{\alpha_{k,i}\}_{i=1}^M$  along with the basis vectors  $\{\mathbf{u}_i\}_{i=1}^M$ .

Conversely, with a large number of UEs, for any given  $\{\alpha_{k,i} \in \mathbb{C}\}_{i=1}^M$ , there exists a UE whose channel corresponds to the chosen coefficients via (6). Since  $\mathbf{h}_{2,k}$  is a Gaussian

<sup>1</sup>To serve UEs with different covariance matrices at the IRS, we first cluster UEs sharing similar covariance matrices as in [10]. Then, we select a cluster in an RR manner, and within the slots allocated for the selected cluster, a UE is served via the opportunistic BF scheme considered in this paper.

**Scheme 1:** Spatial Correlation-aware Opportunistic BF

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**Input:** Correlation values:  $\mathbf{U}$ ,  $\Sigma_\alpha$ ; BS-IRS link:  $\mathbf{h}_1$ .

**1 for** time slot  $t = 1, 2, 3, \dots$  **do**

/\* Random sampling of IRS configurations \*/

Sample the random vector  $\beta \sim \mathcal{CN}(\mathbf{0}, \Sigma_\alpha)$ .

Set  $\theta_{\text{rand}} = \mathcal{F}\left(\left[1, (\mathbf{h}_1 \odot \mathbf{U}\beta)^T\right]^T\right)$ , as per (4).

/\* Towards identifying the best UE \*/

BS broadcasts a common pilot signal to every UE.

All UEs compute their PF metrics & feedback their identities to BS using *timer* schemes [11].

/\* Proportional-fair scheduling of UEs \*/

The BS identifies and schedules UE- $k^*(t)$  for data transmission with  $k^*(t) = \arg \max_{k \in \{1, \dots, K\}} R_k(t)/T_k(t)$ .

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vector,  $\alpha_{k,i} \sim \mathcal{CN}(0, [\Sigma_\alpha]_{i,i})$ , with  $[\Sigma_\alpha]_{i,i} = \mathbb{E}[|\mathbf{u}_i^H \mathbf{h}_{2,k}|^2] = \mathbf{u}_i^H \Sigma \mathbf{u}_i$ , i.e.,  $\Sigma_\alpha = \mathbf{U}^H \Sigma \mathbf{U} = \Lambda$ . We have the following result.

**Theorem 1.** *The probability density function for drawing the random samples of the IRS vector in every time slot to ensure that a PF scheduler achieves the BF-SE as in Lemma 2 is*

$$f_{\theta}^{\text{opt}}(\theta') = \int_{\mathcal{F}^{-1}(\theta')} \delta(\mathbf{h} - \mathcal{F}^{-1}(\theta')) p_{\mathbf{h}_f}(\mathbf{h}) d\mathbf{h},$$

where  $\mathcal{F}^{-1}(\theta')$  denotes the set-inverse under the  $\mathcal{F}(\cdot)$  mapping, i.e.,  $\mathcal{F}^{-1}(\theta') \triangleq \{\mathbf{h} \in \mathbb{C}^{N+1} : \mathcal{F}(\mathbf{h}) = \theta'\}$ , and  $p_{\mathbf{h}_f}(\mathbf{h})$  is the probability density function of  $\mathbf{h}_f = \left[h_d, (\mathbf{h}_1 \odot \mathbf{h}_2)^T\right]^T$  with  $h_d \sim \mathcal{CN}(0, 1)$ ,  $\mathbf{h}_1$  as given in (1), and  $\mathbf{h}_2 \sim \mathcal{CN}(\mathbf{0}, \Sigma)$ .

*Proof.* Suppose the IRS-UEs channel process is jointly stationary and ergodic. With PF scheduling, the optimal sampling distribution at the IRS so that a scheduled UE obtains the BF-SE as  $K \rightarrow \infty$  is the solution to the variational problem:

$$\arg \max_{f_{\theta}(\theta')} \bar{R} \triangleq \mathbb{E}_{\theta, \mathbf{h}_{f,k}} \left[ \log_2 \left( 1 + |\theta'^H \mathbf{h}_{f,k}|^2 P / \sigma^2 \right) \right], \quad (\text{P2})$$

$$\text{s.t. } \int_{\theta' \in \mathbb{U}^N \cup \{\mathbf{1}\}} f_{\theta}(\theta') d\theta' = 1, \quad (\text{C2-1})$$

$$\text{and } \int_{[\theta']_2} \dots \int_{[\theta']_{N+1}} f_{\theta}(\theta') d\theta' = \delta([\theta']_1 - 1), \quad (\text{C2-2})$$

where in (P2), we seek to maximize the achievable throughput with expectation taken over the joint distribution of the UEs, and (C2-1), (C2-2) account for the constraints of a density function and the structure of  $\theta'$  as per (2), respectively. We begin by solving the unconstrained version of (P2) and then assess its feasibility under (C2-1), (C2-2). So, the problem is

$$\begin{aligned} & \max_{f_{\theta}(\theta')} \mathbb{E}_{\mathbf{h}_{f,k}} \left[ \mathbb{E}_{\theta \mid \mathbf{h}_{f,k}} \left[ \log_2 \left( 1 + \left| \theta'^H \mathbf{h}_{f,k} \right|^2 \frac{P}{\sigma^2} \right) \mid \mathbf{h}_{f,k} \right] \right] \\ & \stackrel{(a)}{=} \max_{g_{\theta}(\theta')} \int_{\mathbf{h}} \left( \int_{\theta'} \log_2 \left( 1 + \left| \theta'^H \mathbf{h}_{f,k} \right|^2 \frac{P}{\sigma^2} \right) g_{\theta}(\theta') d\theta' \right) p_{\mathbf{h}_f}(\mathbf{h}) d\mathbf{h}, \end{aligned}$$

where in (a),  $g_{\theta}(\theta') \equiv g_{\theta \mid \mathbf{h}_f}(\theta')$  is the conditional density function of the IRS configurations given the channel realization. It is related to  $f_{\theta}(\theta')$  via the law of total probability:

$$f_{\theta}(\theta') = \int_{\mathbf{h}} g_{\theta}(\theta') p_{\mathbf{h}_f}(\mathbf{h}) d\mathbf{h}. \quad (7)$$

Then, an equivalent functional optimization problem is

$$\mathcal{I} \triangleq \max_{g_{\theta}(\theta')} \int_{\theta'} \log_2 \left( 1 + \left| \theta'^H \mathbf{h}_{f,k} \right|^2 P / \sigma^2 \right) g_{\theta}(\theta') d\theta'. \quad (8)$$

Using the Hölder inequality:  $|\theta'^H \mathbf{h}_{f,k}| \leq \|\mathbf{h}_{f,k}\|_1 \|\theta'\|_\infty$  along with the fact that  $\|\theta'\|_\infty = 1$ , we upper bound (8) as

$$\mathcal{I} \leq \mathcal{I}_U \triangleq \log_2 \left( 1 + \|\mathbf{h}_{f,k}\|_1^2 P / \sigma^2 \right) \max_{g_{\theta}(\theta')} \underbrace{\int_{\theta'} g_{\theta}(\theta') d\theta'}_{=1}.$$

From Lemma 2, since the PF scheduler achieves the BF-SE,  $g_{\theta}(\theta')$  must satisfy the following lower bound:

$$\mathcal{I} \geq \mathcal{I}_L \triangleq \max_{g_{\theta}(\theta')} \int_{\theta'} \log_2 \left( 1 + \left\{ \|\mathbf{h}_{f,k}\|_1^2 + o(K) \right\} \frac{P}{\sigma^2} \right) g_{\theta}(\theta') d\theta'.$$

Now, letting  $K \rightarrow \infty$  and using the sandwich theorem,  $\lim_{K \rightarrow \infty} \mathcal{I} = \mathcal{I}_L = \mathcal{I}_U$ , and the upper bound is achieved if and only if  $\theta' = \mathcal{F}(\mathbf{h}_{f,k})$ . So, the optimal conditional density for a given channel at scheduled UE- $k$  is

$$g_{\theta}^{\text{opt}}(\theta') = \delta(\theta' - \mathcal{F}(\mathbf{h}_{f,k})). \quad (9)$$

Substituting (9) in (7), we get

$$\begin{aligned} f_{\theta}^{\text{opt}}(\theta') & \stackrel{(b)}{=} \int_{\mathbf{h}} \delta(\theta' - \mathcal{F}(\mathbf{h})) p_{\mathbf{h}_f}(\mathbf{h}) d\mathbf{h} \\ & \stackrel{(c)}{=} \int_{\mathcal{F}^{-1}(\theta')} \delta(\mathbf{h} - \mathcal{F}^{-1}(\theta')) p_{\mathbf{h}_f}(\mathbf{h}) d\mathbf{h}, \end{aligned} \quad (10)$$

where, in (b), we dropped the index  $k$  from (9) as  $\mathcal{F}(\mathbf{h}_{f,k})$  are i.i.d. across  $k \in [K] \triangleq \{1, \dots, K\}$ ; and in (c), we used the definition  $\mathcal{F}^{-1}(\cdot)$  and the sifting property of the Dirac-delta function. By construction, since  $f_{\theta}^{\text{opt}}(\cdot)$  in (10) is a valid probability density function obtained via the  $\mathcal{F}(\cdot)$  mapping, (C2-1) and (C2-2) are trivially satisfied. This completes the proof. ■

**Remark 2.** *The IRS-aided OC scheme achieves the BF-SE without incurring the overheads discussed in Remark 1:*

- 1) *The OC scheme requires just one pilot symbol, as the UEs only need to estimate the composite channel.*
- 2) *No phase optimization: The phase optimization procedure is absent since the IRS phases are randomly chosen.*
- 3) *No phase transportation: The IRS autonomously samples a random phase configuration in every slot, so phase transportation is obviated.*

*Further, to help the BS identify the best UE that yields the highest PF metric, efficient and low-complexity feedback schemes like timer/splitting-based methods can be used [11].*

Using Theorem 1, we present the overall protocol of spatial-correlation-aware OC in Scheme 1 on top of this page.<sup>2</sup>

## V. HOW MANY USERS ARE SUFFICIENT IN PRACTICE?

We now consider the success rate of scheme 1 for a practical system with a finite number of UEs. Let  $\mathcal{E}_k^\delta$  denote the  $(1 - \delta)N^2$ -success event that the channel gain at UE- $k$  is at least a  $(1 - \delta)$  factor of the BF gain obtained via the IRS, i.e.,

$$\mathcal{E}_k^\delta \triangleq \left\{ \left| \theta'^H \mathbf{h}_{f,k} \right|^2 \geq (1 - \delta) \|\mathbf{h}_{f,k}\|_1^2 \right\}, \quad \delta \in (0, 1). \quad (11)$$

<sup>2</sup>We absorb the overall phase in the  $\mathcal{F}$ -mapped channel vectors due to the angle of the direct channel into the randomness in the angle of the cascaded channel. So, the 1st entry in the input to  $\mathcal{F}$ -map in line 3 equals 1. We also assume that the spatial correlation matrix  $\Sigma$  is known, as in [2], [10].

In the sequel, we evaluate the probability of  $\mathcal{E}_k^\delta$ . To that end, we require a characterization of the Rayleigh quotient of heteroscedastic Gaussian random vectors, discussed next.

**Lemma 3.** *Let  $\mathbf{A} \in \mathbb{C}^{L \times L}$  be a Hermitian rank-1 matrix, and  $\alpha \in \mathbb{R}$  be such that  $0 < \alpha < \|\mathbf{A}\|_F$ . If  $\mathbf{x} \in \mathbb{C}^L \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$  and  $\mathbf{R}$  has full rank, the Rayleigh quotient of  $\mathbf{A}$  w.r.t.  $\mathbf{x}$  obeys*

$$\Pr\left(\frac{\mathbf{x}^H \mathbf{A} \mathbf{x}}{\mathbf{x}^H \mathbf{x}} \geq \alpha\right) \geq \prod_{l=1}^L \left(1 \bigg/ \left\{1 + \frac{\alpha}{(\|\mathbf{A}\|_F - \alpha)} \cdot \frac{\lambda_{\mathbf{x},l}}{\lambda_{\mathbf{x},L}}\right\}\right),$$

where  $\lambda_{\mathbf{x},1} \geq \dots \geq \lambda_{\mathbf{x},L}$  are the ordered eigenvalues of  $\mathbf{R}$ .

*Proof.* We note that the required probability can be written as

$$P_\alpha \triangleq \Pr(\mathbf{x}^H \mathbf{A} \mathbf{x} \geq \alpha \mathbf{x}^H \mathbf{x}) = \Pr(\mathbf{x}^H (\mathbf{A} - \alpha \mathbf{I}_L) \mathbf{x} \geq 0).$$

Let  $\mathbf{B} \triangleq \mathbf{A} - \alpha \mathbf{I}_L$ ;  $\mathbf{B}$  is a full-rank, Hermitian matrix. Its spectral decomposition can be written as  $\mathbf{B} = \mathbf{V} \mathbf{\Gamma} \mathbf{V}^H$ . Then

$$P_\alpha \stackrel{(a)}{=} \Pr(\tilde{\mathbf{x}}^H \mathbf{\Gamma} \tilde{\mathbf{x}} \geq 0) \stackrel{(b)}{=} \Pr\left(\sum_{l=1}^L \gamma_l |\tilde{\mathbf{x}}_l|^2 \geq 0\right),$$

where in (a),  $\tilde{\mathbf{x}} \triangleq \mathbf{V}^H \mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{V}^H \mathbf{R} \mathbf{V})$ ; in (b),  $\gamma_l$  is the  $l$ th largest eigenvalue of  $\mathbf{B}$ . Since  $\mathbf{A}$  has rank 1,  $\mathbf{A} = \mathbf{a} \mathbf{a}^H$  for some  $\mathbf{a} \in \mathbb{C}^L$ . Then, the eigenvalues of  $\mathbf{B}$  are  $\gamma_1 = \|\mathbf{a}\|_2^2 - \alpha > 0$ , and  $\gamma_2 = \gamma_3 = \dots = \gamma_L = -\alpha < 0$ . Using this, we have

$$\begin{aligned} P_\alpha &= \Pr\left(|\tilde{\mathbf{x}}_1|^2 \geq \frac{\alpha}{\|\mathbf{a}\|_2^2 - \alpha} \sum_{l=2}^L |\tilde{\mathbf{x}}_l|^2\right) \\ &= \mathbb{E}_{\{[\tilde{\mathbf{x}}_l]\}_{l=2}^L} \left[ \Pr\left(|\tilde{\mathbf{x}}_1|^2 \geq \frac{\alpha}{\|\mathbf{a}\|_2^2 - \alpha} \sum_{l=2}^L |\tilde{\mathbf{x}}_l|^2 \middle| [\tilde{\mathbf{x}}_2, \dots, \tilde{\mathbf{x}}_L]\right) \right]. \end{aligned}$$

Now, we decompose  $\mathbf{x} = \mathbf{R}^{1/2} \mathbf{x}'$ , where  $\mathbf{R}^{1/2} \in \mathbb{C}^{L \times L}$  is a square root of  $\mathbf{R}$ , and  $\mathbf{x}' \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_L)$ . In particular, we can write  $\mathbf{R}^{1/2} = \mathbf{U}_\mathbf{x} \mathbf{\Lambda}_\mathbf{x}^{1/2}$  so that  $\tilde{\mathbf{x}} = \mathbf{W}^H \mathbf{\Lambda}_\mathbf{x}^{1/2} \mathbf{x}'$ , where  $\mathbf{W} \triangleq \mathbf{U}_\mathbf{x}^H \mathbf{V} = [\mathbf{w}_1, \dots, \mathbf{w}_L] \in \mathbb{C}^{L \times L}$  is a unitary matrix. We then have  $[\tilde{\mathbf{x}}_1] \sim \mathcal{CN}(0, \sum_{l=1}^L \lambda_{\mathbf{x},l} |\mathbf{w}_1|_l^2)$ . Since  $\|\mathbf{A}\|_F = \|\mathbf{a}\|_2^2$ , and  $\sum_{l=1}^L \lambda_{\mathbf{x},l} |\mathbf{w}_1|_l^2 \geq \lambda_{\mathbf{x},L}$ , we can lower bound  $P_\alpha$  as

$$P_\alpha \geq \mathbb{E}_{[\tilde{\mathbf{x}}_2, \dots, \tilde{\mathbf{x}}_L]} \left[ e^{-\left(\alpha/(\|\mathbf{A}\|_F - \alpha)\lambda_{\mathbf{x},L}\right) \sum_{l=2}^L |\tilde{\mathbf{x}}_l|^2} \right]. \quad (12)$$

Define  $\tilde{\mathbf{x}}_{(1)} \triangleq [\tilde{\mathbf{x}}_2, \dots, \tilde{\mathbf{x}}_L]^T$ ,  $\mathbf{W}_{(1)} \triangleq [\mathbf{w}_2, \dots, \mathbf{w}_L]$ . Then,

$$\sum_{l=2}^L |\tilde{\mathbf{x}}_l|^2 = \|\tilde{\mathbf{x}}_{(1)}\|_2^2 = \mathbf{x}'^H \mathbf{\Lambda}_\mathbf{x}^{1/2} \mathbf{W}_{(1)} \mathbf{W}_{(1)}^H \mathbf{\Lambda}_\mathbf{x}^{1/2} \mathbf{x}' \stackrel{(c)}{\leq} \left\| \mathbf{\Lambda}_\mathbf{x}^{1/2} \mathbf{x}' \right\|_2^2,$$

where in (c), we first noted that  $\mathbf{W}_{(1)} \mathbf{W}_{(1)}^H$  is an orthogonal projector with eigenvalues either 0 or 1 and then used the Rayleigh-Ritz Theorem. So, we further bound (12) as

$$\begin{aligned} P_\alpha &\geq \mathbb{E}_{[\mathbf{x}']_2, \dots, [\mathbf{x}']_L} \left[ e^{-\left(\alpha/(\|\mathbf{A}\|_F - \alpha)\lambda_{\mathbf{x},L}\right) \sum_{l=2}^L \lambda_{\mathbf{x},l} |\mathbf{x}'_l|^2} \right] \\ &\stackrel{(d)}{\geq} \prod_{l=1}^L \mathbb{E}_{[\mathbf{x}']_l} \left[ e^{-\left(\alpha/(\|\mathbf{A}\|_F - \alpha)\lambda_{\mathbf{x},L}\right) \lambda_{\mathbf{x},l} |\mathbf{x}'_l|^2} \right] \\ &\stackrel{(e)}{=} \prod_{l=1}^L \left(1 \bigg/ \left\{1 + \frac{\alpha}{(\|\mathbf{A}\|_F - \alpha)} \cdot \frac{\lambda_{\mathbf{x},l}}{\lambda_{\mathbf{x},L}}\right\}\right), \end{aligned}$$

where in (d), we used the independence of  $\{[\mathbf{x}']_l\}_{l=1}^L$  and included  $\lambda_{\mathbf{x},1} |\mathbf{x}'_1|^2$  term; in (e), we used the moment generating function of the exponential random variables  $\{[\mathbf{x}']_l\}$ ,  $l \in [L]$ . ■

We are now ready to state the main theorem of this section.

**Theorem 2.** *The probability of the  $(1 - \delta)N^2$ -success event at a scheduled UE (as defined in (11)) using a PF scheduler over  $K$  UEs, denoted by  $P_{\text{succ}}$ , with the spatial correlation-aware random IRS configuration as in Theorem 1 is bounded as*

$$P_{\text{succ}} \geq 1 - \left(1 - \prod_{m=1}^M \frac{1}{1 + \frac{1-\delta}{\delta} \cdot \frac{\lambda_m}{\lambda_M}}\right)^K, \quad (13)$$

where  $M = \text{rank}(\mathbf{\Sigma})$  and  $\lambda_1 \geq \dots \geq \lambda_M$  are the ordered non-zero eigenvalues of the channel covariance matrix,  $\mathbf{\Sigma}$ .

*Proof.* With a PF scheduler used over  $K$  UEs, the probability of at least one UE witnessing the  $(1 - \delta)N^2$ -success event is

$$P_{\text{succ}} = \Pr\left(\bigcup_{k=1}^K \mathcal{E}_k^\delta\right) \stackrel{(a)}{=} 1 - \prod_{k=1}^K \left(1 - \Pr(\mathcal{E}_k^\delta)\right), \quad (14)$$

where (a) follows the independence of channels across UEs. Let  $\mathbf{f}' \triangleq \mathbf{h}_1 \odot \mathbf{f}$ , where  $\mathbf{f} = \mathbf{U} \mathbf{\Lambda}^{1/2} \tilde{\mathbf{f}}$  with  $\tilde{\mathbf{f}} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$ . So,  $\boldsymbol{\theta} = [1, e^{j\angle[\mathbf{f}']_1}, \dots, e^{j\angle[\mathbf{f}']_N}]$  is a candidate random IRS phase vector as per Theorem 1. For simplicity, we ignore the BS-UE direct path. Then, in (11),  $\boldsymbol{\theta} = [e^{j\angle[\mathbf{f}']_1}, \dots, e^{j\angle[\mathbf{f}']_N}]$ , and  $\mathbf{h}_{f,k} = \sqrt{\beta_{r,k}} [\mathbf{h}_{r,k}]_1, \dots, [\mathbf{h}_{r,k}]_N]^T$ . Now, we have  $\Pr(\mathcal{E}_k^\delta) = \Pr\left(\left|\sum_{n=1}^N e^{-j\angle[\mathbf{f}']_n} [\mathbf{h}_{r,k}]_n\right|^2 \geq (1 - \delta) \left|\sum_{n=1}^N [\mathbf{h}_{r,k}]_n\right|^2\right) = \Pr\left(\left|\sum_{n=1}^N e^{-j\angle[\mathbf{f}']_n} [\mathbf{h}_{2,k}]_n\right|^2 \geq (1 - \delta) \|\mathbf{h}_{2,k}\|_1^2\right)$ ,

where we used the form of  $\mathbf{f}'$  and  $[\mathbf{h}_1]_n = 1$ . From the decomposition  $\mathbf{h}_{2,k} = \mathbf{U} \mathbf{\Lambda}^{1/2} \tilde{\mathbf{h}}_{2,k}$ ,  $\mathbf{f} = \mathbf{U} \mathbf{\Lambda}^{1/2} \tilde{\mathbf{f}}$ , since the channel and IRS vectors are generated using the same basis  $\mathbf{U}$ , and their distributions are invariant to left multiplication by a unitary matrix, we let  $\mathbf{U} = [\mathbf{e}_1, \dots, \mathbf{e}_M]$  without loss in generality, where  $\mathbf{e}_m$  is  $m$ th column of  $\mathbf{I}_N$ . Thus,  $\Pr(\mathcal{E}_k^\delta) = \Pr\left(\left|\sum_{m=1}^M e^{-j\angle[\tilde{\mathbf{f}}]_m} \sqrt{\lambda_m} [\tilde{\mathbf{h}}_{2,k}]_m\right|^2 \geq (1 - \delta) \|\mathbf{\Lambda}^{1/2} \tilde{\mathbf{h}}_{2,k}\|_1^2\right) \geq \Pr\left(\left|\sum_{m=1}^M \sqrt{\lambda_m} [\tilde{\mathbf{h}}_{2,k}]_m\right|^2 \geq (1 - \delta) M \|\mathbf{\Lambda}^{1/2} \tilde{\mathbf{h}}_{2,k}\|_2^2\right)$ ,

where we dropped  $e^{-j\angle[\tilde{\mathbf{f}}]_m}$  because  $\angle[\tilde{\mathbf{f}}]_m$  is uniformly distributed in  $[0, 2\pi)$  and independent of  $\angle[\tilde{\mathbf{h}}]_m$ , which does not alter the distribution of  $[\tilde{\mathbf{h}}]_m$ . We also used the property:  $\|\mathbf{x}\|_1 \leq \sqrt{M} \|\mathbf{x}\|_2$ . Now, the above can be rewritten as

$$\Pr(\mathcal{E}_k^\delta) \geq \Pr\left(\hat{\mathbf{h}}_k^H \mathbf{E} \hat{\mathbf{h}}_k / (\hat{\mathbf{h}}_k^H \hat{\mathbf{h}}_k) \geq (1 - \delta) M\right),$$

where  $\hat{\mathbf{h}}_k \triangleq \mathbf{\Lambda}^{1/2} \tilde{\mathbf{h}}_{2,k}$ , and  $\mathbf{E} \triangleq \mathbf{1}_M \mathbf{1}_M^H$  with  $\mathbf{1}_M$  being an  $M$ -length all ones vector. Note that  $\|\mathbf{E}\|_F = M > (1 - \delta)M > 0$ , and that  $\mathbb{E}[\hat{\mathbf{h}}_k \hat{\mathbf{h}}_k^H] = \mathbf{\Lambda}$  has full-rank. Then, using Lemma 3,

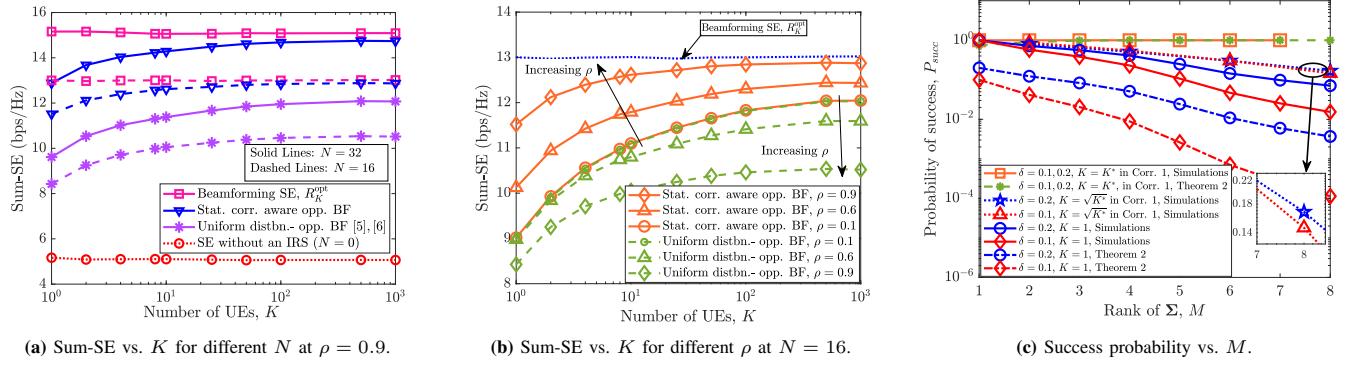
$$\Pr(\mathcal{E}_k^\delta) \geq \prod_{m=1}^M \left(1 \bigg/ \left\{1 + \frac{1-\delta}{\delta} \cdot \frac{\lambda_m}{\lambda_M}\right\}\right). \quad (15)$$

Substituting (15) in (14), we get (13) as desired. ■

The following result is a consequence of Theorem 2.

**Corollary 1.** *Let  $\delta \in (0, 1)$ . With Scheme 1, if  $K$  is at least  $K^* \triangleq -\log(1 - P_{\text{succ}}) \prod_{m=1}^M 1 + \left[\left((1 - \delta)/\delta\right) \left(\lambda_m/\lambda_M\right)\right]$   $\sim \mathcal{O}(-[\log(1 - P_{\text{succ}})]/\delta^M)$ ,*

*then, with probability  $P_{\text{succ}}$ , the channel gain using a randomly configured IRS exhibits a  $(1 - \delta)N^2$  success in every time slot.*



(a) Sum-SE vs.  $K$  for different  $N$  at  $\rho = 0.9$ .

(b) Sum-SE vs.  $K$  for different  $\rho$  at  $N = 16$ .

(c) Success probability vs.  $M$ .

Fig. 2: Performance of spatial correlation-aware opportunistic beamforming in IRS-aided systems.

Corollary 1 shows that a sufficient number of the UEs for  $(1 - \delta)N^2$ -success grows exponentially with the *rank* of the channel covariance matrix and not with the *number of IRS elements*. Thus, if the UE's channel lies in a fixed-dimensional subspace, the number of UEs needed to reap the benefits from the IRS is fixed even if the number of IRS elements grows.

## VI. NUMERICAL RESULTS

We numerically evaluate our results for a setup with the BS at  $(0, 0)$  (in meters), an IRS at  $(1000, 1000)$ , and up to  $K = 1000$  UEs distributed in  $[900, 1100] \times [900, 1100]$ . The path loss is  $\beta = C_0(d/d_0)^\kappa$ , with  $\kappa = 2, 2, 4$  for the BS-IRS, IRS-UE, and BS-UE links, respectively [12]. A PF scheduler with  $\tau = 5000$  is used. The IRS covariance matrix is  $\Sigma = \text{Toeplitz}([1, \rho, \dots, \rho^{N-1}])$ , where  $\text{Toeplitz}(\mathbf{x})$  is a hermitian Toeplitz matrix with  $\mathbf{x}$  as the 1st row, and  $\rho$  is the correlation coefficient between adjacent elements. Since  $\Sigma$  is full-rank when  $\rho \neq 1$ , we use the effective rank [13] for Scheme 1.

In Fig. 2a, we plot the sum-SE vs. the number of UEs,  $K$ , for  $N = 16$  and  $32$ , at  $\rho = 0.9$ . For both values of  $N$ , the sum-SE with the *spatial correlation-aware OC* grows with  $K$  and approaches the SE obtained by optimizing the IRS in every slot using an RR scheduler. Thus, we leverage multi-user diversity and achieve the BF sum-SE in Lemma 1 without incurring the overheads of optimizing the IRS. We also compare the sum-SE with the method in [5], [6], which samples the IRS phases using an i.i.d. uniform distribution. While the SE improves with  $K$ , it is much smaller than the BF-SE, underscoring the importance of sampling IRS phases based on channel statistics. Our scheme also outperforms a system without an IRS.

In Fig. 2b, we plot the sum-SE vs.  $K$  for  $N = 16$  and different correlation values  $\rho$ . For a fixed  $K$ , the gap between the BF-SE and OC-SE decreases as  $\rho$  increases. This is because the effective rank of the channel decreases with  $\rho$ , making it easier for the IRS to achieve a near-BF configuration with fewer UEs, in line with Corollary 1. Conversely, with uniformly sampled IRS phase, the SE gap is large since the configuration spans the full  $N$ -dimensional space, while the channels lie in a lower  $M$ -dimensional subspace. So, as  $\rho$  increases, the mismatch between the IRS phase and the channel phase distribution increases, widening the performance gap.

In Fig. 2c, we plot the success probability,  $P_{succ}$  (see (11)) vs.  $M = \text{rank}(\Sigma)$ . For fixed  $K$ ,  $P_{succ}$  decreases with  $M$  because the effective dimension grows with  $M$ . Also, Theorem 2 is a valid lower bound and succinctly captures the scaling with

$M$ . Finally, we verify that  $P_{succ} = 1$  for any  $M, \delta$  if  $K = K^*$  (see Corollary 1), and  $P_{succ} < 1$  when  $K = \sqrt{K^*} < K^*$ , validating that the scaling we derived is tight.

## VII. CONCLUSIONS

We developed a low-complexity, spatial-correlation-aware opportunistic BF scheme for IRS-aided multi-user systems. Exploiting multi-user diversity, we showed that randomly sampling the IRS phases from an appropriate distribution yields optimal array gains. Interestingly, achieving near BF-SE requires the number of UEs to scale exponentially with the rank of the spatial covariance matrix, rather than the number of IRS elements. Future work could account for UE mobility.

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