Supply Chain Design Optimization With Heterogeneous Risk-Aware Agents

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***Abstract*— Modern supply chain networks (SCN) are becoming increasingly complex, with vulnerable entities exposed to uncertain disruptions that affect local or global supply chain attributes. We model a stochastic mixed-integer program to minimize the overall cost of SCN design and operations, in response to lead-time and demand uncertainties following given probability distributions. We formulate a heterogeneous risk-aware model to trade off between cost and delay/shortage by considering different risk-attitudes amongst supply chain agents. In particular, we employ the Conditional Value-at-Risk (CVaR) as a coherent risk measure for quantifying risk while attaining solution tractability. We derive managerial insights from our numerical studies, finding the most benefit from diversifying agents in the root tier, since their disruptions affect all other tiers in the SCN. We find that as agents become more risk averse, the optimal solutions for key agents (such as assemblers), seek more backup suppliers and allocate extra capacities to achieve resiliency and reliability. Practitioners can use the outcomes of our framework and studies to guide SCN design considering heterogeneous risk attitudes between agents.**

***Note to Practitioners*—With growing uncertainties in global supply chains, inefficient responses to disruptions can lead to large penalties and long-term impacts such as customer dissatisfaction. This research is motivated by the challenges arising during the operations of supply chains under both lead-time and demand uncertainties. We employ optimization and centralized control approaches to optimize supply-chain network design as well as response strategies to disruptions, and our framework can handle heterogeneous risk preferences as it models the risk attitude of each individual entity or agent in supply chains. Our model can be utilized to completely or partially re-design resilient supply chains, to better prepare for unknown features and uncertainties. Our case study provides insights about risk-averse supply-chain designs that can reduce response cost, but increase initial investments on backups and redundancies.**

***Index Terms*— Supply chain network design, risk-averse optimization, conditional value-at-risk (CVaR), stochastic integer programming.**

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I. Introduction

Supply chain networks (SCN) are complex interconnected systems that require coordination between all entities, here called agents. Within a SCN, each agent has specific goals and responsibilities. However, the heterogeneity between agents' risk attitudes can lead to discrepancies in these objectives and can further influence their decisions of choosing suppliers and prioritizing demand. These differences in the agents' risk attitudes can result in SCN designs with an unbalanced structure and ineffective redundancies. When improving the performance and reliability of a SCN, processing capacities and fulfillment times (also called the lead times), are key factors. Not only do they influence SCN operations, but also its layout and design. Often uncertainties arise given the difficulty of predicting changes in demand and labor shortages in global environments adding to the challenge of a SCN with entities that respond differently to such uncertainty [1].

SCN design and capacity allocation often cannot be easily changed during the course of supply chain operations, and therefore, lack of agent redundancy can cause unmet demand and late deliveries. Nevertheless, excessive agent and capacity redundancy increase complexity, setup, and operational costs, as SCN becomes less efficient. As a result, supply chain agents face conflicting objectives to minimize design and planning cost, while maximizing SCN reliability [2]. If a set of agents has different preferences towards their operational costs amidst uncertainty and the associated risk outcomes, cascading disruptions can occur as some agents with low risk aversion might not require redundant capabilities. Undesired disruptions can impact downstream agents in the SCN [3]. Whenever planned operations fail to adapt to disruptions, organizations might incur penalties for not meeting the agreed performance. In other cases, one might impose hard requirements on a minimum amount of units to deliver, or the latest delivery date. Different measures for ensuring high-quality SCN performance under disruptions, may result in diverse supply chain designs [4].

In our prior work [5], we developed a centralized, deterministic optimization model for disruption response with a given SCN design with deterministic lead time. We concluded that for some disruption cases with fixed capabilities, the SCN is unable to respond and adjust its operations to meet demand or on-time delivery, incurring large penalties. If we consider SCN design/redesign decisions (e.g., renegotiating contracts), model in [5] cannot provide such optimal design solutions.

In this paper, we extend the previous work by incorporating SCN design variables and heterogeneous risk-aware objectives of different agents in supply chains. We model uncertain disruptions and incorporate them into the design phase. We aim to design SCNs to maximize efficient disruption responses, whenever layout changes are permitted.

When considering uncertainty, the risk attitudes of supply chain agents influence optimal solutions. Typically, risk-averse designs incur higher costs to protect SCN against extreme disruptions by including redundancies. On the other hand, risk-neutral designs lower the average cost, biasing the design to be long-term efficient [6]. Our formulations consider different risk attitudes for supply chain agents, which we refer to as risk heterogeneity. A SCN where all agents have the same risk aversion standard is considered a homogeneous-risk SCN with a central objective aligning all agents' attitudes. For SCN with multiple tiers, there could exist different risk preferences by the agents, and we call them heterogeneous risk-aware agents.

The three main contributions of our work are as follows. (i) We formulate a centralized SCN design problem with heterogeneous agent risk attitudes, where we model the scheduling dynamics of product flows amidst lead-time uncertainty using stochastic integer programming. (ii) We demonstrate the trade-offs between a heuristic strategy and an optimization model for SCN risk management. (iii) We derive managerial insights for risk-aware SCN design based on numerical results of instances from an automotive SCN.

The remainder of the paper is organized as follows. In Section II, we review the relevant literature and identify the research gaps. In Section III, we develop and compare risk-neutral to risk-aware formulations. We introduce and build an automotive sub supply chain instance under various disruptions, and present the numerical results in Section IV. In Section V, we discuss the results and conclude our work.

II. Literature Review

The existing literature on optimization approaches for SCN management is extensive [7]. In general, the SCN is described by a graph, for which solutions optimize the flow of components through the edges. When considering network design, one can consider variables describing the graph topology [8]. In the deterministic setting, agent-based models [9] and optimization-simulation frameworks [10] have been employed to optimize SCN operations and management. The work in [11] considers an adaptive agent-based simulation framework having heterogeneous agents. This work only evaluates the performance of fixed response strategies and SCN designs, but does not study how to optimize these decisions, nor the effect of the risk attitudes. In this paper, we include heterogeneity in the agents' objectives, but from a centralized, optimization-based perspective, integrating agents' lead-time uncertainties.

When modeling more complex operational decisions, one can consider the scheduling of deliveries, given its relevance for timely processing and delivery of products. When considering lead time as part of the optimization process, for example, in [1], the authors use a mixed-integer program (MIP) to model delivery of products, which involves hard routing and scheduling constraints. In [2], only customer lead-time is modeled, leading to a completely centralized end-customer-driven solution, that cannot capture the heterogeneous nature of each agent's objective. In our prior work [5], we model SCN schedules as soft-constraints, and if delayed, we incur penalty costs in the objective function. We reoptimize the model after disruption realization and find that, as the depth of the disruption increases with respect to the final-customer, SCN redesign is encouraged. The work in [10] considers a multi-stage program by modeling lateness as back-orders that ship in the next period with penalty. We note that to the best of our knowledge, no prior work has modeled the arrival time of product flows explicitly within a stochastic SCN design optimization problem, which is one of our main contributions.

In terms of the SCN design problem, the work by [12] defines a deterministic MIP with binary variables for SCN design and a bi-objective minimizing operational cost and environmental impact. A similar model was developed in [13], minimizing procurement costs considering scheduling decisions. The model in [14] solves a capacitated network design problem by combining transportation lead time with homogeneous fixed penalty on late delivery to final customer. The work in [15] considers fuzzy-logic in a multi-agent supply chain setup. Other works, such as [16] concentrate on the environmental impact for certain SCN designs. The work in [17] studies the topological structure of SCN based on a multi-agent framework. In [18], the authors study the effect of faster communication schemes between agents (RFID technology) in optimal network design. However, all these formulations are deterministic without demand or lead-time uncertainty.

Stochastic optimization approaches incorporate uncertainty in the decision-making process, optimizing the objective with respect to a risk measure that quantifies the probabilistic performance of solutions [19]. This has direct impact on SCN performance and can reduce potential disruption-response cost [5]. When considering tractable formulations, the benefit of these models becomes readily available. For example, the disjunctive MIP in [20] optimizes SCN design by minimizing the expected lead time for each flow with a probabilistic linear model. Risk-neutral attitudes adopted by [20] and [21] minimize an objective in expectation. Similar strategies are used in [22], where the authors develop a risk-neutral SCN design optimization, and quantify SCN resilience in the objective as a function of the response cost and the recovery time given lead-time disruptions. The work in [23] optimizes the cost of a single commodity distribution network ensuring a level of service by means of Lagrangian relaxation. In the risk-averse regime, [21] introduces a two-stage multi-period stochastic robust optimization model with a scenario-based solution approach, to enhance the robustness of SCN design. However, this formulation only models delivery lateness as triggered back-orders. We model delivery lateness with a lateness penalty that is proportional to the lateness magnitude. Another robust-optimization approach in [4] assigns an uncertainty budget for the uncertain parameters, introducing a central tuning parameter for the conservativeness of solutions, without risk heterogeneity. In [24], the authors consider box sets of uncertainty for a risk-homogeneous robust formulation.

This approach does not consider lead time, and thus cannot incorporate lateness at different layers of the SCN.

Some risk measures, such as the Conditional Value-at-Risk (see Section III), can be formulated to combine risk-averse and risk-neutral attitudes, where solutions can be dependent on agents' risk attitudes. The work in [25] shows the challenge of optimizing the CVaR with a central risk attitude in the objective. Similarly, the work in [26] considers a two-stage distributionally robust SCN problem with a central risk parameter for cost uncertainty. The authors highlight the importance of the risk parameters driving the optimal solutions and the challenges of understanding best parameter setting. This challenge increases when we consider that different agents can have different risk preferences. In this work, we allow for a model with different risk attitudes for each agent, which we refer to as a heterogeneous SCN. The work in [27] evaluates the performance in a distributed simulation-optimization framework and describes the importance of a high-quality initial solution, for which this paper can be used to provide the required input of such models.

Following the taxonomy in [7], we consider a two-stage stochastic program, with an objective that incorporates first-stage planning and operation costs. As the second-stage objective we consider SCN responsiveness amidst disruptions by using the conditional value-at-risk measure to quantify its performance. In Table I, we position our work as compared to different approaches in the literature. Given the reviewed works, we identify a gap as an exploration of heterogeneous risk-aware stochastic SCN design problem under both lead-time and demand uncertainties. Our work begins by formulating a risk-neutral problem and then extending it to a heterogeneous risk-aware model, allowing for different risk-attitudes between agents within the SCN.

III. Stochastic Optimization Model

A. Assumptions and Notation

We consider a directed graph $G(V, E)$ representing a SCN with the set V including all candidate agents, and the set E of potential edges (direct shipping or ordering relationships) between them. We denote $V^{\mathrm{c}} \subset V$ as the subset of final-product customers, $V^{\mathrm{d}} \subset V$ as the subset of agents that distribute products, namely, the distributors (with no transformation of components occurring within them), $V^{\mathrm{o}} \subset V$ as the subset of agents in which transformations of products occur, namely, assemblers, and $V^{\mathrm{s}} \subset V$ as the subset of agents who supply products and raw materials, namely, the suppliers who have no upstream flows. Note that $V^{\mathrm{c}} \cup V^{\mathrm{d}} \cup V^{\mathrm{o}} \cup V^{\mathrm{s}} = V$. We denote $U(i) \in V$ and $D(i) \in V$ as the sets of upstream and downstream agents of agent $i \in V$, respectively. We have that $(U(i), i) \subset E,\ \forall i \in V$ and $(i, D(i)) \subset E,\ \forall i \in V$, leading to $\cup_{i \in V}((U(i), i) \cup (i, D(i))) = E$. We denote K as the set of all products and components within the SCN. Additionally, for each agent $i \in V$ and its related downstream products $k \in K(i)$, we denote a subset $K'(i, k) \subset K$ of upstream components $k' \in K'(i, k)$ required for $k \in K(i)$. We define parameter $r_{k'k}$ as the conversion rate from component k' to product k for each component $k' \in K'(i, k)$. We formulate a two-stage stochastic program, where in the first stage, we optimize allocation and order-size decisions. In the second stage, once disruptions occur, we model recourse component flow decisions. We allow managers to quantify the impact of disruptions on the SCN performance given certain design decisions with this stochastic program.

Throughout this paper, we use bold symbols to represent vector forms of parameters or decision variables. For flow of product k through edge (i, j) we consider a unit flow cost c^{f}_{ijk}, and a fixed cost c^{a}_{ijk} for establishing an edge (i, j) to allow positive flows of product k between agents i and j. The decision vector $\mathbf{x} = [x_{ijk}, (i, j) \in E, k \in K]^\top$ represents the amount of planned product k flowing through edge (i, j). Variables in $\mathbf{x}$ can only take positive values if the corresponding binary variables in $\mathbf{y} = [y_{ijk}, (i, j) \in E, k \in K]^\top$ take a value of 1, representing an active contract between agents i and j. We denote q_{ijk} as the maximum capacity for flowing product k through edge (i, j). The total, mixed-product, downstream flow of agent $i \in V$ has a maximum capacity $\bar{q}_i$.

Consider a set Ω of realizations of the uncertain parameters, which include demand and lead-time disruptions. For each scenario $\omega \in \Omega$, $\mathbf{d}^\omega = [d^\omega_{ik}, i \in V, k \in K]^\top$ is the demand for product k at agent $i \in V$ in scenario ω, and we denote $\mathbf{l}^\omega = [l^\omega_{ijk}, (i, j) \in E, k \in K]^\top$ as the lead time of product k flowing through edge (i, j). We denote the overall random vector $\tilde{\boldsymbol{\xi}} = [\mathbf{d}, \mathbf{l}]$, and its realized value for scenario $\omega \in \Omega$ is given by $\boldsymbol{\xi}^\omega = [\mathbf{d}^\omega, \mathbf{l}^\omega]$. Likewise, for each scenario $\omega \in \Omega$, we compute the unmet demand of product $k \in K$ for each agent $i \in V$ via auxiliary variables $\mathbf{u}^\omega = [u^\omega_{ik}, i \in V, k \in K]^\top$. With auxiliary variables $\mathbf{v}^\omega = [v^\omega_{ijk}, (i, j) \in E, k \in K]^\top$ we compute the units of lateness for the flow of product $k \in K$ through edge (i, j). As an adaptive mechanism in the SCN operation, in scenario $\omega \in \Omega$ we allow for backup/emergent flows with decision variables in $\mathbf{z}^\omega = [z^\omega_{ijk}, (i, j) \in E, k \in K]^\top$ to compensate for insufficient flows from the planned flows $\mathbf{x}$. These emergent flows incur unit cost c^{e}_{ijk}, such that $c^{\mathrm{e}}_{ijk} > c^{\mathrm{f}}_{ijk}$, as emergent responses are more expensive. We consider SCN operations that allow partially fulfilled demands, and late deliveries. For each unit of unmet demand we have the penalty cost $\rho^{\mathrm{d}}_{ik}, \forall i \in V, k \in K$. Likewise, for each unit of lateness we have the penalty cost $\rho^{\mathrm{l}}_{ijk}, \forall (i, j) \in E, k \in K$.

Parameter t_{ik} denotes the due date for product $k \in K$ at agent $i \in V$. For each scenario $\omega \in \Omega$ we denote decision variables $\mathbf{a}^\omega = [a^\omega_{ijk}, (i, j) \in E, k \in K]^\top$ representing the scheduled arrival time of product k through edge (i, j). Variables $\mathbf{o}^\omega = [o^\omega_{ik}, i \in V, k \in K]^\top$ model the scheduled time, at which agent $i \in V$ processes product $k \in K$. Auxiliary variables $\mathbf{w}^\omega = [w^\omega_{ijk}, (i, j) \in E, k \in K]^\top$ capture the penalty on lateness for the flow of product $k \in K$ through edge (i, j). We denote the vector of planning decision variables as $\mathbf{X} = [\mathbf{x}, \mathbf{y}]$. Likewise, the vector of decision variables for each scenario $\omega \in \Omega$ is given by $\mathbf{Y}^\omega = [\mathbf{u}^\omega, \mathbf{a}^\omega, \mathbf{o}^\omega, \mathbf{z}^\omega, \mathbf{w}^\omega]$. We make the following assumptions in this paper. Knowledge of disruptions and response time are immediate. Flow of product k through edge (i, j) is treated as an indivisible unit. Each agent waits until all required upstream flows have been received before sending their downstream flows.

TABLE I

CHARACTERIZATION OF RELEVANT WORKS ON OPTIMIZATION FOR SUPPLY CHAIN RISK MANAGEMENT. THIS PAPER FILLS THE IDENTIFIED GAP IN THE LITERATURE FOR A HETEROGENEOUS RISK AVERSE MODEL FOR A SCN DESIGN AND OPERATION MODEL APPLIED TO A MULTI-PRODUCT SUPPLY NETWORK

	SCN type	Risk type	Heterogeneity	Objective	Decisions	Adaptability	Disruption
Petridis [2]	M-pt, S-pd, M-e	Neutral	Centralized	S+O	F, R	BF	D
Zokaee et al. [4]	S-pt, S-pd, M-e	Averse (robust)	Centralized	S+O, UD	F, R	BF	D+C
Park [28]	S-pt, M-pd, M-e	Averse (Mean - risk)	Centralized	S+O, UD	D, O	BF	D+L
Aqlan et al. [10]	M-pt, M-pd, M-e	Averse (probability of undesirable event)	Centralized	Revenue, risk mitigation cost, UD, LT	Y, P, I	BF, RA	D+L
You et al. [20]	M-pt, M-pd, M-e	Neutral	Centralized	Net present value, expected lead time	F, R, S, I	BF	D+L
Sadghiani et al. [29]	M-pt, M-pd, M-e	Robust	Centralized	S+O,	R, S, I	BF,RA	X
Shen et al. [30]	M-pt, M-pd, M-e	Robust	Centralized	S+O,LT,M	R, S, I, H	BF	D+L+M
This work	M-pt, S-pd, M-e	Neutral, averse (CVaR)	Agent risk attitudes	S+O, UD, LT	D, R, S	BF, RA	D+L

SCN type: M-pt: Multi-product , S-pt: Single-product, M-pd: Multi-period , S-pd: Single-period, M-e: Multi-echelon
Objective: S+O: Setup and operation cost, UD: Unmet demand penalty, LT: Lateness penalties, H: Health impact
Adaptability: BF: Backup flows, RA: Redundant agents; **Disruption**: D: demand, L: lead time, C: cost, X: correlated uncertainties, I: infection risks
Decisions: F: facility location/selection, R: routing, S: scheduling, D: SCN design, I: inventory, O: ordering quantity, P: production, Y: response strategy

B. Mixed-Integer Stochastic Linear Program for Risk-Neutral SCN Operation and Design

We first formulate Model (1), a risk-neutral approach to minimize the total costs of SCN design and product flows (1a), the expected cost of emergent flows for every edge (i, j) and product k in (1b). We minimize the expected unmet demand penalty in (1c) and the lateness penalty in (1d). Following the Sample Average Approximation (SAA) approach [31], we assume distributional knowledge of the random vector $\tilde{\boldsymbol{\xi}}$, with a finite set Ω of scenarios of size $N = |\Omega|$, where we model the expectation as the finite weighted summation of scenarios with their respective probabilities $p^\omega, \ \forall \omega \in \Omega$.

$$\min_{\mathbf{X},\mathbf{Y}^\omega} \sum_{(i,j)\in E,\ k\in K} \left(c^{\mathrm{f}}_{ijk} x_{ijk} + c^{\mathrm{a}}_{ijk} y_{ijk}\right) \tag{1a}$$

$$+ \sum_{\omega\in\Omega} p^\omega \left(\sum_{(i,j)\in E,\ k\in K} c^{\mathrm{e}}_{ijk} z^\omega_{ijk} \right) \tag{1b}$$

$$+ \sum_{\omega\in\Omega} p^\omega \left(\sum_{i\in V,\ k\in K} \rho^{\mathrm{d}}_{ik} u^\omega_{ik} \right) \tag{1c}$$

$$+ \sum_{\omega\in\Omega} p^\omega \left(\sum_{(i,j)\in E,\ k\in K} \left(\rho^{\mathrm{l}}_{ijk} v^\omega_{ijk}\right) \right) \tag{1d}$$

$$\text{s.t.} \sum_{j\in U(i)} r_{k'k}(x_{jik'} + z^\omega_{jik'}) \geq \sum_{j\in D(i)} (x_{ijk} + z^\omega_{ijk}),$$

$$\forall\ i \in V \setminus V^c,\ k \in K,\ k' \in K'(k, i),\ \omega \in \Omega, \tag{1e}$$

$$x_{ijk} + z^\omega_{ijk} \leq q_{ijk} y_{ijk}, \forall\ (i, j) \in E,\ k \in K,\ \omega \in \Omega, \tag{1f}$$

$$\sum_{j\in D(i),K} (x_{ijk} + z^\omega_{ijk}) \leq \bar{q}_i, \forall\ i \in V,\ \omega \in \Omega, \tag{1g}$$

$$u^\omega_{ik} + \sum_{j\in U(i)} (x_{jik} + z^\omega_{ijk}) \geq d^\omega_{ik},$$

$$\forall\ i \in V^c,\ k \in K,\ \omega \in \Omega, \tag{1h}$$

$$a^\omega_{ijk} \geq (l^\omega_{ijk} + o^\omega_{ik}) y_{ijk}, \forall\ (i, j) \in E,\ k \in K,\ \omega \in \Omega, \tag{1i}$$

$$o^\omega_{jk} \geq a^\omega_{ijk'},$$

$$\forall\ (i, j) \in E,\ k \in K,\ k' \in K'(k),\ \omega \in \Omega, \tag{1j}$$

$$o^\omega_{ik} = 0, \forall\ i \in V^{\mathrm{p}},\ k \in K,\ \omega \in \Omega, \tag{1k}$$

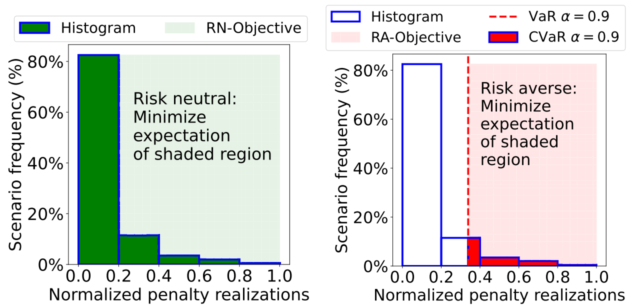
$$a^\omega_{ijk} - v^\omega_{ijk} \leq t_{jk}, \forall\ (i, j) \in E,\ k \in K,\ \omega \in \Omega, \tag{1l}$$

$$x_{ijk}, a^\omega_{ijk}, o^\omega_{ik}, z^\omega_{ijk}, u^\omega_{ik}, v^\omega_{ijk} \geq 0,$$

$$\forall\ i \in V,\ (i, j) \in E,\ k \in K,\ \omega \in \Omega, \tag{1m}$$

$$y_{ijk} \in \{0, 1\}, \forall\ (i, j) \in E,\ k \in K. \tag{1n}$$

Here, constraint (1e) balances the flow of products $x_{ijk} + z^\omega_{ijk}$ at each edge (i, j) in the SCN, considering the units $r_{k'k}$ of downstream components $k' \in K'(k, i)$ consumed to produce one unit of downstream product k. Constraint (1f) sets the capacity q_{ijk} for the flow of product k through edge (i, j). Likewise, constraint (1g) bounds the total production capacity of agent i by $\bar{q}_i$. Using constraint (1h), we compute scenario-based unsatisfied demand u^ω_{ik} for each product k at each customer agent i. Constraint (1i) models scenario-based delivery time a^ω_{ijk} of product k at destination agent j based on the start time o^ω_{ik} and lead time l^ω_{ijk} of the agent of flow origin $i \in V$, whenever we have a contract setup for the edge (i, j) (i.e., $y_{ijk} = 1$). Constraints (1j)–(1k) compute the scenario-based time o^ω_{ik} at which downstream products k are ready to be processed, depending on required upstream products $k' \in K'(k, i)$ and their corresponding arrivals a^ω_{ijk}. In constraint (1l), we compute the scenario-based delivery lateness v^ω_{ijk} at agent j given due date t_{jk} and the time of delivery a^ω_{ijk} from agent i. Constraint (1m) specifies the non-negativity of flow variables $\mathbf{x}$ and second-stage variables $\mathbf{a}^\omega, \mathbf{o}^\omega, \mathbf{z}^\omega, \mathbf{u}^\omega, \mathbf{v}^\omega$. The SCN design variables $\mathbf{y}$ are binary values as expressed in constraint (1n).



(a) Risk neutral objective (b) Risk aware objective

Fig. 1. Visual comparison of the risk-neutral Model (1) and risk-aware Model (4) objectives. 10 bins segment the penalty cost in equally spaced ranges. We overlay the cumulative percentage of scenarios with penalty in the corresponding penalty decile. The risk-neutral objective in Fig. 1a considers the expectation of the full support of the realizations (shaded in green) of the penalties. In contrast, the risk-aware objective in Fig. 1b considers the expectation of the (1)-α) quantile of the penalty distribution, minimizing the impact of rare, but large disruptions (shaded in red).

C. Risk-Aware Stochastic Supply Chain Optimization

The two-stage stochastic optimization approach presented in Section III-B minimizes the expected second-stage cost. It is fitted for repetitive decisions, prioritizing long-term costs, being insensitive to rare but large disruptions. We define the expectation-based objective as a risk-neutral objective. The SCN design problem considers a single-time design of the SCN, as we cannot redesign the SCN with any demand or lead time disruption. We prioritize consequences to disruptions in extreme cases, as we seek SCNs with reliable performance, avoiding excessive unmet demand and lateness. We define to this model as risk aware. To this end, we consider the CVaR, given its coherent characterization [32]. We describe it by first defining the Value-at-Risk (VaR) of a random variable Z at level $\alpha \in (0, 1)$ as the smallest number η such that the probability of Z not exceeding η is at least $1 - \alpha$. VaR is also known as the $(1 - \alpha)$ quantile of the distribution of Z, and CVaR is the expected value of realizations of the random variable Z beyond the VaR. We can reformulate CVaR as a linear minimization problem defined over variable η, the VaR, and auxiliary scenario-based variables s^ω, corresponding to the non-negative difference between the realizations and η, representing the excess of the realization with respect to the VaR as proposed in [33]. We then compute the expectation of variables s^ω as the CVaR. We set the parameter α as the risk-aversion. As α gets closer to zero, we minimize the expectation of the complete distribution equivalent to Model 1. We note that α is associated to each agent, and we can set a different α_i for each agent $i \in V$, modelling a heterogeneous SCN. We minimize over the risk associated to the lateness penalties, and thus, for each flow of component or product $k \in K$ moving through edge (i, j) we have the following linear optimization problem (2).

$$\min_{\eta^{\mathrm{l}}_{ijk}, \mathbf{s}^{\mathrm{l}\omega}_{ijk}} \quad \eta^{\mathrm{l}}_{ijk} + \frac{1}{(1-\alpha_i)} \sum_{\omega \in \Omega} p^\omega s^{\mathrm{l}\omega}_{ijk} \tag{2a}$$

$$\text{s.t. } s^{\mathrm{l}\omega}_{ijk} \geq \rho^{\mathrm{l}}_{ijk} v^\omega_{ijk} - \eta^{\mathrm{l}}_{ijk}, \forall\, \omega \in \Omega, \tag{2b}$$

$$s^{\mathrm{l}\omega}_{ijk} \geq 0, \forall\, \omega \in \Omega, \tag{2c}$$

$$\eta^{\mathrm{l}}_{ijk} \in \mathbb{R}, \tag{2d}$$

where η^{l}_{ijk} is the corresponding VaR for the lateness penalty distribution represented by the realizations $\rho^{\mathrm{l}}_{ijk} v^\omega_{ijk}$ for each scenario. Auxiliary scenario-based variables $s^{\mathrm{l}\omega}_{ijk}$ take positive values whenever a realization is beyond the VaR η^{l}_{ijk}. We illustrate the definition of CVaR, associated with the linear program (LP) (2) where $\alpha = 0.9$ and compare it to a risk neutral objective in Fig. 1. The risk-aware objective minimizes the expected value of the CVaR region, while the risk-neutral objective minimizes the expected value of the full distribution.

Likewise, for the demand penalty CVaR, we have an LP for each agent with demand of product $k \in K$. (We assume that demand comes from final product customers $i \in V^{\mathrm{c}}$).

$$\min_{\eta^{\mathrm{d}}_{ik}, \mathbf{s}^{\mathrm{d}\omega}_{ik}} \quad \eta^{\mathrm{d}}_{ik} + \frac{1}{(1-\alpha_i)} \sum_{\omega \in \Omega} p^\omega s^{\mathrm{d}\omega}_{ik} \tag{3a}$$

$$\text{s.t. } s^{\mathrm{d}\omega}_{ik} \geq \rho^{\mathrm{d}}_{ik} u^\omega_{ik} - \eta^{\mathrm{d}}_{ik}, \forall\, \omega \in \Omega, \tag{3b}$$

$$s^{\mathrm{d}\omega}_{ik} \geq 0, \forall\, \omega \in \Omega, \tag{3c}$$

$$\eta^{\mathrm{d}}_{ik} \in \mathbb{R}. \tag{3d}$$

Here η^{d}_{ik} is the corresponding VaR of the distribution of the unmet demand penalty in realizations $\rho^{\mathrm{d}}_{ik} u^\omega_{ik}$ for each scenario. Auxiliary scenario-based variables $s^{\mathrm{d}\omega}_{ik}$ take positive values whenever a realization is beyond the VaR η^{d}_{ik}.

In our risk-aware formulation, we consider the CVaR as a convex formulation with bounded VaR. A risk-neutral objective with a VaR constraint involves the solution of a chance-constrained program, which is non-convex [34]. We incorporate the CVaR reformulations in (2) and (3) (both being linear programs), to the SCN design and operation Model 1. With the parameter α_i we gain the flexibility to set different risk attitudes amongst the supply chain agents $i \in V$, modeling heterogeneity in the decision making. The risk-aware Model (4) minimizes setup costs (4a) and (4b), which are also present in the risk-neutral Model (1) as we incur them regardless of how we measure risk. We consider total heterogeneous CVaR costs of unmet demand in (4c) and delivery lateness in (4d).

$$\min_{\mathbf{X}, \mathbf{Y}^\omega, \boldsymbol{\eta}, \mathbf{s}^\omega} \quad \sum_{(i,j) \in E,\ k \in K} \left(c^{\mathrm{f}}_{ijk} x_{ijk} + c^{\mathrm{a}}_{ijk} y_{ijk} \right) \tag{4a}$$

$$+ \sum_{\omega \in \Omega} p^\omega \left(\sum_{(i,j) \in E,\ k \in K} c^{\mathrm{e}}_{ijk} z^\omega_{ijk} \right) \tag{4b}$$

$$+ \sum_{i \in V,\ k \in K} \left(\eta^{\mathrm{d}}_{ik} + \frac{1}{(1-\alpha_i)} \sum_{\omega \in \Omega} p^\omega s^{\mathrm{d}\omega}_{ik} \right) \tag{4c}$$

$$+ \sum_{(i,j) \in E,\ k \in K} \left(\eta^{\mathrm{l}}_{ijk} + \frac{1}{(1-\alpha_i)} \sum_{\omega \in \Omega} p^\omega s^{\mathrm{l}\omega}_{ijk} \right) \tag{4d}$$

s.t. (1e)–(1n)

$$s^{\mathrm{d}\omega}_{ik} \geq \rho^{d}_{ik} u^\omega_{ik} - \eta^{\mathrm{d}}_{ik}, \forall\, i \in V,\ k \in K,\ \omega \in \Omega, \tag{4e}$$

$$s^{\mathrm{l}\omega}_{ijk} \geq \rho^{\mathrm{l}}_{ijk} v^\omega_{ijk} - \eta^{\mathrm{l}}_{ijk}, \quad \forall\, (i, j) \in E,\ k \in K,\ \omega \in \Omega, \tag{4f}$$

$$\eta^{\mathrm{d}}_{ik}, \eta^{\mathrm{l}}_{ijk}, s^{\mathrm{d}\omega}_{ik}, s^{\mathrm{l}\omega}_{ijk} \geq 0, \quad \forall\, i \in V, (i, j) \in E,\ k \in K,\ \omega \in \Omega. \tag{4g}$$

TABLE II
A SUMMARY OF PARAMETERS AND VARIABLES

Input Parameters	
Ω	set of scenarios ω corresponding to parameter realizations.
p^ω	probability of scenario $\omega \in \Omega$.
c^{a}_{ijk}	cost of including edge (i, j) flowing product k in the SCN.
c^{f}_{ijk}	cost to plan the flow of one unit of product k through edge (i, j).
c^{e}_{ijk}	cost to flow an emergent unit of product k through edge (i, j).
$r_{k'k}$	conversion rate from component k' to product k.
d^ω_{ik}	demand of product k at agent i at scenario $\omega \in \Omega$
q_{ijk}	flow capacity of product k in edge (i, j).
$\bar{q}_i$	total mixed-flow capacity of agent i
ρ^{d}_{ik}	penalty per unit of unmet demand d_{ik}.
ρ^{l}_{ijk}	penalty per unit of lateness for flow of product k from edge (i, j).
t_{ik}	time at which agent i requires the demand of product k.
l^ω_{ijk}	product k flow lead time of edge (i, j) at scenario $\omega \in \Omega$.
α_i	CVaR risk-attitude of agent $i \in V$.
Decision Variables	
x_{ijk}	units of product k flowing through edge (i, j).
y_{ijk}	binary variables, equal to 1 if edge (i, j) transporting product k is considered as part of the SCN, and 0 otherwise.
z^ω_{ijk}	units of emergency product k flowing through edge (i, j) in scenario ω.
u^ω_{ik}	units of unmet demand of product k at agent i in scenario ω.
a^ω_{ijk}	delivery time of product k to agent j from agent i in scenario ω.
o^ω_{ik}	time at which agent i can process product k in scenario ω.
v^ω_{ijk}	delivery lateness of flow of product k through edge (i, j) in scenario ω.
η^{d}_{ik}	VaR of unmet demand penalty of product k at agent i.
η^{l}_{ijk}	VaR of delivery lateness of flow of product k through edge (i, j).
$s^{\mathrm{d}\omega}_{ik}$	non-negative difference of realization beyond VaR of unmet demand penalty of product k at agent i in scenario ω.
$s^{\mathrm{l}\omega}_{ijk}$	non-negative difference of realization beyond VaR of lateness of flow of product k through edge (i, j) in scenario ω.

Here, for each scenario $\omega \in \Omega$, we compute the value of realization of the unmet demand penalty beyond the VaR in constraint (4e). Likewise, we compute the value of realization beyond the VaR of the lateness penalty in constraint (4f). Constraint (4g) specifies the non-negativity of VaR and CVaR variables given the non-negative nature of penalty costs. Table II provides a summary of all parameters and decision variables used Models (1) and (4).

D. Solution Implementation

We model SCN operations via a flow-based formulation with variable $\mathbf{x}$, generalizing production, inventory holding, among other agent activities. Our framework can be further detailed without loss of generality, such that we can formulate stochastic risk-neutral and risk-aware counterparts to those more granular SCN models, such as our prior work [5] where we consider inventory, production, and open/close facility decisions, having a higher computational complexity.

After solving Models (1) and (4) we attain SCN design variables $\mathbf{y}$ and planned flow variables $\mathbf{x}$ that serve as the here-and-now decisions to be implemented as one initiates the SCN setup and reconfiguration. The scenario-based variables $\mathbf{z}$ represent wait-and-see decisions, such that they should only be considered whenever a similar scenario occurs. Given this structure, at any point in the operation, we fix the layout variables $\mathbf{y}$ and the planned flows $\mathbf{x}$, and optimize over the other scenario-based supply chain operation variables $\mathbf{Y}$ and constraints, see Section III-A. We only consider the realization of the uncertain parameters at that point. This subproblem is smaller and can be efficiently solved. Furthermore, we consider demand and lead-time uncertainty in the form of disruptions, but we can expand the uncertainty vector with any other second-stage element in our model. It is possible to consider cascading or sequential failures in our framework by incorporating joint probabilities in the scenario set used in the optimization models. Scenarios will then consider disruptions in multiple agents. Following the wait-and-see idea, as more disruptions and the response decisions take place over time, we can incorporate these decisions in subsequent subproblems. We then optimize over the remaining variables to obtain the optimal solutions given the occurred disruptions.

IV. Numerical Studies

We first describe the SCN for which we construct our baseline instance. Later we discuss the several instances we test as modifications of the baseline representing different types of disruptions to the SCN. We then describe the greedy heuristic approach we implement to contrast with our optimization Models (1) and (4). Following this, we analyze optimal SCN topologies from the proposed optimization approaches, and measure their performance with respect to SCN agent disruptions. For all computations, we use an Apple M2 Silicon CPU with 8 cores and 8 threads (4 cores @ 3.5 GHz and 4 @ 2.8 GHz), and 16 GB of memory. As the MILP, MIP and LP solvers we use the Gurobi 10 API for Python 3.9.13.

A. SCN Baseline

The SCN instance, for which we develop our numerical studies, is an adapted automotive SCN in [36]. Given the large size of the complete cockpit assembly SCN instance, we consider only one of the components of the cockpit. We concentrate on the In-vehicle infotainment sub-supply chain, where the customers will be the cockpit assemblers. We set their demand based on the demand of cockpits for complete vehicle assembly. We justify the use of this sub-SCN as it is complex enough to be analyzed as a proper SCN, and in the context of the complete automotive SCN, infotainment systems have recently became more popular with their integration with recent technologies that are constantly changing, and have been affected by the semiconductor market disruptions, among others. Furthermore, we can fix many decisions in a large SCN and allow for reconfiguration of a sub-SCN as we show here. The network consists of 30 candidate agents, composed of 3 customers, 19 suppliers, 8 assemblers and distributors:

- four wire suppliers (WR-S_1 – WR-S_4)
- two connector suppliers (CT-S_1 – CT-S_2)
- four wiring assemblers (WN-A_1 – WN-A_4)
- one switch supplier (SW-S_1)
- four touchscreen suppliers (SN-S_1 – SN-S_4)
- two buttons suppliers (BN-S_1 – BN-S_2)
- two chip suppliers (CH-S_1 – CH-S_2)
- two radio suppliers (RD-S_1 – RD-S_2)
- two navigation suppliers (NV-S_1 – NV-S_2)
- four infotainment assemblers (IT-A_1 – IT-A_4)
- three cockpit assemblers (customers) (CP-A_1 – CP-A_3)

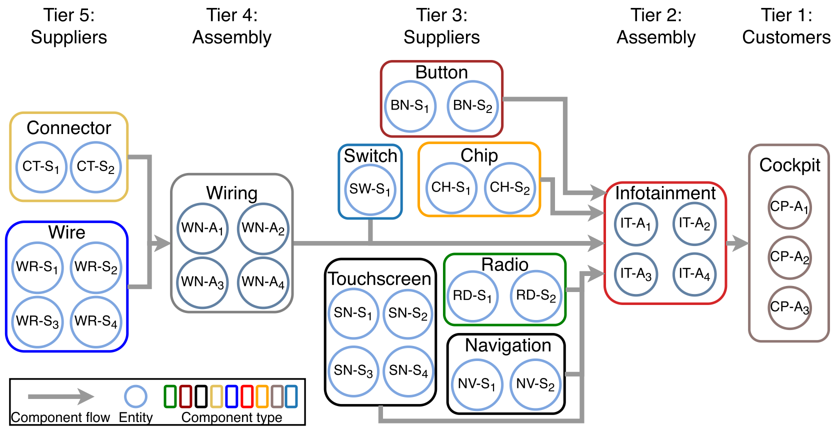


Fig. 2. The SCN for the numerical studies consists of five tiers. Tier 5 consists of two suppliers for connectors and four suppliers for wire. Tier 4 consists of four wiring assemblers, taking connector and wires as the components. The rest of tiers continue this sequential structure up to the cockpit assemblers, considered to be the customers of the supply chain (we refer the interested readers to the online supplement for more details [35]).

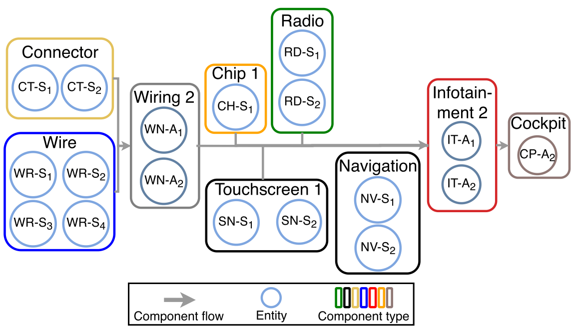


Fig. 3. The product structure for infotainment 2 consists of two possible suppliers of wiring 2, one supplier for chip 1, two touchscreen 1 and two navigation system suppliers, and two possible assemblers to send infotainment 2 to cockpit assembler acting as client. Similar structures exist for the rest of products in the supply chain. (We refer the interested readers to the online supplement for more details [35].)

TABLE III

CHARACTERISTICS OF OPTIMAL SCN DESIGN. THE NUMBER WITH HIGHER CAPACITY IS MARKED IN BOLD. WE OBSERVE THAT THE RISK AVERSE ($\alpha = 0.9$) SOLUTION USES MORE AGENTS AND FLOWS AS A RESPONSE TO DISRUPTIONS. THE RISK NEUTRAL SOLUTION ($\alpha = 0$) HAS LESS REDUNDANCIES AND PLANS FOR AGENTS TO USE MORE OF THEIR CAPACITY, LIMITING THEIR RESPONSE TO INCREASED DEMAND

	Risk neutral	Risk averse
Total agents	21	**23**
Total edges	38	**42**
Total p-flows	30000	30000
Avg. e-flows	3242	**5634**
Avg. edge util. (%)	**19**	17
Avg. agent util. (%)	37	37
Avg. scenarios with e-flows (%)	**4**	3

p-flow: planned flow (variable $\mathbf{x}$) , e-flow: emergent flow (variable $\mathbf{z}$)
Util.: % of total flow capacity utilized

The SC consists of 19 products:

- wire (WR), connector (CNTR)
- four types of wiring (WRN_1,WRN_2,WRN_{3A},WRN_{3B})
- three types of touchscreen (SCR_1,SCR_{2A},SCR_{2B})
- radio (RAD), navigation system (NAV), buttons (BTN)
- switch (SWT), two chip types (CHP_1,CHP_2)
- four infotainment types ($INFT_1$,$INFT_2$,$INFT_{3A}$,$INFT_{3B}$)

We design the instance to include options that process more than one product (with cheaper cost, higher lead time, and shared production capacity), and consider uncertainty from customer demand and lead time. We include agents trading-off cost, lead-time mean and variance, and/or capacity.

Our instance has a sequential, product-processing topology, as shown in Fig. 2. The annotations represent product types that agents can output to downstream agents. Furthermore, Fig. 3 shows the product structure and available entities involved in the assembly of one of the infotainment 2 ($INFT_2$).

To perform our case studies, we consider the cost of emergent flows to be 1.7 times the normal cost for each flow, that is $c^{e}_{ijk} = 1.7\ c^{f}_{ijk}, \forall (i,j) \in E, k \in K$. We set the risk-attitude parameter $\alpha = 0.7, \forall i \in V$, representing a minimization of the expected value of 0.3-quantile for the lateness and unmet demand penalties of all agents in the SCN. We optimize the SCN design instance in Section IV-A with $N = 60$ scenarios. Considering SAA, we perform Monte-Carlo sampling of the distributions of lead-time and demand parameters forming independent, joint samples with equal probabilities $p^{\omega} = \frac{1}{N}, \forall \omega \in \Omega$. As a benchmark, we also consider a deterministic design, which consists of the risk-neutral optimization Model (1) with one scenario equal to the mean of the distributions of the uncertain parameters.

We show the SCN design and flow plans for the deterministic model, the risk-neutral Model (1), and the risk-aware Model (4) in Fig. 4. In Table III, we compare the size of both the risk-averse and risk-neutral designs, and include SCN utilization statistics. We observe that the risk-averse design utilizes a larger number of agents and edges. We identify a strategy to increase the number of redundancies in the SCN, both in the number of agents used, as well as in the number of edges that can flow components and products when needed.

B. Disruption Instances

The three disruptions we consider as a test set include:

- Tier 4 disruption (mean lead-time of WN-A_1 doubles)
- Tier 3 disruption (mean lead-time of CH-S_1 doubles)
- Tier 2 disruption (mean lead-time of IT-A_4 doubles)

We test the reliability of the SCN designs obtained from each approach by following the procedure described in

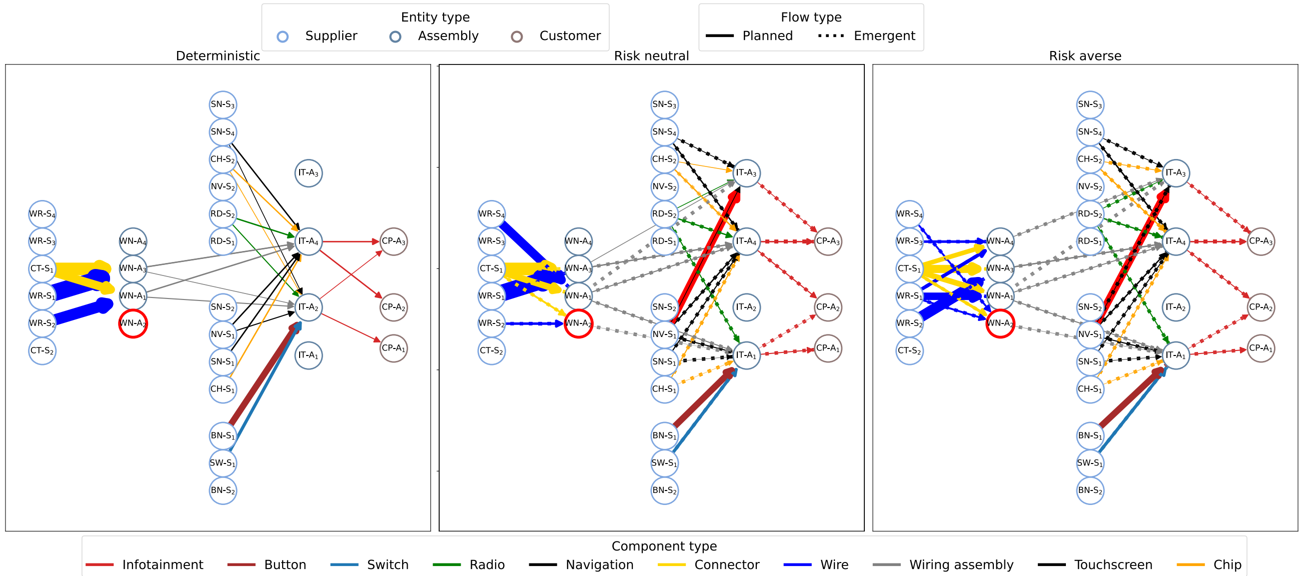


Fig. 4. Optimal SCN designs under different risk attitudes. Arcs can have a combination of planned and emergent flows if they are both solid and dashed lines. The thickness of the line is proportional its flow of product. The deterministic design minimizes the flow volume and number of entities if compared to the stochastic designs. The node with red outline represents an agent that was added in the stochastic solutions. The black line highlighted in red represents backup flows that are added to the risk-averse solution.

Section III-D, by fixing the layout and planned variables to the optimal solutions, and optimizing over the remaining problem, we measure the disruption response of each approach.

C. Benchmark Approaches

In addition to risk-aware Model (4) and risk-neutral Model (1), we consider three greedy heuristics based on cost, capacity and delivery-time criteria. We consider these three criteria to be simplified, risk-neutral approximations, which might be reasonable to use as locally-applied criteria in real environments. In the heuristic algorithms, we start from the uncertain demand average $\bar{\mathbf{d}}$ generated by customers $j \in V^c$, and greedily fulfill it by choosing the best agent $i^* \in U(j)$ based on the selected criterion until we fulfill their demand of product k or the capacity of i^* is finished. When we assign flows, we assign values to variables $x_{i^*,j,k} := \min\{\delta_{j,k}, \bar{q}_{i^*,k}\}$ and $y_{i^*,j,k} := 1$. Then, we generate new demands to be fulfilled for the agents i^* based on their flow and the conversion rate of their upstream products as $\delta_{i^*,k'} \leftarrow \delta_{i^*,k'} + r_{k'k} * x_{i^*jk}, \forall k' \in K'(i^*, k)$. Likewise, we remove the assigned flow from the capacity of the agent, and if the agent has no remaining capacity, we remove it from the set of available upstream agents. We move to the next agent $i \in U(j) \setminus i^*$ if the previous flow did not fully cover the demand. We continue this process until all demand has been met or all capacity has been assigned.

D. SCN Design Performance

We evaluate the performance of the optimal solution to Models (1) (Opt-RN) and (4) (Opt-RA) compared to the greedy heuristics (i.e., cost-minimization (Heu-CST), lead-time minimization (Heu-LT), and capacity-maximization (Heu-CAP), and the deterministic counterpart (DET) of (1).

In terms of delivery performance, the DET and Heu-LT fail to meet the demand, avoiding large lateness penalties. Opt-RN

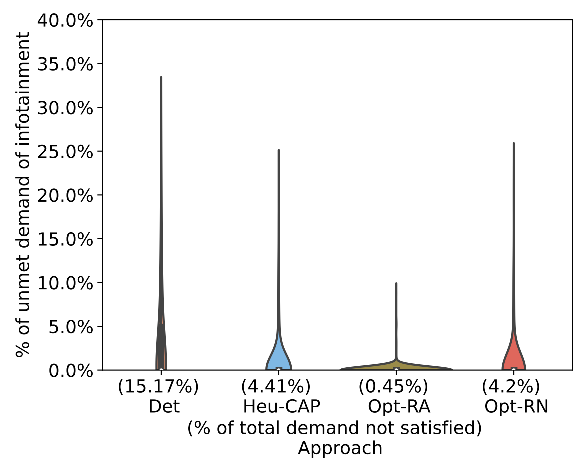


Fig. 5. Violin plot for distribution of unmet demand for entities across all scenarios. Lean and tall distributions represent that a large proportion of customers have a significant amount (y-axis) of their demand not met. Short and thick distributions are desirable, representing meeting demand. Long tails in the distribution represent few cases with large unmet demand. The Opt-RA shows the best performance by having low variance in performance across disruptions with $\alpha = 0.9$. As the cost of each design increases with the robustness to disruptions, we observe a trade off with lateness and unmet demand penalties.

and Opt-RA have minimal unmet demand. With high lateness penalties, all approaches prioritize timely deliveries, trading off high operation cost, and unmet demand. In terms of unmet demand, we observe in Fig. 5 that Opt-RA solution has a more reliable demand meeting performance. By comparing the tail of the violin plots, we observe fewer scenarios with large unmet demand in Opt-RA. In Table IV we evaluate approach costs. DET has the least cost, with high penalties given a minimal SCN design. We observe that the Opt-RN solution fails to always meet the demand, but the penalties are lower in comparison to the heuristics. From this analysis, the multi-objective nature of this problem requires judgment on the priorities that will be set, as is the case with Opt-RA

TABLE IV

SCN Design and Operation Costs Under Tier Disruptions. We Mark in Bold the Highest Quantity of Each Row, Where Ower Costs Are Better. Opt-RA Has a Higher Design Cost (Related to Design Variables $\mathbf{y}$) and More Backup Flow Cost (Related to Emergent Flow Variables $\mathbf{z}$), With Lower Penalties. Following the Discussion in Fig. 5, the Rest of Approaches Minimize Operational Costs. With Extreme Disruptions, Immediate Cost Minimization Incurs Higher Penalties, Becoming an Undesirable Strategy

		Deterministic	Cost heuristic	Risk neutral	Risk averse
Planning	Design ($\mathbf{y}$)	**3.5**	4.6	5.4	6.2
	Flows ($\mathbf{x}$)	**88.2**	107.6	89.1	89.5
Tier 4 disruption (WN-A_1)	Avg. e-flows ($\mathbf{z}$)	**5.8**	13.3	21.5	28.2
	Unmet demand penalty (ρ^d)	29	20	9	**0**
	Lateness penalty (ρ^l)	0	0	0	0
	Total operation cost	**97.5**	125.5	116	123.9
	Total costs	126.5	145.5	125	**123.9**
Tier 2 disruption (IT-A_4)	Avg. e-flows ($\mathbf{z}$)	**5.5**	10.9	19	28.1
	Unmet demand penalty (ρ^d)	27	21	11	**0**
	Lateness penalty (ρ^l)	0	0	0	0
	Total operation cost	**97.2**	123.1	113.5	123.8
	Total costs	124.2	144.1	124.5	**123.8**

All costs ($\times\$10^4$), p-flow: planned flow (variable $\mathbf{x}$) , e-flow: emergent flow (variable $\mathbf{z}$), Util.: % of total flow capacity utilized

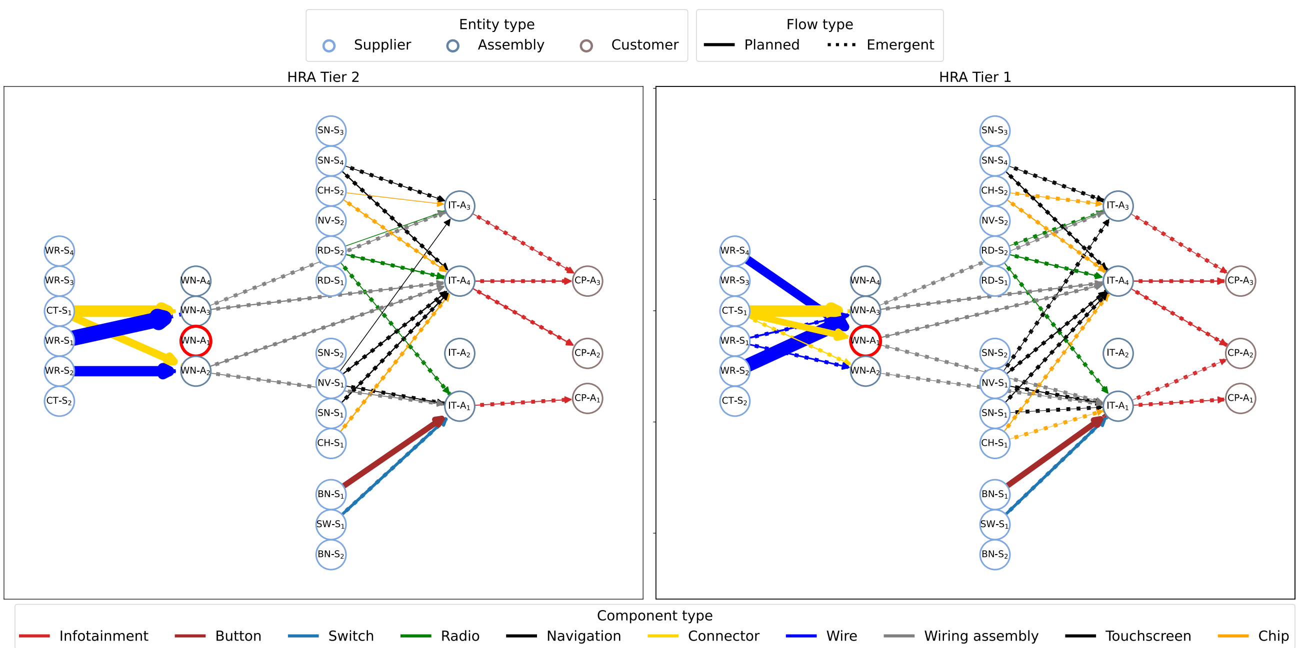


Fig. 6. Optimal SCN designs under heterogeneous risk aversion between tiers. Arcs can have a combination of planned and emergent flows if they are both solid and dashed lines. The thickness of the line is proportional its flow of product. The node outlined in red represents an agent added to the HRA Tier 1 solution, distributing the flow among more agents.

solution, where overly averse attitudes incur larger setup costs for excessive redundancies.

E. Risk Attitudes and Optimal Designs

Given the risk-attitude parameter $\alpha_i, \forall i \in V$, we can represent heterogeneous risk attitudes in the SCN. We study the effect of risk attitude heterogeneity on SCN designs and performance. We consider differences in risk attitudes by tier. We refer to a tier with $\alpha = 0.9$ as highly risk-averse (HRA). Likewise, any tier with $\alpha = 0.5$ is considered moderately risk-averse (MRA). We consider a case where the cockpit assembly (Tier 1) is HRA, and the rest of tiers are MRA. We consider additional cases with tiers 2 and 3 are HRA and the rest are MRA. Figure 6 shows the effect of risk-attitude heterogeneity between tiers on the SCN design. We observe a difference for the suppliers in Tier 5, having less backup flows for HRA Tier 2. On the contrary, the HRA Tier 1 design resembles the design with all tiers with $\alpha = 0.7$. By setting HRA on the customer, we induce higher risk-aversion throughout the SCN, corresponding to the customer-oriented nature of supply chains. We note that in Fig. 7b, when the emergent and planned flow cost ratio $\frac{c^e}{c^f}$ is close to 1, a more risk-averse attitude of $\alpha = 0.7$ does not generate a more conservative SCN (in terms of the number of edges) than an $\alpha = 0.3$. As this ratio increases, the difference in SCN sizes increases. In Fig. 7a, we observe a similar trend in terms of the risk-averse tier, such that highly risk-averse down-stream agents will seek a more conservative SCN as the ratio increases. We note that if the ratio is larger than 2, the optimal SCN reduces its size, as backup flows become more expensive.

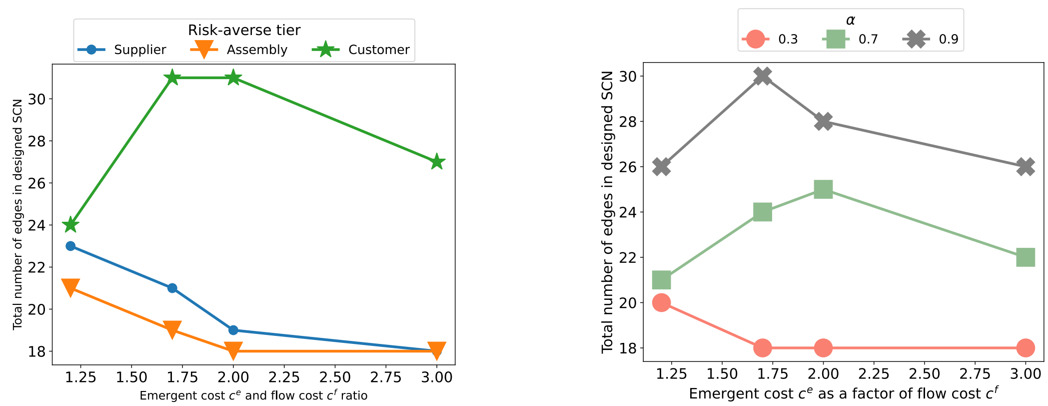
In Table V, we show the performance under disruptions for the heterogeneous risk attitudes. In accordance to the observations of the SCN design, we observe how the highly risk-averse Tier 1 increases the cost of the SCN design as more

TABLE V

SCN DESIGN PERFORMANCE UNDER HRA TIERS. WE MARK IN BOLD THE LOWEST VALUES FOR EACH ROW. IN GENERAL, THE LOWER THE VALUE, BETTER THE PERFORMANCE. HIGH-RISK AVERSE CUSTOMERS (TIER 1) INFLUENCE THE SCN DESIGN INTO A MORE CONSERVATIVE DESIGN, WITH MORE EDGES AND COMPONENT FLOW BETWEEN AGENTS, WITH A HIGHER OPERATION COST, BUT WITH LOWER PENALTIES UNDER DISRUPTIONS IN CONTRAST TO HIGHER RISK AVERSION IN OTHER TIERS

		HRA Tier 1 (Customers)	HRA Tier 2 (Infotainment assembly)	HRA Tier 3 (Suppliers)	HRA Tier 4 (Wiring assembly)
SCN	Total agents	21	21	21	21
	Total edges	41	**38**	**38**	**38**
	Total p-flows	30000	30000	30000	30000
	Avg. edge util. (%)	19	19	19	19
	Avg. agent util. (%)	43	**39**	40	40
Tier 4 disruption (WN-A_1)	Avg. e-flows	5700	**3880**	3950	3950
	Avg. scenarios with e-flows	3	2.5	**2**	**2**
	Unmet demand (%)	**0.9**	3.9	4.7	4.9
	Unmet demand penalty ($\times\$10^4$)	**1**	10	9	9
	Total cost ($\times\$10^4$)	124	**115**	115	115
Tier 3 disruption (CH-S_1)	Avg. e-flows	5120	**3550**	3650	3650
	Avg. scenarios with e-flows	3	2.5	**2**	**2**
	Unmet demand (%)	**0.6**	4.9	4.4	4.4
	Unmet demand penalty ($\times\$10^4$)	**1**	9	8	8
	Total cost ($\times\$10^4$)	124	**114**	115	115
Tier 2 disruption (IT-A_4)	Avg. e-flows	5050	**3460**	3660	3660
	Avg. scenarios with e-flows	3	**2**	2.5	2.5
	Unmet demand (%)	**0.7**	4.0	4.7	4.9
	Unmet demand penalty ($\times\$10^4$)	**1**	10	9	9
	Total cost ($\times\$10^4$)	124	**114**	115	115

p-flow: planned flow (variable $\mathbf{x}$) , e-flow: emergent flow (variable $\mathbf{z}$), Util.: % of total flow capacity utilized



(a) Risk-averse tier analysis

(b) SCN risk-aversion level analysis

Fig. 7. Total number of edges (planned and backup) in the SCN as a function of α and the ratio $\frac{c^e}{c^f}$. Fig. 7a shows the SCN design for different risk-averse tiers, while keeping the rest of tiers risk-neutral. We note that for all ratios, risk-averse downstream agents encourage a larger SCN size in terms of planned edges. Fig. 7b shows the size of SCN for different risk-aversion levels, using the same α for all agents, having a homogeneous risk attitude. We note that for a ratio closer to 1, with cheaper backup flows, the number of edges for different parameters α of 0.3 and 0.7 are similar, 20 and 21, respectively. As the ratio goes beyond two, i.e., backup flows costs are double of planned flows, edges are reduced and replaced by planned flows, trading planned edges' setup cost for expensive backup flows.

TABLE VI

AVERAGE RUNTIME IN SECONDS OF APPROACHES

Scenarios	RN ($\alpha = 0$)	RA ($\alpha = 0.3$)	RA ($\alpha = 0.9$)
200	14.2	33.5	51.4
400	26.7	61.1	183.5
800	54.8	121.3	897.7

RN: Risk neutral, RA: Risk-aware

backups induce a conservative SCN design, incurring less penalties. When the highly risk-averse tier is further back the SCN, its design becomes less customer-oriented, and is biased towards benefiting the specific tier's performance, resulting in more unmet demand penalties. In terms of computation performance, Table VI shows the average runtime for different choices of the scenarios and risk attitudes over 5 replications of the sampling. As we increase the risk-aversion, the problem becomes more difficult to solve by an off-the-shelf solver.

V. DISCUSSION AND CONCLUSION

In this paper, we extended our prior work in [5] by formulating stochastic risk-neutral and risk-aware supply chain optimization models with lead-time awareness. We leveraged the capabilities of stochastic optimization model structure to include new decisions related to the SCN design. We incorporate heterogeneity in the objective of each agent in the supply chain by modifying the risk-attitude for each agent. Through numerical studies, we show that risk-aware objectives are better suited for environments prone to disruption, such as complex supply chains. The presented framework provides an advantage in SCN design as we incorporate both demand and lead-time elements into the decision-making process. Under the possibility of defining the SCN design completely or partially, we are able to introduce disruption response strategies that reduce the need for costly wait-and-see disruption strategies which can incur prohibitively large costs.

We identify that a key feature of risk-averse designs consists of more connected networks. This can be done by leaving idle capacity for agents in case of disruptions, or including agents as backup such that no planned flows exist, but allowing for emergent flows when responding to disruptions. Another form of reliability comes from having fewer indispensable agents. By reducing their relative importance in the supply chain (i.e., by distributing their flow among more agents, or having multiple agents fulfill part of their production), one can set up a backup for the network. In this way, available capacities can be rerouted to adapt to disruptions.

One limitation of our framework is its centralized nature. We could interpret the execution of this optimization framework as a SCN in which all agents communicate their disruptions, share objectives and states to a centralized coordinator which prescribes the optimal actions. This model assumes that all agents act according to the coordinator's prescription. Therefore, next steps in this research consider decentralized optimization approaches, such that we model multi-agent team and game problems, where each agent solves their respective stochastic optimization problem under partial information of the SCN disruptions and other agents' states. We can consider decomposition algorithms for large-scale SCN instances, improving the solution's runtime of off-the-shelf MILP solvers.

In this paper, we consider that disruption risks are independent. Future extensions of this work can model disruptions in lead time and demand as correlated random processes to design response policies and dynamic recovering decisions. By considering a multi-stage extension on this work, we can model dynamic supply chain response to more general disruptions, such as cascading failures over time.

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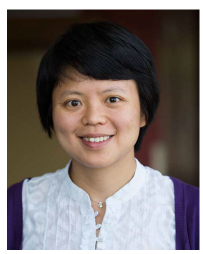

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