A Lead-Time-Aware Decomposition Approach to Optimize Disruption Response in Supply Chains

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***Abstract*—Supply chain (SC) risk management is influenced by both spatial and temporal attributes of different entities (suppliers, retailers, and customers). Each entity has given capacity and lead time to process and transport products to downstream entities. In disruptive events, lead times and capacities may vary, which affects the overall performance of SC. There have been many studies on SC disruption mitigation, but often without considering lead time and the magnitude of lateness. In this paper, we formulate a mixed integer programming (MIP) model to optimize SC operations via a routing and scheduling approach, to model the delivery time of products at different entities as they flow throughout the SC network. We minimize a weighted sum of multiple objectives that involve costs related to transportation, shortages, and delivery lateness. We further develop a Benders decomposition algorithm for speeding up the computation of the NP-hard MIP model. We also develop a discrete-event simulation framework to evaluate the performance of solutions to the MIP model under lead time uncertainty. Through extensive numerical studies, we show how the attributes of SC entities affect the performance, so that we can improve the SC design and operations under various uncertainties.**

***Note to Practitioners*—With increasing uncertainty in global supply chains, inefficient responses to disruptions can lead to large penalties and long-term impacts, such as customer dissatisfaction. This research is motivated by the challenges that arise during supply chain operations under both lead time and demand uncertainties. We employ optimization and centralized control approaches to optimize supply chain network design, as well as response strategies to disruptions, and our framework can handle multiple objectives involving costs related to transportation, shortage, and delivery lateness. We develop a Benders decomposition algorithm for significantly reducing the computational time. We also provide a discrete-event simulation framework to evaluate the performance of solutions in out-of-sample tests, which can be used off-the-shelf by practitioners to evaluate their decisions before realizing the real-world scenarios.**

***Index Terms*—supply chain risk management, multi-objective optimization, mixed-integer programming, lead time uncertainty, discrete-event simulation**

I. Introduction

The design and operations of the supply chain (SC) are impacted by the capacities and production / processing lead times of different entities in an SC network, as well as how the entities are connected and communicate [1]. The global modern SC network has become more complex, leading to more frequent disruptions that affect SC performance [2]. In addition to capacities, the lead time of procurement and production faces increasing uncertainties due to labor shortages and unpredictable global environments in recent years [3], [4].

To cope with lead-time disruptions, enterprises need to incorporate both spatial and temporal attributes of flows and productions into SC design and management [5]. It is crucial to understand how the lead time impacts the response to disruption and SC risk management [6]. In this work, we consider the following decision-making problem: Given a layout of an SC network, existing flows between different entities, and disrupted capacities and/or lead time, how do shortage and lateness penalties, as well as different attributes of the SC entities, affect response decisions?

Furthermore, understanding how different SC attributes impact performance from both the network and the entity levels is critical. Most existing work investigates the effects of network typologies on the resilience of SC, without considering the lead time [7]–[9]. The study in [6] examines the effect of lead time on resilience in different stratification in an SC. However, the authors did not discuss how SC attributes play a role in the response to disruption. Therefore, disruption response models that capture both entity productive capability disruptions and lead-time disruptions are of vital importance to allow efficient adaptation of nominal operations to disruptions. Disruption response models can also be used in practice to perform sensitivity analysis and evaluate investments in capacity expansion or the incorporation of additional backup entities.

To address all the above limitations, the main contributions of this work are as follows. (i) We derive an MIP formulation that tracks temporal attributes of flows through an SC network, while incorporating both fixed and additive penalties on delivery lateness. (ii) We develop a discrete event simulation framework to measure the performance of MIP solutions and the effects of lead-time uncertainty. (iii) We investigate the effects of lateness penalty and network topology on the performance of the lead-time disruption response through several comprehensive SC instances.

The remainder of the paper is organized as follows. In Section III, we introduce detailed notation and the multi-objective MIP model for deriving new flows and response solutions to given SC disruptions. In Section IV, we present the discrete event simulation framework to evaluate the performance of

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the MIP model with uncertain lead time. In Section V, we demonstrate the results of a simulated case study and provide managerial insights. Finally, in Section VI, we conclude the paper and propose future research directions.

II. Literature review

The existing literature on optimization approaches for SCN management are extensive, but mainly focus on the initial SC network planning to achieve network-level lead-time reduction [10]. However, for the disruption response, production and flows that require to be recovered may exist at any entity in an SC network. Enterprises must determine whether and how to vary production and flow plans to meet customer demands. Therefore, delivery times of products are critical elements to be tracked through the entire SC network, so that penalties for product lateness at any point in the SC network can be considered. To the best of our knowledge, no existing work fully addresses such needs and challenges. The work in [11] considers an adaptive agent-based simulation framework having heterogeneous agents. This work only evaluates the performance of fixed response strategies and SCN designs but does not study how to optimize these decisions. In [12], the authors study the effect of faster communication schemes between agents (RFID technology) on the optimal network design. However, all of these formulations do not evaluate SCN performance in the midst of demand or lead-time uncertainty.

Natural disasters, accidents, and road closures are as well as inadequate freight capacities. These problems can cause transportation problems or machine breakdowns, leading to supply shortages that reduce production capabilities, and transportation delays [13]. These events have economic and logistical impacts that are critical to the operation of the SCN and benefit from models that capture the effect of lead time on the original plans. The existing literature models lead-time in SCN operations either as a constraint or as an element of the objective function. In [14], the authors model a multiobjective goal programming approach to optimize the planning of multiperiod SCs given non-negligible lead time; in [15], the authors propose a two-stage multi-period stochastic optimization model with a scenario-based solution approach to enhance the robustness of SC design. However, the formulations in both [14] and [15] only model delivery lateness as backorders that always ship in the next period. In this paper, we include heterogeneity in the agents' objectives from a centralized, optimization-based perspective, integrating agents' lead-time uncertainties.

In [3], the authors use a mixed integer programming (MIP) model for multi-echelon SC considering lead time as hard constraints. They perform extensive numerical experiments to study the feasibility of a network-flow problem with hard lead-time requirements. In [16], authors propose a multi-period, mixed-integer nonlinear program (MINLP) to optimize SC design. They minimize the expected lead time of products by considering the expected lead time of flows. However, these approaches do not consider lead-time disruptions and the corresponding SC response. They also do not model penalties for delivery lateness. The existing literature often models the lead time only for end customers, instead of each entity in the SC [17]. For example, [18] introduces a capacitated network design model that only considers the transport lead time and penalizes late delivery of products to final clients with a fixed homogeneous penalty. Similar strategies are used in [19], where the authors quantify the resilience of the SCN in the objective as a function of the response cost and the recovery time given delays.

In this work, we allow for a model with different penalties for each agent, which we refer to as a heterogeneous SCN. The work in [20] evaluates the performance in a distributed simulation-optimization framework and describes the importance of high-quality initial response plans, for which this paper can be used to provide the input required for such models. We also note that this work can be viewed as a lead-time aware extension [4] of the lead-time neutral centralized MIP formulation presented in our previous work [21]. Following the taxonomy in [10], we present an inventory routing problem, with an objective that incorporates arc and production commitment decisions and operational costs to optimize the response to disruption.

III. Multi-objective MIP

We first introduce an MIP model that tracks product flows and arrival time at each entity in an SC network, such that we can incorporate the magnitude of all late deliveries in the multi-objective function. This model can be viewed as schedule-aware extension to the model proposed in our previous work [21], which only considered a network-flow problem.

A. Notation and assumptions

Consider a directed graph $G(V, A)$ representing an SC network with a vertex set V of all SC entities and an arc set A of all their connections. We denote $V^c \subset V$ as the subset of entities that only demand products, named customers; $V^o \subset V$ denotes the subset of entities where transformations of products occur, named OEMs; $V^s \subset V$ is the subset of entities who supply products and raw materials, named suppliers (i.e., having no upstream flows). The arcs in the set A convey potential flows of products and resources between entities. We denote K as the set of all products and components types within the SC. Furthermore, for each $k \in K$, we have a subset $K'(k) \subset K$ of components k' required for the production of k. We make the following assumptions in this paper: (i) Knowledge of disruptions and response time are immediate. (ii) The flow of the product k from i to j is treated as an indivisible unit. (iii) All entities wait until all required upstream flows have been received before sending their downstream flows. We summarize the parameters and decision variables used in this work in Table I.

B. Multi-objective MIP for SC disruption response with lead time awareness

We compute the objective of our MIP as the total SC operation cost in (1), where $f, I, p, \beta, \zeta, \Delta^l, \Delta^u, a, o, z$ are the

TABLE I
A SUMMARY OF NOTATION FOR PARAMETERS AND VARIABLES.

Input Parameters	
d_{ik}	demand of product k at entity i; by convention, $d_{ik} < 0$.
c^{f}_{ij}	fixed transportation cost from entity i to entity j.
c^{u}_{ijk}	unit transportation cost of flowing product k from entity i to j.
$\overline{f}_{ij}$	mixed-flow capacity of arc (i, j).
$\overline{p}_i$	mixed-product production capacity available at entity i.
e_{ik}	production cost per unit product k at entity i.
$r_{kk'}$	conversion rate from product k to product k', i.e., the units of product k consumed to produce one unit successor product k'.
ϕ_i	fixed cost of opening a production line at entity i.
I^0_{ik}	initial inventory of product k at entity i at each period.
h_{ik}	unit holding cost of product k at entity i.
ρ^d_{ik}	penalty per unit of unsatisfactory of demand d_{ik}.
t_{ik}	time at which entity i requires the demand of product k.
l_{ijk}	lead time of product k flowing from entity i to entity j.
ρ^{f}_{ijk}	fixed late-delivery penalty of product k flowing via arc (i, j)
ρ^{u}_{ijk}	penalty for unit of lateness product k flowing via arc (i, j).
Decision Variables	
f_{ijk}	units of product k flowing from entity i to entity j.
β_{ijk}	arc utilization, 1 if arc $(i, j) \in A$ is used to transport product k, and 0 otherwise.
p_{ik}	units of product k produced at entity i.
ζ_i	binary variable, equal to 1 if entity i produces/assembles materials, and 0 otherwise.
I_{ik}	inventory of product k at entity i by the end of time period.
Δ^d_{ik}	units of unsatisfied demand of product k at entity i.
Δ^l_{ijk}	lateness of product k arriving at entity j from entity i.
a_{ijk}	time of delivery of product k to entity j from entity i.
o_{ik}	time at which entity i can process product k.
z_{ijk}	binary variable, equal to 1 if flow from entity i to entity j of product k is delivered late, and 0 otherwise.

decision variables shown in Table I. We model the cost of transportation, manufacturing, and product holding, respectively in (1a), fixed costs for transportation route setting, manufacturing capacity in (1b), and the penalties for unmet demand and late deliveries in (1c).

$$\mathcal{J}(y, \beta, p, \zeta, I, \Delta^l, \Delta^u, a, o, z)$$
$$= \sum_{(i,j)\in A, k\in K} c^{\mathrm{f}}_{ij} f_{ijk} + \sum_{i\in V, k\in K} h_{ik} I_{ik} + \sum_{i\in V, k\in K} e_{ik} p_{ik} \tag{1a}$$
$$+ \sum_{(i,j)\in A,\ k\in K} c^{\mathrm{u}}_{ij} \beta_{ijk} + \sum_{i\in V} \phi_i \zeta_i + \tag{1b}$$
$$+ \sum_{i\in V, k\in K} \rho^d_{ik} \Delta^d_{ik} + \sum_{(i,j)\in A, k\in K} \rho^{\mathrm{f}}_{ijk} z_{ijk} + \rho^{\mathrm{u}}_{ijk} \Delta^l_{ijk}, \tag{1c}$$

We present the overall MIP model to optimize SC network flows and operations in (2):

$$\min_{\mathbf{x},\mathbf{y}}\ \mathcal{J} \tag{2a}$$
$$\text{s.t.} \sum_{j:(i,j)\in A} \sum_{k'\in K} f_{ijk'} - \sum_{j:(j,i)\in A} f_{jik} + \sum_{k'\in K} r_{k'k} p_{ik'}$$
$$- p_{ik} - \Delta^d_{ik} + I_{ik} = d_{ik} + I^0_{ik},\ \forall\, i \in V,\ k \in K, \tag{2b}$$
$$f_{ijk} \le \overline{f}_{ij} \beta_{ijk},\ \forall\, (i, j) \in A, k \in K, \tag{2c}$$
$$\sum_{k\in K} f_{ijk} \le \overline{f}_{ij},\ \forall\, (i, j) \in A, \tag{2d}$$
$$\sum_{k\in K} p_{ik} \le \bar{p}_i \zeta_i,\ \forall\, i \in V, \tag{2e}$$
$$a_{ijk} = (l_{ijk} + o_{ik})\beta_{ijk}, \forall\, (i, j) \in A, k \in K, \tag{2f}$$
$$o_{jk} \ge a_{ijk'},$$
$$\forall\, (i, j) \in A, i \in V \setminus V^{\mathrm{s}},\ k \in K, k' \in K'(j, k), \tag{2g}$$
$$o_{ik} = 0, \forall\, i \in V^{\mathrm{s}}, k \in K, \tag{2h}$$
$$a_{ijk} - \Delta^l_{ijk} \le t_{jk}, \forall\, (i, j) \in A, k \in K, \tag{2i}$$
$$\Delta^l_{ijk} \le \mathcal{M} z_{ijk}, \forall\, (i, j) \in A, k \in K \tag{2j}$$
$$f_{ijk}, I_{ik}, \Delta^d_{ik}, \Delta^l_{ijk}, a_{ijk}, o_{ik} \ge 0,$$
$$\forall i \in V, (i, j) \in A, k \in K, \tag{2k}$$
$$\zeta_i, \beta_{ijk}, z_{ijk}\ \in \{0, 1\},$$
$$\forall\, i \in V,\ \forall\, (i, j) \in A, k \in K, \tag{2l}$$

We denote the vector of structural decisions as $\mathbf{x} = [\beta, \zeta]^\top$ and the vector of operational decisions as $\mathbf{y} = [f, p, \Delta^l, \Delta^u, a, o, z]^\top$. The constraints include:

1) Flow balance: Constraints (2b) balance the flow of products at each entity in the SC and compute unmet demand.

2) Capacities: In (2c), (2d), (2e), flows on each arc and productions at each entity are restricted by given capacities.

3) Delivery times: With constraints (2f), we model the delivery time of products at different entities. Note that we can model products with different processing and transportation lead times. With constraints (2g)–(2h) we compute the time at which downstream flows are ready to be processed (i.e., variable o_{ik}), depending on the readiness of upstream products the entity requires. In constraints (2i), we compare the delivery time of products a_{ijk} with the due date t_{jk}, such that we can penalize the units of lateness of flow. In constraints (2j) we use an indicator variable to check whenever the delivery is late (via a big-M approach) and impose a fixed penalty. We can introduce heterogeneous delivery deadlines at any entity of the SC, such that we can model SC networks that not only require the on-time delivery of products to final customers but also at intermediate stages of the SC network.

4) Variable domains: Constraints in (2k) and (2l) specify the domain of decision variables.

C. Linearization of bilinear MIP

We note that the size of the MIP (2) can grow significantly with the number of agents, products, and arcs in the SCN we model. In particular, the bilinear constraints (2f) are difficult to solve by MILP solvers. Thus, we linearize these constraints via McCormick convex estimators [22], by introducing auxiliary variables $\mathbf{m} = [m_{ijk}, (i, j) \in A, k \in K]^\top \ge 0$ representing the product of variables o and β via a big-$\mathcal{M}$ approach. We now replace constraints (2f) with the block of constraints (3).

$$m_{ijk} \le \mathcal{M}\beta_{ijk}, \forall\, (i, j) \in A,\ k \in K, \tag{3a}$$
$$m_{ijk} \le o_{ik}, \forall\, (i, j) \in A,\ k \in K, \tag{3b}$$
$$m_{ijk} \ge o_{ik} - (1 - \beta_{ijk})\mathcal{M}, \forall\, (i, j) \in A,\ k \in K, \tag{3c}$$
$$a_{ijk} \ge l_{ijk}\beta_{ijk} + m_{ijk}, \forall\, (i, j) \in A,\ k \in K, \tag{3d}$$
$$m_{ijk} \ge 0, \forall\, (i, j) \in A,\ k \in K. \tag{3e}$$

Considering the linearization in (3), we can use off-the-shelf MILP solvers to solve Model (2) directly. We will now present a decomposition formulation admitting an efficient solution approach.

D. Benders decomposition

Model (2) can be a large MILP to solve, given the number of binary variables β, ζ which grows with the number of arcs and production-capable nodes in the SCN. Our aim is to decompose the problem such that we solve a series of small binary linear programs and many larger linear programming (LP) formulations. If we consider the LP relaxation of (2) with respect to the binary indicator z, relaxing it into a continuous variables bounded on $[0, 1]$, then for a fixed β, ζ associated with the binary decisions of the arc and production commitment, the rest of the variables and their respective components in the objective function and constraints form an LP. Therefore, we can decompose the problem into two stages: a first-stage formulation that is a binary linear program with a relatively lower number of variables and constraints and a second-stage LP with the operational variables. In formulation (5), we capture the first stage model, where we minimize the arc utilization and production commitment decisions $\mathbf{x} = [\beta, \zeta]^\top$. To improve the quality of the first-stage solutions, we combine constraints (2b)–(2c), and projecting out the rest of variables and parameters to have an valid linear constraint of β:

$$\sum_{j:(i,j)\in A} \sum_{k'\in K} \beta_{ijk'} \geq \beta_{lik}, \forall (l,i) \in A, k, k' \in K. \qquad (4)$$

here, constraints (4) ensure that for any incoming arc into a in a non-customer agent, an outgoing arc must exist in an optimal solution. Otherwise, we can reduce the objective by having the corresponding $\beta = 0$, while keeping feasibility of the current product flows. We can encode these constraints, as well as a similar consistency constraints for production commitment decisions ζ, in vectorized form

$$\min_{\mathbf{x}} \left\{ [\mathbf{c}^{\mathbf{x}}]^\top \mathbf{x} + \mathcal{Q}(\mathbf{x}) : D\mathbf{x} \geq \mathbf{b} \right\}. \qquad (5)$$

We consider the second stage model as a function $\mathcal{Q}$ of structure decisions $\mathbf{x}$. We note that given the unmet demand and lateness slack variables, for any first-stage variable values, the second-stage is feasible (relatively complete recourse). We reformulate the second-stage model in matrix form, such that we can have B, T, q as the coefficient matrix for the second-stage variables $\mathbf{y}$, the coefficient matrix for the first-stage variables $\mathbf{x}$, and the right-hand size vector, respectively. These matrices encode constraints (2b)–(2l), as we model a relaxation of (2). Thus, we obtain second-stage model (6)

$$\mathcal{Q}(\mathbf{x}) = \min_{\mathbf{y}} \left\{ [\mathbf{c}^{\mathbf{y}}]^\top \mathbf{y} : B\mathbf{y} \geq \mathbf{q} - T\mathbf{x} \; [\pi] \right\}. \qquad (6)$$

To solve (5), we implement the *L-shaped* method, also known as Benders Decomposition [23] such that we can iteratively lower approximate Model (2) via LP strong duality of the second stage with a lower computational effort. We will introduce an auxiliary variable θ as the epigraph of $\mathcal{Q}$. We represent adding linear constraints with respect to $\mathbf{x}$ representing supporting hyperplanes for $\mathcal{Q}$ at the evaluated solutions $\hat{\mathbf{x}}$ in previous iterations. By LP strong duality, we consider the dual problem (7) of Model (6)

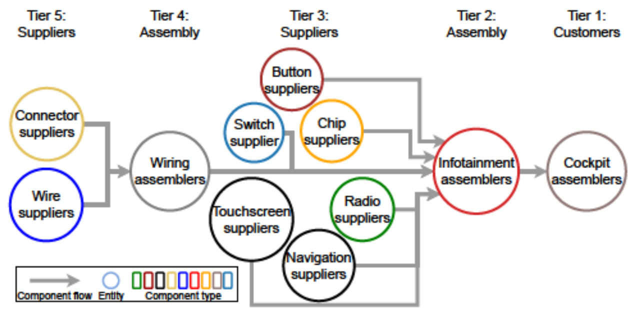


Fig. 1. The SC network for the case study.

$$\max_{\pi} \left\{ \pi^\top (\mathbf{q} - T\mathbf{x}) : \pi^\top B \leq \mathbf{c}^{\mathbf{y}} \right\}. \qquad (7)$$

At iteration i of the algorithm, for any evaluated first-stage solution of the algorithm $\hat{\mathbf{x}}^i$ of (6) and associated second-stage optimal dual π^i maximizing (7), we can add the supporting hyperplane (8) to the first-stage epigraph

$$\theta \geq [\pi^i]^\top (\mathbf{q} - T\mathbf{x}). \qquad (8)$$

By using the current epigraphic approximation, we can solve the problem (9) and obtain a lower bound on (2) at iteration i, given that $\theta \leq \mathcal{Q}$.

$$\begin{aligned} \min_{\mathbf{x}, \theta \geq 0} \quad & [\mathbf{c}^{\mathbf{x}}]^\top \mathbf{x} + \theta && (9a) \\ \text{s.t.} \quad & A\mathbf{x} \geq \mathbf{b} \\ & \theta \geq [\pi^j]^\top (\mathbf{q} - T\mathbf{x}), \forall j \in 1, \ldots, i. && (9b) \end{aligned}$$

We note that we only obtain $\theta = \mathcal{Q}$ in (9) when we have added enough hyperplanes to represent $\mathcal{Q}$ around the optimal solution by solving the second-stage (6). Similarly, given the first-stage solution $\hat{\mathbf{x}}^i$ and the corresponding second-stage solutions $\hat{\mathbf{y}}^i$, we can compute the objective corresponding to (2), which is an upper bound. Given these upper and lower bounds, we can compute an optimality gap that can be employed to define a stopping criteria for the decomposition algorithm.

IV. Case Study Description

A. SC instance

The SCN instance, for which we develop our numerical studies, is an adapted automotive cockpit assembly sub-SCN for the In-vehicle infotainment system introduced in [24]. We consider cockpit assemblers to be the customers in this sub-SCN, considering the demand for cockpits for complete vehicle assembly. This sub-SCN is complex enough to be analyzed on its own, as infotainment systems constantly evolve with the integration of new technologies, making them vulnerable to disruptions of the semiconductor market [25]. In Fig. 1, we show the structure of the SCN with 5 tiers, considering 10 different product categories, considering the components of the wiring and infotainment that require assembly within the SCN. We design the instance to include options that process more than one product (with cheaper cost, higher lead time,

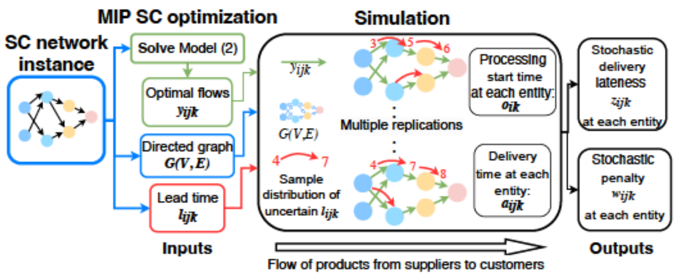


Fig. 2. Simulation framework for out-of-sample testing.

and shared production capacity) and consider uncertainty from customer demand and lead time. We include agents trading-off cost, lead-time mean and variance, and/or capacity.

B. Simulation framework

Simulation models have been used as an evaluation mechanism for resilience to disruption in general supply chains [26], as well as automotive SCN [27]. Thus, we consider a simulation model as our method to evaluate the response solutions and compare them to no-response and lead-time neutral response policies. Fig. 2 depicts inputs for the discrete event simulation model we consider, the general simulation process, and the simulation outputs. We initialize the simulation with the flows from the suppliers (i.e., entities in subset V^s). We draw independent samples of the joint realization of the lead time from a log-normal probability distribution with $\mu = l_{ijk}$ (i.e., the deterministic lead time l_{ijk} used in Model (2)) and $\sigma = 0.3$. We compute the delivery information with variable a_{ijk} considering flows f_{ijk}. We use the product structure information to later compute the starting time for processing the downstream flows, with variable o_{ik}. We continue this process iteratively until the final products are delivered to all end-customers. We run the simulation for multiple replications in parallel to analyze SC performance under different disruption realizations.

C. Case study scenario design

1) Baseline scenario: We solve the MIP model (2) for the instance described in Section IV-A, selecting parameters to ensure a nominal operation. We will compare the performance of this non-disrupted solution to the solutions of disrupted instances that are obtained by re-optimizing model (2) under the new parameter realizations as our baseline results.

2) Delivery lateness policies: We perform a series of studies to analyze the effect of lateness penalties on SC disruption responses under uncertain lead time. We interpret different unit-penalty values as policies chosen by decision makers and model these policies through different ratios between the unit and fixed cost of penalizing lateness. Instances in which lateness does not matter, are considered to have neither unit nor fixed penalty. This lead-time neutral approach is equivalent to the model considered in our prior work [21]. For other instances, we consider a policy that only contains a unit penalty (ratio 1:0), and policies that consider increasing fixed penalties (ratios 1:5 and 1:20), to model situations in which being late by any amount is undesirable. The values 5 and 20 as fixed penalty cost are chosen to be proportional to the volume of flows in the current instance such that the fixed penalty dominates the total delivery lateness penalization as the ratio increases. In practice, these different lateness policies could happen in different industries, e.g., in industries that frequently allow backorders, industries with perishable products, or those that fit in a larger SC with tight due dates.

3) SC entity disruptions: We identify the baseline SC network instance (Fig. 3) as a tiered network structure. Sequential connections between different entities exist [7]. This configuration is justified by the process of assembling different components to yield products for end customers.

We aim to study the effect of lead-time disruptions in different depths of the SCN (i.e., how far back in the product flow process do disruptions occur) on the SC disruption response. We focus on lead-time disruptions, as they represent transportation disruptions that can occur along the SCN. Examples include global disruptions affecting supplier delivery times and output capabilities, technological disruptions considering machine breakdown that affect assembly entities or natural disruptions causing blockages in logistical routes that can affect the distribution of end products. For a review of these and other transportation disruptions, see [28]. We design three scenarios considering the SC network in Fig. 1: (i) Disrupt one wire supplier by reducing their output capacity in half, representing supplier problems to satisfy demand that is external to the current supply chain. (ii) Disrupt two infotainment assembly agents by increasing their lead time by a factor of 1.5 and increasing their transportation costs, as alternative logistic routes are required. (iii) Disrupt the wiring suppliers, by increasing the arc usage costs by 50%, representing greater competition between suppliers that negotiate to deal with higher volumes of orders.

4) SC topology: The baseline SC network has multiple entities capable of providing, assembling, or distributing materials and final products. Its topology follows a "tree-like" structure in which the in-degree of entities reduces as the sequence of entities gets closer to the end customers. This strategy has high cost due to multiple contractual relationships with different suppliers, but can provide the flexibility to respond to disruptions by choosing backup entities [8].

We study the resiliency of different network configurations to understand their performance under entity disruption and lead-time uncertainty. We compare the baseline configuration with a "chain-like" structure with fewer backup entities, without removing productive capacities in the SC (i.e., we remove multiple suppliers and assemblers). This configuration has the least cost, as the economy of scale benefits from having larger contracts with fewer entities. For both topologies, we have the same total capacity between all entities and the same demand. We optimize the non-disrupted instance, and then disrupt similar agents in each topology setting, to compare out-of-sample performance of the response given by the original solution.

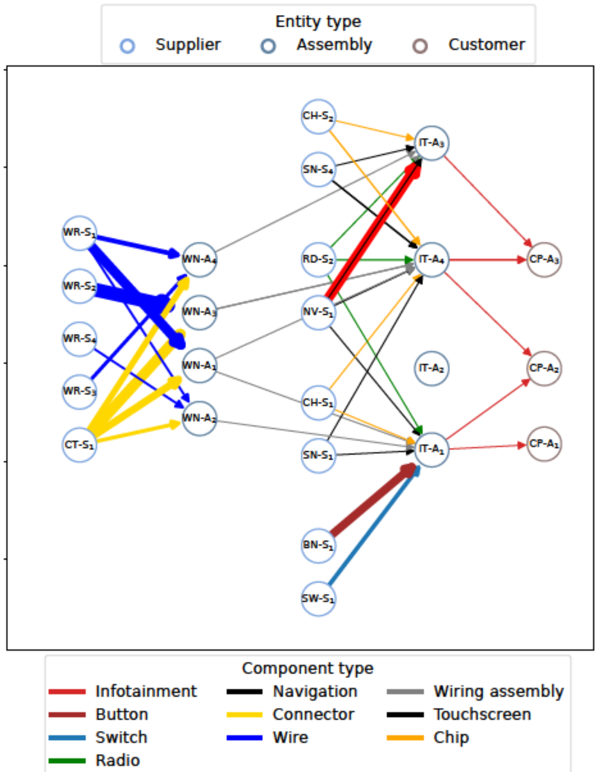


Fig. 3. Optimal flows from solving MIP model (2) for baseline instance.

V. Case Study Results

We perform numerical studies for the instances we describe in Section IV, testing the performance of solutions of Model (2). For each instance, we first solve Model (2) and obtain the optimal flows f_{ijk} and production p_{ik}. Then, we run 300 replications of the simulation framework in which we fix f_{ijk} and p_{ik} to test the average performance given stochastic lead times l_{ijk} and demand d_{ik}. We compare the simulated performance of each solution, given different parameter configuration (disruption depth, lateness policy, and SC network structure). We measure SCN effectivity and timeliness by comparing the delivery lateness of final products and the percentage of unmet demand.

A. Computational results

We implement the solution approaches and run all instances using an Apple M2 Silicon CPU with 8 cores and 8 threads (4 cores 3.5 GHz and 4 2.8 GHz), and 16 GB of memory. We use Gurobi 11.0 API for Python 3.11.8

We compare the runtime of directly solving Model (2) with Gurobi and the relaxations we discussed in Section III-D. We solve the baseline instance and a synthetic instance generated by increasing the making copies of the entities and the arcs in the SCN as a larger-scale instance. We set the corresponding demand and capacities by sampling over a normal distribution centered around the values in the base instance. The size of this instance is five times that of the baseline, and we employ to consider more challenging instances for the computational comparison.

TABLE II
Runtime comparison for direct Gurobi solve vs Benders decomposition

Solution approach	Base instance Gap (%)	Base instance Runtime (s)	Synthetic instance Gap (%)	Synthetic instance Runtime (s)
Gurobi	-	35.4	-	1214.2
Relaxation Gurobi	2.4	25.3	6.5	850.3
Relaxation Benders	2.5	12.2	7.1	246.7

In order to solve the instances in the case study, we use the Benders decomposition to solve the lower bounding problem, and then use those solutions to warm start the solution of (2). This procedure leads to an efficient lower bounding procedure that generates a high-quality initial solution to exactly solve (2).

B. Case study results

1) Solution of baseline: We solve the MIP model (2) with the parameter design described in Section IV-C1. Fig. 3 shows the flow solutions, which we consider as the *optimal* SC operational plan.

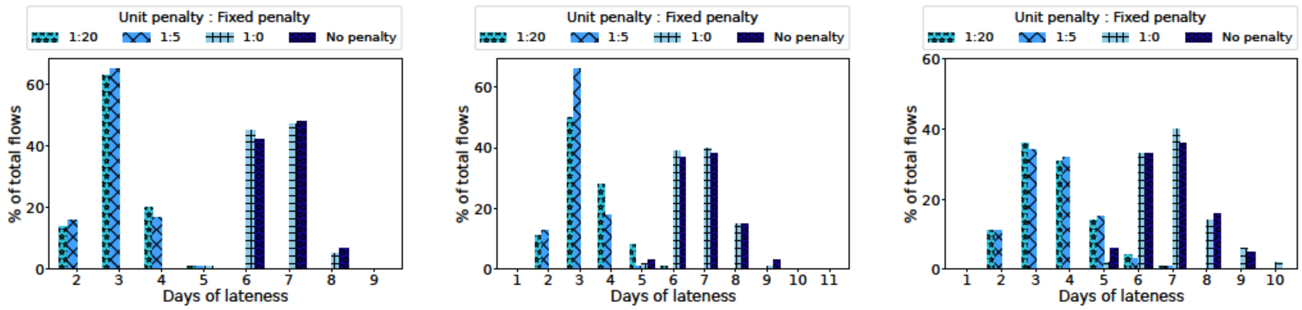
2) SC network redesign for different lateness policies: We consider the disruption scenarios described in Section IV-C3 and apply the different lateness penalties described in Section IV-C2 to disrupt the baseline instance to optimize model (2) under the SCN disruption, obtaining optimal solutions for each scenario. We define three categories to describe how the network changes compared to the baseline scenario in the order of SC redesign severity. (i) K: Keep current SC, flow volumes, and incur the lateness penalty. (ii) E: Interdict arcs connecting the disrupted entity with downstream entities and redesign flow solutions. (iii) V: Interdict entity by redesigning the SC without the disrupted entity. Table III shows that given the hierarchical structure of the SC network [7], as the depth of the disrupted agent increases, SC redesign is encouraged. This explains why in the case of Fig. 4(a) most penalties on lateness cause the interdiction of the entity, and by the contrary, in Fig. 4(c), only with a very high penalty, the SC layout changes.

TABLE III
Identified strategy from MIP model (2) optimal solution to entity disruptions under different network structures and lateness policies

Scenario Disruption	Structure	**(Unit penalty : Fixed penalty)** No penalty	1:0	1:5	1:20
i WR_S	Tree	K	E	V	V
	Chain	K	K	K	R
ii WN_A	Tree	K	K	E	V
	Chain	K	K	K	R
iii IT_A	Tree	K	K	R	E
	Chain	K	K	R	E

K: keep flows; E: interdict arcs; V: interdict entity; R: reduce flows.

Fig. 4 demonstrates the effectiveness of the strategies prescribed by our lead-time-aware MIP model, to mitigate the effect of disruptions in the delivery of final products. From the distribution of days of lateness, MIP model (2) solutions



(a) Disruption at suppliers WR-S_3, WR-S_4.

(b) Disruption at wiring assembler WN-A_2

(c) Disruption at infotainment assembler IT-A_3

Fig. 4. Out-of-sample performance of solutions under different lateness policies and entity disruptions for a final product of the SC.

enjoy shorter lateness, when we simulate the uncertain lead time in the SC network.

3) Network structure in SC performance and disruption response: We consider additional SC network structures described in Section IV-C4. Fig. V shows the optimal flows for each topology without disruptions. The chain structure has lower cost in comparison to the other two structures. The caveat of this topology, as it relies on most entities being used at full capacity.

We perform disruption response studies similar to those in Section V-B2. Fig. 5 illustrates the average lateness and standard deviation as error bars of each network structure over all of the lateness policies. We observe that the tree structure, which establishes backups further back in the SC, shows higher reliability, with a consistent performance invariant to the disrupted tier of the SC. The chain structure performance suffers the most under entity disruptions. Only a limited number of changes to the structure can take place, making almost all entities critical.

In addition to the response strategies discussed in Section V-B2, we identify an additional action shown in the our MIP model (2). When considering a policy without fixed penalties (ratio 1:0), the optimal solution prescribes strategy (R) to reduce the volume of affected flows while keeping the SC layout. We interpret this as a last-resort strategy, when it is inevitable to incur in unmet demand penalties, as a trade-off to use the released capacity to work on other products that still can be delivered on time. Table III shows the type of responses we identify for the different scenarios. Results in Table III demonstrate that the chain structure has the least flexibility, as in most cases only the strategy (K) to absorb lateness penalties is possible. Finally, only the tree structure avoids strategy (R), while meeting demand in all scenarios.

C. Managerial insights

From the previous results shown in Section V, we point out the following managerial insights.

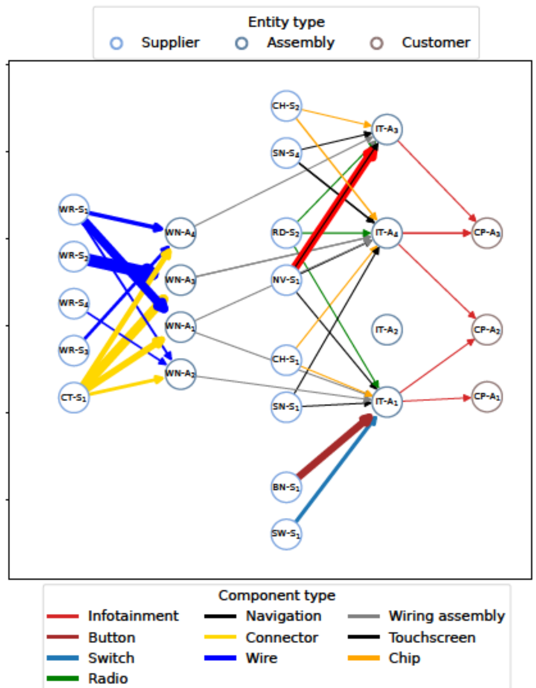
- Lead time is a critical factor to model. The temporal component of material flows can make an optimal solution with the least cost highly undesirable because of lead-time disruption and delivery lateness.
- There is a trade-off between the response to disruption and the mitigation strategies. Fixed penalty policies favor costly network-wide redesign to keep flows on time and limit unmet demand. Unit penalty policies mitigate disruptions with local modifications of the network structure, with a larger change as penalties become larger.
- Network-wide changes include redesigning a network by removing arcs associated with disrupted entities. Local modifications include minimizing the influence of disrupted entities by reducing flows through them and redistributing them to other available entities.
- As the effect of lead-time disruption of entities accumulates over the sequence of flows in the SC network, entity redundancy further back into the SC (e.g., suppliers) yields more reliability. In contrast, redundancy of entities closer to end customers (e.g., assembly) shows less resiliency, since disruptions accumulate less over the remaining sequence of flows, having a lower influence on overall SC performance, as they can be easily replaced by other entities with similar capabilities.

VI. Conclusion

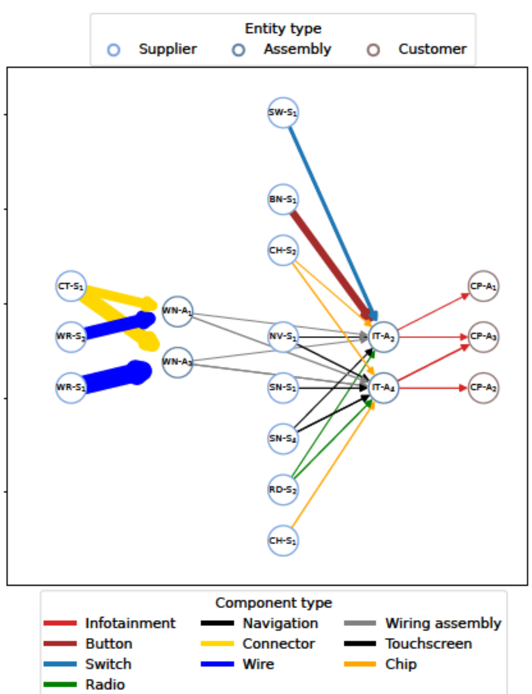
In this paper, we develop an MIP model to track the delivery time of flows throughout an SC network, incorporating different lateness and unmet demand penalties for each agent in the SCN. Via numerical studies, we show that a hierarchical network structure with backups in the initial suppliers yields consistent SC performance given lead-time uncertainty of the SC entities, as opposed to having backups further down the SC. This framework provides an advantage in SCN operations as we incorporate both demand and lead time elements into the decision-making process.

In this paper, we consider the disruption risks to be independent. Future extensions of this work can model disruptions in lead time and demand as correlated random processes to design response policies and dynamic recovery decisions.

In future research, one can consider communication and coordination time lags to incorporate real-life environments which can be modeled by dynamic optimization. Given the centralized nature in this research, consider decentralized optimization approaches such that we model multi-agent team and game problems, where each agent takes actions under



(a) Optimal plan for baseline tree structure.



(b) Optimal plan for chain structure.

partial information of the SCN disruptions and other agents' states.

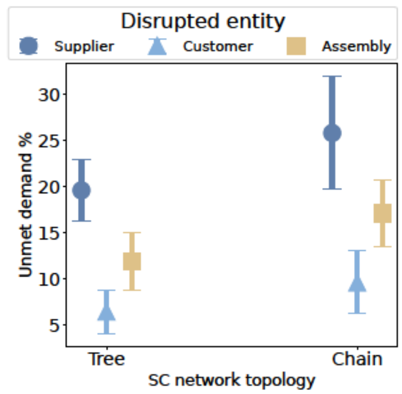


Fig. 5. Average delivery lateness of different SC network typologies under diverse entity disruption.

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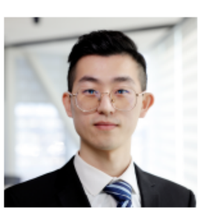

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