Global Product Design Platforming: A Comparison of Two Equilibrium Solution Methods

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Global product platforms can reduce production costs through economies of scale and learning but may decrease revenues by restricting the ability to customize for each market. We model the global platforming problem as a Nash equilibrium among oligopolistic competing firms, each maximizing its profit across markets with respect to its pricing, design, and platforming decisions. We develop and compare two methods to identify Nash equilibria: (1) a sequential iterative optimization (SIO) algorithm, in which each firm solves a mixed-integer nonlinear programming problem globally, with firms iterating until convergence; and (2) a mathematical program with equilibrium constraints (MPEC) that solves the Karush Kuhn Tucker conditions for all firms simultaneously. The algorithms' performance and results are compared in a case study of plug-in hybrid electric vehicles where firms choose optimal battery capacity and whether to platform or differentiate battery capacity across the US and Chinese markets. We examine a variety of scenarios for (1) learning rate and (2) consumer willingness to pay (WTP) for range in each market. For the case of two firms, both approaches find the Nash equilibrium in all scenarios. On average, the SIO approach solves 200 times faster than the MPEC approach, and the MPEC approach is more sensitive to the starting point. Results show that the optimum for each firm is to platform when learning rates are high or the difference between consumer willingness to pay for range in each market is relatively small. Otherwise, the PHEVs are differentiated with low-range for China and high-range for the US.
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1 Introduction

Increased globalization has provided opportunities for companies to reduce costs by creating common platforms across products sold in different countries [1]. Examples of product platforms include the Black & Decker universal motor, which is used across 122 products available globally [2,3], Sony's Walkman portable audio player [4], Intel's microprocessor platform [5], HP's Deskjet printer platform [6], and passenger vehicle models sold in different countries that are built with the same chassis and transmissions but have different engines to better fit differing consumer preferences or regulations across countries [7–9]. Such product platforming can lower production costs by taking advantage of cost reductions that are possible with increased production quantity. A profit-maximizing firm must determine whether the cost reductions of choosing a common platform outweigh the loss of market share that may result from limiting customization for each market.

Existing literature on product platforming has developed methods to solve individual firm platforming problems [1,10–21] but they do not account for the interaction with competing firms, which affect the conditions under which platforming is profit-optimal [22]. In order for companies to determine profit-optimal strategies for global platforming, they will need to weigh the factors of differing consumer preferences, market sizes, and competition with other companies across different countries with the potential cost savings from commonality. Thus, finding profit-optimal platforming decisions in the context of a market (Nash) equilibrium is needed.

Solving an optimal platforming problem for firms in equilibrium is challenging because (1) the mathematical relationships between firm profits, component design decisions, and product attributes of interest to consumers frequently involve continuous variables and nonlinear and non-convex functions, and (2) the decision of whether to create a common platform across products in different markets is discrete. So, choosing whether or not to create a platform and finding optimal design decisions generally constitutes a non-convex mixed-integer nonlinear programming (MINLP) problem for each firm, where each firm's optimization problem depends on the decisions of competing firms.[2] Existing product platform optimization approaches do not determine market equilibrium solutions [1,10–20,23], and existing equilibrium approaches typically assume continuity or linearity and are not applicable to platform problems where each firm's profit optimization is MINLP [24–40].

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[2]Certain product platforming problems can be represented in less complex forms such as INLP problems or those having convex design spaces, which we consider to be special cases. The algorithmic approaches we develop can be used to solve for these special cases as well, but we focus on an MINLP problem in this study.

Table 1 Categories of relevant design optimization studies that address product platforming and/or competing firms in equilibrium

	Single firm	Multiple firms in equilibrium
No platforming decisions	Many examples [45–55]	Product line oligopoly studies [22,26,27,30–40]
Optimal product platform	Product platform optimization [1,10–21,23]	This paper

We develop two approaches of solving for optimal platforming equilibria: (1) an iterative procedure solves each firm's profit maximization problem conditional on the decisions of other firms using a global MINLP solver, and the algorithm iterates each firm's profit optimization sequentially with competitors fixed until no firm can improve their profits given the decisions of competitors (called sequential iterative optimization (SIO)) and (2) the Karush Kuhn Tucker (KKT) conditions of all firms' profit maximization problems are solved directly for all possible combinations of platforming decisions (a mathematical program with equilibrium constraints (MPEC)). We compare the algorithms' performance and results in a case study of plug-in hybrid electric vehicles (PHEV) where each firm chooses whether or not to platform the battery pack design across the US and Chinese markets and chooses the optimal battery capacity, which determines the vehicle's all-electric range (AER). In the PHEV case, economies of scale are small at current battery production volumes [41], so we focus on quantity-based cost reductions from learning-by-doing that decrease costs as the plant gains experience producing the batteries [42–44]. Optimal platforming depends on the learning rate as well as the difference between consumers' willingness to pay (WTP) for AER across countries. Results show that the optimum for each firm is to platform when learning rates are high or the difference between consumer willingness to pay for AER in each market is relatively small. Outside of these cases, the optimum is to not platform and instead to produce a low-range PHEV in China and a high-range PHEV in the US.

1.1 Literature Review. Our problem addresses optimal product platform decisions among competing firms in equilibrium. We reviewed the literature in engineering, economics, operations research, and marketing and could not find any prior work that addresses this scope. The most closely related work can be grouped into two approaches: (1) optimal product platform problems for a single firm or decision-maker that do not account for competing firms [1,10–21,23] and (2) oligopolistic equilibrium of competing firms designing product lines without platforming, often restricted to linear programming problems [26,27] or only continuous decision variables [22,30,40]. See Table 1 for a summary of this literature and the contribution of our approach.

Studies in the first category that optimize product platforms typically either use stochastic methods like genetic algorithms to identify which sets of components should be common across product variants [10,14,20], or nonlinear programming (NLP) formulations in which the decision of whether components should be common across product variants is set exogenously or relaxed to a continuous approximation [1,11–13,15–17,19,21,23]. These studies focus on an individual firm without accounting for the strategic interaction of competing firms in equilibrium. There are a few studies that incorporate an equilibrium solution into platform design, but the equilibrium of interest in these studies is between players internal to the firm, rather than between competing firms [17,18,21]. See Table 2 for a summary of this category of literature.

It is interesting to note that platforming studies in this category of literature do not model quantity-based cost reductions as dependent on sales (which is a function of product design decisions and pricing). Instead, they implicitly model quantity-based cost reductions as resulting from the similarity among product variant designs. This simplification misses an important mechanism of quantity-based cost reductions: all else equal, designing a product so that consumer sales will be higher can lead to cost reductions, e.g., though economies of scale or learning-by-doing. This creates a tradeoff within the platforming problem between the quantity gains from using common components and the quantity losses from lower sales because the product is not customized for each market. Thus, approaches that do not account for the implications of platforming on sales risk recommending platforming when it is not profit-optimal.

In the second category of literature, equilibrium among competing firms is found but the approaches do not include platforming decisions or allow the firms' problems to be MINLP. Methods of solving for market (Nash) equilibrium in this literature either use

Table 2 Examples of product platform and portfolio optimization in the engineering design literature

Study	Case study	Description
Simpson et al. [1]	General aviation aircraft and universal electric motor	Product platform concept exploration method
Simpson and Dsouza [10]	General aviation aircraft	Multiobjective two-level genetic algorithm
Martin and Ishii [11]	Water cooler	Generational variety index between products, coupling index between components
Messac et al. [12]	Universal electric motor	Product family penalty function
Fellini et al. [13]	Race car	Multiobjective Pareto problem
Kumar et al. [14]	Universal electric motor	Segmented market-driven product family design
deWeck et al. [15]	Vehicles	Two-level platform/product formulation
Gonzalez-Zugasti et al. [16]	Spacecraft	Common platform compared against performance and budget constraints
Khire and Messac [23]	Active building envelope system	Relaxes MINLP to NLP using variable segregating mapping function
Khajavirad and Michalek [19]	Bathroom scale	Relaxes MINLP to NLP using commonality index, enforces platforming via analytical target cascading
Khajavirad et al. [20]	Universal electric motor	Multiobjective genetic algorithm determining platform selection and design in addition to variant design
Du et al. [17]	Universal electric motor	Two-level Stackelberg model coupling module selection and module design scaling
Miao et al. [18]	Gear reducer	Two-level genetic algorithm coupling platform and customization optimization
Moon and McAdams [21]	Vehicle design	Cooperative game between products in a single firm

Table 3 Examples of product line optimization methods that find market equilibrium with either genetic algorithm, integer linear programming (ILP), NLP, or formulations of MPECs restricted to continuous variables

Study	Case study	Description
Liu et al. [26]	Generic representative product	Integer nonlinear programming
Liu et al. [27]	Mobile phones	Integer nonlinear programming
Shiau and Michalek [30]	Vehicle design	MPEC restricted to continuous variables
Shiau and Michalek [22]	Vehicle design	MPEC restricted to continuous variables
Whitefoot et al. [31]	Vehicle design	NLP restricted to continuous variables
Whitefoot and Skerlos [32]	Vehicle design	NLP restricted to continuous variables
Whitefoot et al. [33]	Vehicle design	NLP restricted to continuous variables
Michalek et al. [34]	Vehicle design	NLP restricted to continuous variables
Shiau and Michalek [35]	Pain reliever, weight scale, and power grinder	MPEC restricted to continuous variables
Shiau et al. [36]	Vehicle design	MPEC restricted to continuous variables
Choi et al. [37]	Pain reliever	MPEC restricted to continuous variables
Horsky and Nelson [38]	Vehicle design	MPEC restricted to continuous variables
Rhim and Cooper [39]	Liquid detergents	Genetic Algorithm
Luo et al. [40]	Angle grinder	MPEC restricted to continuous variables

a genetic algorithm approach that does not guarantee convergence to an optimum or they restrict the firm's problem so that it is not MINLP. These restrictions include solving the firm's problem as an integer nonlinear programming (INLP) problem [24,26,27] or an NLP problem [31–33] or solving for equilibrium using an MPEC where the first order conditions for optimality of each firm are restricted to continuous variables [22,30–40]. See Table 3 for a summary of this category of literature.

Solving platforming problems in equilibrium is complicated by the lack of equilibrium solution methods (even outside of the context of product design) that allow firm problems to be MINLP. Existing methods of solving for equilibrium with mixed-integer approaches require simplifying assumptions that do not often apply to the platforming problem. For example, equilibrium methods developed for supply chain design and electricity markets allow for integer variables but assume properties like linearity or convexity [24,26,27]. Methods developed to solve for negotiations of competitive players, e.g., in cases of customer allocation or contract negotiations, use MINLP approaches to solve for equilibrium, but these approaches require specific cooperative behavior among players such that the equilibrium conditions can be represented by a "Nash product," which allows them to be solved as a single MINLP problem rather than representing competing firms as solving their own MINLP problems [25].

We extend the prior literature by developing approaches to solve equilibrium for competing firms where each firm's profit maximization problem is MINLP. Specifically, we build upon prior work that developed approaches to solving equilibrium for product design problems (especially [30,33]) to propose two approaches to solving equilibrium problems that involve both mixed-integers and non-convex functions: an SIO that sequentially solves each firm's optimization problem as an MINLP using a global solver (while holding decisions of other firms fixed) until convergence, and an MPEC approach that–for each combination of the integer variables—searches for points that simultaneously satisfy the first order optimality conditions with respect to continuous variables for all firms and checks each candidate point to verify Nash conditions.

2 Platform Optimization Problem

We represent the global platforming problem as a set of firms each deciding design variables and prices for a set of products sold in multiple markets and determining whether or not to produce each product on a common global platform. Once these decisions are made, they are fixed for a time period of T years. Firms choose the decisions that will maximize the net-present value of profits gained from these products over the T years. The profit maximization problem for a single firm is defined as follows in Eq. (1):

$\max\limits_{y_j, \mathbf{x}_{ij}, p_{ij} \forall i, \forall j \in \mathcal{J}_k} \Pi_k = \sum\limits_{j \in \mathcal{J}_k} \sum\limits_{i \in \mathcal{M}_j} \sum\limits_{t=1}^{T} \frac{(p_{ij} - c_{ijt}) q_{ijt}}{(1+r)^t} \quad (1)$	Maximize the net present value of future profit (discounted revenue less cost)
subject to	
$\mathbf{x}_{\mathrm{MIN}} \le \mathbf{x}_{ij} \le \mathbf{x}_{\mathrm{MAX}} \quad \forall i \in \mathcal{M}, j \in \mathcal{J}_k$	Design variables are within simple bounds
$\mathbf{g}(\mathbf{x}_{ij}) \le 0 \quad \forall i \in \mathcal{M}, j \in \mathcal{J}_k$	A vector of engineering constraints is satisfied for each product j in each market i
$[y_j = 0] \vee \begin{bmatrix} (\mathbf{x}_{ij} - \mathbf{x}_{i'j}) = 0 \\ \forall i, i' \in M \end{bmatrix} \forall j \in \mathcal{J}_k$	For each product, either there is no platform or platform variables are common across markets
where	
$q_{ijt} = q_t(p_{ij}, \mathbf{x}_{ij}, p_{ij'}, \mathbf{x}_{ij'} \quad \forall j' \in \mathcal{J})$	Demand for product j in market i depends on the price and design variables of all products
$c_{ijt} = y_j c(\mathbf{x}_{ij}, Q_{ijt})$	Cost for product j in market i in year t depends on design variables and cumulative production volume
$Q_{ijt} = y_j Q_{ijt}^{\mathrm{P}} + (1 - y_j) Q_{ijt}^{\mathrm{NP}}$	Cumulative production volume across all markets is used for products on a common platform, and cumulative production volume within a single market is used for products that do not share a common platform
$Q_{ijt}^{\mathrm{NP}} = \sum_{\tau=1}^{t} q_{ij\tau}$	
$Q_{ijt}^{\mathrm{P}} = \sum\limits_{\tau=1}^{t} \sum\limits_{i' \in \mathcal{M}} q_{i'j\tau}$	
$p_{ij} \in \mathbb{R} \quad \forall i \in \mathcal{M}, j \in \mathcal{J}_k$	Each product has a real-valued price for each market
$\mathbf{x}_{ij} \in \mathbb{R}^n \quad \forall i \in \mathcal{M}, j \in \mathcal{J}_k$	Each product has a vector of n design variables that must match across markets when sharing a common platform (when $y_j = 1$)
$y_j \in \{0, 1\} \quad \forall j \in \mathcal{J}_k$	Each product has the option to build all markets on a common platform $y_j = 1$ or not $y_j = 0$

In this formulation, $i \in \mathcal{M}_j$ indexes the set of markets in which product j is sold; $j \in \mathcal{J}$ indexes the set of products, including the subset of products $\mathcal{J}_k \subset \mathcal{J}$ produced by firm k; $t \in \{1, 2, \ldots, T\}$ indexes time in years; Π_k is the net present value of profit for firm k; p_{ij} is the price of product j in market i; $\mathbf{x}_{ij}$ is a vector of design variables for product j in market i; y_j is a binary variable that is one when product j uses a common platform across markets and zero otherwise; r is the discount rate; $\mathbf{g}(\mathbf{x}_{ij})$ is a vector of engineering design inequality constraints that must be satisfied; q_t is a function that computes demand for one product in one market at time t given the prices and design variables of all products in that market; Q_{ijt} is the cumulative demand for product j at time t for market i, depending on whether the product is on a common platform Q^{P}_{ijt} or not Q^{NP}_{ijt}; and c is a function that computes unit production cost as a function of product design variables and cumulative production volume i. If the product uses a common platform ($y_j = 1$), the cumulative production volume includes production in all markets; if not, only market i is counted toward cumulative production volume.

Straightforward extensions of this core model may include: (1) additional design variables that can be differentiated across markets even when sharing a common platform; (2) price and/or design variables that can vary over time; (3) demand models whose parameters vary over time; (4) cost models that vary over time for reasons other than cumulative production volume; (5) options to use a common platform across a subset of markets rather than a single global platform; (6) platforms that impose constraints other than equality of design variables; and (7) engineering design constraints that vary across products and/or markets.

Note that the disjunction in the formulation in Eq. (1) requires that when a product is built on a common platform, its design variables must match across markets. This constraint can be implemented in a number of ways, including representing it directly as a disjunction (where branching in a branch-and-bound-based algorithm occurs directly on the logical cases); using "the Big M reformulation"; or using "the convex hull reformulation," among other approaches [56]. We adopt the Big M reformulation, which represents the disjunction as the constraint: $-(1-y_j)M \leq (\mathbf{x}_{ij} - \mathbf{x}_{i'j}) \leq (1-y_j)m \quad \forall i, i' \in \mathcal{M},\ j \in \mathcal{J}_k$, where m is a constant large enough such that the constraint is dominated and non-binding when $y_j = 0$. We choose to set $m = 2(\mathbf{x}_{\mathrm{MAX}} - \mathbf{x}_{\mathrm{MIN}})$.

To complete the formulation for a particular instance, the demand function $Q(p_{ij}, \mathbf{x}_{ij}, p_{ij'}, \mathbf{x}_{ij'}\ \ \forall j' \in \mathcal{J})$ and the cost function $c(\mathbf{x}_{ij}, Q_{ijt})$ must be defined. In our case study, consumers in the same market are modeled using a multinomial logit model [57]. Demand is thus computed using Eq. (2)

$$q_{ijt} = q(p_{ij}, \boldsymbol{x}_{ij}, p_{ij'}, \mathbf{x}_{ij'}\ \ \forall j' \in \mathcal{J}) = s_{it}\frac{e^{v_{ij}}}{\theta + \sum_k \sum_{j' \in \mathcal{J}_k} e^{v_{ij'}}} \quad \forall i, j, t \tag{2}$$

where s_{it} is the fixed size of market i in time t; the deterministic portion of consumer utility, $v_{ij} = \alpha_i(p_{ij} + \boldsymbol{\gamma}_i^T \mathbf{z}(\mathbf{x}_{ij}))$, follows Helveston et al. [58]; α is the scale parameter; $\boldsymbol{\gamma}_i$ is a vector of average WTP coefficients per unit increase in each of the product attributes $\mathbf{z}$; each element in $\mathbf{z}$ is a function of $\mathbf{x}_{ij}$, and θ is the exponentiated utility of the outside good, representing all other options beyond the products offered by the oligopoly firms.

Note that production quantity in each year is a function of product prices and design decisions made at $t = 0$. While in general firms may adjust product prices between redesign cycles, in the case of products like automobiles, these changes are small, often on the order of 1% or less [59,60].

2.1 Solution Methods.

There are several possible methods for numerically solving an equilibrium problem in which multiple firms in competition simultaneously maximize Eq. (1). We investigate two: an SIO approach and an MPEC approach. In the SIO approach, each firm in sequence solves Eq. (1) using a global MINLP algorithm, holding the decisions of other firms fixed at their most recent values, until no firm can unilaterally improve their profits. In the MPEC approach, for each permutation of the binary platforming decision variables, an MPEC is solved searching the remaining continuous variables to find points that satisfy the KKT conditions of Eq. (1) for all firms simultaneously. Each candidate solution is then checked with one iteration of a global solver per firm to verify the Nash criteria that the MPEC point found is a global solution for each firm, conditional on the decisions of other firms. In both approaches, we use multistart with a grid of starting points to search for multiple equilibria. A flowchart of the two algorithm approaches can be seen in Fig. 1.

2.1.1 Sequential Iteration Optimization Approach.

For the SIO approach, branch-and-reduce optimization navigator (BARON) software was chosen to solve the inner-loop of each firm's optimization problem. BARON is a global optimization algorithm that solves non-convex MINLP problems using convexification. In contrast to other MINLP algorithms [61,62] and stochastic approaches [63], convexification-based branch and bound algorithms are guaranteed to converge to global optima within termination tolerances even when the problem is non-convex, so long as certain criteria are met (e.g., factorable functions, bounded domains, etc.) [61].

In the SIO algorithm, optimal decisions found using BARON to solve Eq. (1) are then used to calculate demand for products in competition with the products of the next firm. Each firm's optimization problem is then solved in sequence repeatedly until each firm's profits in its latest turn differ from its profits in its previous turn by less than a relative iterative tolerance ϵ. (Pseudocode for the SIO approach for the case study problem is presented in the SI available in the Supplemental Materials on the ASME Digital Collection.)

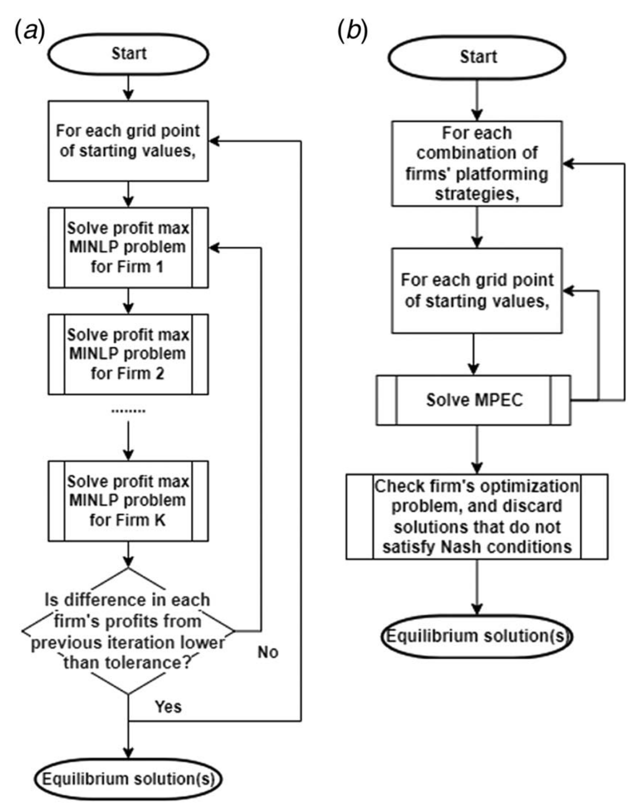

Fig. 1 (*a*) Flowchart of the operation of the SIO algorithm and (*b*) flowchart of the operation of the MPEC algorithm (extended from Shiau and Michalek [30])

A potential drawback of the iterative approach is that the conditions under which it is guaranteed to find multiple equilibria if they exist are not known (to the authors). Because of this potential drawback, we compare results to those of the alternative MPEC approach.

2.1.2 Mathematical Program With Equilibrium Constraints Approach. The MPEC approach solves the first-order necessary KKT conditions of Eq. (1) for each firm simultaneously. A potential advantage of this approach is the potential to search more efficiently in a single run in the multi-dimensional space that includes decision variables of all firms to identify first-order points that are candidates for Nash equilibrium, rather than repeatedly alternating search among orthogonal subspaces that represent each firm's decisions. Potential disadvantages include: (1) KKT conditions apply only to the continuous variables for each fixed combination of discrete variables, so the discrete platform variables must be enumerated exhaustively, resulting in multiple MPEC problem instances; (2) computational effort may be wasted finding first-order points that are not Nash equilibria; (3) searching a higher-dimensional space can introduce computational and memory challenges, and (4) KKT conditions are only necessary conditions under certain constraint qualification conditions, so it is possible that this approach could miss any equilibria at points that do not satisfy constraint qualification.

We analytically derive the KKT conditions of each profit-maximizing firm's platform and design optimization problem that hold at equilibrium (under assumptions of regularity, continuity, and smoothness [64,65]) for each discrete combination of platforming decisions. KKT conditions for a related problem in which a firm has no ability to platform have been derived by Yip et al. [66] and are extended here for the platforming problem in Eq. (1). In this formulation, we derive stationarity conditions separately for each possible value of the platforming variable y.

The first-order Lagrangian stationarity conditions of Eq. (1) with respect to the firm's vehicle prices are

$$\sum_{t=1}^{T}\sum_{j\in J_k}\sum_{i\in\mathcal{M}_j}\frac{\frac{\partial q_{ijt}}{\partial p_{i'j'}}(p_{ij}-c_{ijt})+q_{ijt}\left(\frac{\partial p_{ij}}{\partial p_{i'j'}}-\frac{\partial c_{ijt}}{\partial p_{i'j'}}\right)}{(1+r)^t}=0 \quad \forall j'\in J_k, \forall i'\in\mathcal{M}_j \tag{3a}$$

It should be noted that in most formulations of a firm profit function, cost is not dependent upon price (of either the same product or other products), and thus the $\partial c_{ijt}/\partial p_{ij}$ term drops out [66]. However, when costs reduce with production volume (via learning-by-doing or economies of scale) and quantity is dependent upon price (of the same and other products in the market), this derivative is not equal to zero. Thus, it is necessary for us to retain this term in our formulation.

The first-order Lagrangian stationarity conditions of Eq. (1) with respect to design decisions of the firm's products, using matrix calculus notation, are

$$\sum_{t=1}^{T}\sum_{j\in J_k}\sum_{i\in\mathcal{M}_j}\frac{\frac{\partial q_{ijt}}{\partial \mathbf{x}_{i'j'}}(p_{ij}-c_{ijt})-q_{ijt}\frac{\partial c_{ijt}}{\partial \mathbf{x}_{i'j'}}}{(1+r)^t}-\left(\frac{\partial \mathbf{g}}{\partial \mathbf{x}_{ij}}\right)^{\top}\boldsymbol{\mu}_{ij}=0 \quad \forall j'\in\mathcal{J}_k, \forall i'\in\mathcal{M}_j \tag{3b}$$

where $\boldsymbol{\mu}_{ij}$ is the vector of Lagrange multipliers associated with the constraints $\mathbf{g}$. The partial derivatives of q_{ij} and c_{ijt} shown in the equations are presented in the SI available in the Supplemental Materials.

Finally, feasibility, positivity, and complementarity require the following conditions for all of the firm's products

$$\mathbf{g}(\mathbf{x}_{ij})\le 0 \quad \forall j\in\mathcal{J}_k, \forall i\in\mathcal{M}_j \tag{3c}$$

$$\boldsymbol{\mu}_{ij}\ge 0 \quad \forall j\in\mathcal{J}_k, \forall i\in\mathcal{M}_j \tag{3d}$$

$$\boldsymbol{\mu}_{ij}^{\top}\mathbf{g}(\mathbf{x}_{ij})=0 \quad \forall j\in\mathcal{J}_k, \forall i\in\mathcal{M}_j \tag{3e}$$

At the Nash equilibrium, the KKT conditions defined above in Eqs. (3a)–3(e) will hold simultaneously for every firm [67].

When the complementarity conditions in Eq. (3e) are treated as constraints in an equilibrium problem, they violate constraint qualifications at every feasible point—a fundamental challenge for MPECs. Several solution strategies have been developed in order to address this issue, most commonly by relaxing the constraint [68]. We utilize an approach analyzed by Scholtes [69], in which Eq. (3e) can be relaxed as follows:

$$\boldsymbol{\mu}_{ij}^{\top}\mathbf{g}(\mathbf{x}_{ij})\le \varphi \quad \forall j\in\mathcal{J}_k, \forall i\in\mathcal{M}_j \tag{3f}$$

where ϕ is a relaxation parameter, which allows solutions of Eq. (3) to approach solutions of Eq. (1) as $\varphi\to 0$ while avoiding the constraint qualification implications of the MPEC's complementarity constraint.

In our case study, we solve the MPEC problem using the interior point method implementation of the fmincon function in the MATLAB optimization toolbox.

3 Plug-In Hybrid Battery Case

We model the learning associated with producing a PHEV Li-ion battery with prismatic cells over a 5-year time period, which is a typical time period of automotive design cycles before a major redesign occurs [70]. We model automakers as choosing the battery capacity for each of their vehicle models by scaling up a modular battery pack; this is done implicitly by drawing from a cost model in which electrode dimensions, the number of electrodes per cell, and the number of cells in each pack can be adjusted to reach the desired battery capacity [71]. The chosen battery capacities may be the same or differentiated across the US and Chinese markets depending on the automakers' battery platforming decisions. Once these battery platforming and design decisions are made by the automakers at time $t=0$, they are considered fixed over the 5-year time period. When an automaker chooses to platform battery designs across global markets, production of these batteries is assumed to occur in the same facility (operated by either the automaker or a supplier). When the automaker chooses not to platform and instead customizes the battery designs for different markets, production of the different battery designs is assumed to occur in different facilities. We assume that the different facilities share the same operating conditions—including processing, labor, machine, and facility characteristics and costs—but production cost reductions that occur with increased experience producing a particular battery design does not transfer to a different facility producing a different battery design.[3]

3.1 Sources of Quantity-Based Cost Reductions. In this section, we review PHEV Li-ion battery production and identify potential sources of quantity-based cost reductions in the battery production process. Battery cell and pack production consist of the following sequence of steps: preparation of coated-electrode sheets; electrode slitting and drying; stacking, welding, enclosing, and sealing of electrodes into cells; charge retention testing; assembly of cells into modules; final assembly of modules into packs; pack testing; and shipping [41]. Each of these steps involves labor, energy use, and equipment and maintenance costs; all of which can potentially incur cost reductions as gains in production efficiency accrue.

[3]Prior work investigating cost reductions as a result of learning-by-doing indicates that learning can transfer (or "spillover") between technologies that are sufficiently similar [72,73], but this effect is smaller than cost reductions due to learning within the same product design.

The product platforming algorithms we develop are generally applicable to any source of quantity-based cost reductions of the same subsystem design, including economies of scale and learning-by-doing. However, prior work estimates that the effects of PHEV Li-ion battery economies of scale are small at current production volumes, even when considering only the production volume for vehicles in the US PHEV market alone [41,74] or the Chinese PHEV market alone [41,75]. This implies that over a 5-year period where the battery design parameters are held fixed, economies of scale are not the driving factor of cost reductions that may be possible through platforming. Thus, we focus on another source of quantity-based cost reduction that has the potential to significantly influence PHEV Li-ion battery platforming decisions: learning-by-doing.

Learning-by-doing is the phenomenon by which the unit cost of a product decreases at a decreasing rate as manufacturers produce more of it [44]. The types of learning that can occur within a fixed battery pack design produced over a 5-year time period consist of major changes in the production process, such as changes in equipment or equipment processing parameters requiring significant expense, and minor changes to the production process, such as labor learning, improved use of tooling, and standardization of production steps, requiring comparatively less expense [76,77]. Empirical estimates of these sources of learning have found that, in certain industries, major changes only account for 20–25% of learning while the rest is made up of minor changes requiring significantly less investment, including labor learning and improvements in tooling [78]. Examples of these cheaper efficiency improvements within battery production may include increased efficiency of laborers operating electrode slitting machines and the use of improved tooling that could make enclosing cells easier [41]. Examples of major changes within battery production may include the improvement of capital equipment such as material mixers and cell control laboratories, resulting in cheaper equipment costs or more efficient equipment use [41].

Learning that occurs as a result of major changes to the production process or changes in the battery cell design is out of scope of this work. This includes production changes that require large scale investment, such as significantly increasing automation of production or changing production steps from batch to continuous processing. We also do not consider learning associated with improvements in the battery chemistry or type of cell (i.e., changing from prismatic cells to different geometries) as the battery design is considered fixed over the 5-year time period.

Recent work has highlighted that material costs serve as a floor on pack cost in the near-term that is not itself subjected to the cost effects of learning-by-doing as much as other aspects of pack assembly [79]. As such, we model material costs as not subject to cost-reductions through learning.

3.2 Modeling Cost Reductions From Learning. Following Schmidt et al. [80], learning is modeled as a power curve relating the percentage of cost reductions to the learning rate, κ

$$a = AQ^{log_2(1-\kappa)} \tag{4}$$

where Q is the cumulative quantity produced, A is a parameter that represents the unit cost of producing the first unit, and a is the unit cost of producing the Qth unit. Notice that each individual unit produced incrementally reduces in cost according to the learning curve.

In our case, we are interested in calculating a firm's total production costs over a fixed time period of battery pack production incorporating the effects of learning. This can be approximated as the integral under the curve from the cumulative production quantity of that firm at the beginning of the time period to the end: $\int_{Q_1}^{Q_T} a(Q)dQ = A\int_{Q_1}^{Q_T} Q^{log_2(1-\kappa)}dQ$. To calculate this quantity, we define $\xi = \int_{Q_1}^{Q_T} Q^{log_2(1-\kappa)}dQ$ and the average of this integral, $\bar{\xi} = \xi/(Q_T - Q_1)$, such that $A\bar{\xi}$ is the average unit cost produced over the time period. As such, $\bar{\xi}$ represents the average fraction of unit costs that were cut via learning over the time period relative to the first unit produced.

The average fraction unit cost reduction over the time period t is as follows:

$$\bar{\xi}_{ijt} = \frac{\int_{Q_{ij(t-1)}}^{Q_{ijt}} q^{log_2(1-\kappa)}dq}{(q_{ijt})} \tag{5}$$

where Q_{ijt} is the cumulative production volume of battery pack j for market i at the end of time period t, q_{ijt} is the annual production volume, each product j sells in a set of markets $\mathcal{M}_j$, and κ is the learning rate—i.e., for every doubling of Q_{ijt}, the cost of the battery pack decreases by $100\kappa\%$. We assume that if a firm is platforming, learning transfers perfectly across all production of the same platformed product components, since production is assumed to occur in the same facility, as defined in Eq. (1). In this way, the cost reductions incurred by platforming represent a tradeoff against the increased market share incurred by product customization as described by Eqs. (1) and (2).

Research on EV battery learning following similar formulations has reported learning rates from 4.4% to 21% [79–81].[4] Since platforming across markets allows a firm to sell greater quantities of the same product component, learning-based reductions in cost are an incentive for firms to platform in our case study. Learning is not assumed to cross firms or other battery designs within the firm in the case study—empirical estimates have found that knowledge transfer across firms and products is less significant than learning within the product [44,82].

3.3 Derivation of Cost Function. PHEV production costs depend on battery pack attributes as well as production quantity. Yuksel et al. [71] provide PHEV battery pack cost estimates as a function of PHEV AER based on a detailed process-based model of production costs that used equipment, materials, and labor data from Argonne National Laboratory's BatPaC model [83]. In their model, Yuksel et al. represent the production cost implications of increasing battery capacity by scaling up a modular battery pack where electrode dimensions, the number of electrodes per cell, and the number of cells in each pack can be adjusted to reach the desired battery capacity. We fit a second-order polynomial[5] to these data modeling the battery pack production costs as follows:

$$c_{ij}^{PACK} = \beta_2 z_{ij}^2 + \beta_1 z_{ij} + \beta_0 \quad \forall i, j \tag{6}$$

where z_{ij} is the AER of vehicle model j sold in market i, which is a function of battery capacity, x_{ij}.

The unit cost of the PHEV can then be calculated as follows:

$$c_{ij}(x_{ij}, y_j, Q_{ijt}) = d + mx_{ij} + bc_{ij}^{PACK}\bar{\xi}_{ij}(y_j, Q_{ijt}) \quad \forall i, j \tag{7}$$

where c_{ij} is the unit cost of the PHEV depending on whether or not it has a global battery pack platform, y_j. The parameter d represents the production cost for all non-battery pack vehicle components and assembly. The second term, mx_{ij}, represents the battery material costs that scale with the battery capacity, and are not subject to learning. The final term in Eq. (7) represents the portion of costs of the battery pack that are influenced by learning. We use the normalization factor b to calibrate the total unit battery pack costs estimated

[4]These learning rates had widely ranging methods of estimation, some of which simply tracked aggregate drops in battery costs over time with respect to increasing production across the entire market. We would not anticipate that these estimates would necessarily reflect the estimates of within plant or firm learning. However, because there is a lack of empirical estimates of these learning rates, we take the approach of scenario analysis where we analyze the optimal platform and battery design variables across a wide range of possible learning rates.

[5]Because Yuksel et al. [71] only estimate production costs for three different values of AER, it is difficult to know the functional form of this relationship. Based on evidence that battery production costs can increase at an increasing rate with AER [71,84], we choose a second-order polynomial as our primary specification. We additionally fit a linear form of the relationship to the data from Yuksel et al. and re-ran our model. The estimates of the linear cost model and equilibrium results of the platforming problem using this linear model are presented in the SI available in the Supplemental Materials.

in Yuksel [71] to those of the 2019 Toyota Prius Prime as a case study vehicle (hereafter referred to as the "Prius Prime") [85]. In effect, b positions the battery pack costs on the point on the learning curve that corresponds to the cumulative production quantity of battery packs for the Prius Prime in 2019. The term $\bar{\xi}_{ij}$ then accounts for further cost reductions from moving further down the learning curve as cumulative production quantity continues to increase.

Note that the inclusion of $\bar{\xi}_{ij}$ in the cost model means that firm decisions that increase the production quantity of the same battery design, including product prices and design decisions, put downward pressure on costs through learning-by-doing. If the firm chooses to platform products across global markets, then the magnitude of this effect increases because of the larger combined market size for the global platform. (In practice, platforming across geographic boundaries may incur transport costs and regulatory or other factors complicating learning transfer, however, we leave these out of the scope of our study. These factors could moderate the cost reduction incurred by platforming on products that require extra transportation or regulatory costs to sell the platformed product in multiple markets.)

We calibrate the vehicle base cost d by subtracting an assumed 20% markup from the manufacturers' suggested retail price (MSRP) of \$24,945 for a 2019 Prius Prime [85,86], and further subtracting a contemporary estimate of pack-specific cost for Li-ion EV batteries of \$156/kWh [79] scaled to the vehicle's battery capacity of 8.8 kWh [85] minus material costs of \$50/kWh [79]. Finally, we calibrate the normalization factor of the cost function b by calculating Eq. (7) minus d, dependent on 2019 production quantity and the AER for the example vehicle, and scaling b so that it matches the estimate of pack-specific cost of \$156/kWh [79]. A table summarizing case study parameters can be seen in Table 4. We perform sensitivity analyses on each parameter in the Supplementary Material.

3.4 Simulations. We conduct multiple simulations of the PHEV platforming and design equilibrium under different conditions. In our baseline case, two firms each produce one PHEV model (based on the Prius Prime), which is optimized with respect to its battery capacity in kWh, x_{ij}, and the decision to platform or not, such that the vehicle's profits are maximized across China ($i = 1$) and the US ($i = 2$). This is equivalent to assuming either (1) the PHEV is the only product being produced by each firm, or (2) each firm has chosen to design and price the PHEV to maximize the profits associated with that vehicle, not considering any effects on the profits of the other vehicles they produce. We conduct additional simulations where the number of firms and number of vehicles per firm is increased (see details in the Supplementary Material).

For all simulations, we draw on a demand model from Helveston et al. [58]. The demand model contains estimates for consumers in the US and China of price coefficients and partworth WTPs for multiple values of PHEV AER in miles [58]. We run a linear regression on the partworth estimates to determine WTP (γ_i) as a continuous linear function of AER (z_{ij}), rather than use a partworth model. In this formulation, γ_i is the consumer WTP for an increase in the PHEV's AER by one mile, holding all other vehicle attributes fixed. For example, if γ_i is \$80, vehicle j in market i could increase in price by \$80 for each increase in AER of 1 mile and it would receive the same level of demand. Values for the estimated linear parameters (γ_1 for China and γ_2 for the US) of this function are reported in Table 4.

To determine AER as a function of battery capacity, we use a model from Shiau et al. [86]. In Shiau et al. [86], the AER is formulated as a cubic function of relative engine and electric motor peak power as well as battery capacity and swing, which was estimated from vehicle simulation data. We hold battery swing fixed within the metamodel to values matching the specifications of a 2019 Prius Prime [85]. For simplicity, we also hold engine and motor sizes (both measured by peak power) fixed at the level of the Prius Prime. One might expect that the engine and motor size would be determined jointly with the battery size by automotive manufacturers [87]. To the extent that fixing engine and motor size significantly increases costs or limits AER, our results may underestimate the value of differentiating battery capacities across markets.

PHEVs currently on the market have battery capacities between 3 kWh and 34 kWh due to requirements on PHEV battery capacity including the necessity for the ability to sustain an all-electric mode

Table 4 Base parameters for PHEV case study

Parameter	Description	Value	Source
K	Number of firms	2	—
g_1	Capacity lower bound	3 kWh	[88]
g_2	Capacity upper bound	34 kWh	[88]
s_1	Market size, China	2.37×10^7	[91]
s_2	Market size, US	7.00×10^6	[90]
θ	Exponentiated utility of outside good	251	Calibrated based on [92]
α_1	Price sensitivity, China	−33 USD/USD	[58]
α_2	Price sensitivity, US	−52 USD/USD	[58]
γ_1	WTP for increases in all-electric range, China	79 USD/mi	[58]
γ_2	WTP for increases in all-electric range, US	82 USD/mi	[58]
σ_1	Standard error for all-electric range WTP, China	65 USD/mi	[58]
σ_2	Standard error for all-electric range WTP, US	56 USD/mi	[58]
r	Discount rate	10%	[87]
b	Normalization factor of cost function	1.98	Calibrated based on [79]
κ	Learning rate	0.09	Calibrated based on [79]
β_2	Second-order coefficient of attribute cost function	0.487 USD/mi^2	[71]
β_1	First-order coefficient of attribute cost function	5.38 USD/mi^2	[71]
β_0	Zeroth-order coefficient of attribute cost function	2171.9 USD	[71]
d	Base cost	1.83×10^4 USD	Calibrated based on [79,85,86]
Modify capacity-to-all-electric range metamodel coefficients	Engine peak power	71 kW	[85]
	Motor peak power	53 kW	[85]
	Battery swing	0.43	[85]

at the lower end of capacity, and weight and packaging constraints at the upper end of capacity [88,89], so we impose these as lower and upper bounds respectively on capacity as a decision variable.

The market size is specified as 7 million in the US and 24 million in China per passenger car sales in 2018 [90,91]. We include an outside good in the case study to represent a composite of all other vehicles that consumers could purchase other than the PHEVs of interest. The utility of the outside good is calibrated such that sales calculated following Eq. (1) match sales of the Prius Prime in the US and China in 2018 [92]. Under a homogenous preference logit model, this lumped composite calibration of the outside good is equivalent to modeling all of the other vehicle options available assuming that their designs and prices remain fixed over the time period of interest [66].

Whether or not firms choose to platform their battery packs across the two markets is influenced by the cost and consumer demand implications of producing customized products with different battery capacities. To understand how platforming is affected by these cost and consumer factors, we run each set of simulations across multiple scenarios where we vary two parameters: WTP for PHEV AER in China (which we vary relative to that in the US), and the learning rate. The WTP in China has varied over 12 linearly spaced points within its 95% confidence interval of (0, 208.16)[6] USD/mi [58], and learning-related costs were modified by varying κ from 1% to 35%[7] over 15 linearly spaced points, for a total of 180 scenarios. All other parameters were held fixed across these scenarios at the values reported in Table 4.

Because the MPEC approach cannot guarantee that it finds a global maximum (only KKT points) for each firm's profit maximization problem, multiple start points were used for each platforming decision scenario and these candidate solutions are then tested to verify which ones satisfy the Nash equilibrium criteria. A total of 16 start points for price were generated in a grid across the design space, with a minimum price guess of \$0 and a maximum of \$200,000 for both markets. Start points for battery capacity were similarly generated in a grid between 3 kWh and 34 kWh.

For both algorithms, the KKT conditions were verified for each solution to ensure they are profit maxima for each firm conditional on the other's decisions. All solutions found by both algorithms across all scenarios were found to satisfy all KKT conditions.

4 Results and Discussion

We first compare the results of the two algorithmic approaches to examine whether multiple equilibria exist to the platforming problem, and compare the computational performance of each approach. We then discuss the optimal platforming and battery design solutions.

4.1 Computational Performance of Two Algorithms. To compare the solutions generated by the two algorithmic approaches, differences in a firm's profits at the solution from each algorithm are shown in Fig. 2 across a 15 by 12 grid of different WTPs for AER in China (while holding the same WTP fixed in the US at \$82/mi) and different learning rates. The tolerances and convergence criteria are reported in Table 5 for reference.

The difference in profits between the two approaches is in percentage terms, calculated by subtracting profit of the MPEC solution from profit of the SIO solution and then dividing by profit of the SIO solution. In all scenarios, the profit discrepancy between the two approaches is within $\pm$0.0001%, which is comparable to the convergence criteria and constraint tolerances for both approaches, as can be seen in Table 5. (Profits are typically on the order of \$1 billion, and profit differences between the two

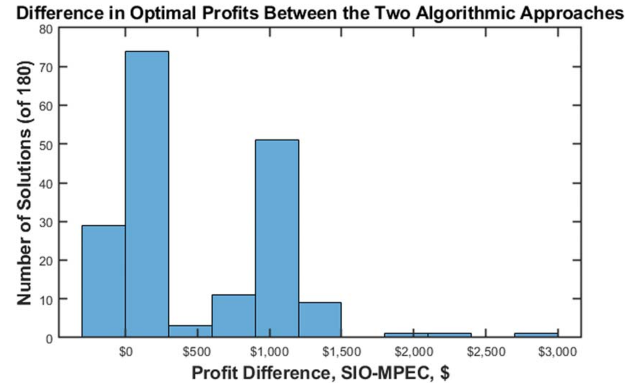

Fig. 2 Difference between total profits calculated using the iterative SIO approach and the MPEC approach over a parameter sweep varying learning rate κ and WTP for all-electric range in China γ_1

approaches are on the order of \$1000 as shown in Fig. 2, resulting in the discrepancy of $\pm$0.0001%.) Sales for each PHEV across the US and China are in the tens of thousands.

For all points on the grid of different learning rates and WTP for AER in China, solutions generated using the two algorithms have identical platforming decisions and the same optimal battery capacities to within $\pm$0.014 kWh.

We additionally compare the CPU time difference across the WTP and learning rate parameter grid between the two approaches when they are each given 16 starting points. These results are shown in Fig. 3.

For the SIO approach, we find that the CPU time to find equilibrium using each of 16 starting points is between roughly 1 s and 7 s in all cases across the parameter grid, while for the MPEC approach, we find that the CPU time to find equilibrium is between 78 s and 126 s. As shown in Fig. 4, this translates to the SIO approach typically being 98.5% faster than the MPEC approach.

We further investigate the performance of the MPEC approach for different types of specifications for the starting points. Because the profit function flattens out at the extremes of vehicle prices, when high prices are used as starting points, the algorithm can converge to spurious solutions that are not equilibrium solutions [94]. Because the likelihood of the algorithm converging to spurious solutions rather than valid equilibrium solutions is influenced by the range of initialized prices and the value of the optimal price, we examine two scenarios: one where the optimal price is relatively low (\$53,000 in China and \$44,000 in the US, which occurs when $\kappa = 35\%$ and WTP for AER in China = \$5/mi), and one where the optimal price is relatively high (\$66,000 in China and \$48,000 in the US, which occurs when $\kappa = 1\%$ and WTP for AER in China = \$208/mi). Algorithm runs for each scenario are repeated for eight different cases where a grid of different starting points with different ranges of starting values for price. In each case, the minimum initial price is \$0. The maximum initial price varies from \$25,000 up to \$200,000. In all cases, the initial battery capacity is set either to 3, 17, or 34 kWh. This results in 576 total runs of the algorithm per scenario.

We first examine the probability of a researcher randomly selecting starting points will get a solution that is a valid equilibrium and how this probability varies with the number of starting points and the maximum possible initial price. This is equivalent to the researcher randomly selecting points from a subgrid of the complete set of starting points where the maximum initial price defines the subgrid. The probability of getting at least one successful equilibrium solution from the selected starting points thus follows a Bernoulli distribution where the probability that o starting points contain at least one valid solution is $1-\left(\frac{h_\omega}{s_\omega}\right)^o$, where h_ω is the number of reported failures in the subgrid ω, and s_ω is the total number of starting points in the subgrid. Note that the equilibrium

[6]The estimated 95% confidence interval in Helveston et al. [58] included WTPs below 0 USD/mi, however these were omitted as consumers are assumed to simply be unwilling to pay for increases in AER at a WTP of 0 USD/mi.

[7]Learning rates of up to 33% have been reported for other electrical energy storage technologies [80].

Table 5 Tolerances and convergence criteria for both approaches

	MPEC approach	SIO approach
Solver absolute tolerance	N/A	1×10^{-6} (on Eq. (1) scaled by $1/1 \times 10^{9}$)
Solver relative tolerance	N/A	1×10^{-9}
Solver constraint tolerance	1×10^{-8}	1×10^{-8}
Convergence criteria	N/A	1×10^{-7} (on relative average firm profits between iterations)
Approach-specific criteria	$\varphi = 1 \times 10^{-8}$ 10,000 maximum function evaluations	$m = 2(\mathbf{x}_{\mathrm{MAX}} - \mathbf{x}_{\mathrm{MIN}})^{8}$

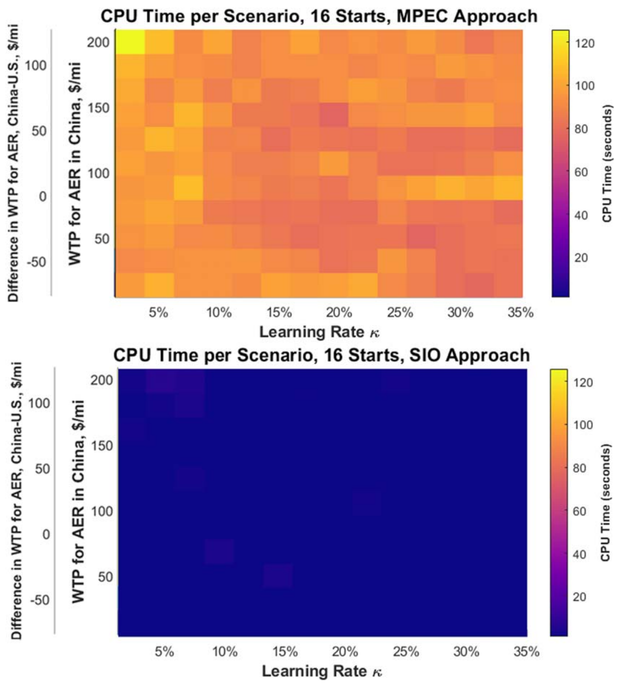

Fig. 3 (*a*) **CPU time to find equilibrium solutions for all scenarios in a parameter sweep varying learning rate κ and WTP for all-electric range in China γ_1, when the MPEC approach was given 16 starting points with varying guesses for battery capacity and vehicle price and (*b*) the same for the SIO approach (All experiments have been run on the same x64 Windows machine with at least 10 GB of memory and a 3.9 GHz processor)**

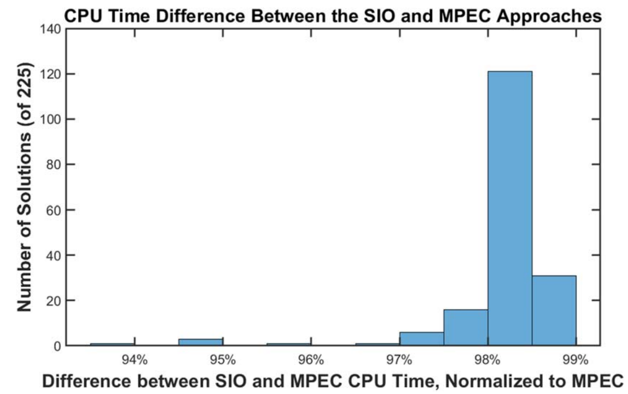

Fig. 4 Histogram of the difference between CPU time to find equilibrium solutions for all scenarios in a parameter sweep varying learning rate κ and WTP for all-electric range in China γ_1 calculated using the iterative SIO approach and the MPEC approach, normalized to the CPU time taken by the MPEC approach

algorithm itself is deterministic and so the probability occurs as a result of the chance of choosing a good starting point, not from any stochasticity in the algorithm.

Results are shown in Fig. 5(*a*). The probability of finding an equilibrium in both cases increases when the starting points have lower values of the maximum price guess. In both scenarios, the probability of finding equilibrium is 100% for price guesses between \$0 and \$75,000 with even only one starting point, but when using initialized prices between \$0–\$100,000 and \$0–\$200,000, the probability of success with one starting point is approximately 60% and 20%, respectively. This probability rises to 98% after using about 5 and 25 starts respectively.

We also show the relationship between the range of initialized prices to the CPU time in Fig. 5(*b*). As the figure shows, shrinking the range of initialized prices increases the chances of finding a valid equilibrium solution, as the maximum initialized prices in the smaller ranges were the closest to the equilibrium prices of this scenario. We find that 81 of 576 points found the equilibrium, or 14%. Of those, starting points having initial prices between \$0 and \$75,000 always found the equilibrium and were also generally the fastest, with CPU times between 1.2 s and 6.7 s.

Because the MPEC approach is instructed to terminate after 10,000 function evaluations if local maxima are not found for firm profits, we would expect that starting points that fail to find equilibria will result in longer CPU times. We plot the CPU time in the low-price scenario separately for runs that successfully find an equilibrium solution and those that do not in Fig. 5(*c*), and find that the median CPU time for successful solutions is 3.4 s with a range between 1.2 s and 14 s whereas the median CPU time for unsuccessful terminations is 6.1 s with a range of 1.4–15 s.

For the high-price scenario, we find that the median CPU time for successful solutions is 3.1 s with a range of 1.4–12 s, whereas the median CPU time for unsuccessful terminations is 6.7 s with a range of 1.2–13 s. As Fig. 5(*b*) shows, shrinking the range of initialized prices decreases the speed of finding a valid equilibrium solution. We find that 102 of 576 points found the equilibrium, or 18%. Of those, starting points having initial prices between \$0 and \$75,000 always found the equilibrium and were also generally the fastest, with CPU times between 1.4 s and 6.7 s. As such, the probability of finding an equilibrium in this scenario is reportedly 100% for price guesses of between \$0 and \$75,000, whereas using initialized prices between \$0–\$100,000 and \$0–\$200,000, the probability increases to 98% after selecting about 3 and 20 starts respectively.

4.2 Optimal Design Decisions. Equilibrium battery capacity and platforming solutions (using the SIO approach) are shown in Figs. 6–8. Figures 6 and 7 show the values of the optimal battery capacity for the US and China at market equilibrium and Fig. 8 shows the optimal platforming decisions at market equilibrium. In each figure, optimal solutions at equilibrium are shown for the different scenarios of learning rates and the difference between the WTP for PHEV AER in China and the US.[8] The horizontal line in the figures represents the fixed US WTP for AER at \$81.8/mi. We find that all equilibria are symmetric, meaning that the

[8]A total of 180 scenarios are considered with the learning rate varying from 1% to 35% in roughly 4% increments and WTP for AER in China varying from about \$5–210 per mile in roughly \$20 per mile increments.

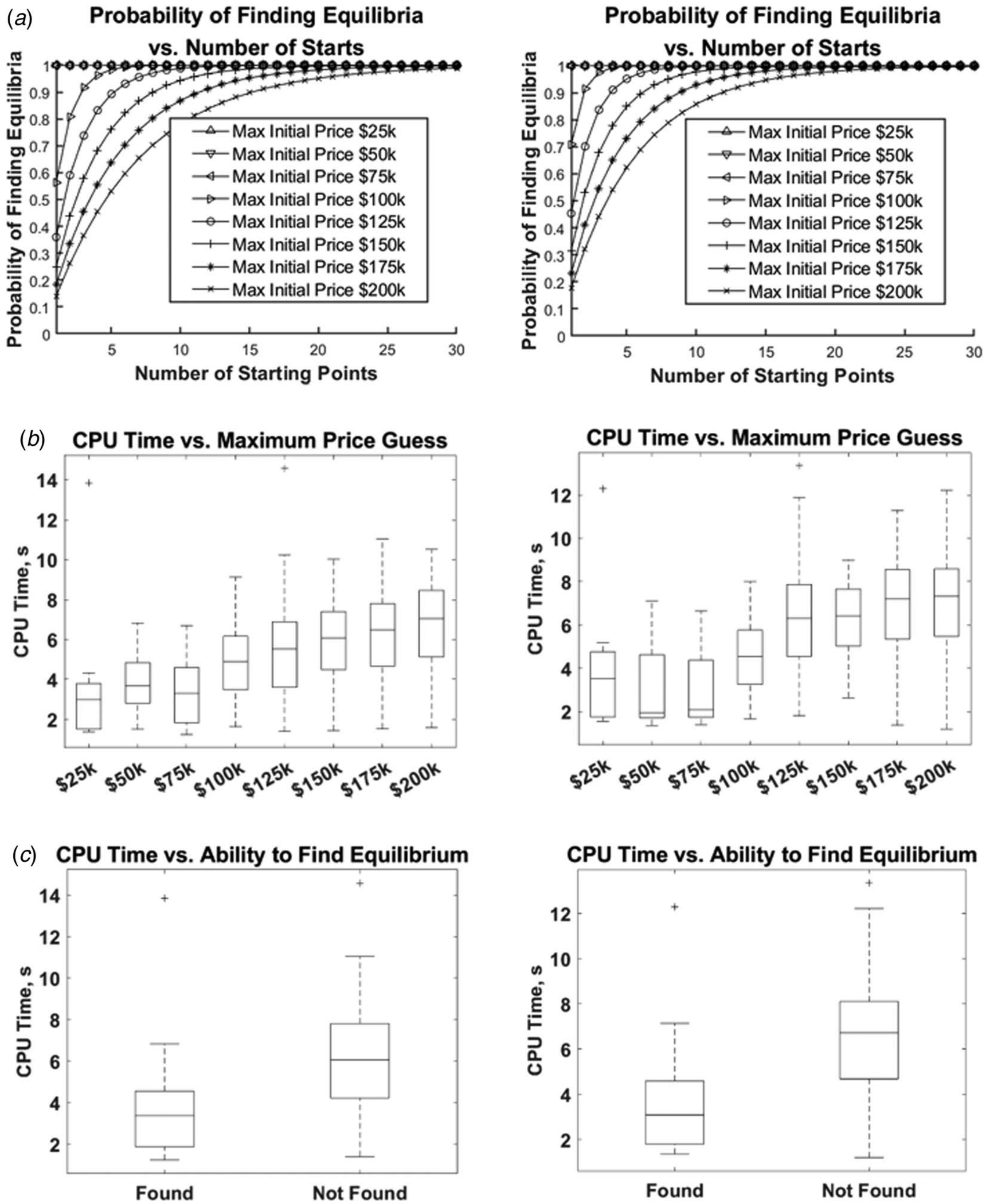

Fig. 5 For the low-price scenario (left) and the high-price scenario (right): (*a*) the probability of finding an equilibrium with an increasing number of starting guesses, (*b*) box plot of CPU time to find an equilibrium as associated with the maximum price guess of each starting point, and (*c*) box plot of CPU time to either find an equilibrium or exit the algorithm for each starting point

optimal solutions for each firm in each scenario are identical.[9] For abbreviation, we present the results for Firm 1 since Firm 2's solutions are the same.

As shown in Figs. 6–8, the solution space can be divided into five different regions: (1) low learning rates and high WTP in China, (2) low learning rates and similar WTP in both markets, (3) low learning rates and low WTP in China, (4) high learning rates and low WTP in China, and (5) high learning rates and high WTP in China. Regions can be distinguished from one another by virtue of having differing optimal platforming solutions, or having interior optimal capacity solutions or solutions that are constrained by the maximum capacity constraint. We discuss each of these regions in more detail below.

[9]As shown in Shiau et al. [22], and Yip et al. [55], the symmetry of firms' optimal solutions is a consequence of using the logit model for demand and identical product design tradeoffs and constraints for all firms. The SIO and MPEC algorithmic approaches are capable of accommodating heterogeneous design tradeoffs and constraints across firms as well as alternate demand models such as mixed-logit, which may result in assymetric solutions, but these are not the focus of this study.

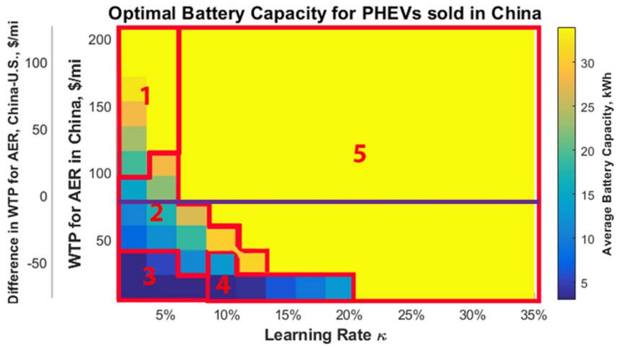

Fig. 6 Optimal battery capacity in China calculated using the SIO approach, and varying learning rate κ and WTP for all-electric range in China γ_1 (for this case study having two firms, both firms made identical decisions)

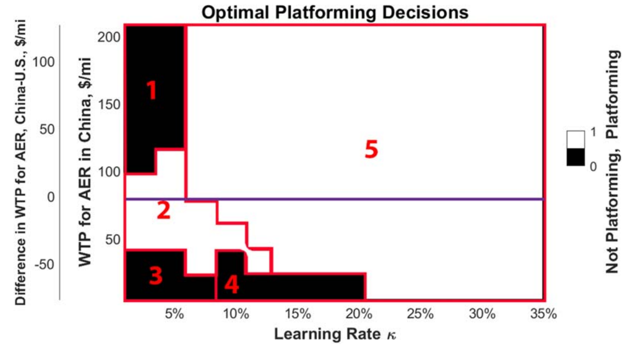

Fig. 8 Optimal platforming decision across markets found using the SIO approach, and varying learning rate κ and WTP for all-electric range in China γ_1

4.2.1 Low Learning Rates and High Willingness to Pay in China. At very low learning rates (close to or below 4%), and WTP for AER in China between \$20 and \$120 above that in the US, the solution is not to platform and to set battery capacity for the US at a smaller level than battery capacity for China. In this case, consumers in China have a WTP for AER sufficient to balance costs for increasing AER to a higher level than consumers in the US. This can be seen more closely in Fig. 9. When WTP in China is approximately \$110/mi or higher than in the US, the optimal capacity solution in the US is 11 kWh, and in China, it is 34 kWh. Thus, gains in profit that occur as a result of tailoring product variants to each specific market outweigh reduced costs due to platforming, so the equilibrium strategy for both firms is not to platform. In this case, optimal battery capacity in the US faces further downward pressure because it has a smaller market size, which means the effects of learning are smaller than for the Chinese market, and so battery production costs per kilowatt hour are higher for the US market than for the Chinese market when firms do not have global platforms.

4.2.2 Low Learning Rates and Similar Willingness to Pay in Both Markets. At very low learning rates (close to or below 4%), and WTP for AER in both markets that are within about ±\$20/mi from each other, the solution is to platform and to set battery capacity for both markets at the same level. In this case, the cost reductions that can occur as a result of platforming outweigh the potential gains in sales made by customizing battery packs across markets because of two factors. First, the benefit of customizing the battery packs is relatively small because the two markets have similar WTP for battery capacity. Second, learning rates are low, and so the extent to which cost reductions are possible for customized products sold within each market alone is limited. Combining the market size by platforming increases the total production quantity, allowing for further cost reductions through learning by experience.

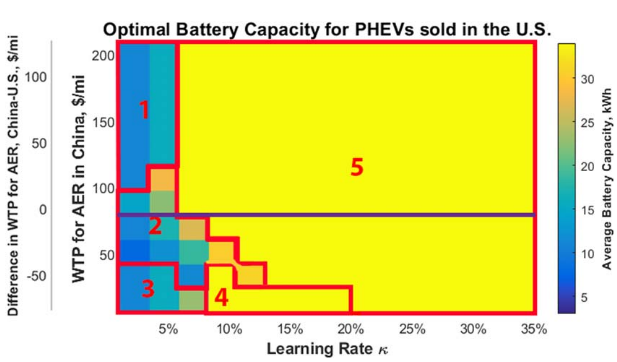

Fig. 7 Optimal battery capacity in the US calculated using the SIO approach, and varying learning rate κ and WTP for all-electric range in China γ_1

4.2.3 Low Learning Rates and Low Willingness to Pay in China. When learning rates are relatively low and the WTP for AER in China is also low (at approximately \$40/mi or lower), the cost of increasing AER is higher than the WTP for the increase in China. In the US, consumers have a high enough WTP for AER sufficient to balance costs for increasing AER up to an intermediate level within the constraint boundaries. However, the optimal battery capacity for China is the smallest capacity possible. This set of scenarios is driven by Chinese consumers' relatively low valuation of battery capacity compared to the increased costs of achieving this capacity, and the relatively high difficulty of reducing battery production costs (indicated by low learning rates). In this region, firms choose a strategy in which they do not platform.

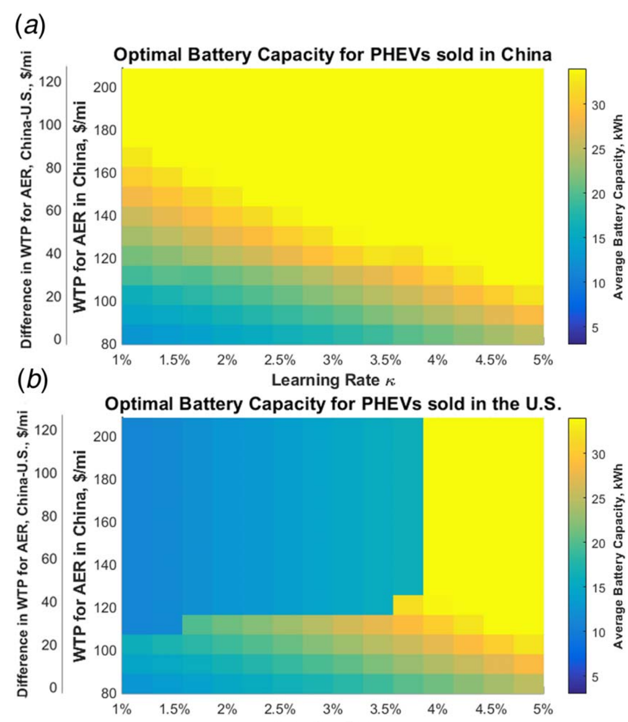

Fig. 9 Firm 1 (*a*) optimal battery capacity in China and (*b*) optimal battery capacity in the US, calculated using the SIO approach and varying learning rate κ and WTP for all-electric range in China γ_1 (for this case study having two firms, both firms make identical decisions). Zoomed-in view of Region 1 in Fig. 8 shows where the optimal decision is not to platform.

4.2.4 High Learning Rates and Low Willingness to Pay in China. When learning rates are high and consumer WTP for AER in China is low relative to the US, firms do not platform, and instead produce a high electric range vehicle for the US and a low electric range vehicle for China. This region is marked by a WTP for AER in China (γ_1) of approximately \$50/mi or lower, which is roughly \$11/mi less than that in the US (γ_2). In this case, consumers in China do not have a WTP that is higher than the costs of increasing AER, but consumers in the US do. In this case, the optimal battery capacity in the US is at or near the upper bound, and the optimal battery capacity in China is at the lower bound. Additionally, because learning rates are relatively high, firms can achieve substantial cost reductions over time even within a single market, lowering the incentive to platform across markets. Therefore, the benefits of customizing the battery capacity for each market outweigh the additional costs of producing separate battery pack designs.

4.2.5 High Learning Rates and High Willingness to Pay in China. When learning rates are high and the WTP for AER in China is approximately equal to the US or higher, equilibrium solutions are to platform and set battery capacity to the largest possible capacity. In this region, consumers' WTP for increased AER in both markets is larger than the cost to increase it. Thus, the optimal battery capacity for each market is the same—at the upper bound—and firms choose to platform these vehicles to further reduce costs.

4.2.6 Sensitivity to the Outside Good and Other Parameters. Problem parameters affect the equilibrium platforming strategy and optimal design in the case study. We conduct sensitivity analyses for all parameters defined in the problem, shown in Table 4. The results are presented in the Supplementary Material. We summarize below sensitivity of the results to four key parameters—the utility of the outside good, the number of firms competing in each market, the number of vehicles per firm, and vehicle markups. Further details are available in the Supplementary Material.

We conduct a sensitivity analysis of results to the exponentiated outside good utility, θ. We vary θ from 0.01 to 546, where 251 is the value of our baseline case, which is the value at which expected sales for each PHEV match the sales of the Prius Prime in 2019. Smaller values of θ occur when the total number of vehicle offerings in the market, including traditional gasoline vehicles as well as electrified vehicles (other than those modeled) decrease, or consumer utility for those vehicles declines. Larger values represent an expansion of competing vehicle offerings or an increase in consumer utility for those vehicles. When θ varies, we see the same general five regions described above but their sizes change. When θ is smaller, both regions where firms do not platform shrink so that firms are platforming in more scenarios of WTP and learning rate compared to when θ is larger. This occurs because the market share of each firm increases in both countries due to the smaller utility of the outside good, and so the gain in production volume from platforming is larger. Thus, the benefits of platforming outweigh the benefits of customizing battery packs across countries in more scenarios.

We also perform sensitivity analyses of the number of competing firms and the number of vehicles per firm in each market. In the former analysis, we increase the number of firms from two up to ten firms, each producing one PHEV each. We do not find that this increase affects the platforming decision, however, the sensitivity analysis on θ indicates that platforming regions may expand when the number of firms is increased further, due to the shrinking market share available to each individual firm.

In the latter analysis, we allow each firm to produce ten PHEVs for which firms may choose the price, platforming, and battery capacity decisions for each vehicle in each market. We find that this affects decision variables very little across the scenarios, except when the learning rate is below 6% and the WTP for AER in China is at least \$100/mi. Outside of this region, design decisions are unchanged, but prices see downward pressure, with a price drop in the US of about \$100 per vehicle and a drop in China of between about \$150 and \$300.

Additionally, we conduct a sensitivity analysis of the vehicle production cost. Specifically, we vary d, the production cost of the vehicle without powertrain components, which was calibrated to \$18,300 in the base case using an assumed markup of 20%. We vary d from \$15,500 to \$100,000 to incorporate uncertainty in the markup rate as well as production costs of a wide-range of vehicle types, including luxury vehicles [95]. In general, for high production costs, the optimal solution is to platform. When d is below roughly \$60,000 and WTP for AER in China is sufficiently low relative to WTP in the US, the cost reductions incurred by platforming are no longer attractive enough to offset the high base cost and the loss in market share due to not customizing vehicles to their respective markets.

Sensitivity analyses for other parameters of the model are described in the Supplementary Material available on the ASME Digital Collection. In general, we find that when any parameter is changed that increases the battery pack cost of the vehicle, the region where the optimum is to platform expands to cover additional scenarios of WTP and learning rate.

4.3 Implications. From the results, we see that when the learning rate is sufficiently high and/or the difference in preferences across markets is sufficiently small, firms have an incentive to platform across the markets to take advantage of the cost reduction of selling common components. In our PHEV case study based on the Prius Prime, we see that this occurs when the WTP for AER in China is no less than \$77/mi below that of the US, when the US WTP is held fixed at \$81/mi.

As expected, because cost savings through technology learning can be obtained by platforming battery packs across the different markets, firms have an incentive to platform whenever consumer preferences affecting battery size in both markets are also similar and within boundary constraints. Given the calculation of AER following Ref. [86] and pack costs following Ref. [71], we find that for a base case with learning rates below 4%, this occurs when WTP for AER in both markets are no different than $\pm$\$20/mi from each other. (Estimates for learning rates for Li-ion battery technologies have ranged from 4.4% to up to 20% by some measurements [80,81].) By comparison, when the aggregated WTP across markets is not sufficient to balance the cost of the optimal attribute level when platforming, the optimum is to not platform.

5 Conclusion

We investigate profit-optimal platforming and design optimization of product components considering market equilibrium. We develop two approaches to solve the problem: an algorithm that iteratively solves each firm's MINLP profit optimization problem until convergence to an equilibrium (SIO algorithm), and an algorithm that solves for the KKT conditions of all firms simultaneously (MPEC algorithm). We compare results for a case study where two firms produce PHEVs sold in China and the US where firms can choose optimal battery capacity and whether to platform the battery packs. In the case study, production costs decrease with increased quantity sold through learning-by-doing.

Solutions calculated using both approaches satisfy KKT conditions in all cases, and solutions calculated using the MPEC algorithm satisfy all post-hoc checks for at least one of 16 starting points in all scenarios. When MPEC results pass all post-hoc checks, they imply a single equilibrium solution for each case. We find that the SIO approach has better computational performance compared to the MPEC algorithm for the MINLP platforming problem formulation in this study, where the MPEC algorithm requires 98.5% more computational time than the SIO approach on average.

Results show that when consumer willingness to pay for PHEV all-electric range in China is low relative to the US, firms choose not to platform. The optimum is to produce a small-range PHEV in China, and a high-range PHEV in the US. With exceptions at low learning rates, the optimum is for firms to platform the battery pack and have the same battery capacity and same range in both markets.

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Conflict of Interest

There are no conflicts of interest.

Data Availability Statement

The datasets generated and supporting the findings of this article are obtainable from the corresponding author upon reasonable request.

Nomenclature

a = unit cost to product Qth unit
b = normalization factor of cost function
d = base cost of vehicle sans battery pack
g = inequality constraint function
h = reported number of failed starts in a subgrid
i = market number
j = index of the set of all products
j = product number of a specific firm
k = firm number
m = materials cost per kilowatt hour
m = big M parameter
n = inequality constraint number
o = number of randomly selected starts from a subgrid
p = price
q = continuous quantity index
t = time period index
v = product utility
w = iteration counter
x = decision variable
y = binary variable indicating platform decision
z = product attribute
A = parameter that calibrates unit cost to produce first unit
I = set of markets
K = total number of firms
M = market size
N = set of inequality constraints
P = probability that consumer chooses a product
R = discount rate
S = number of starts in a subgrid
T = total number of time periods
$\mathcal{M}$ = set of markets
$\mathcal{J}$ = set of all products
c_{ijt} = cost of product j in market i at time t
q_{ij} = rate of production of product j in market i
x_M = non-platformable decision variable
x_L = platformable decision variable
J_k = set of products belonging to firm k
Q_{ijt} = cumulative production quantity of product j in market i at the end of time period t
c_{ij}^{PACK} = cost of battery pack sans material costs
α = price coefficient
$\boldsymbol{\beta}$ = cost function coefficients
γ = willingness to pay
ε = tolerance
ξ = fraction of unit cost reduction achieved via learning over some time period, relative to the first unit produced
θ = exponentiated utility of outside good
κ = learning rate
μ = Lagrange multipliers
Π = profit
τ = continuous time index
ϕ = MPEC relaxation parameter
ω = subgrid index

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