선형 확률 시스템을 위한 모델 예측 최소 비용 분산 제어

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Model Predictive Minimum Cost Variance Control for Linear Stochastic Systems

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Abstract

A new optimal control approach was developed to control stochastic systems which minimizes the variance of the cost function using a model predictive framework. The nominal control was applied to a stochastic linear model for longitudinal aircraft trajectory tracking in simulation to evaluate the effectiveness of the developed control.

I. Introduction

Minimal Cost Variance (MCV) Control is a branch of stochastic optimal control first developed in a 1966 [1] and has been developed and improved over the years [2]. Extensions of this control theory to the multi-cumulant case is presented in [3] and the discrete form of the MCV control is developed in [4]. We extend this MCV control with model predictive approach.

Model Predictive Control (MPC) is also referred to as Receding Horizon Control was initially developed in the 1970s with initial applications in the process industry [5, 6, 7, 8]. The main idea behind MPC is to use an explicit model of the plant to be controlled to predict the future output behavior, and optimize that behavior according to some objective function [8]. This prediction capability allows one to solve an optimal control problem online, where a tracking error is minimized over a future horizon.

In this paper, a new form of the MCV control is developed for stochastic aircraft systems. To do this, two modifications to the MCV control were needed. First, the controller was modified to allow for optimal tracking of a known reference signal. Second, the controller was cast into a model predictive framework to allow for the control solution to be computed continuously online out to prediction horizon.

The new control method, called Model Predictive Minimal Cost Variance (MPMCV) control is developed for a linearized aircraft model around an operating point. The controller is developed and simulated for an aircraft tracking of a prescribed trajectory. Simulated results show the performance of the MPMCV controller for aircraft tracking application.

II. Model Predictive Minimal Cost Variance Control System

For the MPMCV control, the discrete-time stochastic system is of the form,

$$x(k+1) = Ax(k) + Bu(k) + G\omega(k)$$

with cost function

$$J = \sum_{k=1}^{N_p} \left[\left(x(k) - \tilde{x}(k)\right)^T Q \left(x(k) - \tilde{x}(k)\right) + u^T(k-1) R u(k-1) \right]$$

with symmetric weighting matrices Q and R being positive semidefinite and positive definite respectively, and $\tilde{x}(k)$ is the state reference trajectory. The system additive noise $\omega(k)$ is assumed to be zero-mean Gaussian noise with covariance,

$$\mathbb{E}\{\omega(k)\omega^T(k)\} = W.$$

The optimal control solution for the MPMCV control is computed online at each time-step k out to time $k + N_p$, where N_p is the prediction horizon selected a-priori. The first element of the resulting control sequence $u(k+1)$ is then applied to the plant, and the process is repeated at the next timestep. The optimal control sequence $u(k+1)$ is found by solving a sequence of 9 discrete recursion equations for each timestep in the prediction. The resulting optimal control is of the form,

$$u^*(k) = K_0(k)x(k) + u_{ext}(k),$$

where $K_0(k)$ and $u_{ext}(k)$ are found via the following recursion equations,

$$Q_M(k) = Q + M(k+1), \tag{1}$$

$$\Lambda(k) = Q_M(k) G W G^T Q_M(k) + \gamma(k) Q_M(k) + \frac{V_0(k+1)}{4}, \tag{2}$$

$$K_0(k) = -\left(B^T \Lambda(k) B + \gamma(k) R\right)^{-1} B^T \Lambda(k) A, \tag{3}$$

$$u_{ext}(k) = -\left(B^T \Lambda(k) B + \gamma(k) R\right)^{-1} B^T \Lambda(k) \tilde{x}(k+1), \tag{4}$$

$$A_0(k) = A + BK_0(k), \tag{5}$$

$$M(k) = A_0^T(k) Q_M(k) A_0(k) + K_0^T(k) R K_0(k), \tag{6}$$

$$m(k) = m(k+1) + Tr\{G^T Q_M(k) G W\}, \tag{7}$$

$$V_0(k) = A_0^T(k)\left(4 Q_M(k) W Q_M(k) + V_0(k+1)\right) A_0(k), \tag{8}$$

$$v_0(k) = v_0(k+1) + Tr\{V_0(k+1) W\} + \mathbb{E}\left\{\left(\omega^T(k) Q_M(k) \omega(k)\right)^2\right\} - Tr\{Q_M(k) W\}^2, \tag{9}$$

with boundary conditions

$$M(N_p) = m(N_p) = V_0(N_p) = v_0(N_p) = 0.$$

The desired state trajectory is denoted as $\tilde{x}(k)$, and is known from timestep k to prediction horizon $k + N_p$.

In addition to the weighting matrices Q and R, two additional control parameters are available to the designer to tune the tracking response of the system. They are the mean cost constraint γ and prediction horizon N_p.

The MPMCV controller was developed by first deriving the tracking form of the continuous time MCV controller presented in [1]. The discrete form of the tracking MCV controller was then developed to allow for use within a model predictive control framework, as online implementation of MPC is most commonly applied in the discrete time form [9].

Due to the discrete nature of the MPMCV controller, these control parameters may be tuned once and remain fixed for the entire control period or adjusted throughout the process. Continuous adjustment of the control parameters based on schedules or measured values from the plant constitutes an adaptive approach to the MPMCV problem and is considered future work.

Ⅲ. Aircraft Control Simulation

An MPMCV control was developed for the linear longitudinal aircraft model given in [10] to track a desired altitude reference trajectory in the presence of noise. The linear model consists of five states (altitude, forward speed, pitch angle, pitch rate, and vertical speed) and three inputs (spoiler angle, forward acceleration, and elevator angle). The controller was implemented in simulation and tuned using the weighting parameters. The simulation was designed for the aircraft to track a series of step changes to altitude over the course of 60 seconds in the presence of wind gusts modeled as zero-mean Gaussian noise.

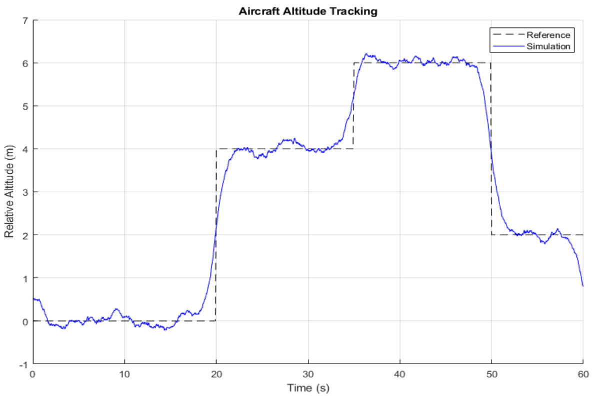

Figure 1, Simulated altitude compared to reference.

The tracking response of the aircraft altitude over time for the MPMCV controller is shown in Figure 1. Simulated trajectories for the other four states, which were desired to track the zero-state as closely as possible, are shown in Figure 2.

Figure 2, Other States

The computed inputs for the controller are shown in Figure 3. The tuned control parameters found for the simulated results presented above are $Q = \boldsymbol{I_{5x5}}$, $R = 0.01\ \boldsymbol{I_{3x3}}$, $\gamma = 0.05$, and $N_p = 5$. The developed controller successfully tracks the altitude reference input in the presence of noise and minimizes the deviation of the other states from the zero-state.

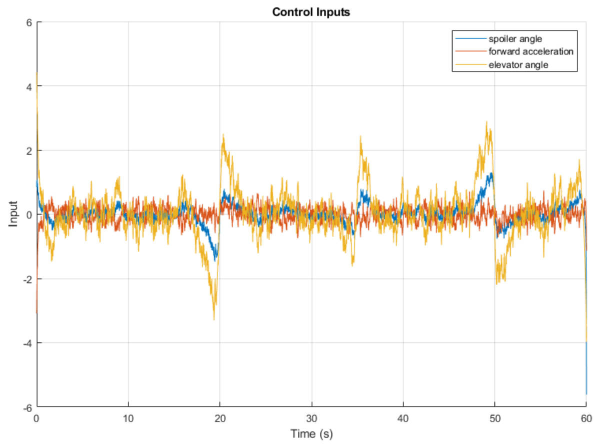

Figure 3. Control Input versus time

IV. Conclusion and Future Research Direction

The results presented above indicate the proposed control approach can accurately track reference inputs for stochastic systems by minimizing the variance of the cost function. We conclude that MPMCV tracks aircraft altitude accurately despite of the stochastic noises.

This MPMCV controller is also appropriate for use in the control of compliant robotic arms applied to medical diagnostics. In this application, the compliant joints used in the robot arm to improve safety when working in proximity to humans also introduce noise into the system in the form of zero-mean Gaussian disturbances. Ongoing work by the authors has shown that the MPMCV controller presented here can improve control in these robotic applications. This is an area of active research.

Future work in this area will focus on the implementation of the proposed MPMCV controller on physical systems, as well as exploring extensions of the MPMCV controller into areas such as constrained MPMCV and adaptive MPMCV controllers.

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