Optimal Detection in MIMO Systems using Spatial Sigma-Delta ADCs

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***Abstract*—The spatial Sigma-Delta architecture can be used to reduce the quantization noise and thus improve the effective resolution of few-bit analog-to-digital converters (ADCs) for certain spatial frequencies of interest. This paper proposes a novel data detection scheme based on the variational Bayes (VB) inference framework for multiple-input multiple-output (MIMO) systems that utilize first-order spatial Sigma-Delta ADCs. We derive a closed-form expression to approximate the posterior distributions of the transmitted data symbols, which are then used for their estimation. Simulation results show that the proposed detection scheme achieves a detection performance comparable to unquantized systems and has a lower symbol error rate (SER) than the conventional quantized VB and linear minimum mean-squared error (LMMSE) methods. The effects of the azimuth range, and the antenna spacing and wavelength on the SER performance of all detection algorithms are also extensively analyzed.**

***Index Terms*—Data detection, few-bit ADCs, MIMO detection, $\Sigma\Delta$ ADCs, variational Bayesian inference.**

I. Introduction

The next generation of wireless networks will require substantial bandwidth in both the millimeter (mmWave) and terahertz bands to deliver high data throughput [1]. Signals at these frequencies are hindered by low penetration capabilities and high propagation loss, which restrict their practical communication range [2]. Massive multiple-input multiple-output (MIMO) arrays have been used to compensate for the propagation loss while simultaneously achieving high capacity through spatial multiplexing [3]. However, exploiting the full benefits of beamforming and multiplexing in massive MIMO can be challenging due to the need for dedicated high-resolution analog-to-digital converters (ADCs)/digital-to-analog converters (DACs) for each antenna element [4], [5]. This results in high hardware complexity and increased power consumption, especially with larger bandwidths and sampling rates [6]. To address these concerns, the use of low-resolution ADCs with, e.g., 1–3 bits of precision, has emerged as an energy-efficient and low-complexity solution for massive MIMO systems [7]–[9].

Low-resolution ADCs have recently become an important feature in massive MIMO systems due to their simple design and low power consumption. However, the inherent high nonlinearity of low-resolution ADCs degrades the performance of MIMO systems such as achievable rate and symbol error rate (SER), especially at medium-to-high signal-to-noise ratios (SNRs) [10]–[12]. However, this negative impact can be mitigated by increasing the number of antennas, which indicates that massive MIMO can be effectively operated with few-bit ADCs. However, attaining a good tradeoff between the system performance and the number of quantization bits requires advanced signal processing algorithms and architectural innovations tailored for the specific characteristics of the quantized signals [9], [13].

Spatial Sigma-Delta ($\Sigma\Delta$) quantization is used in such systems to improve the effective resolution of low-resolution ADCs [14]. In spatial $\Sigma\Delta$ architecture, the difference between the input and the quantized output represents the quantization noise at each antenna, which is fed back and compared with an adjacent antenna. This feedback stage enables more aggressive noise-shaping, wherein the spatial spectrum of the quantization noise is pushed beyond the angular sector of the signal of interest [14]. In [15], the spectral efficiency of 1-bit $\Sigma\Delta$ massive MIMO was analyzed, revealing that one-bit $\Sigma\Delta$ scales down the quantization noise power proportionally to the square of the spatial oversampling rate. Spatial few-bit $\Sigma\Delta$ ADCs have also been used in massive MIMO to shape the quantization noise away from users in certain angular sectors for improved channel estimation [16].

Most of the existing works on MIMO detection have focused on Gaussian channel models with conventional few-bit quantization [9], [10]. Moreover, achieving optimal MIMO detection with spatial $\Sigma\Delta$ ADCs under the maximum-a-posteriori (MAP) probability criterion may not be feasible. These observations prompt the development in this paper of novel data detection algorithms based on the VB inference framework for MIMO systems with spatial $\Sigma\Delta$ ADCs.

The contributions of the paper are summarized as follows:

- We develop a 1^{st}-order Sigma-Delta Variational Bayes (SD-VB) detection algorithm, where few-bit $\Sigma\Delta$ quantizers are employed in a massive MIMO system. Using the VB framework, variational distributions of latent variables are derived to enable efficient updates for the developed algorithm. We also present an LMMSE detector for MIMO detection with 1^{st}-order $\Sigma\Delta$ quantization.

- We demonstrate through simulation results that the 1^{st}-order SD-VB algorithm achieves the lowest SER compared with state-of-the-art detection algorithms such as the matched-filter quantized VB (MF-QVB) and linear minimum mean-squared error (LMMSE) detectors for certain angular sectors.

Notation: Boldface lowercase and boldface uppercase variables denote vectors and matrices, respectively. The symbols $\mathbb{C}$ and $\mathbb{R}$ stand for the sets of complex and real numbers, respectively. The L_2-norm and the absolute value are indicated by $\|\cdot\|$ and $|\cdot|$, respectively. Real and imaginary parts are denoted by $\Re\{\cdot\}$ and $\Im\{\cdot\}$, respectively, with $j=\sqrt{-1}$. A circularly symmetric complex Gaussian (CSCG) distribution with mean $\boldsymbol{\eta}$ and covariance matrix $\mathbf{Z}$ is indicated by $\mathcal{CN}(\boldsymbol{\eta},\mathbf{Z})$. The identity matrix is denoted by $\mathbf{I}$, the trace operator by $\mathrm{Tr}(\cdot)$, and the expectation operator by $\mathbb{E}\{\cdot\}$. The transpose, complex conjugate, and complex conjugate transpose operators are denoted by $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$, respectively.

II. System Model

We consider an uplink MIMO system with K single-antenna users and a base station (BS) with N antennas, where the received data at the BS before quantization is expressed as

$$\mathbf{x}=\mathbf{H}\mathbf{s}+\mathbf{n}, \tag{1}$$

where $\mathbf{H}=[\mathbf{h}_1,\ldots,\mathbf{h}_N]^T\in\mathbb{C}^{N\times K}$ is the uplink channel matrix, $\mathbf{s}=[s_1,\ldots,s_K]^T\in\mathbb{C}^{K\times 1}$ is the uplink data symbol vector, and $\mathbf{n}\sim\mathcal{CN}(\mathbf{0},N_0\mathbf{I}_N)$ denotes a zero-mean Gaussian noise vector at the BS, with variance N_0.

We assume a geometric channel model typical of mmWave communications systems in which the channel for each user is composed of a linear combination of L propagation paths [2], [17], i.e.,

$$\bar{\mathbf{h}}_k=\sqrt{\frac{\beta_k}{L}}\mathbf{A}_k\mathbf{g}_k, \tag{2}$$

where $\bar{\mathbf{h}}_k$ is the kth column of $\mathbf{H}$ and the uplink channel from user-k, the columns of $\mathbf{A}_k\in\mathbb{C}^{N\times L}$ represent the array response for L propagation paths, $\mathbf{g}_k\sim\mathcal{CN}(\mathbf{0},\mathbf{I}_L)$ represents the small-scale fading, and β_k models the geometric attenuation and slow fading. We assume that $\mathbf{A}_k$ is a full rank matrix whose ℓ-th column is the array steering vector corresponding to the angle of arrival (AoA) $\theta_{k\ell}$ of the ℓ-th path, as given by

$$\mathbf{a}(\theta_{k\ell})=\left[1,\mathrm{e}^{-j2\pi\frac{d}{\lambda}\sin\theta_{k\ell}},\ldots,\mathrm{e}^{-j(N-1)2\pi\frac{d}{\lambda}\sin\theta_{k\ell}}\right]^T, \tag{3}$$

where d is the inter-antenna spacing of the ULA and λ denotes the wavelength. For simplicity, we denote $\omega_{k\ell}=2\pi\frac{d}{\lambda}\sin\theta_{k\ell}$ as the spatial frequency of the ℓ-th path from user-k. We assume that the AoAs for all users are situated within a specific angular sector $\mathcal{S}_{\theta_0}\triangleq[\theta_0-\Theta/2,\theta_0+\Theta/2]$, where θ_0 is the center angle of the sector and Θ is the azimuth angular spread.

The first-order spatial $\Sigma\Delta$ quantization architecture is depicted in Fig. 1, where a few-bit quantizer is used in each step.

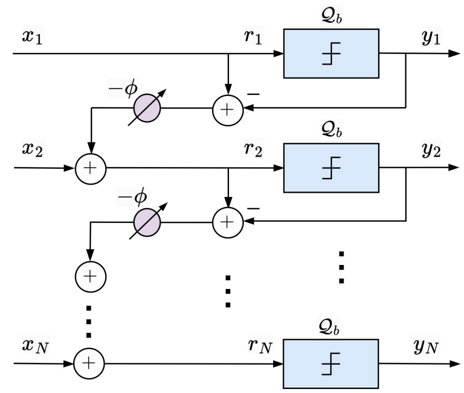

Fig. 1: A first-order spatial $\Sigma\Delta$ architecture at an N-antenna receiver.

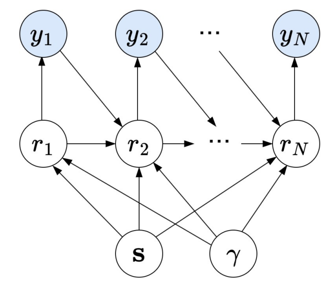

Fig. 2: The Bayesian network for the 1^{st}-order $\Sigma\Delta$ receiver.

The pre-quantized signal at stage i (i.e., antenna i), denoted by r_i, which consists of the unquantized received signal x_i and the difference between the input and output of the previous quantizer, is given by [16]

$$r_i=x_i+\mathrm{e}^{-j\phi}(r_{i-1}-y_{i-1}),\quad\forall i=1,\ldots,N, \tag{4}$$

where $x_i\triangleq\mathbf{h}_i^T\mathbf{s}+n_i$. At the BS, the quantized observation at antenna i is $y_i=\mathcal{Q}_b(r_i)$, where $\mathcal{Q}_b(.)$ denotes the b-bit ADC. Here, r_0 and y_0 are set to 0. Considering the input-output relationship of the $\Sigma\Delta$ quantizer in (4), the conditional distributions between the observed quantized signal y_i and latent variables, i.e., r_i, x_i, $\mathbf{s}$, are given by $p(r_i|r_{i-1},y_{i-1},\mathbf{s};\mathbf{H},N_0)=\mathcal{CN}(r_i;\mathbf{h}_i^T\mathbf{s}+\mathrm{e}^{-j\phi}(r_{i-1}-y_{i-1}),N_0)$ and $p(y_i|r_i)=\mathbb{1}(r_i\in[y_i^{\text{low}},y_i^{\text{up}}])$, with $\mathbb{1}$ representing the indicator function, y_i^{low} and y_i^{up} are the lower and upper bounds of the bins to which r_i belongs, respectively.

III. Variational Bayes for Data Detection

We consider the residual interference-plus-noise as an unknown parameter N_0^{post}, which must be estimated using the VB method. The dependency between random variables under spatial $\Sigma\Delta$ processing can be graphically modeled through the Bayesian network in Fig. 2, where we denote $\gamma\triangleq 1/N_0^{\text{post}}$ as the precision that is floated as an unknown random variable. The main goal is to infer the distribution of data $\mathbf{s}$ given the observation $\mathbf{y}$. To accomplish this, we employ the mean-field variational distribution $q(\mathbf{s},\mathbf{r},\gamma)$ to optimize the variational distribution $q(\mathbf{s},\mathbf{r},\gamma)$ such that:

$$p(\mathbf{s},\mathbf{r},\gamma|\mathbf{y};\mathbf{H}) \approx q(\mathbf{s},\mathbf{r},\gamma) = \prod_{k=1}^{K} q(s_k) \prod_{i=1}^{N} q(r_i) q(\gamma). \quad (5)$$

Based on VB framework, to obtain the optimal solution of the variational densities in (5), we need the joint distribution $p(\mathbf{y},\mathbf{r},\mathbf{s},\gamma;\mathbf{H})$ which can be factorized as

$$p(\mathbf{y},\mathbf{r},\mathbf{s},\gamma;\mathbf{H}) = p(\mathbf{s}) \prod_{i=1}^{N} p(y_i|r_i) p(r_i|r_{i-1}, y_{i-1}, \mathbf{s}, \gamma; \mathbf{H}) p(\gamma), \quad (6)$$

where $p(r_i|r_{i-1}, y_{i-1}, \mathbf{s}, \gamma; \mathbf{H}) = \mathcal{CN}(r_i; \mathbf{h}_i^T\mathbf{s} + \mathrm{e}^{-j\phi}(r_{i-1} - y_{i-1}), \gamma^{-1})$.

By applying the VB framework, all the considered variational densities can be derived in closed form as follows:

1) Updating r_i: For $i = 1, \ldots, N-1$, the variational distribution $q(r_i)$ can be obtained by taking the expectation of the conditional in (6) w.r.t. $q(\mathbf{s},\gamma)$ as

$$\begin{aligned} q(r_i) &\propto \exp\Big\{ \Big\langle \ln p(y_i|r_i) + \ln p(r_{i+1}|r_i, y_i, \mathbf{s}, \gamma; \mathbf{H}) \\ &\quad + \ln p(r_i|r_{i-1}, y_{i-1}, \mathbf{s}, \gamma; \mathbf{H}) \Big\rangle \Big\} \\ &\propto \mathbb{1}(r_i \in [y_i^{\mathrm{low}}, y_i^{\mathrm{up}}]) \exp\big\{ -\langle\gamma\rangle\big(|r_i - a_i|^2 + |r_i - b_i|^2\big)\big\} \\ &\propto \mathbb{1}(r_i \in [y_i^{\mathrm{low}}, y_i^{\mathrm{up}}]) \mathcal{CN}\big(r_i; (a_i + b_i)/2, 1/(2\langle\gamma\rangle)\big), \end{aligned} \quad (7)$$

where $a_i \triangleq \mathbf{h}_i^T\langle\mathbf{s}\rangle + \mathrm{e}^{-j\phi}(\langle r_{i-1}\rangle - y_{i-1})$ and $b_i \triangleq y_i + \mathrm{e}^{j\phi}\langle r_{i+1}\rangle - \mathrm{e}^{j\phi}\mathbf{h}_{i+1}^T\langle\mathbf{s}\rangle$. Here, $\langle x\rangle$ denotes the variational mean of a variable x. Let $v_i \triangleq (a_i + b_i)/2$, so that the variational distribution $q(r_i)$ in (7) is the truncation of $r_i \sim \mathcal{CN}\big(v_i, 1/(2\langle\gamma\rangle)\big)$ onto the interval $[y_i^{\mathrm{low}}, y_i^{\mathrm{up}}]$. Thus, the mean and variance of r_i can be calculated as $\langle r_i\rangle = \mathsf{F}_r(v_i, 2\langle\gamma\rangle, y_i^{\mathrm{low}}, y_i^{\mathrm{up}})$ and $\tau_{r_i} = \mathsf{G}_r(v_i, 2\langle\gamma\rangle, y_i^{\mathrm{low}}, y_i^{\mathrm{up}})$, respectively, where $\mathsf{F}_r(.)$ and $\mathsf{G}_r(.)$ were calculated as (66) and (67) in [9].

For $i = N$, while the factor $p(r_{N+1}|r_N, y_N, \mathbf{s}, \gamma; \mathbf{H})$ does not exist, the variational distribution $q(r_N)$ can be computed similarly to (7) and is given by

$$q(r_N) \propto \mathbb{1}(r_N \in [y_N^{\mathrm{low}}, y_N^{\mathrm{up}}])\, \mathcal{CN}\big(r_N; a_N, 1/\langle\gamma\rangle\big). \quad (8)$$

2) Updating s_k: The variational distribution $q(s_k)$ is obtained by taking the expectation of the conditional in (6) w.r.t. $q(\mathbf{r},\gamma)$:

$$\begin{aligned} q(s_k) &\propto \exp\Big\{ \Big\langle \ln p(s_k) + \sum_{i=1}^{N} \ln p(r_i|r_{i-1}, y_{i-1}, \mathbf{s}, \gamma; \mathbf{H}) \Big\rangle \Big\} \\ &\propto p(s_k) \exp\Big(-\langle\gamma\rangle \sum_{i=1}^{N} |z_{i,k} - h_{i,k}s_k|^2 \Big) \\ &\propto p(s_k) \prod_{i=1}^{N} \mathcal{CN}\big(z_{i,k}; h_{i,k}s_k, \langle\gamma\rangle^{-1}\big), \end{aligned} \quad (9)$$

where

$$z_{i,k} = \langle r_i\rangle - \mathrm{e}^{-j\phi}(\langle r_{i-1}\rangle - y_{i-1}) - \mathbf{h}_i^T\langle\mathbf{s}\rangle + h_{i,k}\langle s_k\rangle, \quad (10)$$

using the current estimate $\langle s_k\rangle, \forall k$. The variational distribution $q(s_k)$ is equivalent to the posterior distribution $p(s_k|\mathbf{z}_k, \langle\gamma\rangle; \mathbf{h}_k)$ of s_k in a decoupled single-input multiple-output (SIMO) system as below:

$$\mathbf{z}_k = \mathbf{h}_k s_k + \mathcal{CN}(\mathbf{0}, \langle\gamma\rangle^{-1}\mathbf{I}_N). \quad (11)$$

The variational mean and variance of s_k can be updated as $\langle s_k\rangle = \mathsf{F}_s(\mathbf{z}_k, \mathbf{h}_k, \langle\gamma\rangle)$ and $\tau_{s_k} = \mathsf{G}_s(\mathbf{z}_k, \mathbf{h}_k, \langle\gamma\rangle)$, respectively, and their computations can be obtained by the same steps in Appendix B of [18].

3) Updating γ: We assume a conjugate prior Gamma distribution $\mathrm{Gamma}(\alpha,\beta)$ for the precision γ, where α and β present the shape and rate parameters of the distribution, respectively. The variational distribution $q(\gamma)$ is obtained by taking the expectation of the conditional distribution in (6) w.r.t. $q(\mathbf{s},\mathbf{r})$:

$$\begin{aligned} q(\gamma) &\propto \exp\Big\{ \Big\langle \sum_{i=1}^{N} \ln p(r_i|r_{i-1}, y_{i-1}, \mathbf{s}, \gamma; \mathbf{H}) + \ln p(\gamma) \Big\rangle \Big\} \\ &\propto \exp\Big\{ -\gamma \sum_{i=1}^{N} \big\langle |r_i - \mathbf{h}_i^T\mathbf{s} - \mathrm{e}^{-j\phi}(r_{i-1} - y_{i-1})|^2 \big\rangle \\ &\quad + N\ln\gamma + (\alpha - 1)\ln\gamma - \beta\gamma \Big\} \\ &\propto \exp\Big\{ -\gamma \sum_{i=1}^{N} \big(\big\langle |r_i - \mathbf{h}_i^T\mathbf{s}|^2 \big\rangle + \big\langle |r_{i-1} - y_{i-1}|^2 \big\rangle \\ &\quad - 2\,\Re\big\{ \big\langle (r_i - \mathbf{h}_i^T\mathbf{s})^*(r_{i-1} - y_{i-1})\mathrm{e}^{-j\phi} \big\rangle \big\} \big) \\ &\quad + (N + \alpha - 1)\ln\gamma - \beta\gamma \Big\}. \end{aligned} \quad (12)$$

Using the expansion $\big\langle |r_i - \mathbf{h}_i^T\mathbf{s}|^2 \big\rangle = |\langle r_i\rangle - \mathbf{h}_i^T\langle\mathbf{s}\rangle|^2 + \tau_{r_i} + \mathbf{h}_i^T\boldsymbol{\Sigma}_{\mathbf{s}}\mathbf{h}_i$, we arrive at

$$\begin{aligned} q(\gamma) &\propto \exp\Big\{ -\gamma \sum_{i=1}^{N} \Big[|\langle r_i\rangle - \mathbf{h}_i^T\langle\mathbf{s}\rangle - (\langle r_{i-1}\rangle - y_{i-1})\mathrm{e}^{-j\phi}|^2 \\ &\quad + \tau_{r_i} + \mathbf{h}_i^H\boldsymbol{\Sigma}_{\mathbf{s}}\mathbf{h}_i + \tau_{r_{i-1}} \Big] + (N + \alpha - 1)\ln\gamma - \beta\gamma \Big\} \\ &\propto \exp\Big\{ -\gamma \sum_{i=1}^{N} \Big[|u_i|^2 + \tau_{r_i} + \mathbf{h}_i^H\boldsymbol{\Sigma}_{\mathbf{s}}\mathbf{h}_i + \tau_{r_{i-1}} \Big] \\ &\quad + (N + \alpha - 1)\ln\gamma - \beta\gamma \Big\} \\ &\propto \exp\Big\{ -\gamma \Big[\beta + \|\mathbf{u}\|^2 + 2\mathrm{Tr}\{\boldsymbol{\Sigma}_{\mathbf{r}}\} - \tau_{r_N} \\ &\quad + \mathrm{Tr}\big\{ \mathbf{H}\boldsymbol{\Sigma}_{\mathbf{s}}\mathbf{H}^H \big\} \Big] + (N + \alpha - 1)\ln\gamma \Big\}, \end{aligned} \quad (13)$$

where $u_i \triangleq \langle r_i\rangle - \mathbf{h}_i^T\langle\mathbf{s}\rangle - \mathrm{e}^{-j\phi}(\langle r_{i-1}\rangle - y_{i-1})$, $\mathbf{u} = [u_1, \ldots, u_N]^T$, and $\Sigma_{\mathbf{r}} = \mathrm{diag}(\tau_{r_1}, \ldots, \tau_{r_N})$ and $\Sigma_{\mathbf{s}} = \mathrm{diag}(\tau_{s_1}, \ldots, \tau_{s_K})$ are the covariance matrices of $\mathbf{r}$ and $\mathbf{s}$, respectively, α and β are the parameters of the prior distribution of $p(\gamma)$ that was assumed to be a Gamma distribution with parameters α and β. Here, u_i denotes the residual term at antenna i, which reconciles with the noise term n_i when r_i, r_{i-1}, and $\mathbf{s}$ are perfectly estimated.

Algorithm 1 – VB Algorithm for MIMO Detection with 1st-Order $\Sigma\Delta$ Quantization

Input: $\mathbf{y}, \mathbf{H}$
Output: $\hat{\mathbf{s}}$
Initialize $\hat{r}_i^1 = y_i^1$, $\tau_{r_i}^1 = 0$, $\forall i$, $\hat{s}_k^1 = 0$, $\tau_{s_k}^1 = \mathrm{Var}_{p(s_k)}[s_k]$, $\forall k$, $\mathbf{u} = \hat{\mathbf{r}}^1 - \mathbf{H}\hat{\mathbf{s}}^1$.
for $t = 1, 2, \ldots$ **do**
  $\hat{\gamma}^t \leftarrow (N+\alpha)/\big(\beta + \|\mathbf{u}\|^2 + 2\mathrm{Tr}\{\mathbf{\Sigma_r}\} - \tau_{r_N}^t + \mathrm{Tr}\{\mathbf{H\Sigma_s H}^H\}\big)$
  for $i = 1, \ldots, N$ **do**
    if $i = N$ **then**
      $v_i^t = \hat{r}_N^t - u_N$, $\epsilon = 1$
    else
      $v_i^t \leftarrow \hat{r}_i^t - (u_i - e^{j\phi} u_{i+1})/2$, $\epsilon = 2$
    end if
    $\hat{r}_i^{t+1} \leftarrow \mathsf{F}_r(v_i^t, \epsilon\hat{\gamma}^t, y_i^{\mathrm{low}}, y_i^{\mathrm{up}})$
    $\tau_{r_i}^{t+1} \leftarrow \mathsf{G}_r(v_i^t, \epsilon\hat{\gamma}^t, y_i^{\mathrm{low}}, y_i^{\mathrm{up}})$
    $u_i \leftarrow u_i - \hat{r}_i^t + \hat{r}_i^{t+1}$
    $u_{i+1} \leftarrow u_{i+1} + e^{-j\phi}(\hat{r}_i^t - \hat{r}_i^{t+1})$ only for $i < N$
  end for
  for $k = 1, \ldots, K$ **do**
    $\mathbf{z}_k^t \leftarrow \mathbf{h}_k \hat{s}_k^t + \mathbf{u}$
    $s_k^{t+1} \leftarrow \mathsf{F}_s(\mathbf{z}_k^t, \mathbf{h}_k, \hat{\gamma}^t)$
    $\tau_{s_k}^{t+1} \leftarrow \mathsf{G}_s(\mathbf{z}_k^t, \mathbf{h}_k, \hat{\gamma}^t)$
    $\mathbf{u} \leftarrow \mathbf{u} + \mathbf{h}_k(\hat{s}_k^t - \hat{s}_k^{t+1})$
  end for
end for
$\forall k : \hat{s}_k \leftarrow \arg\max_{a\in\mathcal{S}} p_a \mathcal{CN}\big(\mathbf{z}_k^t; \mathbf{h}_k a, (1/\hat{\gamma}^t)\mathbf{I}_M\big)$

The variational distribution $q(\gamma)$ is Gamma with mean

$$\langle\gamma\rangle = \frac{N+\alpha}{\beta + \|\mathbf{u}\|^2 + 2\mathrm{Tr}\{\Sigma_{\mathbf{r}}\} - \tau_{r_N} + \mathrm{Tr}\big\{\mathbf{H}\Sigma_{\mathbf{s}}\mathbf{H}^H\big\}}. \quad (14)$$

The proposed VB method for MIMO detection with 1st-order $\Sigma\Delta$ quantization is summarized in Algorithm 1, where the parameter ϵ is the numerator of $\hat{\gamma}$ in the variational distributions of (7) and (8) for updating r_i and r_N, respectively.

IV. Simulation Results

This section presents illustrative numerical results for the performance of the proposed SD-VB algorithm based on the 1st-order $\Sigma\Delta$ architecture compared with state-of-the-art data detection methods such as the MF-QVB in [9] and LMMSE-based detection for various scenarios. We implement all VB-based algorithms with a maximum of 50 iterations and consider scenarios with 100 transmitted data symbols. The noise variance N_0 is set based on the SNR, which is defined as $\mathrm{SNR} = \mathbb{E}[\|\mathbf{Hs}\|^2]/\mathbb{E}[\|\mathbf{n}\|^2] = K/(NN_0)$. Unless otherwise stated, all cases assume the number of paths as $L = 20$, the width of the angular sector as $\Theta = 40°$ and assume it is centered at $\theta_0 = 0°$. We assume all users lie within the same azimuth angular range, with AoAs drawn uniformly from the interval $[-20°, 20°]$.

To highlight the effectiveness of the proposed approach, we compare the performance of the SD-VB algorithm with that of three benchmark approaches: (i) The LMMSE receiver implemented with the 1st-order $\Sigma\Delta$ architecture and 1-bit quantizers, (ii) the MF-QVB algorithm developed in [9] implemented with conventional few-bit quantizers, and (iii) the MF-VB algorithm developed in [19] implemented with ideal/infinite quantizers.

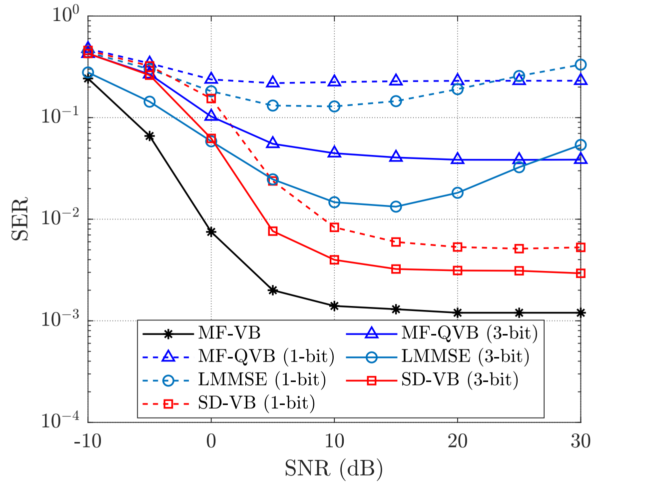

Fig. 3: SER performance vs. SNR with 1-bit and 3-bit quantizers, azimuth angular spread $\Theta = 40^0$, $\phi = 20^0$, $N = 128$, and $K = 16$.

The data detection performance of the proposed SD-VB algorithm is shown in Fig. 3. The SERs of MF-QVB, proposed in [19], and LMMSE are evaluated based on the conventional quantizer and the linearized model of the $\Sigma\Delta$ quantizer, respectively. The SER of the proposed SD-VB is closest to the unquantized system, i.e., MF-VB, and outperforms MF-QVB and LMMSE in both 1-bit and 3-bit quantization. MF-QVB is inefficient for the channels with relatively few paths that are confined to some angular sector since the quantization noise in a conventional quantizer is uniform across all spatial frequencies. LMMSE is based on the approximation of the $\Sigma\Delta$ quantizing output. The $\Sigma\Delta$ receiver shapes the quantization noise and pushes it out of the spatial frequencies of interest. A proper signal processing algorithm, e.g., SD-VB, can help improve SER performance without linearizing the $\Sigma\Delta$ quantized output as in LMMSE.

Fig. 4 shows the effect of the azimuth angular spread on SER performance. For narrow azimuth ranges, all approaches experience high SER due to the extreme channel correlation that results in $K = 16$ users densely packed together. The 3-bit SD-VB implementation performs best among all the quantized approaches for sectors smaller than 80°. The SD-VB implementation achieves its best performance for $\Theta \in [60°, 80°]$, but its SER degrades as the sectors become wider since the noise-shaping effect is limited. MF-VB and MV-QVB have the best performance for large sectors since the $\Sigma\Delta$-based approaches have reduced spatial correlation to exploit. The advantage of using VB, in general, is evident in the superior performance of the VB algorithms compared with LMMSE in all cases.

Fig. 5 presents the effect of antenna spacing and wavelength on the SER performance of all detection algorithms for an array with a fixed number of antennas ($N = 128$). Observations similar to those in the previous example can

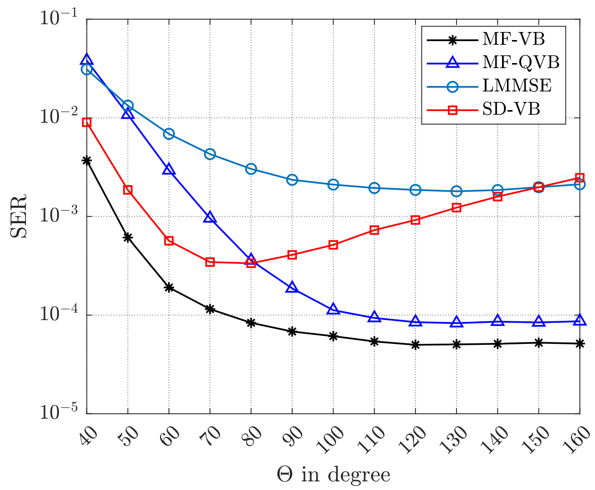

Fig. 4: SER performance vs. azimuth angular spread Θ, with 3-bit quantizers, SNR $= 5$ dB, $\theta_0 = 0^\circ$, and $d = \lambda/6$.

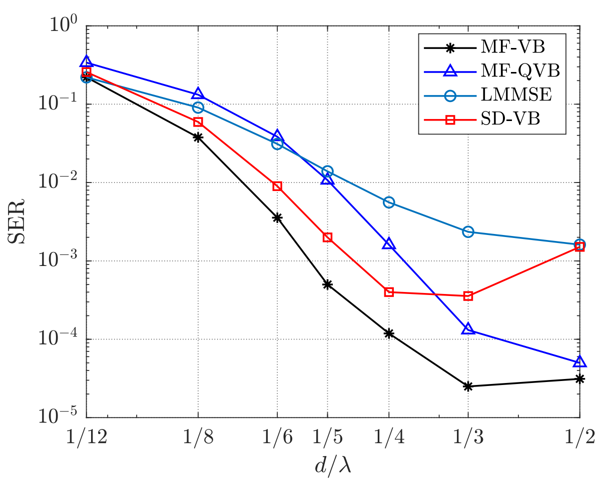

Fig. 5: SER performance vs. d/λ, with 3-bit quantizers, $N = 128$, $\Theta = 40^\circ$, $\theta_0 = 0^\circ$, and SNR $= 5$ dB.

be made here. In particular, when d is very small, the array aperture is significantly reduced and none of the methods are able to counteract the extreme channel correlation that results from the narrow angular sector of $\Theta = 40^\circ$. The SER of all algorithms improves with increasing d, although the proposed SD-VB algorithm provides the best performance for values of d around $1/4$ to $1/3$, and degrades for larger d since the benefit of oversampling is lost. As the antenna spacing increases, the reduced channel correlation benefits MF-VB and MF-QVB.

V. Conclusion

In this paper, we have developed a MIMO detection algorithm based on the VB inference framework in massive MIMO systems with few-bit $\Sigma\Delta$ ADCs. The variational distributions of the latent variables were obtained in closed form, based on which an iterative algorithm was developed for MIMO detection with efficient updates. Simulation results showed that the proposed SD-VB algorithm achieved higher detection performance than the MF-QVB and LMMSE under 1-bit and 3-bit quantization. Moreover, the SD-VB achieved its best MIMO detection performance when the users are confined to angular sectors less than 80° wide. The consideration of second-order $\Sigma\Delta$ ADCs and additional numerical results are given in the extended version of this work in [18].

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