Achieving Synchronized Fresh Communication Over Broadcast Channels

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***Abstract*—We consider a scenario whereby the state of a common source is being updated at multiple distributed devices. We are particularly interested in the tradeoff that exists between the *freshness* of the updates at the distributed devices and the *synchrony* of the updates across them. In this paper, we explore this tradeoff in a wireless downlink setting whereby the transmitter can choose between unicast transmissions (with given success probabilities) to particular users and broadcast transmissions (with a smaller success probability) to all users. After discussing the Linear Programming (LP)-based optimal design and extreme choices of "always-unicasting" and "always-broadcasting" policies, we note that the optimal design is not scalable and the extreme policies are inefficient. This motivates us to develop two classes of policies, namely a "mixed randomized policy" and a "feature-based learning policy", which have desirable performance and computational-complexity characteristics. Additionally we manage to provide complete analysis for the mixed randomized policy under the two-user case, which provides interesting insights and can be partially extended to general cases. We perform extensive numerical studies to compare the performance of these designs over the benchmarks to reveal their gains.**

***Index Terms*—Age asynchrony, age-of-information, broadcasting channels, wireless downlink system, linear programming, renewal process.**

I. Introduction

IN RECENT years, the exponential growth of connected devices for the next-generation wireless networks and the advent of latency-sensitive applications, such as industrial automation, vehicular networks, and the Internet of Things (IoT), have shifted the focus from traditional communication metrics like throughput to more nuanced performance indicators. Age of Information (AoI) is one such metric that quantifies the freshness of information by measuring the time elapsed since the generation of the most recent update received by a user (see, for example, [1], [2], [3]).

Since the introduction of the AoI metric, numerous related studies emerged in various networking scenarios, including wireless random access networks (e.g., [4], [5]), content distribution networks (e.g., [6], [7]), scheduling (e.g., [8], [9], [10]) and queuing networks (e.g., [11], [12]). More recently, various extensions and variants of the AoI metric have been proposed to address different aspects of information freshness. Peak Age of Information (PAoI in [13]) is one such metric that captures the worst-case AoI by considering the maximum value of AoI over a time window and is especially important in applications where information staleness could lead to severe consequences. The AoI violation rate metric (see [14]) describes the time ratio of AoI violating a fixed level and is used in scenarios where the AoI for each source can tolerate occasional violations. The Age of Synchronization (AoS) metric (see [15]) describes the age difference between the source and the destination for the same source, which is totally different from the synchronization we are going to introduce.

In this paper, we will introduce and study the measure of *age asynchrony* among distributed users in a wireless downlink system, which measures how similar the age (and hence the freshness) levels are at the users. In particular, we explore the trade-off between the freshness and the synchrony of the updates under different transmission policies. Synchronization is a critical aspect of future wireless communication systems since accurate synchronization is essential for coordinating time-sensitive operations among different users (see [16]). There are many scenarios where synchronization among users takes precedence over AoI, such as in distributed control systems, cooperative communication networks ([17]) and so on (e.g., Vehicular Networks [18], Wireless Sensor Networks [19] and Precision Agriculture [20]). For example, in industrial automation and process control applications, distributed control systems involve multiple sensors, actuators, and controllers that need to coordinate their actions in real-time. Accurate synchronization among these users is critical for maintaining the stability and efficiency of the system, while the AoI may be of secondary importance (see [21]).

Thus, achieving a balance between AoI and synchronization is therefore of paramount importance for the effective functioning of these systems. The remainder of this paper is organized as follows:

- In Section II, we build our system model in a discrete-time wireless downlink setting whereby the transmitter can choose between unicasting and broadcasting with different transmission success probabilities. We formulate our problem as minimizing the weighted sum of the AoI and Age Asynchrony to study the trade-off between the freshness and the synchrony.
- In Section III, we study the optimal solutions via Linear programming for small number of users n due to the computational complexity of the optimal solution for

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large n. In Section IV, we analyze the performance of two extreme policies: always unicasting and always broadcasting, and make comparisons.

- In Section V and Section VI, we propose a mixed randomized policy and a feature-based learning policy, both with good scalability characteristics and non-negligible performance gains compared with extreme policies with meaningful success probabilities. Additionally, in Section V we managed to overcome the difficulties that arise in the analysis of the mixed randomized policy to obtain: (i) complete results on the average age and age asynchrony for the two-user setting, and (ii) gave a procedure of extending average age analysis to general $n-$user case.
- In Section VII, we execute simulations and compare all the mentioned policies. We observe that, with different number of users, success probability and weights, we may prefer different policies for optimizing the tradeoff. Counter-intuitively, we note that unicasting can be more preferable to broadcasting when aiming to minimize the age asynchrony under an unreliable communication environment. And in Section VIII, we conclude the paper and mention the potential future works.

In related literature, many works (e.g., [22], [23], [24]) have studied different types of the clock synchronization in a decentralized system, such as reference-broadcast synchronization (RBS) and time-stamp synchronization (TSS), but they focus on the structure of the protocols instead of considering transmission successes and failures. In [25] and [26] and many other works, the authors have aimed to decrease the synchronization and other metrics with time-sensitive 5G networks, but by the means of improving the transmission architecture and mechanisms to provide ultra-reliability and low-latency communications (URLLC). More recently, [27] have presented an efficient window-based resource allocation method for the end-to-end time-sensitive network scheduling problem under the uncertainty of the channel. This work aims to reduce large-scale fading correlation across the devices which is different from our scope. There are other works that aim at minimizing other AoI metrics under fading channels which is different from our focus. To our best knowledge, there is no prior work considering the trade-off between the freshness and the synchrony among the users in an unreliable communication environment under different transmission strategies.

II. System Model

In this paper, we will consider the operation of a discrete-time wireless communication system, whereby a Base Station (BS) sends information updates to n users at the beginning of every time slot $t \in \{1, 2, 3, \cdots\}$ either by broadcasting the information to all the users with a relatively lower individual success probability (that are generated independently for each user) or by unicasting the information to a specific user with a relatively higher success probability.

We assume that the BS refreshes its status and creates a new packet at the beginning of every time slot t. Accordingly, the BS always sends the freshest status to all the users. This assumption is especially reasonable for the scenarios where the state of the source is observable or accessible at the BS. More complicated models, such as randomly generated new packets, add more complexity and can be considered in the future extensions. Our goal is to find an effective strategy that can keep the information at the users fresh as well as the age of information amongst the users as synchronized as possible. We describe the key terminology and the essential system dynamics in the rest of this section. Then, in the following sections we formulate the problem and propose different strategies with different performance and complexity characteristics for its solution.

A. AoI and Age Asynchrony Metrics

First, define the Age-of-Information (AoI) of User i as $U_i[t]$, which is the number of time slots elapsed at time t since User i last received a successful update from the station. The AoI is updated as follows:

$$U_i[t+1] = \begin{cases} 0, & \text{if transmission of source i succeeds} \\ U_i[t]+1, & \text{otherwise.} \end{cases}$$

Define $A[t]$ as the average AoI of all the users at time t, $A[t] = \frac{1}{n}\sum U_i[t]$. To study the information freshness difference between users, we will additionally define the metric $S^1[t]$ as the $1^{st}-$order average Age Asynchrony[1],

$$S^1[t] = \frac{1}{\binom{n}{2}} \sum_{i \neq j} |U_i[t] - U_j[t]| \,.$$

For example, consider a group of autonomous drones performing a coordinated task and the control system requires up-to-date location information from all drones with a maximum AoI difference to avoid potential collisions. The goal of the Age Asynchrony is to measure such age differences.

B. Broadcast/Unicast Model

We assume that in our model the base station will choose one of the actions $x[t] \in \mathscr{X}$ at every time slot t, where $\mathscr{X} = \{0, 1, \cdots, n\}$, $x[t] = 0$ represents that the station chooses to broadcast to all n users and $x[t] = i$ means that the station chooses to unicast with the i^{th} user.

Under the broadcasting model, we let $\mathbb{P}\{U_i[t+1] = 0\} = p_b$ be the probability of success, whereby the success/failure outcomes of each user is independently determined. [2] Under the unicasting model, when $x[t] = i$, $\mathbb{P}\{U_i[t+1] = 0\} = p_d$, $\mathbb{P}\{U_j[t+1] = 0\} = 0$ for $j \neq i$. Broadcasting success probability is likely to be smaller than the unicast success probability in our setting since the PHY-layer signalling that is needed to perform a simultaneous broadcast transmission to n users over independently fading channels is significantly more difficult than the signalling that can be optimized for an individual unicast channel.

[1]All the theoretical results that we will obtain on $S^1[t]$ can be extended to $r^{th}-$order average Age Asynchrony cases $S^r[t] = \sum_{i \neq j} |U_i[t] - U_j[t]|^r / \binom{n}{2}$, $r \in \mathscr{N}$, by using the Faulhaber's formula and Arithmetico-geometric sequence formula. Here, we focus on the case of $r = 1$ in order to keep the results cleaner.

[2]In reality, users may have different success probabilities under the broadcasting model due to user locations, which can be discussed in future works.

C. Objective

In this paper, we focus on minimizing the weighted sum of the long-term average of Age Asynchrony and Age-of-Information, which allows us to study the trade-off between the information freshness of the users and the information synchronization between users. Define the cost at t to be a function of the weight $\alpha \in [0,1)$:

$$C_\alpha[t] \triangleq (1-\alpha)A[t] + \alpha S^1[t].$$

Notice that α is not allowed to be 1 in our model, since only minimizing Age Asynchrony can push the system into an unstable operating mode where none of the users wants to get updates when their Age Asynchrony is small. We will study the optimal solution to the problem of minimizing the long-term average:

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\left[C_\alpha[t]\right]$$

via Linear Programming in Section III, extreme policies with either always broadcasting or always unicasting with the oldest users in Section IV, a mixed randomized policy in Section V, and feature-based learning policies in Section VI. We will compare the theoretical and simulation performance of these designs in Section VII.

III. Optimal Design

In this section, we formulate the minimization problem under the Markov Decision Process (MDP) setup. Let the state be the current age of n users: $\mathbf{U} = [U_i[t]]_{i=1}^n \in [0,D]^n$ where D is an upper bound on the ages [3] and $P(\mathscr{X})$ is the probabilistic policy on set $\mathscr{X}$. Then, the MDP problem for n users can be formulated as:

$$\min_{x[t]\in P(\mathscr{X})} \lim_{T\to\infty} (1-\alpha)\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\left[A[t]\right] + \alpha\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\left[S^1[t]\right]$$

Theoretically, this problem can be solved by transforming the MDP in an appropriate Linear Program (LP). However, this approach is not scalable due to the exponential growth of the problem size the n. Nevertheless, using this solution for small n values will allow us to use it as a benchmark for our designs to compare against. As such, for completeness, we provide the optimal solution LP for $n = 2$ users, which can be generalized to $n > 2$ with increasing notational complexity.

Theorem 1: The solution to the 2 users minimization problem can be obtained by solving the following linear programming problem:

$$\begin{aligned}
\min_{y^k_{a_1,a_2}} \quad & \sum_{a_1,a_2=0}^{D}\sum_{k=0}^{2}\left[\frac{1-\alpha}{2}(a_1+a_2)+\alpha|a_1-a_2|\right]y^k_{a_1,a_2}\\
\text{s.t: } & 0 \le y^k_{a_1,a_2} \le 1 \quad \forall 0\le a_1,a_2\le D, 0\le k\le 2,\\
& \sum_{a_1,a_2=0}^{D}\sum_{k=0}^{2} y^k_{a_1,a_2} = 1,\\
& \mathbf{Q}\boldsymbol{y} = \mathbf{0},
\end{aligned}$$

where $\boldsymbol{y}$ is a column vector of size $3(D+1)^2$ with $\boldsymbol{y} = (y^0_{0,0}, y^1_{0,0}, y^2_{0,0}, \cdots, y^0_{D,D}, y^1_{D,D}, y^2_{D,D})^T$ as its components, D is an upper bound on the age state in the system which can be set sufficiently large so that the probability of reaching D is vanishing. And $\mathbf{Q}\boldsymbol{y} = \mathbf{0}$ is the matrix representation of the (Markov balance) equations that describes the probability flux associated with the Markov chain in and out of states $(a_1,a_2) \in [0,D]^2$, which can specified based on the transition probabilities in Appendix A. If this LP is feasible and $\boldsymbol{y}$ is an optimal solution, where $y^k_{a_1,a_2}$ represents the stationary probability of being in state (a_1,a_2) and choosing the action k, then the optimal policy is a probabilistic policy $P(\mathscr{X})$, whereby the probability $f^k_{a_1,a_2}$ of choosing $x[t] = k$ given the age is at state (a_1,a_2) equals:

$$f^k_{a_1,a_2} = \begin{cases} \frac{y^k_{a_1,a_2}}{\sum_{k=0}^{2} y^k_{a_1,a_2}}, & \text{if} \sum_{k=0}^{2} y^k_{a_1,a_2} \ne 0 \\ \frac{1}{3}, & \text{if} \sum_{k=0}^{2} y^k_{a_1,a_2} = 0 \end{cases}$$

for $(a_1,a_2) \in [0,D]^n$.

Proof: The proof follows directly from the equivalency between MDP and LP problem and is omitted here (refer to [28]). ■

Since our solution vector $\boldsymbol{y}$ is a column vector of size $(n+1)(D+1)^n$ for n user scenario, eventually the time complexity of implement LP solution [4] can be approximated as $O((D+1)^{3.5n})$ which will increase exponentially as the number of users n increases and makes the LP solution not implementable for large n. Thus, We will study more policies with better scalability in following sections.

IV. Extreme Policies

To develop an understanding of the broadcasting and unicasting decisions, in this section, we study the performance of age and synchronization metrics for two extreme policies as a function of n for different p_b and p_d, and compare the theoretical results at the end of this section. The related simulation performance can be found in Section VII.

A. Always-Broadcasting Policy

In this section, we study the policy that selects $x[t] = 0$ for all t, i.e., the BS always chooses to broadcast the current information to n users. We will analyze the long-term average of $A[t]$ in Theorem 2 and the long-term average of $S^1[t]$ in Theorem 3.

Theorem 2: The long-term average of $A[t]$ for always-broadcasting equals to:

$$\frac{\sum_{k=0}^{\infty}(1-p_b)^k\cdot\frac{1}{2}k(k+1)}{\sum_{k=0}^{\infty}(1-p_b)^k(k+1)} = \frac{(1-p_b)}{p_b}$$

which remains constant as n increases.

Proof: Since each user is statistically identical with respect to age and user successes are independent, it is sufficient to calculate the long-term average of $U_1[t]$. The result follows from the characteristics of the associated geometric distribution. Details are omitted due to limited space. ■

[3] In reality, D can be viewed as an upper bound where ages older than D make no difference to the system. Theoretically, as D approaches infinity, the solution approaches the solution of the infinite CMDP where ages are unbounded ([28]).

[4] In our paper, we implemented the LP solution through the MATLAB 'linprog' function, which uses the interior-point method and the time complexity can be approximated as $O(N^{3.5})$, where N is the dimensionality of the solution vector.

Theorem 3: The long-term average of $S^1[t]$ for always-broadcasting equals to:

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{S^1[t]\} = \lim_{t\to\infty}\mathbb{E}\{S^1[t]\}$$
$$= \sum_{k=0}^{\infty}\frac{2p_b(1-p_b)}{1-(1-p_b)^2}p_b(1-p_b)^k(k+1) = \frac{2(1-p_b)}{1-(1-p_b)^2}.$$

Proof: Since each user pairs are identical, it is sufficient to calculate the long-term average of the absolute age gap between any two users. Details are omitted due to space. ■

B. Always-Unicasting Policy

In this section, we study the policy that selects $x[t] = \arg\max_i U_i[t]$ for all time slots t, i.e., the BS always chooses to unicast information to the user with the oldest age. We will analyze the long-term average of $A[t]$ in Theorem 4 and the long-term average of $S^1[t]$ in Theorem 5. This policy will be more complex than the previous always-broadcasting policy since this is a state-dependent policy.

Theorem 4: The long-term average of $A[t]$ for always-unicasting equals to:

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{A[t]\} = \lim_{t\to\infty}\mathbb{E}\{A[t]\} =$$
$$\frac{\sum_{k=0}^{\infty}p_d^n(1-p_d)^k\binom{n+k-1}{n-1}\frac{1}{2}(n+k)(n+k-1)}{\sum_{k=0}^{\infty}p_d^n(1-p_d)^k\binom{n+k-1}{n-1}(n+k)}$$
$$= \frac{\frac{1}{2}\cdot -np_d^{-n-2}(-n+2p_d-1)}{np_d^{-n-1}} = \frac{n-2p_d+1}{2p_d}.$$

Note that the average AoI is linear in n for a fixed success probability p_d.

Proof: By symmetry, we only need to calculate the long-term average of User 1. Since the Markov Chain $U_1[t]$ is positive recurrent, thus the sequence of entry times to state $U_1[t] = 0$ can be viewed as the arrival epochs of a renewal process. Define T_N as the time of the N^{th} entries to state 0 with $T_0 = 0$, $N \in \mathcal{N}$ and define $\Delta_N = T_{N+1} - T_N$ to be the time interval between two entries.

Since we always perform direct transmission, after a success at User 1, $U_1[t]$ will keep increasing by one until another success happens at User 1, and since we always choose the user with the oldest age to transmit, between two successes at User 1, all the other $n-1$ users must succeed once. Based on the above description, independent of the starting points, the steady-state probability distribution of the interarrival time $P(\Delta_N = k) = 0$ when $k = 0, 1, \cdots, n-1$; $P(\Delta_N = n+k) = p_d^n(1-p_d)^k\binom{n+k-1}{n-1}$, when $k = 0, 1, \cdots$. And when $\Delta_N = n+k$, $U_1[t+\tau] = \tau$ for $\tau = 0, \cdots, n+k-1$ in this renewal, so by the Wald's identity ([29]), we get the claimed formula of the average age. ■

Theorem 5: The long-term average of $S^1[t]$ equals to: $\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{S^1[t]\} = \lim_{t\to\infty}\mathbb{E}\{S^1[t]\} = \frac{n+1}{3p_d}$, which is linear in n for a fixed probability p_d.

Proof: The proof is more involved and is moved to Appendix B to avoid disrupting the flow of the main text. ■

C. Discussion on the Performance of Extreme Policies

By comparing the long-term average of the average AoI $A[t]$ of always-broadcasting and always-unicasting policies, we can see that when the broadcasting success probability $p_b > \frac{2p_d}{n+1}$, always-broadcasting policy provides better average AoI performance. Assume that the expected number of successes is unchanged for always-broadcasting when n increases, i.e., assume that $p_b = \frac{\mu}{n} \in [0,1]$ where μ is a positive constant, then the average Age Asynchrony under the broadcasting policy will become:

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\left\{S^1[t]\right\} = \frac{2\left(1-p_b\right)}{\left(1-p_b\right)^2} = \frac{2(n-\mu)n}{(2n-\mu)\mu}$$

Recall that for always unicasting policy, the average Age Asynchrony equals $\frac{n+1}{3p_d}$. Therefore, asymptotically speaking, average Age Asynchrony approaches $\frac{n}{\mu}$ and $\frac{n}{3p_d}$ respectively for always-broadcasting and always-unicasting. Combining both metrics together to minimize the long-term average of $C(\alpha) = (1-\alpha)A[t] + \alpha S[t]$ for a given α, we get for alwaysbroadcasting policy:

$$\lim_{n\to\infty}\frac{1}{n}\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\left\{C_\alpha[t]\right\} = (1-\alpha)\frac{1}{\mu} + \alpha\frac{1}{\mu} = \frac{1}{\mu}$$

and for always-unicasting policy:

$$\lim_{n\to\infty}\frac{1}{n}\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\left\{C_\alpha[t]\right\} = \frac{(1-\alpha)}{2p_d} + \frac{\alpha}{3p_d} \quad (1)$$

Hence, when (1) is larger than $\frac{1}{\mu}$ or p_b decays slower than $\frac{\mu}{n}$, we would prefer always-broadcasting eventually, otherwise, always-unicasting policy eventually becomes better, see Section VII for simulation results. We also observe that, when one of the success probabilities p_b and p_d is large enough so that the average performance of one extreme policies is much better than the other one, extreme policies perform good enough compared to LP solutions. It is because this case implies that broadcasting or unicasting dominates the other on most of the age states. However, when $2p_d \le np_b \le 3p_d$ (from (1)), the average performance of the extreme policies are comparable, in which case we need to find other policies to achieve better performance. This motivates us to develop new policies in the following sections.

V. Mixed Randomized Policy

In the previous section, we were able to provide a complete analysis of two extreme policies of always-broadcasting and always-unicasting to the user with the oldest age. An important weakness of these policies is their limitation in trading-off the age and synchronization performances. In this section, we introduce a mixed randomized policy that can explore this tradeoff by employing a randomization parameter to decide between broadcasting or unicasting the oldest age user. This policy is more capable by encapsulating the previous two cases as extremes, but is also far more difficult to analyze, even for a two-user setting, due to the mixture of unicasting and broadcasting decisions made in its operation.

In this section, we are able to overcome these difficulties for the two-user setting to obtain complete results on the average age and synchronization performance of this policy in terms of the system parameters and the algorithm parameter p. These, subsequently, allow us to characterize and optimize the tradeoff between the two opposing metrics. Finally, we discuss the extension of the analysis to the general case of n users, whereby the analysis is still possible but becomes more complex due to the emergence of growing number of cases in the recursive process.

A. Average Age and Age Asynchrony Analysis for Two Users

The next theorem provides the average age performance of the mixed randomized policy for the two-user case.

Theorem 6: The long-term average AoI performance under the mixed randomized policy for $n = 2$ is given by

$$\frac{\sum_{i=1}^{\infty} \frac{i(i-1)}{2}\rho(i)}{\sum_{i=1}^{\infty} \rho(i) \cdot i}, \tag{2}$$

where $\rho(1) = \beta_p \eta_p + \gamma_p \theta_p$, and for $i \geq 2$,

$$\rho(i) = \beta_p \left(\zeta_p^{i-1} \eta_p + \mu_p \lambda_p \frac{\psi_p^{i-1} - \zeta_p^{i-1}}{p \cdot p_b(1-p_b)} \right) + \gamma_p \left(\zeta_p^{i-1} \theta_p + \xi_p \lambda_p \frac{\psi_p^{i-1} - \zeta_p^{i-1}}{p \cdot p_b(1-p_b)} \right),$$

where the definitions of the terms in the expression are provided in Appendix C.

Proof: Same as in Theorem 4, the sequence of entry times to state 0 for $U_1[t]$ can be viewed as the arrival epochs of a renewal process. In this section, define T_N as the time of the N^{th} entries to state 0 with $T_0 = 0$, $N \in \mathscr{N}$ for User 1 and define $\Delta_N = T_{N+1} - T_N$ to be the time interval between two entries. For simplicity, use Δ to denote the inter-arrival times under the steady-state distribution.

Under the event where $\Delta = i$, there are two cases, $U_2[t] = 0$ and $U_2[t] \neq 0$. Then $P(\Delta = i) = \beta_p P(\Delta = i | U_2[t] = 0) + \gamma_p P(\Delta = i | U_2[t] \neq 0)$. Since the relationship between the age of the two users will affect the success probability of each user, in each case, there are two sub-cases: User 2 never succeeds in time slots $t+1$ to $t+\Delta-1$ and User 2 succeeds at least once in time slots $t+1$ to $t+\Delta-1$ (the probability of the second sub-case equals 0 when $i = 1$). So, $P(\Delta = 1) = \beta_p \eta_p + \gamma_p \theta_p$. And for $i \geq 2$, $P(\Delta = i | U_2[t] = 0) = \zeta_p^{i-1} \eta_p + \mu_p \lambda_p \sum_{j=0}^{i-2} \zeta_p^j \psi_p^{i-2-j}$, where $\sum_{j=0}^{i-2} \zeta_p^j \psi_p^{i-2-j} = \frac{\psi_p^{i-1} - \zeta_p^{i-1}}{\psi_p - \zeta_p} = \frac{\psi_p^{i-1} - \zeta_p^{i-1}}{p \cdot p_b(1-p_b)}$. Similarly, we can calculate $P(\Delta = i | U_2[t] \neq 0) = \zeta_p^{i-1} \theta_p + \xi_p \lambda_p \sum_{j=0}^{i-2} \zeta_p^j \psi_p^{i-2-j}$ for $i \geq 2$. And finally, since when $\Delta = i, i = 1, 2, \cdots$, we will have $U_2[t+\tau] = \tau$ for $\tau = 0, 1, \cdots, i-1$ in this renewal, so the long-term average AoI equals to (2) followed by the Wald's identity ([29]), where $\rho(i)$ denotes $P(\Delta = i)$. ■

The performance of the policy can be seen in Figure 4 and Figure 7. Notice that all the terms in (2) are in the form of arithmetico-geometric series (see [30]), so the result can be simplified as an explicit expression without summations. This is omitted here due to the complex form that they will result in.

Having provided the average age performance, we now turn to the synchronization performance, which is an even more complex metric under the mixed randomized policy. Next, we are able to obtain the result for the two-user case. To that end,

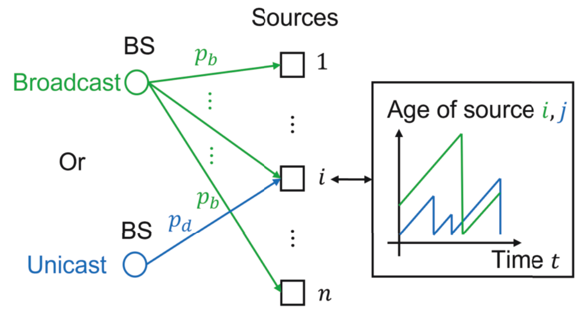


Fig. 1. Base Station updates its status to n users by either broadcasting or unicasting to keep the age levels at users low and synchronized.

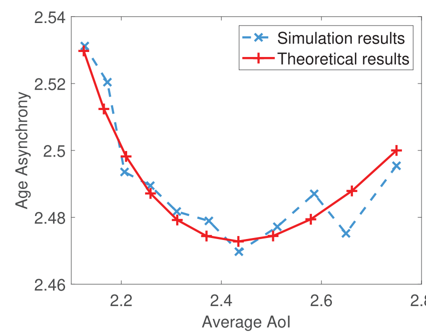
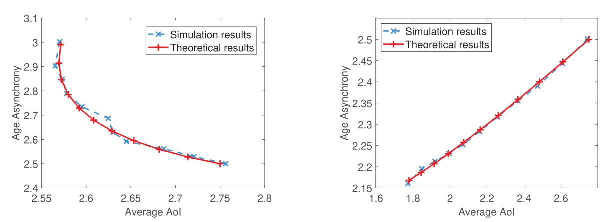


Fig. 2. The trade-off between average age and Age Asynchrony for mixed randomized policy under simulation and by theorem when $n = 2, p_b = 0.32, p_d = 0.4$.



(a) When $n = 2$, $p_b = 0.28$, $p_d = 0.4$.

(b) When $n = 2$, $p_b = 0.36$, $p_d = 0.4$.

Fig. 3. The trade-off between average age and Age Asynchrony for mixed randomized policy under simulation and by theorem with other channel successful probabilities.

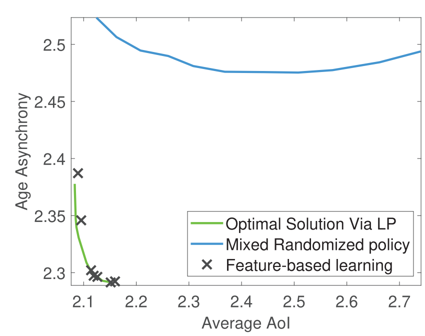


Fig. 4. The trade-off between average age and Age Asynchrony for all policies when $n = 2, p_b = 0.32, p_d = 0.4$.

let us define T_N as the time of the $N^{th} \in \mathscr{N}$ visit to the subset of states $[u_1, u_2]$ such that $u_1 \cdot u_2 = 0$, with $T_0 = 0$.

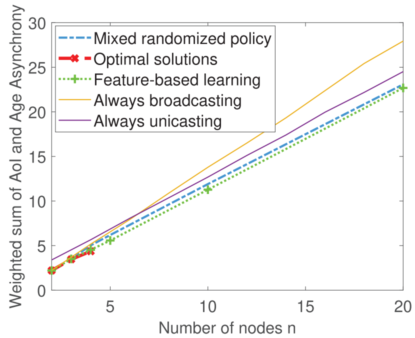


Fig. 5. Performance comparison against increasing n when $p_d = 0.3$, $p_b = 0.7/n, \alpha = 0.9$.

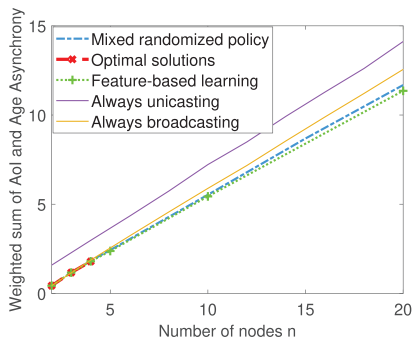
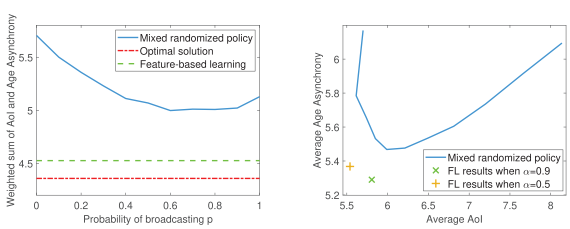


Fig. 6. Performance comparison against increasing n when $p_d = 0.6$, $p_b = 1.5/n, \alpha = 0.5$.



(a) Weighted sum of AoI and Age Asynchrony when $\alpha = 0.9$, $n = 4$, $p_d = 0.3$, $p_b = 0.7/n$.

(b) AoI and Age Asynchrony performance trade-off when $n = 10$, $p_d = 0.6$, $p_b = 1.5/n$.

Fig. 7. Performance comparisons between mixed randomized policy, feature-based learning and Optimal solution for given n.

Also let $\Delta_N = T_{N+1} - T_N$ to be the time interval between two such visits. Same as in Theorem 4, the sequence of visits to those states can be viewed as the arrival epochs of a renewal process.

For the long-term average of Age Asynchrony performance when $n = 2$, we utilize the steady-state distribution of $S^1[T_N]$. Through the steady-state balance equation, it is possible to find a recursive formula of $P(S^1[T_N] = i), i \in \mathcal{N}$ and get the general formulas by induction as shown in the following theorem. Since the time interval between two entries are independently and identically distributed, and the value of $S^1[t]$ does not change in one renewal interval, the long-term average of the Age Asynchrony equals the mean of the steady-state distribution of $S^1[T_N]$, i.e., $\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\{S^1[t]\} = \lim_{N\to\infty} \mathbb{E}[S^1[T_N]]$ and thus can be calculated after we get the steady-state distribution $\overline{S^1}$ given in the next theorem.

Theorem 7: The steady-state distribution of the average Age Asynchrony under $n = 2$ will be,

$$P(\overline{S^1} = i) = \frac{p \cdot p_b^2}{P_{\mathrm{R}}},$$

when $i = 0$,

$$P\left(\overline{S^1} = i\right) = \frac{2\mu_p\left(p \cdot p_b^2 + \xi_p\right)}{P_{\mathrm{R}}}\left(\zeta_p + \phi_p\right)^{i-1}$$

when $i \geq 1$, where $P_{\mathrm{R}} = p \cdot p_b^2 + 2p \cdot p_b\left(1 - p_b\right) + (1 - p)p_d$ denotes the steady-state probability of the event $U_1[t]U_2[t] = 0$. Thus, the long-term average of Age Asynchrony will be:

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\{S^1[t]\} = \frac{2\mu_p}{P_{\mathrm{R}}(p \cdot p_b + (1 - p)p_d)}. \quad (3)$$

Other definitions of the notations are available in Appendix C.

Proof: First, it is easy to calculate that:

$$\begin{aligned} &P(\overline{S^1} = 0) = P(S^1[T_N] = 0) \\ &= P(U_1[T_N] = 0, U_2[T_N] = 0 | U_1[T_N]U_2[T_N] = 0) \\ &= \frac{p \cdot p_b^2}{p \cdot p_b^2 + 2p \cdot p_b(1 - p_b) + (1 - p)p_d}. \end{aligned}$$

For $i = 1$, we separate the events into two sub-events since the relationship between $U_1[t]$ and $U_2[t]$ will affect their own successful probabilities: $\{S^1[T_N] = 1, S^1[T_{N-1}] = 0, T_{N-1} = T_N - 1\}$ and $\{S^1[T_N] = 1, S^1[T_{N-1}] > 0, T_{N-1} = T_N - 1\}$. Since we have $\lim_{t\to\infty} P(S^1[t] = 0) = p \cdot p_b^2$ and $\lim_{t\to\infty} P(S^1[t] > 0, U_1[t]U_2[t] = 0) = 2p \cdot p_b(1 - p_b) + (1 - p)p_d$. So,

$$\begin{aligned} &P(\overline{S^1} = 1) \\ &= \frac{2p \cdot p_b^2\mu_p}{P_{\mathrm{R}}} + \frac{(2p \cdot p_b(1 - p_b) + (1 - p)p_d)\xi_p}{P_{\mathrm{R}}} \\ &= \frac{2\mu_p(p \cdot p_b^2 + \xi_p)}{P_{\mathrm{R}}} \end{aligned}$$

For $i \geq 2$, we need to separate the events into three sub-events: $\{S^1[T_N] = i\} = \{S^1[T_N] = i, T_{N-1} = T_N - i, S^1[T_{N-1}] = 0\} \cup \{S^1[T_N] = i, T_{N-1} = T_N - i, S^1[T_{N-1}] > 0, U_1[T_{N-1}] + U_1[T_N] > 0, U_2[T_{N-1}] + U_2[T_N] > 0\} \cup \{S^1[T_N] = i, T_{N-1} = T_N - i, S^1[T_{N-1}] > 0, (U_1[T_{N-1}] + U_1[T_N])(U_2[T_{N-1}] + U_2[T_N]) = 0\}$. Specifically, the first sub-event represents those cases where $U_1[T_{N-1}] = U_1[T_{N-1}] = 0$; the second sub-event represents those cases where a different user succeeds at the $N - 1^{th}$ entry from the user succeeds at the N^{th} entry; the third sub-event represents those cases where at the N^{th} entry and $N - 1^{th}$ entry, the same user succeeded. So,

$$\begin{aligned} P(\overline{S^1} = i) &= \frac{p \cdot p_b^2\zeta_p^{i-1}2\mu_p}{P_{\mathrm{R}}} \\ &\quad + \frac{(2p \cdot p_b(1 - p_b) + (1 - p)p_d)\zeta_p^{i-1}\xi_p}{P_{\mathrm{R}}} \\ &\quad + \frac{P_{\mathrm{R}}\sum_{k=1}^{i-1}\zeta_p^{i-k-1}\phi_p P(\overline{S^1} = k)}{P_{\mathrm{R}}} \\ &= \frac{2\mu_p\zeta_p^{i-1}}{P_{\mathrm{R}}}\left(p \cdot p_b^2 + \xi_p\right) \end{aligned}$$

$$+ \phi_p \sum_{k=1}^{i-1} \zeta_p^{i-k-1} P(\overline{S^1} = k)$$
$$= P(\overline{S^1} = 1)\zeta_p^{i-1} + \phi_p \sum_{k=1}^{i-1} \zeta_p^{i-k-1} P(\overline{S^1} = k).$$

Based on the observation in the last equality, we can prove by induction that for $i \geq 1$,

$$P(\overline{S^1} = i) = \frac{2\mu_p(p \cdot p_b^2 + \xi_p)}{P_{\mathrm{R}}}(\zeta_p + \phi_p)^{i-1}.$$

The probability sums up to one since $p \cdot p_b^2 + \xi_p = 1 - (\zeta_p + \phi_p) = p \cdot p_b + (1-p)p_d$ and $2\mu_p + p \cdot p_b^2 = P_R$. Finally, since for $i \geq 1$, the steady-state distribution is a geometric progression, so the long-term average Age Asynchrony can be calculated explicitly as:

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\{S^1[t]\} = \sum_{i=0}^{\infty} iP(\overline{S^1} = i)$$
$$= \frac{2\mu_p(p \cdot p_b^2 + \xi_p)}{P_{\mathrm{R}}} \sum_{i=1}^{\infty} i(\zeta_p + \phi_p)^{i-1}$$
$$= \frac{2\mu_p(p \cdot p_b^2 + \xi_p)}{P_{\mathrm{R}}} \cdot \frac{1}{(1-\zeta_p-\phi_p)^2}$$
$$= \frac{2\mu_p(p \cdot p_b^2 + \xi_p)}{P_{\mathrm{R}}(p \cdot p_b + (1-p)p_d)^2} = \frac{2\mu_p}{P_{\mathrm{R}}(p \cdot p_b + (1-p)p_d)}.$$

■

The complete performance analysis of the mixed policy for two-users allows us to study their tradeoff from a theoretical standpoint. In the next subsection, we make such an excursion.

B. Discussion on the Average AoI Vs. Age Asynchrony Tradeoff

Based on Theorems 6 and 7, for fixed channel successful probabilities p_b and p_d, both the long-term average AoI and Age Asynchrony can be expressed explicitly as a function of p where both numerator and denominator are polynomials. Thus, by finding the derivative of $\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\{C_\alpha[t]\}$, we can get the value of the optimal $p^* = \arg\min_{p\in[0,1]} \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\{C_\alpha[t]\}$ that lead to the following three different cases based on the value of p_b and p_d: i) $p^* = 0$ and $\frac{d}{dp}\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\{C_\alpha[t]\} \geq 0$, which means that broadcasting dominates unicasting under this α; ii) $p^* = 1$ and $\frac{d}{dp}\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\{C_\alpha[t]\} \leq 0$, which means that unicasting dominates broadcasting under this α; (iii) $p^* \in (0,1)$, which means that neither unicasting nor broadcasting is always better and thus we can benefit from applying the mixed-randomized policy under this α. Given different weights α, mixed randomized policy benefits under different channel successful probability ranges of p_b and p_d.

In Figure 2 and Figure 3, we compare the simulation results and the theoretical results of the mixed randomized policy under different channel successful probabilities to validate Theorem 6 and Theorem 7 as well as to show the impact of p_b and p_d on the mixed randomized policy. As can be seen in all three figures, the simulation results align with the theoretically results for both long-term average AoI and Age Asynchrony. Figure 3 shows the cases where either unicasting or broadcasting dominates the other when minimizing only the Age Asynchrony $S^1[t]$ while in Figure 2 the mixed randomized policy can provides non-negligible improvements on Age Asynchrony compared to two extreme policies. More specifically, theoretically we have $p^* = \frac{600\sqrt{\frac{2}{43}}}{11} - \frac{125}{11} \approx 0.3999$ when $p_b = 0.32$, $p_d = 0.4$ by setting the derivative of the long-term average of Age Asynchrony in (3) to 0 with respect to p, which perfectly matches the numerical results in Figure 2.

C. Discussion on the Extension to General n-User Case

In Theorems 6 and 7 provide complete results of the mixed randomized policy for two-users. While their calculation is relatively straight-forward, even these results have fairly intricate expressions to be stated fully. The general case of n users are increasingly more involved to expressed, and thereby not feasible for complete statements. In this subsection, we provide a partial discussion on how the above results can be extended to the general case or what challenges exist in such an extension.

We first start by noting that, the results of Theorem 6 can still be calculated for general n as in (2) where $\rho(i) = P(\Delta = i)$ are in the form of the sum of finitely many geometric progressions. To clarify this extension, we take $n = 3$ as an example to explain the procedure with which the two-user case approach can be extended, albeit with increasing notational complexity.

The difficulty for higher dimensions comes from the fact that, conditioned on the case where $U_1[t] = 0$ and $U_2[t], U_3[t] \neq 0$, we cannot easily calculate the probability of the sub-cases of $U_2[t] \neq U_3[t]$ or $U_2[t] = U_3[t]$. And whether $U_2[t] = U_3[t]$ or not will affect the success probability of User 2 and 3 under the unicasting model and further affects the success probability of User 1. However, by carefully calculating the conditional probabilities, we find that whether $U_2[t] = U_3[t]$ or not will not affect the probability distribution of the slots until User 1 succeeds next time, so the same methods can be applied to higher dimensions as well. Next we explain the major steps to calculate the probability $\rho(i)$ for $n = 3$.

- First of all, we will explicitly calculate $P(Z[t] = z|U_1[t] = 0)$ for $z = 1, \cdots, n$ based on the Bayesian rule, where random variable $Z[t]$ denotes the number of users with age $U_i[t] = 0$ in $i = 1, \cdots, n$ at time t. Under n users case, $P(Z[t] = n) = p \cdot p_b^n$, $P(Z[t] = 1) = p \cdot np_b(1-p_b)^{n-1} + (1-p) \cdot p_d$, for $z = 2, \cdots, n-1$, $P(Z[t] = z) = p \binom{n}{z} p_b^z (1-p_b)^{n-z}$;
- Next we will illustrate that, given $U_1[t] = 0$ and $Z[t] = 1$ in $n = 3$ case, the probability of both User 2 and User 3 succeeded before $t + \Delta$ (i.e., User 1 succeeded at time slot $t + \Delta$ as the oldest user with successful probability $p \cdot p_b + (1-p) * p_d$) are the same under $U_2[t] = U_3[t]$ and $U_2[t] \neq U_3[t]$ conditions, so without knowing $P(U_2[t] = U_3[t]|U_1[t] = 0, Z[t] = 1)$ or $P(U_2[t] \neq U_3[t]|U_1[t] = 0, Z[t] = 1)$, $P(\Delta = i|U_1[t] = 0, Z[t] = 1)$ can be calculated by discussing whether U_1 succeeded at time slot $t + \Delta$ as the oldest, one of the oldest, or non-oldest users. Particularly, $P(U_2$ and U_3 succeeded before $t + \Delta|U_2[t] = U_3[t]) = P(U_2$ and U_3 first succeeded together at one slot before $t + \Delta|U_2[t] = U_3[t]) + 2\sum_{k\neq r, k<r<\Delta} P($one of two users succeeded first at slot k as one of the oldest user$)P($the other user succeeded first at slot r as the only oldest

user). $P(U_2$ and U_3 succeeded before $t + \Delta | U_2[t] \neq U_3[t]) = P(U_2$ and U_3 first succeeded together at one slot before $t + \Delta | U_2[t] = U_3[t]) + \sum_{k \neq r, k<r<\Delta}$ {P(the older user succeeded first at slot *k* as the only oldest user)P(the other user succeeded first at slot *r* as the only oldest user) + P(the smaller succeeded first at slot *k* as the non-oldest user)P(the other user succeeded first at slot *r* as the only oldest User)}. Notice the fact that $2\left(p(1-p_b)^2 p_b + \frac{1}{2}(1-p)p_d\right) = (p(1-p_b)^2 p_b + (1-p)p_d) + (p(1-p_b)^2 p_b)$, the above two probabilities are the same with each other. Similar equalities hold in general cases for $n \geq 3$ and $Z[t] = 1, \cdots, n$;

- Based on that, we can calculate $P(\Delta = i) = \sum_{z=1}^{n} P(Z[t] = z | U_1[t] = 0) P(\Delta = i | Z[t] = z)$, where $P(\Delta = i | Z[t] = z)$ can be calculated in the form of the sum of finitely many geometric progressions in the same way as in Theorem 6.

However, the results of Theorem 7 are difficult to extend for general *n* even for $n = 3$. In the two-user case, we calculated the average Age Asynchrony by conditioning on whether the same user succeeds at both T_N and T_{N-1}, or not. In the three-user scenario, under the case $U_1[T_N] = 0$, we do not only care about whether $U_1[T_{N-1}] = 0$ or not, but also care about whether $U_2[T_{N-1}] = 0$ or $U_3[T_{N-1}] = 0$ or both. Furthermore, we also need to know the time slot that User 3 last succeeded if $U_2[T_{N-1}] = 0$. There are many different cases to cover even in three user scenarios, which makes it impossible to give a clear procedure to calculate average Age Asynchrony for higher dimensional cases.

VI. Feature-Based Learning Policy

Since formulating the Linear Programming problem in Theorem 1 is very complicated even for $n > 2$, in Figure 5 and Figure 6, we perform first visit Monte Carlo tabular reinforcement learning method on state space $\mathbf{U} = [U_i[t]]_{i=1}^{n} \in [0, D]^n$ for $n = 3, 4$ cases to compute the performance of the optimal solution that minimizes the discounted total cost. For higher dimension $n > 4$, as Monte Carlo tabular learning algorithm becomes much slower, we next introduce a feature-based learning algorithm which is based on the feature state $(A[t], S^1[t])$ and compare the performance with other policies.

Before we start with the feature-based learning algorithm, we would like to mention that one popular method to handle high-dimensional state/action space MDP problems is the value approximation and Deep Reinforcement Learning (DRL) methods. However, such pure data-driven methods are best-suited when there is a lack of knowledge about the critical features or the essential characteristics of the system. Accordingly, implementing them will prevent us from understanding the system better. As such, the feature states have physical meanings and the feature-based learning algorithm provides a more interpretable model, making it easier to understand the underlying decision-making process. Additionally, as the number of users *n* increases, the state space grows exponentially as $(D+1)^n$. This exponential growth in state space can make it challenging for DRL algorithms to converge within a sufficiently small time horizon (days). Feature-based learning, on the contrary, is less computationally intensive and can provide quicker insights, allowing us to conduct extensive experiments. As such, we have considered feature-based methods in our comparisons, instead of DRL methods.

Algorithm 1 Feature-Based Monte Carlo Learning

Initialize $Q(y, x)$ arbitrarily for all feature state-action pairs (y, x)
Initialize the empty list for all (y, x)
Initialize policy π_0 randomly
for $i = 0, 1, \ldots,$ number of episode **do**
  Generate an episode following the current policy π_i, observe sequence of states $\{\mathbf{U}\}_{0:T}$, actions $\{x\}_{0:T}$, costs $\{C_\alpha\}_{0:T}$ and extract the feature $\{Y\}_{0:T}$ from $\{\mathbf{U}\}_{0:T}$
  Calculate the first visit discounted total cost *G* with a discounted factor close to 1
  Calculate $Q_i(y, x)$ as the average of all discounted total costs *G*
  Update policy greedily for all feature states by $\pi_{i+1}(y) = \arg\min Q_i(y, x)$
end for
for Sufficiently many episodes **do** // Evaluation of the policy π
  Initialize and evaluate the average cost in a new environment
end for

A. Feature-Based Learning Policy

Intuitively speaking, when one of the users $U_i[t]$ is much higher, and all the other users are at a much lower age level, the Age Asynchrony is relatively high and the average AoI is relatively low. Then, to reduce the Age Asynchrony metric, we would intuitively unicast with User *i*. In contrast, when all the users have relatively high age levels, but their age is closer to each other, we would prefer broadcasting to all of the users. This observation implies that the average AoI and Age Asynchrony might be able to guide the broadcasting or unicasting decisions well. Thus, we define the feature state $Y[t] \triangleq (A[t], S^1[t])$ and then perform the following first visit Monte Carlo feature-based learning algorithm.

In the next section, we will see that when *n* is small, the feature-based learning policy provides near-optimal performance, for moderate *n*, the feature-based learning policy still performs better than the mixed randomized policy.

VII. Simulation

First of all, we compare the AoI and Age Asynchrony performance trade-off of all policies (optimal solutions via LP, two extreme policies, mixed randomized policy, and the feature-based learning policy) under a two-user scenario in Figure 4. In this simulation, we set the broadcasting success probability to $p_b = 0.32$ and the unicasting success probability to $p_d = 0.4$. The ends of the mixed randomized policy represent the performance of two extreme policies (the left end is always-broadcasting). The black dots are the feature-based learning results with different α values, since the feature state $\mathbf{Z}[t]$ and age state $\mathbf{U}[t]$ is a one-to-one mapping in the two-user case, the black dots are very close to the optimal solutions. Through the figure, the minimum long-term average of $C(\alpha)$ can be easily found by finding the lowest line intersects with the policy curve with the slope being $-\frac{1-\alpha}{\alpha}$.

Secondly, we plot $C(\alpha)$ the weighted sum of long-term average AoI and Age Asynchrony under different policies against an increasing number of users n. In Figure 5, we set $\alpha = 0.9$, $p_d = 0.3$ and $p_b = 0.7/n$ with different n. In Figure 6, we set $\alpha = 0.5$, $p_d = 0.6$ and $p_b = 1.5/n$ with different n. In figures, the performance of the mixed randomized policy is with the best choice of p for different n, and the optimal solutions are solved via LP for $n = 2$, and from the Monte Carlo Tabular learning algorithm for $n = 3, 4$.

Before making observations about these two figures, notice that in both cases, we set np_b in the range of $2p_d$ to $3p_d$ in order to make a meaningful comparison for large n as explained in Section IV-C after (1). For this range of values, the performances of optimal solutions, mixed randomized policy, and feature-based learning policy overlap with the broadcasting policy when n is small. This choice of probabilities also makes the performances of policies to be comparable, because in those cases the performance of two extreme policies does not differ significantly from each other. See Figure 7 as a reference. Figure 7(a) shows that the feature-based learning policy outperforms the mixed randomized policy by 10% and is much closer to the optimal level while mixed randomized policy outperforms always-broadcasting by 2.5% when $n = 4$. Figure 7(b) shows that the feature-learning based policy can outperform the mixed randomized policy (left ends of blue curve is $p = 1$) for $n = 10$ where the optimal solution is unknown.

With the comparison of the weighted sum against n, we find: (i) Matched with our finding in Section IV-C, when $\alpha = 0.9$, $p_d = 0.3$ and $p_b = 0.7/n$, (1) $= 7/6 < 10/7 = 1/\mu$, we see in Figure 5 that unicasting eventually performs better than broadcasting. In constrast, when $\alpha = 0.5$, $p_d = 0.6$ and $p_b = 1.5/n$, (1) $= 25/36 > 2/3 = 1/\mu$, Figure 6 illustrates that broadcasting eventually performs better than unicasting. (ii) From the theoretical results of extreme policies, the AoI and Age Asynchrony trade-off comparison for mixed randomized policy, the action learned from feature-based learning policy for different α values, and Figure 5, 6, we can see that unicasting is more preferable than broadcasting if we are more focused on minimizing the Age Asynchrony (i.e., α is closer to 1) when np_b is in the range of $2p_d$ to $3p_d$. This is somewhat counter-intuitive as without careful thinking we may expect broadcasting to help synchronization more than unicasting. (iii) For a moderate number of users n in the system, the feature-based learning policy performs non-negligibly better than the mixed randomized policy. However, the advantage of feature-based learning will be diminishing with n since when the number of users is large, two features $A[t]$ and $S^1[t]$ cannot accurately distinguish which action is better anymore. So, for moderate n and under the case that the computational power is enough, we can apply a feature-based learning policy to achieve better performance while for larger n, the mixed randomized policy has good scalability as well as a gain compared with extreme policies that grow linearly with n. (iv) We also notice that when the success probabilities of broadcasting and unicasting are both small, the gains for mixed randomized policy and feature-learning policy are more obvious, which implies that in a bad communication environment, we should act more carefully to benefit more.

VIII. CONCLUSION

In this paper, we considered a time-sensitive scenario whereby the state of a common source is being updated at n distributed devices over unreliable channels. We studied the tradeoff between the Age of Information (AoI) and the Age Asynchrony whereby the transmitter can choose between unicast transmissions and broadcast transmissions for the updates.

We first posed and solved the optimal solution of the associated constrained MDP problem via Linear Programming, which is tractable only for small n values. Then, we analyzed the AoI and Age Asynchrony performance under two extreme policies (i.e., always-unicasting and always-broadcasting), where we pointed out how the success probabilities for unicasting and broadcasting along with n would affect the performance of both extreme policies. Motivated by the observations from the extreme policies, we proposed a mixed randomized policy and a feature-based learning policy, both with good scalability characteristics and non-negligible performance gains compared with the extreme policies. We successfully obtained complete results for average age and Age Asynchrony for the mixed randomized policy under the two-user case. Based on that, we were able to find the exact value of $p*$ for a given objective function $C_\alpha[t]$, which also reveals the range of α, p_b, p_d that the mixed randomized policy can outperform the extreme policies. Subsequently, we performed extensive numerical studies and observe that, for different number of users, success probabilities, and weights, we prefer different policies for optimizing the AoI- Age Asynchrony tradeoff. Counter-intuitively, we noted that unicasting is preferable to broadcasting when aiming to minimize the Age Asynchrony under an unreliable communication environment.

Throughout the study, we noticed that the synchronization between users in an unreliable communication environment is a very complicated but interesting metric. We proposed several different classes of policies with desired properties. Yet, the optimal structure of state-dependent choice to minimize the Age Asynchrony and stabilize the system with increasingly large n is still unknown and requires further investigation.

APPENDIX A
THE BALANCE EQUATION OF LP IN THM 1

For $i = 1, \cdots, D-1$,

$$\sum_{k=0}^{2} y_{0,i}^k = \sum_{a_1=0}^{D} y_{a_1,i-1}^0 p_b (1-p_b) + y_{a_1,i-1}^1 p_d,$$

$$\sum_{k=0}^{2} y_{i,0}^k = \sum_{a_2=0}^{D} y_{i-1,a_2}^0 p_b (1-p_b) + y_{i-1,a_2}^2 p_d,$$

$$\sum_{k=0}^{2} y_{i,D}^k = (y_{i-1,D}^0 + y_{i-1,D-1}^0)(1-p_b)^2 + (y_{i-1,D}^1 + y_{i-1,D-1}^1 + y_{i-1,D}^2 + y_{i-1,D-1}^2)(1-p_d),$$

$$\sum_{k=0}^{2} y_{D,i}^k = (y_{D,i-1}^0 + y_{D-1,i-1}^0)(1-p_b)^2 + (y_{D-1,i}^1 + y_{D-1,i-1}^1 + y_{D-1,i}^2 + y_{D-1,i-1}^2)(1-p_d),$$

for $i, j = 1, \cdots, D-1$,

$$\sum_{k=0}^{2} y_{i,j}^k = y_{i-1,j-1}^0 (1-p_b)^2$$

$$
\begin{aligned}
&+ \sum_{k=1}^{2} y_{i-1,j-1}^{k} \left(1-p_d\right), \sum_{k=0}^{2} y_{D,D}^{k} \\
&= \left(y_{D-1,D-1}^{0} + y_{D-1,D}^{0} + y_{D,D-1}^{0} + y_{D,D}^{0}\right)\left(1-p_b\right)^2 \\
&+ \sum_{k=1}^{2} \left(y_{D-1,D-1}^{1} + y_{D-1,D}^{1} + y_{D,D-1}^{1} + y_{D,D}^{1}\right)\left(1-p_d\right).
\end{aligned}
$$

Appendix B
Proof of Thm 5

Based on the symmetry,

$$
\lim_{t\to\infty} \mathbb{E}\left\{S^1[t]\right\} = \lim_{t\to\infty} \mathbb{E}\left[\frac{1}{n-1}\sum_{j=2}^{n} |U_1[t] - U_j[t]|\right]
$$

Define function $f\left(i, p_d\right)$ as the expectation of the number of slots until the next i^{th} successes happen among all users under steady state when the success probability for unicasting is p_d. In the following analysis, we use $f(i)$ instead of $f\left(i, p_d\right)$ for simplification. Similarly as in Theorem 4, define T_i as the time of the i^{th} successes among all the users with $T_0 = 0, i \in \mathcal{N}$ and define $\Delta_i = T_i - T_{i-1}$ to be the time interval between two successes. Then,

$$
f(i) = \mathbb{E}\left[T_i\right] = \mathbb{E}\left[\sum_{j=1}^{i} \Delta_j\right] = \sum_{j=1}^{i} \mathbb{E}\left[\Delta_j\right] = i\mathbb{E}\left[\Delta_1\right] = \frac{i}{p_d},
$$

where the last step is by the Blackwell renewal theorem.

Similarly as in Theorem 4, the sequence of entry times to state 0 for User 1 can be viewed as the arrival epochs of a renewal process. In each such renewal period, each of the other users will succeed once sequentially. Based on the elementary renewal theorem for renewal reward processes, the expected long-term average of the age difference between U_1 and U_j (for $j \neq 1$) equals the ratio of the expected summation of the age difference between U_1 and U_j over a renewal period to the expected length of a renewal interval. Then,

$$
\begin{aligned}
&\lim_{t\to\infty} \mathbb{E}\{S^1[t]\} = \frac{1}{n-1}\sum_{j=2}^{n} \mathbb{E}\left|U_1 - U_j\right| \\
&= \frac{1}{n-1}\sum_{j=1}^{n-1} \frac{f(j)\times f(n-j) + f(n-j)\times f(j)}{f(j)+f(n-j)} \\
&= \frac{2}{n-1}\cdot\sum_{j=1}^{n-1} \frac{f(j)\times f(n-j)}{f(n)} \\
&= \frac{2}{n-1}\times\frac{\sum_{j=1}^{n-1}\frac{j}{p_d}\times\frac{n-j}{p_d}}{\frac{n}{p_d}} = \frac{2}{n-1}\times\frac{\sum_{j=1}^{n-1} j(n-j)}{np_d} \\
&= \frac{2}{n-1}\times\frac{1}{np_d}\times\frac{1}{6}n\cdot(n-1)(n+1) = \frac{n+1}{3p_d}. \qquad (4)
\end{aligned}
$$

The equation (4) follows from the facts that: (i) Before User j succeeds in the renewal interval, the age difference between User 1 and User j equals the number of time slots from the last success of User j to the beginning of the current renewal when User 1 succeeds and the age difference between User 1 and User j remains unchanged until User j succeeds in this renewal; (ii) After User j succeeds, the age difference equals the number of time slots from the beginning of this renewal to the success of User j, and the age difference between these two users remains unchanged until the end of this renewal; (iii) Due to the behavior of Always-Unicasting Policy, all users succeed sequentially, then the time slots from the last success of User j to the success of User 1 equals to the number of time slots the system takes for $n-j+1$ successes happen among all users, the number of time slots from the beginning of this renewal to the success of User j equals to the number of time slots the system takes for $j-1$ successes happen among all users; (iv) the number of time slots until the next success is independent of the successes happened before. Thus, for $j = 2, \cdots, n$, $\mathbb{E}\left|U_1 - U_j\right| = \frac{f(j-1)f(n-j+1)+f(n-j+1)f(j-1)}{f(j-1)+f(n-j+1)}$.

Appendix C
Notations in Theorem 6

Let us use β_p to denote $P(U_1[t] = 0|U_2[t] = 0)$ under the steady-state values of $U_i[t]$ under the above policy.[5] Similarly, all the probability notations below are under the steady state distribution of $(U_1[t], U_2[t])$. Let γ_p denotes $P(U_1[t] \neq 0|U_2[t] = 0)$. Since $P(U_1[t] = 0, U_2[t] = 0) = p \cdot p_b^2$, $P(U_1[t]U_2[t] = 0, U_1[t] + U[t]_2 \neq 0) = 2p \cdot p_b(1-p_b) + (1-p)p_d$, by symmetry, $P(U_1[t] \neq 0, U_2[t] = 0) = p \cdot p_b(1-p_b) + \frac{1}{2}(1-p)p_d$. Then,

$$
\begin{aligned}
\beta_p &= \frac{p\cdot p_b^2}{p\cdot p_b^2 + p\cdot p_b(1-p_b) + \frac{1}{2}(1-p)p_d}; \\
\gamma_p &= \frac{p\cdot p_b(1-p_b) + \frac{1}{2}(1-p)p_d}{p\cdot p_b^2 + p\cdot p_b(1-p_b) + \frac{1}{2}(1-p)p_d}.
\end{aligned}
$$

Additionally, denote $\zeta_p = P(U_1[t] \neq 0, U_2[t] \neq 0)$, $\eta_p = P(U_1[t] = 0|U_1[t-1] = U_2[t-1])$, $\theta_p = P(U_1[t] = 0|U_1[t-1] < U_2[t-1]$, $\lambda_p = P(U_1[t] = 0|U_1[t-1] > U_2[t-1]$, then,

$$
\begin{aligned}
\zeta_p &= p(1-p_b)^2 + (1-p)(1-p_d); \\
\eta_p &= p\cdot p_b + \frac{1}{2}(1-p)p_d; \\
\theta_p &= p\cdot p_b; \\
\lambda_p &= p\cdot p_b + (1-p)p_d.
\end{aligned}
$$

Denote $\mu_p = P(U_1[t] = 0, U_2[t] \neq 0|U_1[t-1] = U_2[t-1])$, $\xi_p = P(U_1[t] = 0, U_2[t] \neq 0|U_1[t-1] > U_2[t-1])$, $\phi_p = P(U_1[t] = 0, U_2[t] \neq 0|U_1[t-1] < U_2[t-1])P_{:,+|\text{La}} = P(U_2[t] \neq 0|U_2[t-1] > U_1[t-1])$, then,

$$
\begin{aligned}
\mu_p &= p\cdot p_b(1-p_b) + \frac{1}{2}(1-p)p_d; \\
\xi_p &= p\cdot p_b(1-p_b) + (1-p)p_d; \\
\phi_p &= p\cdot p_b(1-p_b); \\
\psi_p &= p\cdot(1-p_b) + (1-p)(1-p_d).
\end{aligned}
$$

References

[1] S. Kaul, M. Gruteser, V. Rai, and J. Kenney, "Minimizing age of information in vehicular networks," in *Proc. 8th Annu. IEEE Commun. Soc. Conf. Sens. Mesh Ad Hoc Commun. Netw.*, Jun. 2011, pp. 350–358.

[2] S. Kaul, R. Yates, and M. Gruteser, "Real-time status: How often should one update?," in *Proc. IEEE INFOCOM*, Mar. 2012, pp. 2731–2735.

[3] A. Kosta, N. Pappas, and V. Angelakis, "Age of information: A new concept, metric, and tool," *Found. Trends Netw.*, vol. 12, no. 3, pp. 162–259, 2017.

[5] It can be shown that the system is stable under the proposed policy, which is omitted here.

[4] H. Chen, Y. Gu, and S.-C. Liew, "Age-of-information dependent random access for massive IoT networks," in *Proc. IEEE Conf. Comput. Commun. Workshops (INFOCOM WKSHPS)*, Jul. 2020, pp. 930–935.

[5] X. Zhou, I. Koprulu, A. Eryilmaz, and M. J. Neely, "Efficient distributed MAC for dynamic demands: Congestion and age based designs," *IEEE/ACM Trans. Netw.*, vol. 31, no. 1, pp. 74–87, Feb. 2023.

[6] B. Abolhassani, J. Tadrous, A. Eryilmaz, and E. Yeh, "Fresh caching of dynamic content over the wireless edge," *IEEE/ACM Trans. Netw.*, vol. 30, no. 5, pp. 2315–2327, Oct. 2022.

[7] R. Liu, E. Yeh, and A. Eryilmaz, "Proactive caching for low access-delay services under uncertain predictions," *Proc. ACM Meas. Anal. Comput. Syst.*, vol. 3, no. 1, pp. 1–46, 2019.

[8] H. Tang, J. Wang, L. Song, and J. Song, "Scheduling to minimize age of information in multi-state time-varying networks with power constraints," in *Proc. 57th Annu. Allerton Conf. Commun., Control, Comput. (Allerton)*, Sep. 2019, pp. 1198–1205.

[9] B. Sombabu and S. Moharir, "Age-of-Information aware scheduling for heterogeneous sources," in *Proc. 24th Annu. Int. Conf. Mobile Comput. Netw.*, Oct. 2018, pp. 696–698.

[10] P. R. Jhunjhunwala and S. Moharir, "Age-of-information aware scheduling," in *Proc. Int. Conf. Signal Process. Commun. (SPCOM)*, Jul. 2018, pp. 222–226.

[11] L. Hu, Z. Chen, Y. Dong, Y. Jia, L. Liang, and M. Wang, "Status update in IoT networks: Age-of-Information violation probability and optimal update rate," *IEEE Internet Things J.*, vol. 8, no. 14, pp. 11329–11344, Jul. 2021.

[12] N. Pappas and M. Kountouris, "Delay violation probability and age of information interplay in the two-user multiple access channel," in *Proc. IEEE 20th Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, Jul. 2019, pp. 1–5.

[13] Y. Sun, E. Uysal-Biyikoglu, R. D. Yates, C. E. Koksal, and N. B. Shroff, "Update or wait: How to keep your data fresh," *IEEE Trans. Inf. Theory*, vol. 63, no. 11, pp. 7492–7508, Nov. 2017.

[14] C. Li, Q. Liu, S. Li, Y. Chen, Y. T. Hou, and W. Lou, "On scheduling with AoI violation tolerance," in *Proc. IEEE INFOCOM Conf. Comput. Commun.*, May 2021, pp. 1–9.

[15] J. Zhong, R. D. Yates, and E. Soljanin, "Two freshness metrics for local cache refresh," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Jun. 2018, pp. 1924–1928.

[16] T. Adame, M. Carrascosa-Zamacois, and B. Bellalta, "Time-sensitive networking in IEEE 802.11be: On the way to low-latency WiFi 7," *Sensors*, vol. 21, no. 15, p. 4954, Jul. 2021.

[17] Z. Ding, X. Lei, G. K. Karagiannidis, R. Schober, J. Yuan, and V. K. Bhargava, "A survey on non-orthogonal multiple access for 5G networks: Research challenges and future trends," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 10, pp. 2181–2195, Oct. 2017.

[18] S. Al-Sultan, M. M. Al-Doori, A. H. Al-Bayatti, and H. Zedan, "A comprehensive survey on vehicular ad hoc network," *J. Netw. Comput. Appl.*, vol. 37, pp. 380–392, Jan. 2014.

[19] J. Yick, B. Mukherjee, and D. Ghosal, "Wireless sensor network survey," *Comput. Netw.*, vol. 52, no. 12, pp. 2292–2330, Aug. 2008.

[20] Y. Mekonnen, S. Namuduri, L. Burton, A. Sarwat, and S. Bhansali, "Machine learning techniques in wireless sensor network based precision agriculture," *J. Electrochem. Soc.*, vol. 167, no. 3, Jan. 2020, Art. no. 037522.

[21] H. Kopetz and W. Ochsenreiter, "Clock synchronization in distributed real-time systems," *IEEE Trans. Comput.*, vol. C-36, no. 8, pp. 933–940, Aug. 1987.

[22] G.-S. Tian, Y.-C. Tian, and C. Fidge, "Precise relative clock synchronization for distributed control using TSC registers," *J. Netw. Comput. Appl.*, vol. 44, pp. 63–71, Sep. 2014.

[23] K. F. Hasan, C. Wang, Y. Feng, and Y.-C. Tian, "Time synchronization in vehicular ad-hoc networks: A survey on theory and practice," *Veh. Commun.*, vol. 14, pp. 39–51, Oct. 2018.

[24] M. Pahlevan, B. Balakrishna, and R. Obermaisser, "Simulation framework for clock synchronization in time sensitive networking," in *Proc. IEEE 22nd Int. Symp. Real-Time Distrib. Comput. (ISORC)*, May 2019, pp. 213–220.

[25] M. Khoshnevisan, V. Joseph, P. Gupta, F. Meshkati, R. Prakash, and P. Tinnakornsrisuphap, "5G industrial networks with CoMP for URLLC and time sensitive network architecture," *IEEE J. Sel. Areas Commun.*, vol. 37, no. 4, pp. 947–959, Apr. 2019.

[26] A. Mahmood, M. I. Ashraf, M. Gidlund, and J. Torsner, "Over-the-air time synchronization for URLLC: Requirements, challenges and possible enablers," in *Proc. 15th Int. Symp. Wireless Commun. Syst. (ISWCS)*, Aug. 2018, pp. 1–6.

[27] D. Ginthör, R. Guillaume, M. Schüngel, and H. D. Schotten, "Robust end-to-end schedules for wireless time-sensitive networks under correlated large-scale fading," in *Proc. 17th IEEE Int. Conf. Factory Commun. Syst. (WFCS)*, Jun. 2021, pp. 115–122.

[28] E. Altman, *Constrained Markov Decision Processes*, vol. 7. Boca Raton, FL, USA: CRC Press, 1999.

[29] A. Wald, "On cumulative sums of random variables," *Ann. Math. Statist.*, vol. 15, no. 3, pp. 283–296, Sep. 1944.

[30] A. Carr, "2663. on the arithmetico-geometric series," *Math. Gazette*, vol. 41, no. 335, pp. 44–46, Feb. 1957.

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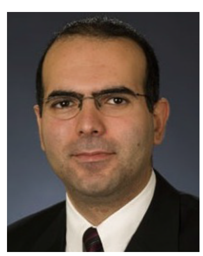

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