Proceedings of the ASME 2024
Pressure Vessels & Piping Conference
PVP2024
July 28-August 2, 2024, Bellevue, Washington

PVP2024-123054

TILTING MANHOLE COVER: A NONLINEAR SPRING-MASS SYSTEM

Niels F. van de MEULENHOF
Department of Mathematics and Computer Science
Eindhoven University of Technology
P.O. Box 513, 5600 MB Eindhoven
The Netherlands
E-mail: n.f.v.d.meulenhof@student.tue.nl

Arris S. TIJSSELING *(corresponding author)*
Department of Mathematics and Computer Science
Eindhoven University of Technology
P.O. Box 513, 5600 MB Eindhoven
The Netherlands
E-mail: a.s.tijsseling@tue.nl

Jose G. VASCONCELOS
Department of Civil Engineering
Auburn University
Auburn, AL 36849
U.S.A.
E-mail: jgv0001@auburn.edu

Qingzhi HOU
State Key Laboratory of Hydraulic Engineering
Intelligent Construction and Operation
Tianjin University
Tianjin 300350
China
E-mail: qhou@tju.edu.cn

Zafer BOZKUŞ
Hydraulics Laboratory
Department of Civil Engineering
Middle East Technical University
Ankara 06800
Turkey
E-mail: bozkus@metu.edu.tr

ABSTRACT

Manhole covers are potential "dancers". They may leave their resting state and start "dancing". They may hover, move up and down, tilt, rotate, bounce, make noise, flip over, or even fly up into the air. In general, their motion looks chaotic, probably due to the nonlinear dynamics governing the system.

The authors have previously derived basic models of dancing manhole covers covering the translational vertical motion of free covers and the rotational motion of hinged covers. In the current contribution the basic model is extended with tilting (without hinge) and bouncing behavior. Some fundamental problems and assumptions are discussed. Preliminary numerical results are shown together with 3D visualizations.

Scientific curiosity into a mysterious phenomenon has been the motivation for this study. The obtained equations governing the manhole cover's motion may serve as boundary conditions in hydraulic-pneumatic models of sewer-manhole systems (think of geysering and ventilation).

Key words
manhole cover, orifice, vent, gas flow, fluid-structure interaction (FSI), sewer, Formula 1

INTRODUCTION

Manhole covers are heavy, in the order of 100 kg, and their coming out of position forms a danger to pedestrians and road traffic. Most displacement events occur under heavy (wet) weather conditions when sewers are overloaded, and large amounts of air and water need a way to escape. Geysering, where spectacular fountains of water come out of the manholes, is another – even more extreme – event occurring under these circumstances. The dancing motion of manhole covers is studied herein, because it is a mysterious phenomenon that is not yet fully understood. It is something difficult to prevent: gluing or locking are not realistic options, because it is expensive and restricts access. Under heavy storm conditions the pressures in the sewer system will exist regardless of the manhole being "loose" or "fixed". Fixing would trap the high pressures and potentially cause other problems like damage to surrounding pavement or a more explosive ejection of the cover if the restraints are broken.

The first extensive study of the subject known to the authors is by Satoshi Yamamoto [1] who showed the danger and damage caused by displaced and ejected manhole covers. Jue Wang and our third author presented pioneering work on the dislodging of manhole covers [2, 3].

Air flows from a sewer into a covered manhole and compresses the air. A high-enough pressure will dislodge the cover, which – under certain conditions – may start to dance (vertical motion, tilting, rotating, bouncing). The dancing is modeled by pure vertical motion of a free cover [4, 5] or by pure rotation of a hinged cover [6]. These are one-dimensional models of the cover (governing either displacement x_c or rotation θ_c). When the free cover is allowed to tilt, we have a two-dimensional model (with both x_c and θ_c). Tilting of a free cover was added in [7] and this model is extended herein with bouncing behavior. The two-dimensional model leads to interesting complications, most of them related to the cover hitting its support.

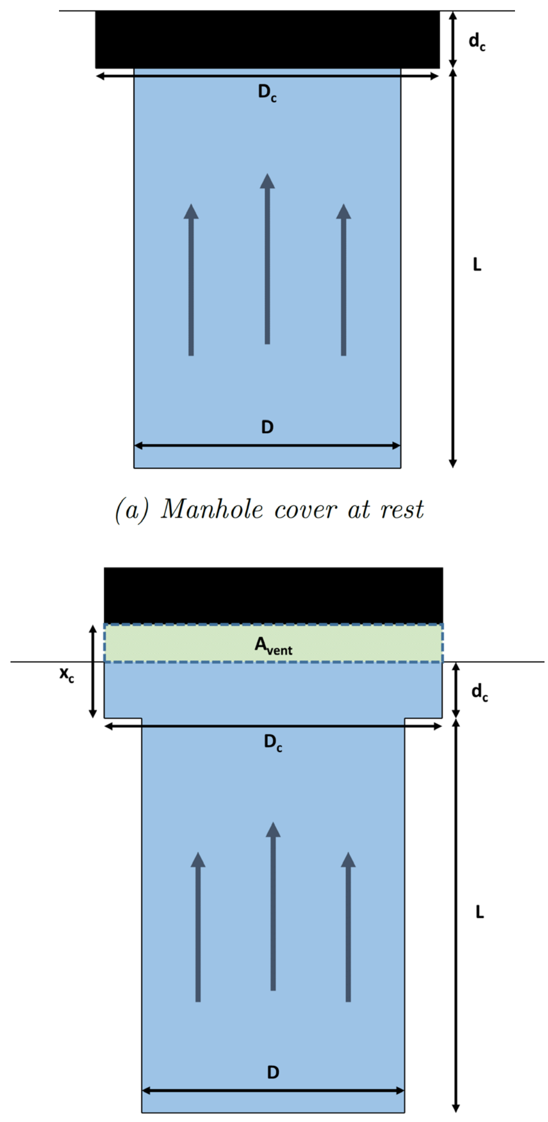
MODEL DESCRIPTION

The model (see Fig. 1) consists of 6 ordinary differential equations for the 6 main unknowns: the air mass m in the manhole and its uniform (in axial direction) pressure P, the cover's vertical displacement x_c and velocity v_c, together with tilting angle θ_c and angular velocity η_c. We start with the driving force behind the system, i.e. the incoming air mass and its (ideal gas) absolute pressure (see Nomenclature for symbols):

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \rho A v_{in} - A_{vent}\sqrt{\rho C_d\left(P - P_{atm}\right)} \tag{1}$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{nP}{m}\frac{\mathrm{d}m}{\mathrm{d}t} \tag{2}$$

The vent area A_{vent} is assumed to be small compared to the manhole area A, which means that cover displacement and rotation are small. In the current model, the net force exerted by the air inside the manhole acts on the cover, which is a uniform circular disk, but slightly out of the center of mass (gravity). This can be seen as a representation of the fact that the air pressure acts non-uniformly (in radial direction) on the cover. The vector $\boldsymbol{r}_c$ identifies the position of the center of mass and is therefore the pivot of rotational motion. Gravity acts at position $\boldsymbol{r}_c$. We define another vector $\boldsymbol{r}_a$ to be the point of application of the net air force; this is – to keep things simple – a fixed point on the cover. Since the pressure is assumed to be distributed non-uniformly on the cover's surface, these two vectors will in general not be the same. The distance between them, that is $\|\boldsymbol{r}_a - \boldsymbol{r}_c\|$, plays a crucial role in the model and its sensitivity to tilting.



(a) Manhole cover at rest

(b) Manhole cover lifted above the ground

Fig. 1 Figure 1 from Ref. [8].

The net force resulting from air pressure and gravity causes a vertical displacement (x_c) of the manhole cover and a rotation (of angle θ_c) around pivot $\boldsymbol{r}_c$. The vertical displacement is described similarly to the pure-vertical-motion model [4-6] but gains an extra cosine factor due to the variable angle θ_c (because, neglecting shear forces, the air force is perpendicular to the cover). This also means that there will be a horizontal component (with a sine factor) of the air force and corresponding horizontal motion, which can indeed be seen in some of the videos of dancing manhole covers (in which the cover bumps sideways against its supporting ring). This horizontal component is – to keep things simple – disregarded herein. Thus:

$$\frac{\mathrm{d}x_c}{\mathrm{d}t} = v_c \tag{3}$$

$$\frac{\mathrm{d}v_c}{\mathrm{d}t} = -g + \frac{A_c}{m_c}\left(P - P_{atm}\right)\cos\theta_c \tag{4}$$

The torque acting on the cover is defined similarly to the one-dimensional rotational model for a hinged cover [6] but differs in two aspects. First, the distance between pivot and point of application of the air force is (much) smaller. Second, the gravitational force acts on the pivot (by definition), so it does not result in any torque. Since the pivot itself is now at point $\boldsymbol{r}_c$, the cover's moment of inertia is:

$$I_1 = \frac{1}{16} m_c D_c^2 + \frac{1}{12} m_c d_c^2 \tag{5}$$

This leads to the ODEs:

$$\frac{\mathrm{d}\theta_c}{\mathrm{d}t} = \eta_c \tag{6}$$

$$\frac{\mathrm{d}\eta_c}{\mathrm{d}t} = \frac{\left\|\mathbf{r}_a - \mathbf{r}_c\right\|}{I_1} A_c\left(P - P_{atm}\right) \tag{7}$$

Equation (7) (which has an incorrect factor $\cos(\theta_c)$ in [6]) only holds when the cover can rotate freely, which is not always the case as the cover's tip cannot rotate below its supporting ring, see Fig. 2. This rotational constraint is linked to the vertical constraint $x_c \geq 0$, namely:

$$-x_c \leq \frac{D_c}{2}\sin\theta_c \leq x_c \tag{8}$$

The vertical side of the triangle made by the cover can never be larger than the vertical displacement x_c, for otherwise the cover would be partially underground. This argument should also hold when the cover is rotated the other way around, hence the negative part of Eq. (8).

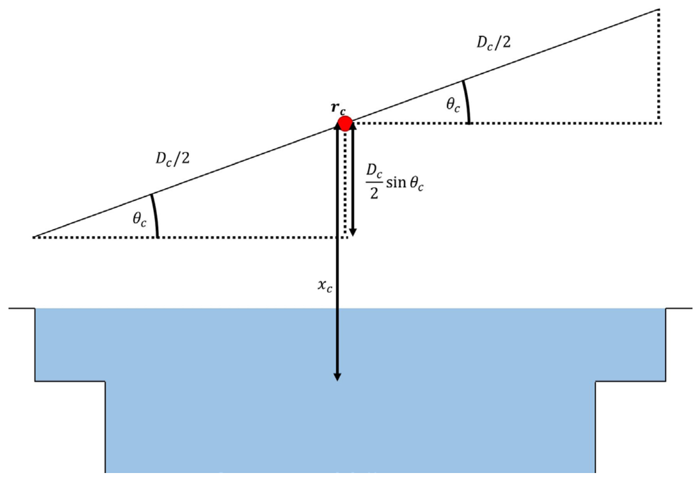


Fig. 2 Schematic representation of the situation from side-view. The bold (red) dot represents the geometric center of the cover, which is also the pivot point herein. The blue part represents the manhole with outflowing air.

When the angle θ_c is such that one of the inequalities in Eq. (8) becomes an equality, the cover touches the ground and something interesting happens. Assume that the lowest point of the cover remains in contact with the ground for some time. In this case, the cover actually rotates about the point in contact with the ground (like in the model for hinged covers). The pivot point is then the point at which the cover makes contact with the ground. The moment of inertia of the cover is different for this different pivot, as it is now the same one as for pure rotational motion around a hinge:

$$I_2 = \frac{5}{16} m_c D_c^2 + \frac{1}{12} m_c d_c^2 \tag{9}$$

This situation only exists for a small period of time (because the cover bounces back), i.e. as long as the constraint in Eq. (8) is not satisfied numerically. Although such a temporary hinge may be considered, it is not expected to have a significant effect on the overall behavior of the system.

It is an interesting situation, when the system reaches one of the limits of constraint (8) and the cover is about to cross it, as this means that its edge hits the ground or supporting ring. At this point, the energy that the cover has does not disappear all at once (as in the pure vertical motion presented in [4-6] where the cover reaches its resting position without bouncing back, i.e. inelastic collision). Instead, (a part of) the energy remains in the system as the cover bounces back after the collision. What follows are the first steps in the modeling of a complex matter (snooker ball analogy).

Theoretically, we first assume and apply conservation of energy in the system and see how the linear and angular momentum of the cover are distributed before and after collision. The energy of the cover is defined by the sum of the linear (vertical) and angular kinetic energy:

$$E = \frac{1}{2} m_c v_c^2 + \frac{1}{2} I_1 \eta_c^2 \tag{10}$$

When the cover hits the ground, a vertical impulse J is delivered onto the cover by the upward normal force $\boldsymbol{F}_n$ with size $\|\boldsymbol{F}_n\| = F_n$, resulting in a change in momentum and energy. We assume this happens instantaneously, i.e. within one time-step numerically, but a Dirac pulse theoretically. Since the mass m_c is constant we get the following equation for the applied linear momentum:

$$J = m_c \tilde{v}_c - m_c v_c \tag{11}$$

where $\tilde{v}_c$ is the vertical velocity of the cover after impact. The normal force also results in a torque $\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}_n$, where we define $\boldsymbol{r} = \boldsymbol{r}_i - \boldsymbol{r}_c$ as the vector from geometric center $\boldsymbol{r}_c$ to point of contact $\boldsymbol{r}_i$. Due to the geometry of the cover, we assume that, when tilted, it can only hit the ground at its outer side. Therefore, with the cover's geometric center in the middle, the length of the vector is always $\|\boldsymbol{r}\| = D_c / 2$, except when the cover falls down flat, in which case the distance is taken 0. Hence the torque $\tau = \|\boldsymbol{\tau}\|$ for collision at angle θ_c is:

$$\tau = -\frac{D_c}{2} \operatorname{sign}(\theta_c) \cos(\theta_c) F_n := r(\theta_c) F_n \tag{12}$$

This equation also holds for $\theta_c = 0$, as $r(0) = 0$ due to the sign function. We use Eq. (12) in the resulting equation for the angular momentum:

$$r(\theta_c) J = I_1 \tilde{\eta}_c - I_1 \eta_c \tag{13}$$

where $\tilde{\eta}_c$ is the angular velocity of the cover after impact. To complete the system, we assume elastic collision and conservation of energy:

$$E = \frac{1}{2} m_c v_c^2 + \frac{1}{2} I_1 \eta_c^2 = \frac{1}{2} m_c \tilde{v}_c^2 + \frac{1}{2} I_1 \tilde{\eta}_c^2 = \tilde{E} \tag{14}$$

Now we have Eqs (11), (13) and (14) governing the three unknowns J, $\tilde{v}_c$ and $\tilde{\eta}_c$. Substituting the first two equations into the latter gives an equation for J:

$$J \left[J \left(\frac{1}{2m_c} + \frac{(r(\theta_c))^2}{2I_1} \right) + v_c + \eta_c \, r(\theta_c) \right] = 0 \tag{15}$$

So, we either get the arbitrary solution $J = 0$ for a situation without impulse, i.e. no impact, which we are not interested in, or we find:

$$J = -\frac{2\, m_c I_1 (v_c + \eta_c \, r(\theta_c))}{I_1 + m_c (r(\theta_c))^2} \tag{16}$$

and consequently, once we have found J, from Eqs (11) and (13):

$$\tilde{v}_c = v_c + \frac{1}{m_c} J \tag{17}$$

$$\tilde{\eta}_c = \eta_c + \frac{r(\theta_c)}{I_1} J \tag{18}$$

In the case of pure vertical motion, these equations hold as well. We then have by definition $\theta_c = \eta_c = 0$. Consequently $r(0) = 0$ and $J = -2m_c v_c$, which yields $\tilde{\eta}_c = 0$ and more importantly $\tilde{v}_c = -v_c$. The cover bounces back up with the same speed after hitting the ground. This is a nice verification to see that the derived equations make sense.

We do not expect manhole covers to bounce like a rubber ball would. Furthermore, due to unevenness, dirt and roughness of the cover and its supporting ring, friction and damping cannot be ignored during collision. Therefore, as a gross simplification, we introduce 'damping factors' α and β in Eqs (17) and (18):

$$\tilde{v}_c = \alpha \left(v_c + \frac{1}{m_c} J \right) \tag{19}$$

$$\tilde{\eta}_c = \beta \left(\eta_c + \frac{r(\theta_c)}{I_1} J \right) \tag{20}$$

where $0 \le \alpha \le 1$ and $0 \le \beta \le 1$. This is a significant assumption, and the values of α and β need to be researched more in depth. However, for now it is key to the realism of the proposed model. Perhaps other more elaborate ways of modeling friction and inelasticity can be considered, but this requires more advanced analysis.

ASSUMPTIONS, CONSTRAINTS AND NUMERICAL DIFFICULTIES

Numerically, the proposed model comes with some inherent difficulties. To start with, we have the constraint that the cover cannot drop below the ground level, i.e. $x_c \ge 0$ all the time, as well as the constraint that $v_c \ge 0$ when $x_c = 0$.

Furthermore, the vertical velocity v_c changes only like described by Eq. (4) when the cover $x_c > 0$ or if the air pressure P inside the manhole is large enough to lift the cover. If these conditions both do not hold, the cover is at rest on the ground and the velocity does not change at all because the normal force from the supporting ring counterbalances the gravitational force.

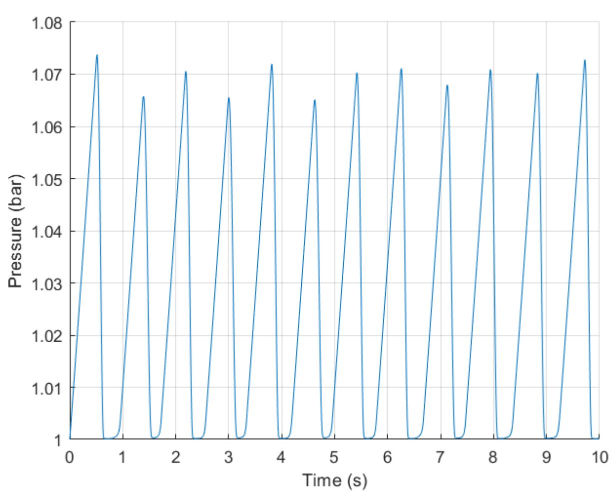
Something similar is the case for the angular velocity. Equation (7) only holds when the cover is able to rotate freely, i.e. when $x_c > 0$ and the inequality in Eq. (8) holds. Otherwise, the cover cannot gain any angular velocity.

Lastly, after evaluating the new values for x_c and θ_c, we check if the constraint from Eq. (8) is violated in the new time-step while it was not violated in the old time-step. This would imply the cover is hitting the ground somewhere between the two time-steps. If this is the case, we use Eqs (19) and (20) to reevaluate the new values for variables v_c and η_c.

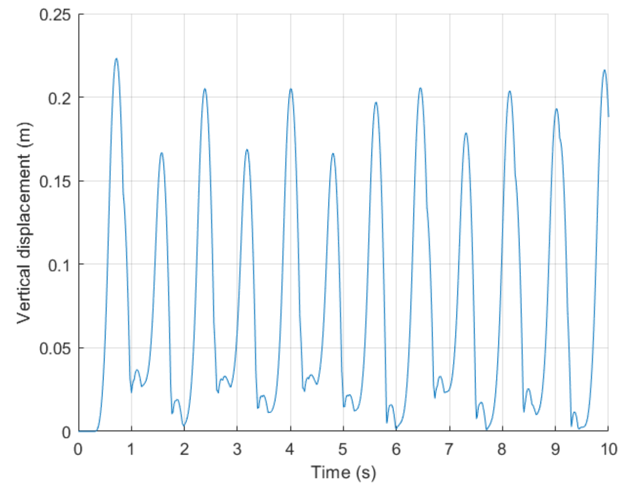
For simplicity and to be able to maintain control over all these different constraints and conditions, the forward Euler method has been used for the tilting cover model. This method is not preferable due to low accuracy and stability concerns, but with a time-step of 0.00001 s the method yields acceptable results (Fig. 3) and corresponding animations (Fig. 4). Gauge pressures up to 0.07 bar (Fig. 3a) are sufficient to make a 100 kg cover dance with a displacement amplitude of 0.1 m (Fig. 3b) and whimsical tilting behavior (Fig. 3c).

An important choice that was made for this implementation was to keep the distance $\|\boldsymbol{r}_a - \boldsymbol{r}_c\|$ fixed at all times. Only 3D CFD simulations may shed light on this issue, i.e. the unsteady pressure distribution on the cover.

(a)



(b)



(c)

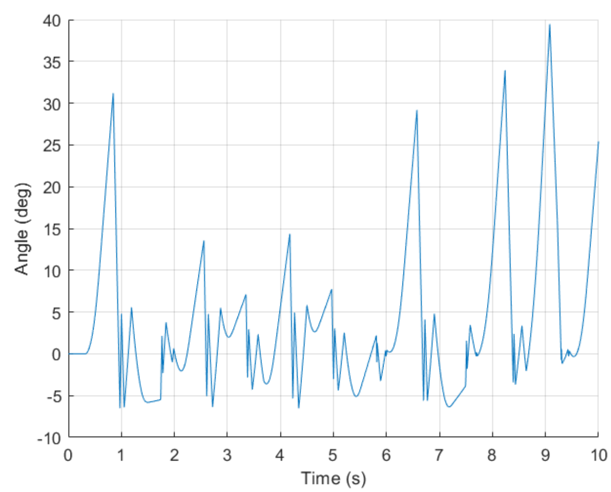


Fig. 3 Simulated dancing manhole cover: **(a)** driving pressure, **(b)** vertical displacement, **(c)** angle of rotation.
Input parameters
Constant inflow velocity $v_{in} = 5$ m/s, $\|\boldsymbol{r}_a - \boldsymbol{r}_c\| = D_c / 80$, $\alpha = \beta = 0.6$, $D = 0.5$ m, $D_c = 0.55$ m, $d_c = 0.055$, $m_c = 100$ kg, $P_{atm} = 1$ bar, $L = 50$ m, $g = 9.81$ kg/m^3, $C_d = 1$; $n = 1.4$.

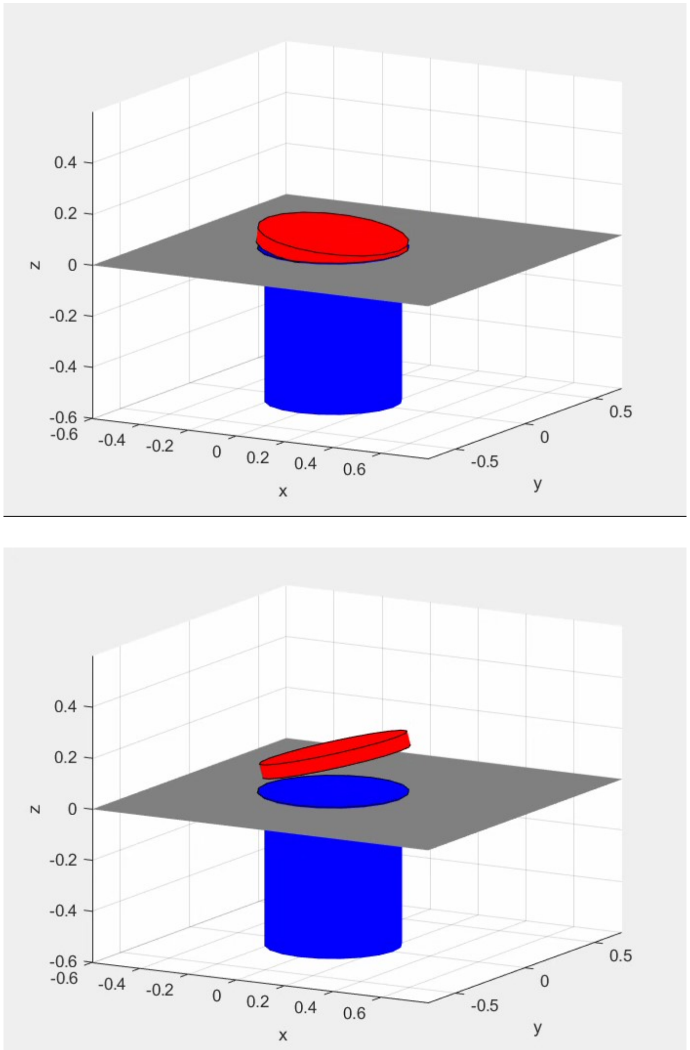


Fig. 4 Snapshots from model animation.

DISCUSSION

The proposed model is work in progress, and there are still difficulties and problems to overcome. Most of them are related to catching 3D phenomena in a 1D model. For example, when the cover is rotated 90 degrees, the air pressure has no effect on the vertical displacement of the cover anymore as the cosine factor in Eq. (4) amounts to 0. However, the angular velocity in Eq. (7) is still impacted in the same way as previously. Perhaps, this equation should include a term taking into account the angle as well. Furthermore, the vertical displacement of the cover does not affect the differential equations themselves directly. This is odd, because it is unlikely that the air coming out of the manhole has any effect on the cover when it is blown up a meter in the sky. Actually, in this case the area A_{vent} would simply be the gap left by the cover. The current assumptions may be justified by noting that the model only holds for small displacements, both angular and vertical.

FUTURE WORK

Sideways behavior, introducing a y-axis. Like mentioned earlier, the air pressure exerts a force which also has a horizontal component when the cover is tilted. It might be worth the effort to try and implement this into the model, as it is something that is indeed seen in real life footage. The difficulty lies in the lack of understanding how the cover somehow does get back on the supporting ring. This definitely needs some more research, perhaps also practical testing, but a preliminary model might already give some insight into the mechanism behind this behavior.

Rotation of the cover around the vertical x_c-axis, as seen in many of the videos. It is difficult to initiate this behavior in a model, as the cover will probably start in an unstable equilibrium. Therefore, some additional feature is necessary to get the cover out of the equilibrium and initiate the rotation. Perhaps an option would be to define a center of mass that is different from the geometric center, but this results in new other problems. We might also need a y-axis to model this.

Point of application $\mathbf{r}_a(t)$ and therefore distance $\left\| \mathbf{r}_a(t) - \mathbf{r}_c \right\|$ changes over time, instead of maintaining fixed value. This makes sense physically with the behavior of the air inside, and this idea can be extended to 3D, possibly resulting in the rotation mentioned in the previous point.

PREVENTION

Of course, the main question is how to prevent “dancing” or displacement in general. Large-enough permanent vents (orifices) may help [2, 3]. Limiting the displacement by means of a strap, string or chain may be another option; at least, this will prevent complete ejection. Locking the cover seems a sensible option, as this is already being done to prevent theft (Fig. 5). However, Klaver et al. [9] reported “displacement of manhole covers at two locations and, at one location where the manhole cover was bolted down, by pavement deformation”. Under heavy storm conditions the pressures in the sewer system will exist regardless of the manhole being open or not. "Fixing" it would trap the high pressures potentially causing other problems – or worse, a more "explosive" ejection of the cover if the restraints are broken. The most extreme solution is to glue the covers to their supporting rings, as done on Formula 1 street racing circuits (see Appendix).

Fig. 5 Manhole cover with lock (patent Dmitry Rod, Ukraine).

ACKNOWLEDGMENTS

The third author would like to acknowledge the support from the US National Science Foundation under Grant No. 2048607. The fourth author is grateful for the support by the National Natural Science Foundation of China (No. 52079090) and the Belt and Road Innovation Talent Exchange Foreign Expert Project (No. DL2023126001L). The authors thank both reviewers for their criticism and helpful suggestions.

NOMENCLATURE

A = cross-sectional manhole area (m^2)
A_c = manhole cover area (m^2)
A_{car} = car floor area (m^2)
A_{vent} = venting area (m^2)
C_d = orifice discharge coefficient
D = manhole diameter (m)
D_c = manhole cover diameter (m)
d_c = depth of manhole cover (m)
d_{car} = clearance (m)
E = energy (J)
F = force (N)
g = acceleration due to gravity (m/s^2)
I = moment of inertia (kg m^2)
J = impulse (N s)
L = length of gas column in manhole (m)
L_{car} = length of car (m)
m_c = mass of manhole cover (kg)
m_{car} = mass of car (kg)
m = mass of gas (kg)
n = constant polytropic exponent
P = absolute pressure (cross-sectional average) (Pa)
r = function of θ_c defined in Eq. (12)
$\mathbf{r}$ = position vector
t = time (s)
v_c = vertical velocity of manhole cover (m/s)
v_{car} = horizontal velocity of car (m/s)
v_{in} = inflow velocity at bottom of manhole (m/s)
w_{car} = width of car (m)
x_c = vertical displacement of manhole cover (m)
η_c = rotational velocity of cover (rad/s)
α, β = damping coefficients
γ = downforce factor
θ_c = angle of rotation of cover (rad)
ρ = mass density of gas (kg/m^3)
ρ_c = mass density of cover (kg/m^3)
$\boldsymbol{\tau}$ = torque (N m)

Subscripts and Overbar

a = application point of force
atm = atmospheric
c = cover, center
car = racing car
$down$ = down force
i = impact
in = inflow
n = normal
$vent$ = orifice, opening
~ = after impact

REFERENCES

[1] Yamamoto S (2002) Mechanism behind manhole cover ejection phenomenon and its prevention measures. Global Solutions for Urban Drainage, pp. 1-16. Ninth International Conference on Urban Drainage (9ICUD), Portland, Oregon, USA.
[2] Wang J, Vasconcelos JG (2018) Manhole cover displacement caused by the release of entrapped air pockets. Journal of Water Management Modeling 26, art. no. C444, pp. 1-6.
[3] Wang J., Vasconcelos JG (2020) Investigation of manhole cover displacement during rapid filling of stormwater systems. Journal of Hydraulic Engineering 146 (4), art. no. 04020022, pp. 1-11.
[4] Tijsseling AS, Vasconcelos JG, Hou Q, Bozkuş Z (2019) Dancing manhole cover: a nonlinear spring-mass system. Proc. of the ASME 2019 Pressure Vessels and Piping Division Conf., San Antonio, USA, Paper PVP2019-93086.
[5] Tijsseling AS, Vasconcelos JG, Hou Q, Bozkuş Z (2020) Venting manhole cover: a nonlinear spring-mass system. Proc. of the ASME 2020 Pressure Vessels and Piping Division Conf., Virtual, Online, Paper PVP2020-21084.
[6] Meulenhof NF van de, Tijsseling AS, Vasconcelos JG (2022) Dancing manhole-cover basics. Proc. of the ASME 2022 Pressure Vessels and Piping Division Conf., Las Vegas, USA, Paper PVP2022-84546.
[7] Meulenhof NF van de, Tijsseling AS, Vasconcelos JG, Hou Q, Bozkuş Z (2023) Tilting manhole covers. Proc. of the 40th IAHR World Congress, Vienna, Austria, pp. 1515-1519.

[8] Meulenhof NF van de (2021) Dancing Manhole Covers. B.Sc. thesis, Department of Mathematics and Computer Science, Eindhoven University of Technology, Eindhoven, The Netherlands.

[9] Klaver P, Collins D, Robinson K, Bell S (2016) Modeling of transient pneumatic events in a combined sewer overflow storage tunnel system. Journal of Water Management Modeling 25, art. no. C409, pp. 1-7.

APPENDIX *FORMULA*.1

A manhole cover made it to the headlines during the Formula 1 race weekend in Las Vegas, November 2023. It came loose and hit the car of Carlos Sainz. The same happened to George Russell in Baku, April 2019.

Formula 1 cars have a lot of "down force", which includes low pressure underneath them. When racing over a manhole on a street circuit, the cover may come loose and hit the car; such an event causes a lot of expensive damage. The solution in Las Vegas was to glue the loose manhole covers (Fig. 6). In Baku they had to fix 300 covers. Both solutions were expensive and not durable.

Cars, i.e. "ordinary traffic", driving over covers keep them "loose" thereby increasing their potential to dance.

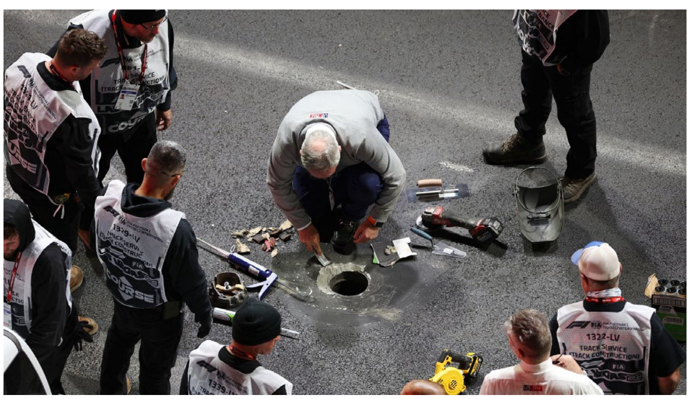


Fig. 6 Manhole cover with glue (Las Vegas 2023 F1).

Let us do a basic calculation assuming uniform acceleration, based on the data (found on the internet) listed in Table A.1. The key parameter is the very short time Δt that the car is located above the manhole cover. Within this time of L_{car} / v_{car} = 0.05 s the cover moves upwards 2.5 mm, which is much too short to hit the car's front clearance of 30 mm (let alone the back clearance of 80 mm), unless the cover has tilted by an angle θ_c = arcsin(0.0275 m / (D_c / 2)) = 3.9°, which seems not impossible.

The key formula in dimensionless form is:

$$\frac{x_c}{d_{car}} = \left(\gamma \frac{A_c}{A_{car}} \frac{m_{car}}{m_c} - 1 \right) \frac{g\,(\Delta t)^2}{2 d_{car}} \qquad \text{(A.1)}$$

This number must be smaller than 1 for safe racing.

A similar formula for the tilting angle θ_c would complete the model.

Table A.1 Input and derived data.

F1 racing car	manhole cover
v_{car} = 360 km/h = 100 m/s	g = 10 m/s^2
m_{car} = 800 kg	m_c = 100 kg
L_{car} = 5 m	D_c = 0.8 m
w_{car} = 2 m	$A_c = \pi D_c^2 / 4$ = 0.5 m^2
$A_{car} = L_{car}\, w_{car}$ = 10 m^2	d_c = 0.025 m
d_{car} = 0.03 m – 0.08 m	ρ_c = 8000 kg/m^3
γ = 3	$a_c = P_{down} A_c / m_c - g$ = = 2 m/s^2
$F_{down} = \gamma\, m_{car}\, g$ = 24 kN	
$P_{down} = F_{down} / A_{car}$ = 2.4 kPa	$v_c = a_c\, \Delta t$ = 0.1 m/s
$\Delta t = L_{car} / v_{car}$ = 0.05 s	$x_c = v_c\, \Delta t / 2$ = 0.0025 m

