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Beyond inherent robustness: strong stability of MPC despite plant-model mismatch

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***Abstract*— In this article, we establish the asymptotic stability of MPC under plant-model mismatch for problems where the origin remains a steady state despite mismatch. This class of problems includes, but is not limited to, inventory management, path-planning, and control of systems in deviation variables. Our results differ from prior results on the inherent robustness of MPC, which guarantee only convergence to a neighborhood of the origin, the size of which scales with the magnitude of the mismatch. For MPC with quadratic costs, continuous differentiability of the system dynamics is sufficient to demonstrate exponential stability of the closed-loop system despite mismatch. For MPC with general costs, a joint comparison function bound and scaling condition guarantee asymptotic stability despite mismatch. The results are illustrated in numerical simulations, including the classic upright pendulum problem. The tools developed to establish these results can address the stability of offset-free MPC, an open and interesting question in the MPC research literature.**

***Index Terms*— Model predictive control (MPC), plant-model mismatch, inherent robustness, optimal control, robust control.**

I. Introduction

PLANT-MODEL mismatch is an ever-present challenge in model predictive control (MPC) practice. In industrial implementations, the main driver of MPC performance is model quality [1, 2]. There has been recent progress on improving model quality and MPC performance through disturbance modeling and estimator tuning [3, 4, 5], simultaneous state and parameter estimation [6, 7, 8], and data-driven MPC design and analysis [9, 10, 11, 12] to name a few methods. However, there is not yet a sharp theoretical understanding of the robustness of MPC to plant-model mismatch.

Before discussing MPC robustness, let us first define *robustness*. In the stability literature, *robust asymptotic stability* has been used to refer to both (i) input-to-state stability (ISS) [13] and (ii) asymptotic stability despite disturbances [14]. To avoid confusion, we reserve the term *robust asymptotic stability* for (i) and use *strong asymptotic stability* to refer to (ii).[1] When such properties are given by a nominal MPC,[2] we call it *inherently robust* or *inherently strongly stabilizing*. Robust and strong exponential stability are defined similarly.

It is well-known that MPC is stabilizing under certain assumptions on the terminal ingredients (cf. [17, Ch. 2]). To achieve robust stability in the presence of parameter errors, estimation errors, and exogenous perturbations, a disturbance model can be included (cf. [17, Ch. 1, 3]). Even in the absence of a disturbance model, a wide range of nominal MPC designs are inherently robust to disturbances. Continuity of the control law was first proven to be sufficient for inherent robustness [18, 19]. Later, [20] proved continuity of the optimal value function is sufficient for inherent robustness, and stated MPC examples with discontinuous optimal value functions that are nominally stable but otherwise not robust to disturbances. A special class of time-varying terminal constraints were proven to confer robust stability to nominal MPC by [21], and to suboptimal MPC by [22]. In [23, 24], the inherent robustness of optimal and suboptimal MPC, using a class of time-invariant terminal constraints, was proven. The inherent stochastic robustness (in probability, expectation, and distribution) of nominal MPC was shown by [25, 26, 27]. Finally, direct data-driven MPC was shown to be inherently robust to noisy data [11].

If the origin remains a steady state under mismatch, we might expect strong asymptotic stability. While this assumption may seem strong, it includes a wide class of problems, including inventory management, path-planning, and control of systems that can be recast in deviation variables. For linear systems, unconstrained optimal control stabilizes the origin despite bounded perturbations to the system gain [28, 29, 30]. In the nonlinear setting, we might expect similar behavior under such disturbances. To the best of our knowledge, the inherent strong stability of nominal MPC to plant-model mismatch has been discussed by only [31, 32]. For unconstrained systems with a sufficiently small bound on the mismatch, nominal MPC is shown to stabilize the plant to the origin. While exact penalty functions are considered for handling constraints, there is no guarantee of recursive feasibility.

In this article, we extend the work of [32] to include input

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[1] The latter term is borrowed from the differential inclusion literature [15]. Some authors [13, 16] use the term *uniform asymptotic stability* to refer to (ii), but we wish to avoid confusion with the time-varying case.

[2] *Nominal* MPC refers to MPC designed without a disturbance model.

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constraints and stabilizing terminal constraints. We show in Theorem 8 that MPC with quadratic costs achieves strong exponential stability given (i) a fixed steady state, (ii) a mild differentiability condition, and (iii) standard stabilizing terminal ingredients (cf. [24]). For MPC with general, positive definite cost functions, a fixed steady state, and stabilizing terminal ingredients, we show a *joint $\mathcal{K}$-function* bound holds on the increase in the optimal value function (Proposition 9), but strong stability is implied only if this bound decays sufficiently quickly near the origin (Theorem 7). A counterexample (Section VI-A) shows this property does *not* hold in general.

The theory in this article can be extended to address the open problem of offset-free MPC stability [33]. In offset-free MPC, an integrating disturbance model is used to effectively estimate the steady states as a function of the disturbances. This guarantees (in the absence of estimation errors) the steady state is uniform in the parameters, and strong stability can be established (for quadratic costs and differentiable plants).

For brevity, complete proofs of Theorems 1 to 4, an additional nondifferentiable example, and additional remarks throughout are deferred to an extended technical report [34].

Notation: Let $\overline{\mathbb{R}}_{\geq 0} := \mathbb{R}_{\geq 0} \cup \{\infty\}$ denote the extended nonnegative reals. For any function $V : \mathbb{R}^n \to \overline{\mathbb{R}}_{\geq 0}$ and finite $\rho \geq 0$, we define the sublevel set $\text{lev}_\rho V := \{x \in \mathbb{R}^n \mid V(x) \leq \rho\}$. We say $V : \mathbb{R}^n \to \overline{\mathbb{R}}_{\geq 0}$ is lower semicontinuous (l.s.c.) if $\text{lev}_\rho V$ is closed for each $\rho \geq 0$. We say a symmetric matrix $P = P^\top \in \mathbb{R}^{n\times n}$ is positive definite if $x^\top P x > 0$ for all $x \in \mathbb{R}^n \setminus \{0\}$. We define the Euclidean and Q-weighted norms by $|x| := \sqrt{x^\top x}$ and $|x|_Q := \sqrt{x^\top Q x}$ for each $x \in \mathbb{R}^n$, where Q is positive definite. Moreover, $|\cdot|_Q$ has the property $\underline{\sigma}(Q)|x|^2 \leq |x|_Q^2 \leq \overline{\sigma}(Q)|x|^2$ for all $x \in \mathbb{R}^n$, where $\underline{\sigma}(Q)$ and $\overline{\sigma}(Q)$ denote the smallest and largest singular values of Q. For any signal $a(k)$, we denote both infinite and finite sequences in bold font as $\mathbf{a} := (a(0), \ldots, a(k))$ and $\mathbf{a} := (a(0), a(1), \ldots)$. We define the infinite and length-k signal norm as $\|\mathbf{a}\| := \sup_{k\geq 0} |a(k)|$ and $\|\mathbf{a}\|_{0:k} := \max_{0\leq i\leq k} |a(i)|$. Let $\mathcal{PD}$ be the class of functions $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ such that $\alpha(0) = 0$ and $\alpha(s) > 0$ for all $s > 0$. Let $\mathcal{K}$ be the class of $\mathcal{PD}$-functions that are continuous and strictly increasing. Let $\mathcal{K}_\infty$ be the class of $\mathcal{K}$-functions that are unbounded. Let $\mathcal{KL}$ be the set of functions $\beta : \mathbb{R}_{\geq 0} \times \mathbb{I}_{\geq 0} \to \mathbb{R}_{\geq 0}$ such that $\beta(\cdot, k) \in \mathcal{K}$, $\beta(r, \cdot)$ is nonincreasing, and $\lim_{i\to\infty} \beta(r, i) = 0$ for all $(r, k) \in \mathbb{R}_{\geq 0} \times \mathbb{I}_{\geq 0}$.

II. Problem statement

Consider the following discrete-time plant:

$$x^+ = f(x, u, \theta) \tag{1}$$

where $x \in \mathbb{R}^n$ is the plant state, $u \in \mathbb{R}^m$ is the plant input, and $\theta \in \mathbb{R}^{n_\theta}$ is an *unknown* parameter vector. We denote the parameter estimate by $\hat{\theta} \in \mathbb{R}^{n_\theta}$ and the modeled system by

$$x^+ = f(x, u, \hat{\theta}). \tag{2}$$

We assume the parameter estimate is time-invariant, while the parameter vector itself may be time-varying. For simplicity, let $\hat{\theta} = 0$ and denote the model as

$$x^+ = \hat{f}(x, u) := f(x, u, 0). \tag{3}$$

In this article, we study the behavior of an MPC designed with the model (2), but applied to the plant (1). Under the assumption $\hat{\theta} = 0$, θ takes the role of an estimate residual. In the language of inherent robustness, the model (3) is the nominal system, and the plant (1) is the uncertain system.

A. Nominal MPC and basic assumptions

We consider an MPC problem with control constraints $u \in \mathbb{U} \subseteq \mathbb{R}^m$, a horizon length of $N \in \mathbb{I}_{>0}$, a stage cost $\ell : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}_{\geq 0}$, a terminal constraint $\mathbb{X}_f \subseteq \mathbb{R}^n$, and a terminal cost $V_f : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$. For an initial state $x \in \mathbb{R}^n$, we define the set of admissible $(x, \mathbf{u})$ pairs (4), admissible input sequences (5), and admissible initial states (6) by

$$\mathcal{Z}_N := \{(x, \mathbf{u}) \in \mathbb{R}^n \times \mathbb{U}^N \mid \hat{\phi}(N; x, \mathbf{u}) \in \mathbb{X}_f\} \tag{4}$$

$$\mathcal{U}_N(x) := \{\mathbf{u} \in \mathbb{U}^N \mid (x, \mathbf{u}) \in \mathcal{Z}_N\} \tag{5}$$

$$\mathcal{X}_N := \{x \in \mathbb{R}^n \mid \mathcal{U}_N(x) \text{ is nonempty}\} \tag{6}$$

where $\hat{\phi}(k; x, \mathbf{u})$ denotes the solution to (3) at time k, given an initial state x and a sufficiently long input sequence $\mathbf{u}$. For each $(x, \mathbf{u}) \in \mathbb{R}^{n+Nm}$, we define the MPC objective by

$$V_N(x, \mathbf{u}) := \sum_{k=0}^{N-1} \ell(\hat{\phi}(k; x, \mathbf{u}), u(k)) + V_f(\hat{\phi}(N; x, \mathbf{u})) \tag{7}$$

and for each $x \in \mathcal{X}_N$, we define the MPC problem by

$$V_N^0(x) := \min_{\mathbf{u}\in\mathcal{U}_N(x)} V_N(x, \mathbf{u}). \tag{8}$$

Using the convention of [35] for infeasible problems, we take $V_N^0(x) := \infty$ for all $x \notin \mathcal{X}_N$.

Throughout, we use the standard assumptions for inherent robustness of MPC from [24].

Assumption 1 (Continuity): The functions $f : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^{n_\theta} \to \mathbb{R}^n$, $\ell : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}_{\geq 0}$, and $V_f : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ are continuous and $\hat{f}(0, 0) = 0$, $\ell(0, 0) = 0$, and $V_f(0) = 0$.

Assumption 2 (Constraint properties): The set $\mathbb{U}$ is compact and contains the origin. The set $\mathbb{X}_f$ is defined by $\mathbb{X}_f := \text{lev}_{c_f} V_f$ for some $c_f > 0$.

Assumption 3 (Terminal control law): There exists a terminal control law $\kappa_f : \mathbb{X}_f \to \mathbb{U}$ such that

$$V_f(\hat{f}(x, \kappa_f(x))) \leq V_f(x) - \ell(x, \kappa_f(x)), \quad \forall\, x \in \mathbb{X}_f.$$

Assumption 4 (Stage cost bound): There exists a function $\alpha_1 \in \mathcal{K}_\infty$ such that

$$\ell(x, u) \geq \alpha_1(|(x, u)|), \qquad \forall\, (x, u) \in \mathbb{R}^n \times \mathbb{U}. \tag{9}$$

Remark 1: Assumptions 2 and 3 imply $V_f(\hat{f}(x, \kappa_f(x))) \leq V_f(x) \leq c_f$ for all $x \in \mathbb{X}_f$ and therefore $\mathbb{X}_f$ is positive invariant for $x^+ = \hat{f}(x, \kappa_f(x))$.

Under Assumptions 1 and 2, the existence of solutions to (8) follows from [17, Prop. 2.4]. To ensure uniqueness, we assume some selection rule has been applied and denote the solution by $\mathbf{u}^0(x) = (u^0(0; x), \ldots, u^0(N-1; x))$, denote

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the corresponding optimal state sequence by $\hat{x}^0(k;x) := \hat{\phi}(k;x,\mathbf{u}^0(x))$ for each $k \in \mathbb{I}_{0:N}$, and define the MPC control law $\kappa_N : \mathcal{X}_N \to \mathbb{U}$ by $\kappa_N(x) := u^0(0;x)$. Note that the subsequent analyses do not depend on the chosen selection rule, so the results hold no matter what solutions are selected at a particular time. It is also useful to define the following suboptimal input sequence:

$$\tilde{\mathbf{u}}(x) := (u^0(1;x), \dots, u^0(N-1;x), \kappa_f(\hat{x}^0(N;x))).$$

Quadratic stage and terminal costs are of particular interest in this work. Throughout, we call an MPC satisfying the following assumption a *quadratic cost MPC*.

Assumption 5 (Quadratic cost): We have

$$\ell(x,u) := |x|_Q^2 + |u|_R^2, \qquad V_f(x) := |x|_{P_f}^2 \tag{10}$$

for all $(x,u) \in \mathbb{R}^n \times \mathbb{R}^m$ and positive definite Q, R, and P_f.

Consider the *modeled* closed-loop system

$$x^+ = \hat{f}_c(x) := \hat{f}(x, \kappa_N(x)). \tag{11}$$

From Assumptions 1 to 4, it can be shown $x^+ = \hat{f}_c(x)$ is asymptotically stable in $\mathcal{X}_N$ with the Lyapunov function V_N^0 [17, Thm. 2.19]. Similarly, it is shown in [17, Sec. 2.5.5] that, under Assumptions 1 to 3 and 5, the quadratic cost MPC *exponentially* stabilizes the closed-loop system (11) on any sublevel set of the optimal value function $\mathcal{S} := \mathrm{lev}_\rho V_N^0$. These facts are stated as special cases of the inherent robustness results in Section IV.

To show strong stability of the MPC with mismatch, we eventually require one or both of the following assumptions.

Assumption 6 (Steady state): The origin is a steady state, uniformly in $\theta \in \mathbb{R}^{n_\theta}$, i.e., $f(0,0,\theta) = 0$ for all $\theta \in \mathbb{R}^{n_\theta}$.

Assumption 7 (Differentiability): The derivative $\partial_{(x,u)} f$ exists and is continuous on $\mathbb{R}^{n+m+n_\theta}$.

Remark 2: State constraints were not considered, as there is no way to guarantee robust feasibility of state-constrained nominal MPC [24]. Soft state constraints (cf. [32, 33]) are compatible with our general cost MPC assumptions, but using them in the quadratic cost MPC would require some modifications to our analysis.

Remark 3: Assumption 6 limits our results to problems where the steady state is known and fixed (e.g., path-planning and inventory problems). If the steady state depends on θ, i.e., $x_s(\theta) = f(x_s(\theta), u_s(\theta), \theta)$, we can still work with deviation variables $(\delta x, \delta u) := (x - x_s(\theta), u - u_s(\theta))$, but (i) we have to estimate the steady-state pair $(x_s(\theta), u_s(\theta))$ (e.g., via an integrating disturbance model [17, Ch. 1]), and (ii) strong stability is only achieve when the steady-state map is continuous, the parameters are asymptotically constant, and the estimation errors converge [33].

Remark 4: Assumption 7 effectively requires the plant (1) to be continuous in θ. Continuous differentiability of f is sufficient, but not necessary, for guaranteeing Assumption 7.

Remark 5: The linear parameter-varying (LPV) system $x^+ = A(\theta)x + B(\theta)u$, where (A,B) are continuous in θ, is a simple example satisfying both Assumptions 6 and 7. If (A,B) are known, one could treat $\hat{\theta}$ as a fitted parameter

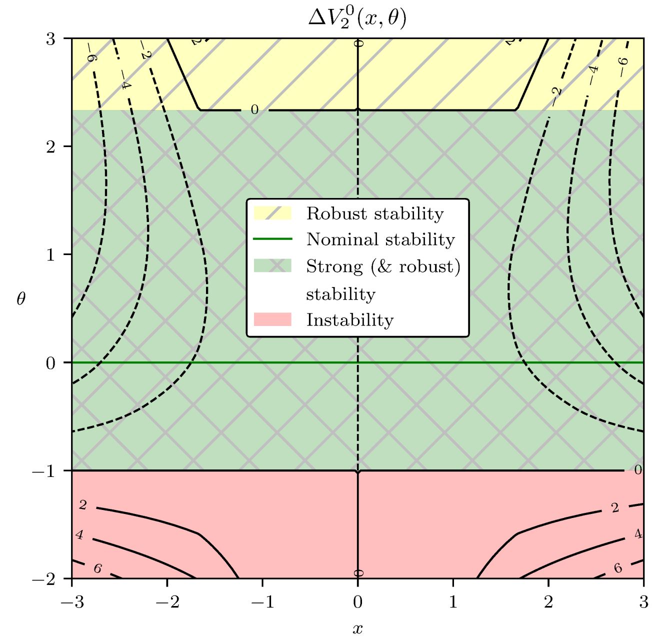

Fig. 1. Contours of the cost difference as a function of the initial state $\boldsymbol{x}$ and the parameter $\boldsymbol{\theta}$.

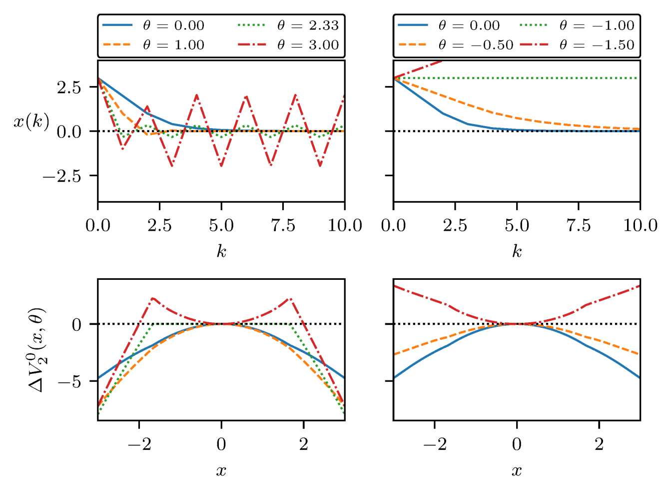

Fig. 2. For (left) positive and (right) negative values of $\boldsymbol{\theta}$, the (top) closed-loop trajectories with initial state $\boldsymbol{x} = \mathbf{3}$, and (bottom) cost differences as a function of $\boldsymbol{x}$, along with the nominal values.

estimate, construct the MPC from the data-driven surrogate model $x^+ = A(\hat{\theta})x + B(\hat{\theta})u$, and use the theory herein to demonstrate stability given sufficiently accurate estimates $\hat{\theta}$.

B. Motivating example

We close this section with a motivating example exhibiting many types of stability under persistent mismatch. Recall from the introduction we define *robust stability* as an ISS property for parameter errors, and *strong stability* as convergence to the origin despite mismatch. While precise definitions are given in Section III, these informal definitions suffice for the example.

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Consider the scalar linear system

$$x^+ = f(x,u,\theta) := x + (1+\theta)u. \tag{12}$$

The plant (12) is a prototypical integrating system, such as a storage tank or vehicle on a track, with an uncertain input gain. As usual the system is modeled with $\hat{\theta} = 0$,

$$x^+ = \hat{f}(x,u) := f(x,u,0) = x + u. \tag{13}$$

We define a nominal MPC with $\mathbb{U} := [-1,1]$, $\ell(x,u) := (1/2)(x^2+u^2)$, $V_f(x) := (1/2)x^2$, $\mathbb{X}_f := [-1,1]$, and $N := 2$. Notice that the terminal set can be reached in $N = 2$ moves if and only if $|x| \leq 3$, so we have the set of admissible initial states $\mathcal{X}_2 = [-3,3]$. *Without* the terminal constraint (i.e., $\mathbb{X}_f = \mathbb{R}$), the optimal control sequence is

$$\mathbf{u}^0(x) = \begin{cases} (-3x/5, -x/5), & |x| \leq 5/3 \\ (-\mathrm{sgn}(x), -x/2 + \mathrm{sgn}(x)/2), & 5/3 < |x| \leq 3 \end{cases}$$

and the control law is $\kappa_2(x) := -\mathrm{sat}(3x/5)$ [17, p. 104]. However, the optimal input sequence gives

$$\hat{x}^0(2;x) = \begin{cases} x/5, & |x| \leq 5/3 \\ x/2 - \mathrm{sgn}(x)/2, & 5/3 < |x| \leq 3 \end{cases}$$

so the terminal constraint $\mathbb{X}_f = [-1,1]$ is automatically satisfied for all $|x| \leq 3$. Therefore $\kappa_2(x) = -\mathrm{sat}(3x/5)$ is also the control law of the problem *with* the terminal constraint.

In Figure 1 we plot contours of the cost difference $\Delta V_2^0(x,\theta) := V_2^0(f(x,\kappa_2(x),\theta)) - V_2^0(x)$, and in Figure 2, we plot closed-loop trajectories and the cost difference curve $\Delta V_2^0(\cdot,\theta)$ for several values of θ. For $-1 < \theta < 7/3$, we have strong stability as $\Delta V_2^0(\cdot,\theta)$ is negative definite. For $\theta \geq 7/3$, the sign of $\Delta V_2^0(\cdot,\theta)$ is ambiguous, and we have robust stability. For $\theta < -1$, the trajectories are unbounded as $\Delta V_2^0(\cdot,\theta)$ is positive definite. We point out the existing literature on inherent robustness is not sufficient to predict strong stability whenever $-1 < \theta < 7/3$.

III. Robust and Strong Stability

Consider the closed-loop system

$$x^+ = f_c(x,\theta) := f(x,\kappa_N(x),\theta), \qquad \theta \in \Theta \tag{14}$$

where $\Theta \subseteq \mathbb{R}^{n_\theta}$. Let $\phi_c(k;x,\boldsymbol{\theta})$ denote solutions to (14) at time k, given an initial state $x \in \mathcal{X}_N$ and a sufficiently long parameter sequence $\theta \in \Theta$. If $\Theta := \{\, \theta \in \mathbb{R}^{n_\theta} \mid |\theta| \leq \delta \,\}$, it is convenient to write (14) as $x^+ = f_c(x,\theta), |\theta| \leq \delta$. We define robustly positive invariant (RPI) sets for (14) as follows.

Definition 1 (Robust positive invariance): A set $X \subseteq \mathbb{R}^n$ is *robustly positive invariant* for the system $x^+ = f_c(x,\theta), \theta \in \Theta$ if $f_c(x,\theta) \in X$ for all $x \in X$ and $\theta \in \Theta$.

In this section, we present stability definitions and results for (14). For brevity, asymptotic and exponential definitions and results are consolidated into the same statement. The main difference between our definitions and results and existing ones in the literature is the restriction of the state to the RPI set X and the disturbance to the arbitrary set Θ.

A. Robust stability

We define robust asymptotic stability (RAS) similarly to input-to-state stability (ISS) from [13]. Likewise, we define robust exponential stability (RES) similarly to input-to-state exponential stability (ISES) from [36].

Definition 2 (Robust stability): A system $x^+ = f_c(x,\theta), \theta \in \Theta$ is *robustly asymptotically stable* (in a RPI set $X \subseteq \mathbb{R}^n$) if there exists $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$ such that

$$|\phi_c(k;x,\boldsymbol{\theta})| \leq \beta(|x|,k) + \gamma(\|\boldsymbol{\theta}\|_{0:k-1}) \tag{15}$$

for all $k \in \mathbb{I}_{\geq 0}$, $x \in X$, and $\boldsymbol{\theta} \in \Theta^k$. If, additionally, $\beta(s,k) = cs\lambda^k$ for some $c > 0$ and $\lambda \in (0,1)$, we say $x^+ = f_c(x,\theta), \theta \in \Theta$ is *robustly exponentially stable* (in X).

Definition 3 (ISS/ISES Lyapunov function): A function $V : X \to \mathbb{R}_{\geq 0}$ is an *ISS Lyapunov function* (in an RPI set $X \subseteq \mathbb{R}^n$, for the system $x^+ = f_c(x,\theta), \theta \in \Theta$) if there exist functions $\alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty$ and $\sigma \in \mathcal{K}$ such that

$$\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|) \tag{16a}$$

$$V(f_c(x,\theta)) \leq V(x) - \alpha_3(|x|) + \sigma(|\theta|). \tag{16b}$$

for all $x \in X$ and $\theta \in \Theta$. If, additionally, $\alpha_i(\cdot) := a_i(\cdot)^b$ for some $a_i, b > 0$ and each $i \in \mathbb{I}_{1:3}$, we say V is an *ISES Lyapunov function* (in X, for $x^+ = f_c(x,\theta), \theta \in \Theta$).

The result below is a generalization of [24, Prop. 19] to include arbitrary disturbance sets and the exponential case.

Theorem 1 (ISS/ISES Lyapunov theorem): The system $x^+ = f_c(x,\theta), \theta \in \Theta$ is RAS (RES) in an RPI set $X \subseteq \mathbb{R}^n$ if it admits an ISS (ISES) Lyapunov function in X.

Proof: While a self-contained proof can be found in [34], we provide a sketch of the proof as follows.

As in [24], the asymptotic case follows identically to the proof [13, Lem. 3.5], noting that it is unnecessary to invoke continuity of f_c and V, and, without loss of generality, we can restrict the state and disturbance to X and Θ, respectively.

For the exponential case, we immediately have

$$\begin{aligned} a_1|x(k)|^b &\leq V(x(k)) \leq \lambda_0 V(x(k-1)) + \sigma(|\theta(k-1)|) \\ &\leq \lambda_0^k V(x) + \sum_{i=1}^k \lambda_0^{i-1}\sigma(\theta(k-i)) \\ &\leq a_2\lambda_0^k|x|^b + \frac{\sigma(\|\boldsymbol{\theta}\|_{0:k-1})}{1-\lambda_0} \end{aligned} \tag{17}$$

for all $k \in \mathbb{I}_{\geq 0}$, $x \in X$, and $\boldsymbol{\theta} \in \Theta^k$, where $\lambda_0 := 1 - \frac{a_2}{a_3} \in (0,1)$ and $x(j) := \phi_c(k;x,\boldsymbol{\theta})$ for all $j \in \mathbb{I}_{0:k}$. The desired bound follows by applying the function $((\cdot)/a_1)^{1/b}$ to both sides and using the triangle inequality when $b \leq 1$ and the definition of convexity when $b > 1$, appropriately defining $c > 0$ and $\lambda \in (0,1)$ for each case. ■

B. Strong stability

We take strong asymptotic stability (SAS) as a time-invariant version of the conclusion of [16, Prop. 2.2]. Strong exponential stability (SES) is defined similarly.

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Definition 4 (Strong stability): A system $x^+ = f_c(x,\theta), \theta \in \Theta$ is *strongly asymptotically stable* (in a RPI set $X \subseteq \mathbb{R}^n$) if there exists $\beta \in \mathcal{KL}$ such that

$$|\phi_c(k; x, \boldsymbol{\theta})| \leq \beta(|x|, k) \tag{18}$$

for all $k \in \mathbb{I}_{\geq 0}$, $x \in X$, and $\boldsymbol{\theta} \in \Theta^k$. If, additionally, $\beta(s,k) := cs\lambda^k$ for all $s \geq 0$ and $k \in \mathbb{I}_{\geq 0}$, and some $c > 0$ and $\lambda \in (0,1)$, we say $x^+ = f_c(x,\theta), \theta \in \Theta$ is *strongly exponentially stable* (in X).

Definition 5 (Lyapunov function): A function $V : X \to \mathbb{R}_{\geq 0}$ is a *Lyapunov function* (in a RPI set $X \subseteq \mathbb{R}^n$, for the system $x^+ = f(x,\theta), \theta \in \Theta$), if there exist functions $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$ and a continuous function $\alpha_3 \in \mathcal{PD}$ such that

$$\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|) \tag{19a}$$

$$V(f_c(x,\theta)) \leq V(x) - \alpha_3(|x|) \tag{19b}$$

for all $x \in X$ and $\theta \in \Theta$. If, additionally, $\alpha_i(\cdot) := a_i(\cdot)^b$ for some $a_i, b > 0$ and each $i \in \mathbb{I}_{1:3}$, we say V is an *exponential Lyapunov function* (in X, for $x^+ = f_c(x,\theta), \theta \in \Theta$).

The following theorem generalizes [24, Prop. 13] and [23, Lem. 15] to include arbitrary disturbance sets.

Theorem 2: The system $x^+ = f_c(x,\theta), \theta \in \Theta$ is SAS (SES) in a RPI set $X \subseteq \mathbb{R}^n$ if it admits a Lyapunov function (an exponential Lyapunov function) in X.

Proof: See [34] for a self-contained version of the following proof sketch.

For the asymptotic case, since the proof of [16, Lem. 2.8] still holds when parts relating to continuity of f_c and V are dropped (cf. [34, Prop. 12]), there exist functions $\alpha, \rho \in \mathcal{K}_\infty$ such that $W(f_c(x,\theta)) \leq W(x) - \alpha(|x|)$ for all $x \in X$ and $\theta \in \Theta$, where $W := \rho \circ V$. Moreover $\hat{\alpha}_1(|x|) \leq W(x) \leq \hat{\alpha}_2(|x|)$ for all $x \in X$, where $\hat{\alpha}_i := \rho \circ \alpha_i \in \mathcal{K}_\infty, i \in \mathbb{I}_{1:2}$. In other words, W is a Lyapunov function on X for the system (14), but with a $\mathcal{K}_\infty$-function cost decrease. The rest of the proof of the asymptotic part follows identically to the proof of the relevant part of [16, Thm. 1(1)].

Following the proof of Theorem 1, the exponential case is established by setting $\sigma \equiv 0$ in (17) and applying the function $((\cdot)/a_1)^{1/b}$ to both sides. ∎

IV. Inherent robustness of MPC

In this section, we review results on the inherent robustness of nominal MPC. See [34] for direct proofs of the results in this section. Theorem 3 follows as a special case of the suboptimal MPC robustness result [24, Thm. 21].

Theorem 3: Suppose Assumptions 1 to 4 hold. Let $\rho > 0$ and $\mathcal{S} := \text{lev}_\rho V_N^0$. Then there exist $\delta > 0$, $\alpha_2 \in \mathcal{K}_\infty$, and $\sigma \in \mathcal{K}$ such that

$$\alpha_1(|x|) \leq V_N^0(x) \leq \alpha_2(|x|) \tag{20a}$$

$$V_N^0(f_c(x,\theta)) \leq V_N^0(x) - \alpha_1(|x|) + \sigma(|\theta|) \tag{20b}$$

for all $x \in \mathcal{S}$ and $|\theta| \leq \delta$, and the system $x^+ = f_c(x,\theta), |\theta| \leq \delta$ is RAS in the RPI set $\mathcal{S}$.

A key step of the proof of Theorem 3 and the main results is to establish the following robust descent property:

$$V_N^0(f_c(x,\theta)) \leq V_N^0(x) - \ell(x, \kappa_N(x)) + V_N(f_c(x,\theta), \tilde{\mathbf{u}}(x)) - V_N(\hat{f}_c(x), \tilde{\mathbf{u}}(x)). \tag{21}$$

In fact, it is shown (21) can be achieved on any sublevel set of V_N^0 and a sufficiently small neighborhood $|\theta| \leq \delta$. We restate this intermediate result in the following proposition.

Proposition 1: Suppose Assumptions 1 to 4 hold. Let $\rho > 0$ and $\mathcal{S} := \text{lev}_\rho V_N^0$. There exists $\delta > 0$ such that (21) holds for all $x \in \mathcal{S}$ and $|\theta| \leq \delta$ and $\mathcal{S}$ is RPI for $x^+ = f_c(x,\theta), |\theta| \leq \delta$.

With quadratic costs (Assumption 5), Assumptions 1 to 3 also imply inherent *exponential* robustness of MPC. Theorem 4 follows as a special case of the suboptimal MPC robustness result [23, Thm. 18].

Theorem 4: Suppose Assumptions 1 to 3 and 5 hold. Let $\rho > 0$ and $\mathcal{S} := \text{lev}_\rho V_N^0$. There exist $\delta, c_2 > 0$ and $\sigma \in \mathcal{K}$ such that

$$c_1|x|^2 \leq V_N^0(x) \leq c_2|x|^2 \tag{22a}$$

$$V_N^0(f_c(x,\theta)) \leq V_N^0(x) - c_1|x|^2 + \sigma(|\theta|) \tag{22b}$$

for all $x \in \mathcal{S}$ and $|\theta| \leq \delta$, where $c_1 := \underline{\sigma}(Q)$, and the system $x^+ = f_c(x,\theta), |\theta| \leq \delta$ is RES in the RPI set $\mathcal{S}$.

V. Stability of MPC despite mismatch

In this section, we investigate two approaches to guarantee strong stability of the closed-loop system (14). First, we take a direct approach and assume the existence of an ISS Lyapunov function that achieves a certain maximum increase due to mismatch. In general, an additional scaling condition is required for the mismatch term, although it is automatically satisfied for quadratic cost MPC. While these new Lyapunov assumptions are difficult to check, we can easily construct error bounds that imply the maximum Lyapunov increase for V_N^0 via the standard MPC assumptions (Assumptions 1 to 5) and one or both of Assumptions 6 and 7.

A. Maximum Lyapunov increase

We begin with the direct approach. The goal here is not (necessarily) to provide the means to check if a given MPC is strongly stabilizing, but to (i) identify a set of conditions for which an ISS Lyapunov function also guarantees strong stability (not only for the closed-loop MPC but for a general class of systems) and (ii) provide a path towards proving certain classes of nominal MPCs are strongly stabilizing.

1) Asymptotic case: For inherent robustness, a maximum increase of the form (20b) is proven for the optimal value function V_N^0. However, since the perturbation term $\sigma(|\theta|)$ is uniform in $|x|$, strong stability is not demonstrated for nonzero θ. Under Assumption 6, we might assume the perturbation vanishes in either of the limits $|x| \to 0$ or $|\theta| \to 0$. In this sense, the perturbation should be class-$\mathcal{K}$ in $|x|$ whenever $|\theta|$ is fixed, and vice versa. We call these functions *joint $\mathcal{K}$-functions* or *$\mathcal{K}^2$-functions* and define them as follows.

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Definition 6 (Class $\mathcal{K}^2$): The class of *joint $\mathcal{K}$-functions*, denoted $\mathcal{K}^2$ is the class of continuous functions $\gamma : \mathbb{R}^2_{\geq 0} \to \mathbb{R}_{\geq 0}$ such that $\gamma(s, \cdot), \gamma(\cdot, s) \in \mathcal{K}$ for all $s > 0$.

To achieve strong stability, we assume the existence of an ISS Lyapunov function with a $\mathcal{K}^2$-function perturbation term, rather than the standard $\mathcal{K}$-function perturbation term. Moreover, we require the perturbation to decay faster than the nominal cost decrease in the limit $|x| \to 0$ so that the descent property of Definition 5 is achieved for sufficiently small θ.

Assumption 8: There exists a l.s.c. function $V : \mathbb{R}^n \to \overline{\mathbb{R}}_{\geq 0}$ such that, for each $\rho > 0$, there exist $\delta_0 > 0, \alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty$, and $\gamma_V \in \mathcal{K}^2$ satisfying the following:

(a) $\mathcal{S} := \mathrm{lev}_\rho V \subseteq \mathcal{X}_N$,

(b) for each $x \in \mathcal{S}$ and $|\theta| \leq \delta_0$, we have

$$\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|) \tag{23a}$$

$$V(f_c(x, \theta)) \leq V(x) - \alpha_3(|x|) + \gamma_V(|x|, |\theta|); \tag{23b}$$

(c) and there exists $\tau > 0$ such that

$$\limsup_{s \to 0^+} \frac{\gamma_V(s, \tau)}{\alpha_3(s)} < 1. \tag{24}$$

Remark 6: We assume V is l.s.c. to ensure $\mathcal{S}$ is closed and can be used as a domain of attraction. Note l.s.c. of V_N^0 is compatible with the jump to $V_N^0(x) = \infty$ when $x \notin \mathcal{X}_N$.

With Assumption 8, we have our first main result.

Theorem 5: Suppose Assumption 8 holds with $V : \mathbb{R}^n \to \overline{\mathbb{R}}_{\geq 0}$. For each $\rho > 0$, there exists $\delta > 0$ for which $x^+ = f_c(x, \theta), |\theta| \leq \delta$ is SAS in the RPI set $\mathcal{S} := \mathrm{lev}_\rho V$.

To prove Theorem 5, we require a preliminary result related to the ability of a given $\mathcal{K}^2$-function to lower bound another given $\mathcal{K}$-function (see Appendix A for proof).

Proposition 2: Let $\alpha \in \mathcal{K}_\infty$ and $\gamma \in \mathcal{K}^2$. If there exists $\tau > 0$ such that $\limsup_{s \to 0^+} \gamma(s, \tau)/\alpha(s) < 1$, then, for each $\rho > 0$, there exists $\delta > 0$ such that $\gamma(s, t) < \alpha(s)$ for all $s \in (0, \rho]$ and $t \in [0, \delta]$.

Finally, we prove Theorem 5.

Proof of Theorem 5: By Assumption 8(a,b) there exists $\delta_0 > 0$, $\alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty$, and $\gamma_V \in \mathcal{K}^2$ such that $\mathcal{S} \subseteq \mathcal{X}_N$ and (23) holds for each $x \in \mathcal{S}$ and $|\theta| \leq \delta_0$. Let $\varepsilon_0 := \sup_{x \in \mathcal{S}} |x| > 0$.[3] By Assumption 8(c) and Proposition 2, there exists $\delta_1 > 0$ such that $\alpha_3(s) > \gamma_V(s, t)$ for all $s \in (0, \varepsilon_0]$ and $t \in [0, \delta_1]$. With $\delta := \min\{\delta_0, \delta_1\}$, the function

$$\sigma(s) := \begin{cases} \alpha_3(s) - \gamma_V(s, \delta), & 0 \leq s \leq \varepsilon_0 \\ \alpha_3(\varepsilon_0) - \gamma_V(\varepsilon_0, \delta), & s > \varepsilon_0 \end{cases}$$

is both class-$\mathcal{PD}$ and continuous. By (23b), we have

$$V(f_c(x, \theta)) - V(x) \leq -\alpha_3(|x|) + \gamma_V(|x|, \delta) = -\sigma(|x|)$$

for all $x \in \mathcal{S}$ and $|\theta| \leq \delta$. Moreover, $V(x) \leq \rho$ implies

$$V(f_c(x, \theta)) \leq V(x) - \sigma(|x|) \leq \rho$$

so $\mathcal{S} = \mathrm{lev}_\rho V$ must be RPI. Finally, $x^+ = f_c(x, \theta), |\theta| \leq \delta$ is SAS in $\mathcal{S}$ by Theorem 2. ∎

[3] If $\mathcal{S} = \{0\}$, the conclusion would hold trivially, so we can assume $\mathcal{S} \neq \{0\}$ without loss of generality.

Remark 7: One might naïvely assume that the closed-loop system (14) is SAS under only Assumption 8(a,b). However, if the scaling condition Assumption 8(c) does not hold, then it may be the case that we cannot shrink t small enough to make $\alpha_3(\cdot) - \gamma_V(\cdot, t)$ positive definite in a sufficiently large neighborhood of the origin, let alone any neighborhood at all. Thus Assumption 8(a,b) alone are insufficient to show V is a Lyapunov function for the closed-loop system (14). This is illustrated in the example of Section VI-A.

Remark 8: To achieve Assumption 8(a), it is necessary to have $V(x) = \infty$ for all $x \notin \mathcal{X}_N$. Under Assumptions 1 to 4, this is automatically achieved by the optimal value function V_N^0, since, according to the convention of [35], we have $V_N^0(x) = \infty$ for infeasible problems.

2) Exponential case: To achieve strong *exponential* stability, Assumption 8 is strengthened to require power law versions of the bounds in (23). Since identical exponents are required, the scaling condition Assumption 8(c) is automatically satisfied.

Assumption 9: There exists a l.s.c. function $V : \mathbb{R}^n \to \overline{\mathbb{R}}_{\geq 0}$ such that, for each $\rho > 0$, there exist $\delta_0, a_1, a_2, a_3, b > 0$ and $\sigma_V \in \mathcal{K}_\infty$ satisfying the following:

(a) $\mathcal{S} := \mathrm{lev}_\rho V \subseteq \mathcal{X}_N$; and

(b) for each $x \in \mathcal{S}$ and $|\theta| \leq \delta_0$, we have

$$a_1|x|^b \leq V(x) \leq a_2|x|^b \tag{25a}$$

$$V(f_c(x, \theta)) \leq V(x) - a_3|x|^b + \sigma_V(|\theta|)|x|^b. \tag{25b}$$

With Assumption 9, we have our second main result.

Theorem 6: Suppose Assumption 9 holds with $V : \mathbb{R}^n \to \overline{\mathbb{R}}_{\geq 0}$. For each $\rho > 0$, there exists $\delta > 0$ for which $x^+ = f_c(x, \theta), |\theta| \leq \delta$ is SES in the RPI set $\mathcal{S} := \mathrm{lev}_\rho V$.

Proof: Assumption 9 gives $\delta_0, a_1, a_2, a_3, b > 0$ such that $\mathcal{S} \subseteq \mathcal{X}_N$ and (25) holds for each $x \in \mathcal{S}$ and $|\theta| \leq \delta_0$. Let $\delta_1 \in (0, \sigma_V^{-1}(a_3))$ and $\delta := \min\{\delta_0, \delta_1\} > 0$. Then, by (25b),

$$V(f_c(x, \theta)) - V(x) \leq -[a_3 - \sigma_V(\delta)]|x|^b = -a_4|x|^b$$

for all $x \in \mathcal{S}$ and $|\theta| \leq \delta$, where $a_4 := a_3 - \sigma_V(\delta) \geq a_3 - \sigma_V(\delta_1) > 0$. But this means that $V(x) \leq \rho$ implies

$$V(f_c(x, \theta)) \leq V(x) - a_4|x|^b \leq \rho$$

so $\mathcal{S} = \mathrm{lev}_\rho V$ must be RPI. Finally, $x^+ = f_c(x, \theta), |\theta| \leq \delta$ is SES in $\mathcal{S}$ by Theorem 2. ∎

B. Error bounds

While the maximum Lyapunov increases (23b) and (25b) are difficult to verify directly, they are in fact satisfied for the optimal value function (i.e., $V := V_N^0$) under fairly general conditions, as we show in Section V-C. First, however, we require bounds on the error due to mismatch.

1) Model error bounds: Stability of MPC under mismatch was first investigated by [31, 32], who considered, for a fixed parameter $\theta \in \mathbb{R}^{n_\theta}$, the following power law bound:

$$|f(x, u, \theta) - \hat{f}(x, u)| \leq c|x| \tag{26}$$

for some $c > 0$ and all $(x, u) \in \mathbb{R}^n \times \mathbb{R}^m$. However, the bound (26) does not account for changing or unknown $\theta \in \mathbb{R}^{n_\theta}$ and is uniform in $u \in \mathbb{R}^m$, thus ruling out the motivating example

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from Section II-B. To handle the former issue, we can take $c = \sigma_f(|\theta|)$ for some $\sigma_f \in \mathcal{K}_\infty$. For the latter issue, it suffices to either replace $|x|$ with $|(x,u)|$, i.e.,

$$|f(x,u,\theta) - \hat{f}(x,u)| \leq \sigma_f(|\theta|)|(x,u)| \tag{27}$$

or consider a bound on the closed-loop error, i.e.,

$$|f_c(x,\theta) - \hat{f}_c(x)| \leq \tilde{\sigma}_f(|\theta|)|x| \tag{28}$$

for all $x \in \mathcal{S}$, $u \in \mathbb{U}$, and $\theta \in \mathbb{R}^{n_\theta}$, where $\sigma_f, \tilde{\sigma}_f \in \mathcal{K}_\infty$ and $\mathcal{S} \subseteq \mathbb{R}^n$ is an appropriately chosen compact set.

In the following propositions, we derive the bounds (27) and (28) using Taylor's theorem and Assumptions 1 to 3 and 5 to 7 (see Appendices B and C for proofs).

Proposition 3: Suppose Assumptions 1, 2, 6, and 7 hold. For each compact set $\mathcal{S} \subseteq \mathbb{R}^n$, there exists $\sigma_f \in \mathcal{K}_\infty$ such that (27) holds for all $x \in \mathcal{S}$, $u \in \mathbb{U}$, and $\theta \in \mathbb{R}^{n_\theta}$.

Proposition 4: Suppose Assumptions 1 to 3 and 5 to 7 hold. For each compact set $\mathcal{S} \subseteq \mathcal{X}_N$, there exists $\tilde{\sigma}_f \in \mathcal{K}_\infty$ such that (28) holds for all $x \in \mathcal{S}$ and $\theta \in \mathbb{R}^{n_\theta}$.

More generally, we could consider $\mathcal{K}^2$-function bounds,

$$|f(x,u,\theta) - \hat{f}(x,u)| \leq \gamma_f(|(x,u)|, |\theta|) \tag{29}$$

$$|f_c(x,\theta) - \hat{f}_c(x)| \leq \tilde{\gamma}_f(|x|, |\theta|) \tag{30}$$

for all $x \in \mathcal{S}$ and $\theta \in \Theta$, where $\gamma_f, \tilde{\gamma}_f \in \mathcal{K}^2$, and $\mathcal{S} \subseteq \mathbb{R}^n$ and $\Theta \subseteq \mathbb{R}^{n_\theta}$ are appropriately chosen compact sets. In the following propositions, we derive the bounds (29) and (30) using Assumptions 1 to 3, 5, and 6 (see Appendices D and E for proofs).

Proposition 5: Suppose Assumptions 1, 2, and 6 hold. For any compact sets $\mathcal{S} \subseteq \mathbb{R}^n$ and $\Theta \subseteq \mathbb{R}^{n_\theta}$, there exists $\gamma_f \in \mathcal{K}^2$ satisfying (29) for all $x \in \mathcal{S}$, $u \in \mathbb{U}$, and $\theta \in \Theta$.

Proposition 6: Suppose Assumptions 1 to 4 and 6 hold. For any compact sets $\mathcal{S} \subseteq \mathcal{X}_N$ and $\Theta \subseteq \mathbb{R}^{n_\theta}$, there exists $\tilde{\gamma}_f \in \mathcal{K}^2$ satisfying (30) for all $x \in \mathcal{S}$ and $\theta \in \Theta$.

2) Suboptimal cost error bounds: Ultimately, we require a maximum Lyapunov increase of the form (23b) or (25b). The robust descent property (21) suggests a path through imposing an error bound on the suboptimal cost function $V_N(f_c(x,\theta), \tilde{\mathbf{u}}(x))$, i.e.,

$$|V_N(f_c(x,\theta), \tilde{\mathbf{u}}(x)) - V_N(\hat{f}_c(x), \tilde{\mathbf{u}}(x))| \leq \sigma_V(|\theta|)|x|^2 \tag{31}$$

where $\sigma_V \in \mathcal{K}_\infty$. In Proposition 7, we establish (31) under Assumptions 1 to 3 and 5 to 7 (see Appendix F for proof).

Proposition 7: Suppose Assumptions 1 to 3 and 5 to 7 hold and let $\mathcal{S} \subseteq \mathcal{X}_N$ be compact. Then there exists $\sigma_V \in \mathcal{K}_\infty$ such that (31) holds for all $x \in \mathcal{S}$ and $\theta \in \mathbb{R}^{n_\theta}$.

Similarly, we can derive a $\mathcal{K}^2$-function version of (31) under Assumptions 1 to 4 and 6 (see Appendix G for proof).

Proposition 8: Suppose Assumptions 1 to 4 and 6 hold. Let $\mathcal{S} \subseteq \mathcal{X}_N$ and $\Theta \subseteq \mathbb{R}^{n_\theta}$ be compact. Then there exists $\gamma_V \in \mathcal{K}^2$ such that, for each $x \in \mathcal{S}$ and $\theta \in \Theta$,

$$|V_N(f_c(x,\theta), \tilde{\mathbf{u}}(x)) - V_N(\hat{f}_c(x), \tilde{\mathbf{u}}(x))| \leq \gamma_V(|x|, |\theta|). \tag{32}$$

C. Stability despite mismatch

1) General costs: Finally, we are in a position to construct a maximum Lyapunov increase (23b) or (25b). For general costs, this is accomplished in the following proposition.

Proposition 9: Suppose Assumptions 1 to 4 and 6 hold. Then Assumption 8(a,b) hold with $V := V_N^0$.

Proof: Let $\rho > 0$, $\mathcal{S} := \text{lev}_\rho V_N^0$, and $V := V_N^0$. Then $\mathcal{S} \subseteq \mathcal{X}_N$ trivially. Since V_N^0 is l.s.c. [37, Lem. 7.18], $\mathcal{S}$ is closed. By Theorem 3, there exists $\alpha_2 \in \mathcal{K}_\infty$ satisfying (23a) for all $x \in \mathcal{S}$. Then $|x| \leq \alpha_1^{-1}(V(x)) \leq \alpha_1^{-1}(\rho)$ for all $x \in \mathcal{S}$, so $\mathcal{S}$ is compact.

By Proposition 1, there exists $\delta_0 > 0$ such that $\mathcal{S}$ is RPI for $x^+ = f_c(x,\theta), |\theta| \leq \delta_0$ and (21) holds for all $x \in \mathcal{S}$ and $|\theta| \leq \delta_0$. Moreover, for each $x \in \mathcal{S}$ and $|\theta| \leq \delta_0$, (32) holds for some $\gamma_V \in \mathcal{K}^2$ by Proposition 8. Finally, combining (9), (21), and (32) gives (23b) with $\alpha_3 := \alpha_1$. ■

Assumption 8(a,b) alone do not guarantee strong stability. However, we can strengthen the hypothesis of Proposition 9 with a scaling requirement to guarantee strong stability.

Theorem 7: Suppose Assumptions 1 to 4 and 6 hold. Let $\rho > 0$ and $\mathcal{S} := \text{lev}_\rho V_N^0$. Then (23) holds for all $x \in \mathcal{S}$ and $|\theta| \leq \delta_0$ with $V := V_N^0$ and some $\delta_0 > 0$, $\alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty$, and $\gamma_V \in \mathcal{K}^2$. If, additionally, there exists $\tau > 0$ satisfying (24), then there exists $\delta > 0$ such that $x^+ = f_c(x,\theta), |\theta| \leq \delta$ is SAS in the RPI set $\mathcal{S}$.

Proof: The first part follows from Proposition 9, and the second part follows from Theorem 5. ■

2) Quadratic costs: For quadratic costs, we construct (25b) in the following proposition.

Proposition 10: Suppose Assumptions 1 to 3 and 5 to 7 hold. Then Assumption 9 holds with $b := 2$ and $V := V_N^0$.

Proof: Let $\rho > 0$, $V := V_N^0$, and $\mathcal{S} := \text{lev}_\rho V$. Since Assumption 5 implies Assumption 4, we have from the first paragraph of the proof of Proposition 9 that $\mathcal{S}$ is compact.

Theorem 4 also implies (25a) holds for all $x \in \mathcal{S}$, with $a_1, a_2 > 0$ and $b := 2$. By Proposition 1, there exists $\delta_0 > 0$ such that $\mathcal{S}$ is RPI for $x^+ = f_c(x,\theta), |\theta| \leq \delta_0$ and (21) holds for all $x \in \mathcal{S}$ and $|\theta| \leq \delta_0$. Moreover, for each $x \in \mathcal{S}$ and $|\theta| \leq \delta_0$, (31) holds for some $\sigma_V \in \mathcal{K}_\infty$ by Proposition 8, and combining (21) and (32) gives (25b). ■

Our third and final main result follows immediately from Theorem 5 and Proposition 10.

Theorem 8: Suppose Assumptions 1 to 3 and 5 to 7 holds. For each $\rho > 0$, there exists $\delta > 0$ for which $x^+ = f_c(x,\theta), |\theta| \leq \delta$ is SES in the RPI set $\mathcal{S} := \text{lev}_\rho V_N^0$.

Proof: By Proposition 10, Assumption 9 holds with $V := V_N^0$, and by Theorem 6, there exists $\delta > 0$ for which $\mathcal{S}$ is RPI and $x^+ = f_c(x,\theta), |\theta| \leq \delta$ is SES in $\mathcal{S}$. ■

VI. Examples

In this section, we illustrate the results through several examples. First, we consider a non-differentiable system that satisfies Assumption 8(a,b) but not Assumption 8(c), and is not SAS. Finally, we consider the inverted pendulum system to showcase how the nominal MPC handles different types of mismatch. Notably, we consider (i) discretization errors,

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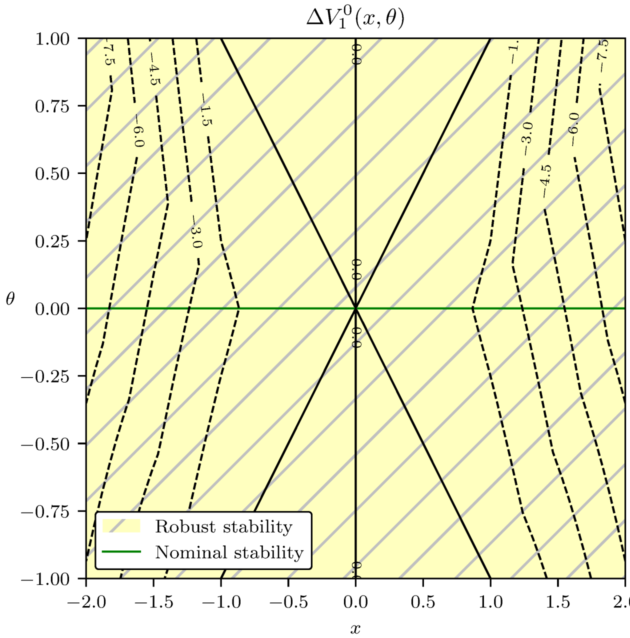

Fig. 3. Contours of the cost difference for the MPC of (33).

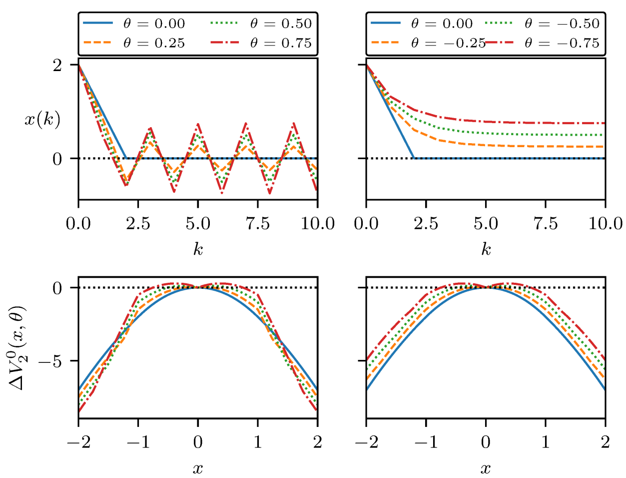

Fig. 4. For (left) nonnegative and (right) nonpositive values of $\boldsymbol{\theta}$, the (top) closed-loop trajectories for the MPC of (33) with initial state $\boldsymbol{x} = \mathbf{2}$, and (bottom) cost differences of the same MPC as a function of $\boldsymbol{x}$.

(ii) unmodeled dynamics, and (iii) incorrectly estimated input gains. See [34] for an example of a non-differentiable system that nonetheless satisfies Assumption 9 and is therefore SES.

A. Strong asymptotic stability counterexample

Consider the scalar system

$$x^+ = f(x,u,\theta) := \sigma(x + (1+\theta)u) \tag{33}$$

where σ is the *signed square root* defined as $\sigma(y) := \mathrm{sgn}(y)\sqrt{|y|}$ for each $y \in \mathbb{R}$. We define a nominal MPC with $\mathbb{U} := [-1,1]$, $\ell(x,u) := x^2 + u^2$, $V_f(x) := 4x^2$, $\mathbb{X}_f := [-1,1]$, and $N := 1$.

In Appendix H, it is shown the closed-loop system $x^+ = f(x,\kappa_1(x),\theta), |\theta| \leq 3$ is RES on $\mathcal{X}_1 = [-2,2]$ with the nominal control law $\kappa_1(x) := -\mathrm{sat}(x)$. Additionally, it is shown Assumption 8(a,b) is satisfied with $V := V_1^0$, and (23b) holds for all $x \in \mathcal{S} := \mathrm{lev}_2 V_1^0 = [-1,1]$ and $|\theta| \leq \delta_0 := 3$ with $\alpha_3(s) := 2s^2$, and $\gamma_V(s,t) := st + 4\sqrt{st}$. But this implies $\lim_{s\to 0^+} \gamma_V(s,t)/\alpha_3(s) = \infty$ for each $t > 0$, so Assumption 8(c) is not satisfied.

However, Assumption 8 is only sufficient, not necessary, for establishing strong stability. But we have $V_1^0(x) = 2x^2$ and

$$\begin{aligned}\Delta V_1^0(x,\theta) &:= V_1^0(f(x,\kappa_1(x),\theta)) - V_1^0(x)\\ &= 2[\sigma(\theta x)]^2 - 2x^2 = 2(|\theta| - |x|)|x| > 0.\end{aligned}$$

for each $0 < |x| < |\theta| \leq 1$, so the state always gets pushed out of $(-|\theta|, |\theta|)$ unless it starts at the origin or $\theta = 0$. In other words, the MPC only provides inherent robustness, not strong stability, even though Assumption 8(a,b) is satisfied.

In Figure 3, we plot contours of the cost difference $\Delta V_1^0(x,\theta)$, and in Figure 4 we plot closed-loop trajectories and the cost difference curve $\Delta V_1^0(\cdot,\theta)$ for several values of θ. Only with $\theta = 0$ does the trajectory converge to the origin and the cost difference curve remain negative definite. For each $\theta \neq 0$, the cost difference is positive definite near the origin, and the trajectory does not converge to the origin.

B. Upright pendulum

Consider the nondimensionalized pendulum system

$$\dot{x} = F(x,u,\theta) := \begin{bmatrix} x_2 \\ \sin x_1 - \theta_1^2 x_2 + (\hat{k} + \theta_2)u \end{bmatrix} \tag{34}$$

where $x_1, x_2 \in \mathbb{R}$ are the angle and angular velocity, $u \in [-1,1]$ is the (signed and normalized) motor voltage, $\theta_1 \in \mathbb{R}$ is an air resistance factor, $\hat{k} > 0$ is the estimated gain of the motor, and $\theta_2 \in \mathbb{R}$ is the error in the motor gain estimate. Let $\psi(t;x,u,\theta)$ denote the solution to the differential equation (34) at time $t \geq 0$ given an initial condition $x(0) = x$, constant input signal $u(t) = u$, and parameters θ. We model the continuous-time system (34) as

$$x^+ = f(x,u,\theta) := x + \Delta F(x,u,\theta) + \theta_3 r(x,u,\theta) \tag{35}$$

where r is a residual function given by

$$r(x,u,\theta) := \int_0^\Delta [F(\psi(t;x,u,\theta),u,\theta) - F(x,u,\theta)]dt.$$

Assuming a zero-order hold on the input u, the system (34) is discretized (exactly) as (35) with $\theta_3 = 1$. Since we model the system with $\theta = 0$ as

$$x^+ = \hat{f}(x,u) := f(x,u,0) = x + \Delta \begin{bmatrix} x_2 \\ \sin x_1 + \hat{k}u \end{bmatrix} \tag{36}$$

we do not need access to r to design the nominal MPC.

For the following simulations, let the model gain be $\hat{k} = 5\ \mathrm{rad/s^2}$, the sample time be $\Delta = 0.1$ s, and define a nominal MPC with $N := 20$, $\mathbb{U} := [-1,1]$, $\ell(x,u) := |x|^2 + u^2$, $V_f(x) := |x|_{P_f}^2$, $\mathbb{X}_f := \mathrm{lev}_{c_f} V_f$, and $c_f := \underline{\sigma}(P_f)/8$, where $P_f = \left[\begin{smallmatrix} 31.133\ldots & 10.196\ldots \\ 10.196\ldots & 10.311\ldots \end{smallmatrix}\right]$ is shown, in Appendix I, to satisfy

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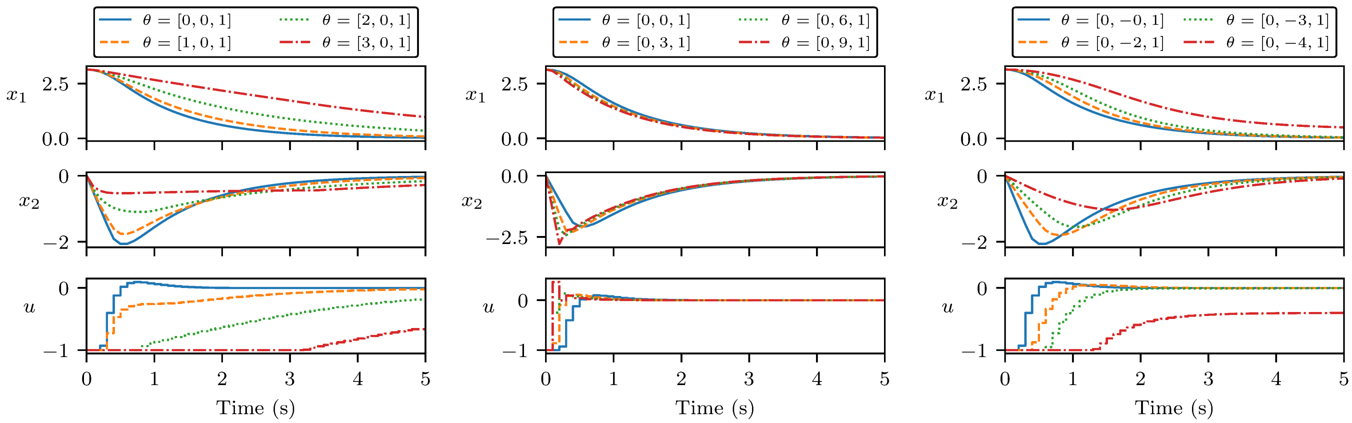

(a) Unmodeled dynamics (b) Underestimated gain (c) Overestimated gain

Fig. 5. Simulated closed-loop trajectories for the MPC of (34) from the resting position $\boldsymbol{x(0)} = \boldsymbol{(\pi, 0)}$ to the upright position $\boldsymbol{x_s} = \boldsymbol{(0, 0)}$ for various values of $\boldsymbol{(\theta_1, \theta_2)} \in \mathbb{R}^{\mathbf{2}}$.

Assumption 3 with the terminal law $\kappa_f(x) := -2x_1 - 2x_2$. Assumptions 1, 2, 5, and 6 are satisfied trivially, and Assumption 7 is satisfied since continuous differentiability of F implies continuous differentiability of ψ (and therefore also r and f) [38, Thm. 3.3]. Thus, the conclusion of Theorem 8 holds for some $\delta > 0$, and if we can take $\delta > 1$, the nominal MPC is inherently strongly stabilizing with $(\theta_1, \theta_2) \in \mathbb{R}^2$ sufficiently small.

In Figure 5, we simulate the closed-loop system $x^+ = f(x, \kappa_{20}(x), \theta)$ for some fixed $(\theta_1, \theta_2, 1) \in \mathbb{R}^3$. Note that all of these simulations include discretization errors. Figure 5a showcases the ability of MPC to handle unmodeled dynamics (i.e., a missing air resistance term). In Figure 5b, the gain of the motor is increased until the nominal controller is severely underdamped. In Figure 5c, the gain of the motor is decreased until the motor cannot overcome the force of gravity and strong stability is not achieved.

VII. Conclusion

We establish conditions under which MPC is strongly stabilizing despite plant-model mismatch in the form of parameter errors. Namely, it suffices to assume the existence of a Lyapunov function with a maximum increase, suitably bounded level sets, and a scaling condition (Assumptions 8 and 9). While we are not able to show the assumptions hold in general, when the MPC has quadratic costs it is possible to show that continuous differentiability of the dynamics implies strong stability (Theorem 6). When the $\mathcal{K}^2$-function bound is not properly scaled, the MPC may not be stabilizing, as illustrated in the examples. In this sense, while MPC is not inherently stabilizing under mismatch *in general*, there is a common class of cost functions (quadratic costs) for which nominal MPC is inherently stabilizing under mismatch.

Several questions about the strong stability of MPC remain unanswered. While quadratic costs are used in many control problems, it may be possible to generalize Theorem 8 to other useful classes of stage costs, such as q-norm costs, or costs with exact penalty functions for soft state constraints (cf. [32, 33]). We propose the direct approach to strong exponential stability (Assumption 9 and Theorem 6) provides a path to generalizing Theorem 8 to other classes of stage costs, output feedback, or semidefinite costs. Nonlinear MPC is computationally difficult to implement online. Therefore it would be worth extending this work to include the suboptimal MPC algorithm from [24] using the approach therein. Finally, while our analysis only considers discrete-time MPC and an effective zero-order hold on the parameter variations, the continuous-time extensions (both continuous-time MPC are discrete-time MPC with continuously varying θ) are worthy pursuits.

While systems with fixed and known setpoints are a useful and interesting class of problems, many systems have parameter-dependent, time-varying setpoints. To track setpoints under mismatch, offset-free MPC can be used. Theory on nonlinear offset-free MPC is limited, typically relying on stability of the closed-loop system to guarantee offset-free performance [39]. In [33], we extend the theory in this article to consider offset-free MPC with quadratic costs, and we establish sufficient conditions for closed-loop stability and guaranteed offset-free performance for a general class of differentiable systems subject to time-varying setpoints, persistent disturbances, and plant-model mismatch. However, many issues in offset-free MPC theory, including necessary conditions for offset-free performance, nonquadratic costs, and nondifferentiable systems, are not yet well understood.

Appendix

The following preliminary results are required throughout.

Proposition 11 (Prop. 20 of [24]): Let $C \subseteq D \subseteq \mathbb{R}^n$, with C compact, D closed, and $f : D \to \mathbb{R}^m$ continuous. Then there exists $\alpha \in \mathcal{K}_\infty$ such that $|f(x) - f(y)| \leq \alpha(|x - y|)$ for all $x \in C$ and $y \in D$.

Proposition 12: Suppose Assumptions 1 to 3 and 5 hold.

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Let $\rho > 0$ and $\mathcal{S} := \mathrm{lev}_\rho V_N^0$. There exist $c_x, c_u > 0$ such that

$$|\hat{x}^0(k;x)| \leq c_x|x|, \qquad \forall\, x \in \mathcal{S},\ k \in \mathbb{I}_{0:N}. \tag{37}$$

$$|u^0(k;x)| \leq c_u|x|, \qquad \forall\, x \in \mathcal{S},\ k \in \mathbb{I}_{0:N-1}. \tag{38}$$

Proof: By Theorem 4, we have the upper bound (22a) for all $x \in \mathcal{S}$ and some $c_2 > 0$. Moreover, since Q, R, P_f are positive definite, we can write, for each $x \in \mathcal{S}$ and $k \in \mathbb{I}_{0:N-1}$,

$$\underline{\sigma}(Q)|\hat{x}^0(k;x)|^2 \leq |\hat{x}^0(k;x)|_Q^2 \leq V_N^0(x) \leq c_2|x|^2$$
$$\underline{\sigma}(P_f)|\hat{x}^0(N;x)|^2 \leq |\hat{x}^0(N;x)|_{P_f}^2 \leq V_N^0(x) \leq c_2|x|^2$$
$$\underline{\sigma}(R)|u^0(k;x)|^2 \leq |u^0(k;x)|_R^2 \leq V_N^0(x) \leq c_2|x|^2.$$

Thus, with $c_x := \max\{\sqrt{c_2/\underline{\sigma}(Q)}, \sqrt{c_2/\underline{\sigma}(P_f)}\}$ and $c_u := \sqrt{c_2/\underline{\sigma}(R)}$, we have (37) and (38). ∎

Proposition 13: For each $\alpha \in \mathcal{K}$ and $\gamma \in \mathcal{K}^2$, let $\gamma_1(s,t) := \alpha(\gamma(s,t))$, $\gamma_2(s,t) := \gamma(\alpha(s),t)$, and $\gamma_3(s,t) := \gamma(s,\alpha(t))$ for each $s,t \geq 0$. Then $\gamma_1, \gamma_2, \gamma_3 \in \mathcal{K}^2$.

Proof: These facts follow directly from the closure of $\mathcal{K}$ under composition [40]. For example, for each $s \geq 0$, we have $\gamma_2(\cdot,s) = \gamma(\alpha(\cdot),s) \in \mathcal{K}$ by closure under composition, $\gamma_2(s,\cdot) = \gamma(\alpha(s),\cdot) \in \mathcal{K}$ trivially, and γ_2 is continuous as it is a composition of continuous functions. ∎

A. Proof of Proposition 2

Let $\alpha \in \mathcal{K}_\infty$, $\gamma \in \mathcal{K}^2$, and $\tau > 0$. Define $\tilde{\gamma}(s,t) := \sup_{\tilde{s}\in(0,s)} \gamma(\tilde{s},t)/\alpha(\tilde{s})$ for each $s,t > 0$, so that $L := \limsup_{s\to 0^+} \gamma(s,\tau)/\alpha(s) = \lim_{s\to 0^+} \tilde{\gamma}(s,\tau)$. Suppose the hypothesis holds, i.e., $L < 1$. Then there exists $\rho_0 > 0$ such that $|\tilde{\gamma}(s,\tau) - L| < 1 - L$ for all $s \in (0,\rho_0]$. But $\tilde{\gamma}(s,t) \geq 0$ and $L \geq 0$ for all $s,t > 0$, so $\tilde{\gamma}(s,\tau) < 1$ for all $s \in (0,\rho_0]$ by the reverse triangle inequality. Therefore

$$\frac{\gamma(s,t)}{\alpha(s)} \leq \frac{\gamma(s,\tau)}{\alpha(s)} \leq \tilde{\gamma}(s,\tau) < 1$$

and $\gamma(s,t) < \alpha(s)$ for all $s \in (0,\rho_0]$ and $t \in [0,\tau]$.

Fix $\rho > 0$. If $\rho \leq \rho_0$, the proof is complete with $\delta := \tau$. Otherwise, we must enlarge the interval in s by shrinking the interval in t. For each $t \in (0,\tau]$, let

$$\gamma_0(t) := \inf\{\, s > 0 \mid \gamma(s,t) \geq \alpha(s)\,\}.$$

Since $\gamma(s,t) < \alpha(s)$ for each $s \in (0,\rho_0]$ and $t \in [0,\tau]$, we have $\gamma_0(t) > 0$ for all $t \in (0,\tau]$. Then, by continuity of α and γ, the first nonzero point at which α and γ intersect, if it exists, must be equal to $\gamma_0(t)$. Otherwise, $\gamma_0(t)$ is infinite. Note that γ_0 is a strictly decreasing function on $(0,\tau]$ since, for any $t \in (0,\tau]$, we have $\gamma(\gamma_0(t),t') < \gamma(\gamma_0(t),t) = \alpha(\gamma_0(t))$ for all $t' \in (0,t)$. Moreover, $\lim_{t\to 0^+} \gamma_0(t) = \infty$ since, if γ_0 was upper bounded by some $\overline{\gamma} > 0$, we could take $\gamma(\overline{\gamma},t) \geq \alpha(\overline{\gamma}) > 0$ for all $t \in (0,\tau]$, a contradiction of the fact that $\gamma(s,\cdot) \in \mathcal{K}$ for all $s > 0$. Then there must exist $\delta \in (0,\tau]$ such that $\gamma_0(\delta) > \rho$, which implies $\gamma(s,t) < \alpha(s)$ for all $s \in (0,\rho]$ and $t \in [0,\delta]$. ∎

B. Proof of Proposition 3

Suppose Assumptions 1, 2, 6, and 7 hold, let $\mathcal{S} \subseteq \mathbb{R}^n$ be compact, and define $z := (x,u)$. By Proposition 11, for each $i \in \mathbb{I}_{1:n}$, there exists $\sigma_i \in \mathcal{K}_\infty$ such that

$$|\partial_z f_i(z,\theta) - \partial_z \hat{f}_i(\tilde{z})| \leq \sigma_i(|(z-\tilde{z},\theta)|) \tag{39}$$

for all $z, \tilde{z} \in \mathcal{S}\times\mathbb{U}$ and $\theta \in \mathbb{R}^{n_\theta}$. Next, let $\mathcal{Z}$ denote the convex hull of $\mathcal{S}\times\mathbb{U}$. Then $tz \in \mathcal{Z}$ for all $t \in [0,1]$ and $z \in \mathcal{Z}$. By Taylor's theorem [41, Thm. 12.14], for each $i \in \mathbb{I}_{1:n}$ and $(z,\theta) \in \mathcal{Z}\times\Theta$, there exists $t_i(z,\theta) \in (0,1)$ such that

$$f_i(z,\theta) - \hat{f}_i(z) = [\partial_z f_i(t_i(z,\theta)z,\theta) - \partial_z \hat{f}_i(t_i(z,\theta)z)]z. \tag{40}$$

Combining (39) and (40) gives, for each $(z,\theta) \in \mathcal{S}\times\mathbb{U}\times\mathbb{R}^{n_\theta}$,

$$|f(z,\theta) - \hat{f}(z)| \leq \sum_{i=1}^n |f_i(z,\theta) - \hat{f}_i(z)| \leq \sum_{i=1}^n \sigma_i(|\theta|)|z|$$

and therefore (27) holds with $\sigma_f := \sum_{i=1}^n \sigma_i$. ∎

C. Proof of Proposition 4

Suppose Assumptions 1 to 3 and 5 to 7 hold. Let $\mathcal{S} \subseteq \mathcal{X}_N$ be compact. By Proposition 12, there exists $c_u > 0$ such that $|\kappa_N(x)| = |u^0(0;x)| \leq c_u|x|$, and therefore $|(x,\kappa_N(x))| \leq |x| + |\kappa_N(x)| \leq (1+c_u)|x|$, for all $x \in \mathcal{S}$. Moreover, by Proposition 3, there exists $\sigma_f \in \mathcal{K}_\infty$ such that

$$|f_c(x,\theta) - \hat{f}_c(x)| \leq \sigma_f(|\theta|)|(x,\kappa_N(x))| \leq \tilde{\sigma}_f(|\theta|)|x|$$

for all $x \in \mathcal{S}$ and $\theta \in \mathbb{R}^{n_\theta}$, where $\tilde{\sigma}_f := \sigma_f(1+c_u)$. ∎

D. Proof of Proposition 5

Suppose Assumptions 1, 2, and 6 hold. Let $\mathcal{S} \subseteq \mathbb{R}^n$ and $\Theta \subseteq \mathbb{R}^{n_\theta}$ be compact. Without loss of generality, assume $\mathcal{S}$ and Θ contain the origin. Then $C := \mathcal{S}\times\mathbb{U}\times\Theta$ is compact, and by Proposition 11, there exists $\sigma_f \in \mathcal{K}_\infty$ such that

$$|f(x,u,\theta) - f(\tilde{x},\tilde{u},\tilde{\theta})| \leq \sigma_f(|(x,u,\theta) - (\tilde{x},\tilde{u},\tilde{\theta})|) \tag{41}$$

for all $(x,u,\theta), (\tilde{x},\tilde{u},\tilde{\theta}) \in C$. Specializing (41) to $(\tilde{x},\tilde{u},\tilde{\theta}) = (x,u,0) \in C$ gives

$$|f(x,u,\theta) - \hat{f}(x,u)| \leq \sigma_f(|\theta|) \tag{42}$$

for all $(x,u,\theta) \in C$. On the other hand, specializing (41) to $(\tilde{x},\tilde{u},\tilde{\theta}) = (0,0,\theta) \in C$ gives

$$|f(x,u,\theta)| = |f(x,u,\theta) - f(0,0,\theta)| \leq \sigma_f(|(x,u)|)$$

and therefore

$$|f(x,u,\theta) - \hat{f}(x,u)| \leq |f(x,u,\theta)| + |\hat{f}(x,u)| \leq 2\sigma_f(|(x,u)|) \tag{43}$$

for all $(x,u,\theta) \in C$. Combining (42) and (43) gives

$$|f(x,u,\theta) - \hat{f}(x,u)| \leq \min\{2\sigma_f(|(x,u)|), \sigma_f(|\theta|)\}$$

for all $(x,u,\theta) \in C$, which is an upper bound that is clearly continuous, nondecreasing in each $|x|$ and $|\theta|$, and zero if either $|x|$ or $|\theta|$ is zero. To make the upper bound strictly increasing, pick any $\sigma_1, \sigma_2 \in \mathcal{K}$ and let $\gamma_f(s,t) := \min\{2\sigma_f(s), \sigma_f(t)\} + \sigma_1(s)\sigma_2(t)$ for each $s,t \geq 0$. Then $\gamma_f \in \mathcal{K}^2$ satisfies (29) for all $(x,u,\theta) \in C$. ∎

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E. *Proof of Proposition* 6

Suppose Assumptions 1 to 4 and 6 hold. Let $\mathcal{S} \subseteq \mathcal{X}_N$ and $\Theta \subseteq \mathbb{R}^{n_\theta}$ be compact. Using the bounds (9) and (20a) with $u = \kappa_N(x)$, we have, for each $x \in \mathcal{X}_N$,

$$\alpha_1(|\kappa_N(x)|) \leq \ell(x, \kappa_N(x)) \leq V_N^0(x) \leq \alpha_2(|x|).$$

Thus, $|(x, \kappa_N(x))| \leq |x| + |\kappa_N(x)| \leq \alpha(|x|)$ for all $x \in \mathcal{X}_N$, where $\alpha(\cdot) := (\cdot) + \alpha_1^{-1}(\alpha_2(\cdot)) \in \mathcal{K}_\infty$. By Proposition 5, there exists $\gamma_f \in \mathcal{K}^2$ such that, for all $x \in \mathcal{S}$ and $\theta \in \Theta$,

$$\begin{aligned}|f_c(x,\theta) - \hat{f}_c(x)| &\leq \gamma_f(|(x, \kappa_N(x))|, |\theta|) \\ &\leq \gamma_f(\alpha(|x|), |\theta|) =: \tilde{\gamma}_f(|x|, |\theta|)\end{aligned}$$

where $\tilde{\gamma}_f \in \mathcal{K}^2$ by Proposition 13. ■

F. *Proof of Proposition* 7

Suppose Assumptions 1 to 3 and 5 to 7 hold and let $\mathcal{S} \subseteq \mathcal{X}_N$ be compact. Throughout, we fix $x \in \mathcal{S}$ and $\theta \in \Theta$, making any constructions independently of x and θ. For brevity, let $\hat{x}^+ := \hat{f}_c(x)$, $x^+ := f_c(x,\theta)$, $\hat{x}^+(k) := \hat{\phi}(k; \hat{x}^+, \tilde{\mathbf{u}}(x))$, and $x^+(k) := \hat{\phi}(k; x^+, \tilde{\mathbf{u}}(x))$. First, we can write

$$\begin{aligned}e_{V_N}^+ &:= V_N(x^+, \tilde{\mathbf{u}}(x)) - V_N(\hat{x}^+, \tilde{\mathbf{u}}(x)) \\ &= e_{V_f}^+ + \sum_{k=0}^{N-1} 2[e_x^+(k)]^\top Q \hat{x}^+(k) + |e_x^+(k)|_Q^2 \qquad (44)\end{aligned}$$

$$\begin{aligned}e_{V_f}^+ &:= V_f(x^+(N)) - V_f(\hat{x}^+(N)) \\ &= 2[e_x^+(N)]^\top P_f \hat{x}^+(N) + |e_x^+(N)|_{P_f}^2 \qquad (45)\end{aligned}$$

where $e_x^+(k) := x^+(k) - \hat{x}^+(k)$.

Next, we establish bounds on the individual terms in (44) and (45). By Proposition 12, there exists $c_x > 0$ such that, for each $k \in \mathbb{I}_{0:N-1}$, we have

$$|\hat{x}^+(k)| = |\hat{x}^0(k+1; x)| \leq c_x |x|. \qquad (46)$$

By Assumptions 3 and 5, whenever $x \in \mathbb{X}_f$ we have

$$\begin{aligned}\underline{\sigma}(P_f)|\hat{f}(x, \kappa_f(x))|^2 &\leq V_f(\hat{f}(x, \kappa_f(x))) \\ &\leq V_f(x) - \underline{\sigma}(Q)|x|^2 \leq [\overline{\sigma}(P_f) - \underline{\sigma}(Q)]|x|^2\end{aligned}$$

and therefore $|\hat{f}(x, \kappa_f(x))| \leq \gamma_f |x|$ where $\gamma_f := \sqrt{[\overline{\sigma}(P_f) - \underline{\sigma}(Q)]/\underline{\sigma}(P_f)}$. Then $\hat{x}^0(N; x) \in \mathbb{X}_f$ gives

$$\begin{aligned}|\hat{x}^+(N)| &= |\hat{f}(\hat{x}^0(N; x), \kappa_f(\hat{x}^0(N; x)))| \\ &\leq \gamma_f |\hat{x}^0(N; x)| \leq \gamma_f c_x |x|. \qquad (47)\end{aligned}$$

Since $(\mathcal{S}, \mathbb{U}, \Theta)$ are each compact and f is continuous, $\mathcal{S}_0 := f(\mathcal{S}, \mathbb{U}, \Theta)$ and $\mathcal{S}_{k+1} := \hat{f}(\mathcal{S}_k, \mathbb{U})$ are compact for all $k \in \mathbb{I}_{\geq 0}$ (by induction). Then $\overline{\mathcal{S}} := \bigcup_{k=0}^N \mathcal{S}_k$ is compact, and, since $\hat{f}$ is Lipschitz continuous on bounded sets, there exists $L_f > 0$ such that $|\hat{f}(x_1, u_1) - \hat{f}(x_2, u_2)| \leq L_f |(x_1, u_1) - (x_2, u_2)|$ for all $x_1, x_2 \in \overline{\mathcal{S}}$ and $u_1, u_2 \in \mathbb{U}$. Then $|e_x^+(k+1)| \leq L_f |e_x^+(k)|$ for each $k \in \mathbb{I}_{0:N-1}$, and, for each $k \in \mathbb{I}_{0:N}$, we have

$$|e_x^+(k)| \leq L_f^k |x^+ - \hat{x}^+|. \qquad (48)$$

Combining (45), (47), and (48) gives

$$|e_{V_f}^+| \leq c_{a,1}|x||x^+ - \hat{x}^+| + c_{a,2}|x^+ - \hat{x}^+|^2. \qquad (49)$$

with $c_{a,1} := 2L_f^N \gamma_f c_x \overline{\sigma}(P_f)$ and $c_{a,2} := L_f^{2N} \overline{\sigma}(P_f)$. Combining (44), (46), (48), and (49) gives

$$|e_{V_N}^+| \leq c_{b,1}|x||x^+ - \hat{x}^+| + c_{b,2}|x^+ - \hat{x}^+|^2 \qquad (50)$$

with $c_{b,1} := c_{a,1} + 2\overline{\sigma}(Q)\sum_{k=0}^{N-1} L_f^k c_x$ and $c_{b,2} := c_{a,2} + \overline{\sigma}(Q)\sum_{k=0}^{N-1} L_f^{2k}$. By Proposition 3, there exists $\tilde{\sigma}_f \in \mathcal{K}_\infty$ such that $|x^+ - \hat{x}^+| \leq \tilde{\sigma}_f(|\theta|)|x|$, and combining this inequality with (50), we have (31) with $\sigma_V := c_{b,1}\tilde{\sigma}_f + c_{b,2}\tilde{\sigma}_f^2$. ■

G. *Proof of Proposition* 8

Suppose Assumptions 1 to 4 and 6 hold. Let $\mathcal{S} \subseteq \mathcal{X}_N$ and $\Theta \subseteq \mathbb{R}^{n_\theta}$ be compact. By Proposition 11, there exists $\alpha_b \in \mathcal{K}_\infty$ such that

$$V_N(x_1, \mathbf{u}_1) - V_N(x_2, \mathbf{u}_2) \leq \alpha_b(|(x_1, \mathbf{u}_1) - (x_2, \mathbf{u}_2)|) \quad (51)$$

for all $(x_1, \mathbf{u}_1), (x_2, \mathbf{u}_2) \in f(\mathcal{S}, \mathbb{U}, \Theta) \times \mathbb{U}^N$. Specializing (51) to $x_1 = x^+ := f_c(x, \theta)$, $x_2 = \hat{x}^+ := f_c(x)$, and $\mathbf{u}_1 = \mathbf{u}_2 = \tilde{\mathbf{u}}(x)$ gives

$$|V_N(x^+, \tilde{\mathbf{u}}(x)) - V_N(\hat{x}^+, \tilde{\mathbf{u}}(x))| \leq \alpha_b(|x^+ - \hat{x}^+|) \qquad (52)$$

for each $x \in \mathcal{S}$ and $\theta \in \Theta$. By Proposition 6 there exists $\tilde{\gamma}_f \in \mathcal{K}^2$ satisfying (30) for all $x \in \mathcal{S}$ and $\theta \in \Theta$. Finally, combining (30) and (52) gives (32) with $\gamma_V(s,t) := \alpha_b(\tilde{\gamma}_f(s,t))$ for all $s, t \geq 0$, where $\gamma_V \in \mathcal{K}^2$ by Proposition 13. ■

H. *Strong asymptotic stability counterexample*

Consider the plant (33) and MPC defined in Section VI-A. We aim to show the closed-loop system $x^+ = f(x, \kappa_1(x), \theta), |\theta| \leq \delta$ is RES with $\delta = 3$, but not inherently strongly stabilizing for any $\delta > 0$. By Lipschitz continuity of x^2 on bounded sets and 1/2-Hölder continuity of $\sqrt{|x|}$,

$$|x^2 - y^2| \leq 4|x - y|, \qquad \forall\, x, y \in [-2, 2], \qquad (53)$$

$$|\sigma(x) - \sigma(y)| \leq 2\sqrt{|x - y|}, \qquad \forall\, x, y \in \mathbb{R}. \qquad (54)$$

First, we derive the control law. The terminal set can be reached in a single move if and only if $|x| \leq 2$, so we have the steerable set $\mathcal{X}_1 = [-2, 2]$. Consider the problem *without* the terminal constraint. The objective is

$$V_1(x, u) = x^2 + u^2 + 4|x + u|$$

which is increasing in u if $x > 1$ and $|u| \leq 1$, and decreasing in u if $x < -1$ and $|u| \leq 1$. Thus $V_1(x, \cdot)$ is minimized (over $|u| \leq 1$) by $\mathbf{u}^0(x) = -\mathrm{sgn}(x)$ for all $x \notin [-1, 1]$. On the other hand, if $|x| \leq 1$, then $V_1(x, \cdot)$ is decreasing on $[-1, -x)$ and increasing on $(-x, 1]$. Thus $V_1(x, \cdot)$ is minimized (over $|u| \leq 1$) by $\mathbf{u}^0(x) = -x$ so long as $|x| \leq 1$. In summary, we have the control law $\kappa_1(x) := -\mathrm{sat}(x)$. But

$$|\hat{f}(x, \kappa_1(x))| = \begin{cases} 0, & |x| \leq 1 \\ |x - \mathrm{sgn}(x)| = |x| - 1, & 0 < |x| \leq 2 \end{cases}$$

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so $u = \kappa_1(x)$ drives each state in $\mathcal{X}_1 = [-2, 2]$ to the terminal constraint $\mathbb{X}_f = [-1, 1]$. Therefore κ_1 is also the control law of the problem *with* the terminal constraint.

It is easy to check that Assumptions 1 to 4 hold, so by Theorem 3, the closed-loop system $x^+ = f(x, \kappa_1(x), \theta), |\theta| \leq \delta$ is RAS on $\mathcal{X}_1 = [-2, 2]$ with ISS Lyapunov function V_1^0 for some $\delta > 0$. Our next goal is to find such a $\delta > 0$.

For robust positive invariance, let $|x| \leq 2$, $\theta \in \mathbb{R}$, $x^+ := f(x, \kappa_1(x), \theta)$, $\hat{x}^+ := \hat{f}(x, \kappa_1(x))$ and note that

$$|x^+| \leq \sqrt{|\hat{x}^+|^2 + |\theta||\text{sat}(x)|} \leq \sqrt{1 + |\theta|}.$$

Then $|x^+| \leq 2$ so long as $|\delta| \leq 3$, so $\mathcal{X}_1 = [-2, 2]$ is RPI for $x^+ = f(x, \kappa_1(x), \theta), |\theta| \leq 3$.

By continuity of f, V_1^0, and κ_1 and Proposition 11, there exists $\sigma \in \mathcal{K}_\infty$ such that $|V_1^0(x^+) - V_1^0(\hat{x}^+)| \leq \sigma(|\theta|)$ and therefore $V_1^0(x^+) \leq V_1^0(\hat{x}^+) + |V_1^0(x^+) - V_1^0(\hat{x}^+)| \leq V_1^0(x) - x^2 + \sigma(|\theta|)$ for all $|x| \leq 2$ and $|\theta| \leq 3$, where $x^+ := f(x, \kappa_1(x), \theta)$ and $\hat{x}^+ := \hat{f}(x, \kappa_1(x))$. Therefore $x^+ = f(x, \kappa_1(x), \theta), |\theta| \leq 3$ is not only RAS, but *RES* on $\mathcal{X}_1$ by Theorem 1.

We now aim to show strong stability is *not* achieved. For simplicity, we consider $\mathcal{S} := \text{lev}_2 V_1^0 = [-1, 1] = \mathbb{X}_f$ as the candidate basin of attraction. Let $|x| \leq 1$, $|\theta| \leq 3$, $x^+ := f(x, \kappa_1(x), \theta)$, and $\hat{x}^+ := \hat{f}(x, \kappa_1(x))$. Moreover, $\ell(x, \kappa_1(x)) \geq 2|x|^2 =: \alpha_3(|x|)$. Next, we have $\kappa_1(x) = -x$, $x^+ = \sigma(x\theta)$, and $\hat{x}^+ = 0$. Therefore

$$\begin{aligned}
&|V_1(x^+, \tilde{\mathbf{u}}(x)) - V_1(\hat{x}^+, \tilde{\mathbf{u}}(x))| \\
&\quad = |(x^+)^2 + 4|x^+|| \leq |x^+|^2 + 4|x^+| \\
&\quad \leq |x||\theta| + 4\sqrt{|x||\theta|} =: \gamma_V(|x|, |\theta|)
\end{aligned}$$

where $\gamma_V \in \mathcal{K}^2$. For each $t > 0$, we have $\gamma_V(s, t)/\alpha_3(s) = t/(2s) + 2\sqrt{t}/s^{3/2}$, so $\lim_{s \to 0^+} \gamma_V(s, t)/\alpha_3(s) = \infty$ for all $t > 0$, and (24) is not satisfied.

As mentioned in the main text, (24) is sufficient but not necessary. The cost difference curve is positive definite, as

$$\Delta V_1^0(x, \theta) = 2[\sigma(\theta x)]^2 - 2x^2 = 2(|\theta| - |x|)|x| > 0$$

for any $0 < |x| < |\theta| \leq 1$. In other words, θ can be arbitrarily small but nonzero, and the cost difference curve will remain positive definite near the origin.

I. Upright pendulum

Consider the plant (35) and MPC defined in Section VI-B. It is noted in the main text that Assumptions 1, 2, and 5 to 7 are automatically satisfied. To design P_f and show Assumption 3 holds, consider the linearization

$$x^+ = \underbrace{\begin{bmatrix} 1 & 0.1 \\ 0.1 & 1 \end{bmatrix}}_{=:A} x + \underbrace{\begin{bmatrix} 0 \\ 5 \end{bmatrix}}_{=:B} u \tag{55}$$

and the feedback gain $K := \begin{bmatrix} 2 & 2 \end{bmatrix}$, which stabilizes (55) because $A_K := A - BK = \left[\begin{smallmatrix} 1 & 0.1 \\ -0.9 & 0 \end{smallmatrix}\right]$ has eigenvalues of 0.9 and 0.1. Numerically solving the Lyapunov equation

$$A_K^\top P_f A_K - P_f = -2Q_K$$

where $Q_K := Q + K^\top RK = \left[\begin{smallmatrix} 5 & 4 \\ 4 & 5 \end{smallmatrix}\right]$, we have a unique positive definite solution $P_f := \left[\begin{smallmatrix} 31.133... & 10.196... \\ 10.196... & 10.311... \end{smallmatrix}\right]$. Using the inequality $|\sin x_1 - x_1| \leq (1/6)|x_1|^3$ for all $x_1 \in \mathbb{R}$, we have

$$\begin{aligned}
&|V_f(\hat{f}(x, -Kx)) - V_f(A_K x)| \\
&\quad = 2x^\top A_K^\top P_f \begin{bmatrix} 0 \\ \Delta(\sin x_1 - x_1) \end{bmatrix} + [P_f]_{22}\Delta^2(\sin x_1 - x_1)^2 \\
&\quad \leq b|x|^4 + a|x|^6
\end{aligned}$$

for all $x \in \mathbb{R}^2$, where $a := [P_f]_{22}\Delta^2/36 = 2.8643\ldots \times 10^{-3}$ and $b := \Delta|A_K^\top P_f \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]|/3 = 0.045675\ldots$. Moreover, $\underline{\sigma}(Q_K) = 1$, so

$$\begin{aligned}
&V_f(\hat{f}(x, -Kx)) - V_f(x) + \ell(x, -Kx) \\
&\quad \leq -[1 - b|x|^2 - a|x|^4]|x|^2
\end{aligned}$$

for all $x \in \mathbb{R}^2$. The polynomial inside the brackets has roots at $x_* = -1.0231\ldots$ and $x^* = 0.9774\ldots$ and is positive in between. Recall $c_f := \underline{\sigma}(P_f)/8$. Then $\underline{\sigma}(P_f)|x|^2 \leq V_f(x) \leq c_f = \underline{\sigma}(P_f)/8$ implies $|x| \leq 1/(2\sqrt{2}) < x^*$ and $|u| = |Kx| = 2(|x_1| + |x_2|) \leq 2\sqrt{2}|x| \leq 1$, so Assumption 3 is satisfied with $\kappa_f(x) := -Kx = -2x_1 - 2x_2$, and P_f and $\mathbb{X}_f$ as defined.

References

[1] S. J. Qin and T. A. Badgwell, "A survey of industrial model predictive control technology," *Control Eng. Pract.*, vol. 11, no. 7, pp. 733–764, 2003.
[2] M. L. Darby and M. Nikolaou, "MPC: Current practice and challenges," *Control Eng. Pract.*, vol. 20, no. 4, pp. 328 – 342, 2012.
[3] S. J. Kuntz and J. B. Rawlings, "Maximum likelihood estimation of linear disturbance models for offset-free model predictive control," in *American Control Conference*, Atlanta, GA, June 8–10, 2022, pp. 3961–3966.
[4] ——, "Maximum likelihood identification of uncontrollable linear time-invariant models for offset-free control," *arXiv preprint arXiv:2406.03760*, 2024.
[5] L. Simpson, J. Asprion, S. Muntwiler, J. Köhler, and M. Diehl, "Parallelizable Parametric Nonlinear System Identification via tuning of a Moving Horizon State Estimator," 2024, arXiv:2403.17858 [math].
[6] K. Baumgärtner, R. Reiter, and M. Diehl, "Moving Horizon Estimation with Adaptive Regularization for Ill-Posed State and Parameter Estimation Problems," in *2022 IEEE 61st Conference on Decision and Control (CDC)*, 2022, pp. 2165–2171.
[7] S. Muntwiler, J. Köhler, and M. N. Zeilinger, "MHE under parametric uncertainty – Robust state estimation without informative data," 2023, arXiv:2312.14049 [cs, eess]. [Online]. Available: http://arxiv.org/abs/2312.14049
[8] J. D. Schiller and M. A. Müller, "A moving horizon state and parameter estimation scheme with guaranteed robust convergence," *IFAC–P. Online*, vol. 56, no. 2, pp. 6759–6764, 2023.
[9] M. Yin, A. Iannelli, and R. S. Smith, "Maximum Likelihood Estimation in Data-Driven Modeling and Control," *IEEE Trans. Auto. Cont.*, vol. 68, no. 1, pp. 317–328, 2023.
[10] F. Dorfler, J. Coulson, and I. Markovsky, "Bridging Direct & Indirect Data-Driven Control Formulations via Regularizations and Relaxations," *IEEE Trans. Auto. Cont.*, vol. 68, no. 2, pp. 883–897, 2022.
[11] J. Berberich, J. Köhler, M. A. Müller, and F. Allgöwer, "Data-Driven Model Predictive Control With Stability and Robustness Guarantees," *IEEE Trans. Auto. Cont.*, vol. 66, no. 4, pp. 1702–1717, 2021.
[12] ——, "Linear Tracking MPC for Nonlinear Systems—Part II: The Data-Driven Case," *IEEE Trans. Auto. Cont.*, vol. 67, no. 9, pp. 4406–4421, 2022.
[13] Z.-P. Jiang and Y. Wang, "Input-to-state stability for discrete-time nonlinear systems," *Automatica*, vol. 37, pp. 857–869, 2001.
[14] C. M. Kellett and A. R. Teel, "On the robustness of $\mathcal{KL}$-stability for difference inclusions: Smooth discrete-time Lyapunov functions," *SIAM J. Cont. Opt.*, vol. 44, no. 3, pp. 777–800, 2005.

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[15] F. Clarke, Y. Ledyaev, and R. Stern, “Asymptotic Stability and Smooth Lyapunov Functions,” *J. Diff. Eq.*, vol. 149, no. 1, pp. 69–114, 1998.
[16] Z.-P. Jiang and Y. Wang, “A converse Lyapunov theorem for discrete-time systems with disturbances,” *Sys. Cont. Let.*, vol. 45, pp. 49–58, 2002.
[17] J. B. Rawlings, D. Q. Mayne, and M. M. Diehl, *Model Predictive Control: Theory, Computation, and Design*, 2nd ed. Santa Barbara, CA: Nob Hill Publishing, 2020, 770 pages, ISBN 978-0-9759377-5-4.
[18] G. De Nicolao, L. Magni, and R. Scattolini, “On the robustness of receding-horizon control with terminal constraints,” *IEEE Trans. Auto. Cont.*, vol. 41, no. 3, pp. 451–453, Mar 1996.
[19] P. O. M. Scokaert, J. B. Rawlings, and E. S. Meadows, “Discrete-time stability with perturbations: Application to model predictive control,” *Automatica*, vol. 33, no. 3, pp. 463–470, 1997.
[20] G. Grimm, M. J. Messina, S. E. Tuna, and A. R. Teel, “Examples when nonlinear model predictive control is nonrobust,” *Automatica*, vol. 40, pp. 1729–1738, 2004.
[21] ——, “Nominally robust model predictive control with state constraints,” *IEEE Trans. Auto. Cont.*, vol. 52, no. 10, pp. 1856–1870, Oct 2007.
[22] M. Lazar and W. P. M. H. Heemels, “Predictive control of hybrid systems: Input-to-state stability results for sub-optimal solutions,” *Automatica*, vol. 45, no. 1, pp. 180–185, 2009.
[23] G. Pannocchia, J. B. Rawlings, and S. J. Wright, “Conditions under which suboptimal nonlinear MPC is inherently robust,” *Sys. Cont. Let.*, vol. 60, pp. 747–755, 2011.
[24] D. A. Allan, C. N. Bates, M. J. Risbeck, and J. B. Rawlings, “On the inherent robustness of optimal and suboptimal nonlinear MPC,” *Sys. Cont. Let.*, vol. 106, pp. 68 – 78, 2017.
[25] R. D. McAllister and J. B. Rawlings, “Inherent stochastic robustness of model predictive control to large and infrequent disturbances,” *IEEE Trans. Auto. Cont.*, vol. 67, no. 10, pp. 5166–5178, 2022.
[26] ——, “The stochastic robustness of nominal and stochastic model predictive control,” *IEEE Trans. Auto. Cont.*, 2022, online Early Access.
[27] ——, “On the inherent distributional robustness of stochastic and nominal model predictive control,” *IEEE Trans. Auto. Cont.*, vol. 69, no. 2, pp. 741–754, 2024.
[28] J. C. Doyle, “Guaranteed margins for LQG regulators,” *IEEE Trans. Auto. Cont.*, vol. 23, pp. 756–757, 1978.
[29] N. Lehtomaki, N. Sandell, and M. Athans, “Robustness results in linear-quadratic Gaussian based multivariable control designs,” *IEEE Trans. Auto. Cont.*, vol. 26, no. 1, pp. 75–93, 1981.
[30] C. Zhang and M. Fu, “A revisit to the gain and phase margins of linear quadratic regulators,” *IEEE Trans. Auto. Cont.*, vol. 41, no. 10, pp. 1527–1530, 1996.
[31] L. O. Santos and L. T. Biegler, “A tool to analyze robust stability for model predictive control,” *J. Proc. Cont.*, vol. 9, pp. 233–245, 1999.
[32] L. O. Santos, L. T. Biegler, and J. A. A. M. Castro, “A tool to analyze robust stability for constrained nonlinear MPC,” *J. Proc. Cont.*, vol. 18, no. 3, pp. 383–390, 2008.
[33] S. J. Kuntz and J. B. Rawlings, “Offset-free model predictive control: stability under plant-model mismatch,” 2024, arXiv:2412 [eecs, math]. [Online]. Available: https://arxiv.org/abs/2412.08104
[34] ——, “Beyond inherent robustness: asymptotic stability of MPC despite plant-model mismatch,” 2024, arXiv:2411.15452 [eecs, math]. [Online]. Available: https://arxiv.org/abs/2411.15452
[35] R. T. Rockafellar and R. J.-B. Wets, *Variational Analysis*. Springer-Verlag, 1998.
[36] L. Grüne, E. D. Sontag, and F. R. Wirth, “Asymptotic stability equals exponential stability, and ISS equals finite energy gain — if you twist your eyes,” *Sys. Cont. Let.*, vol. 38, no. 2, pp. 127–134, 1999.
[37] D. P. Bertsekas and S. E. Shreve, *Stochastic Optimal Control: The Discrete Time Case*. New York: Academic Press, 1978.
[38] J. Hale, *Ordinary Differential Equations*, 2nd ed. Robert E. Krieger Publishing Company, 1980.
[39] G. Pannocchia, M. Gabiccini, and A. Artoni, “Offset-free MPC explained: novelties, subtleties, and applications,” *IFAC–P. Online*, vol. 48, no. 23, pp. 342–351, 2015.
[40] C. M. Kellett, “A compendium of comparison function results,” *Math. Contr. Sign. Syst.*, vol. 26, no. 3, pp. 339–374, 2014.
[41] T. M. Apostol, *Mathematical analysis*. Addison-Wesley, 1974.

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