CRB Optimization for Joint Array Partitioning and Beamforming Design in ISAC Systems

Rang Liu[†], A. Lee Swindlehurst[†], and Ming Li[‡]
[†]University of California Irvine, CA, USA
[‡]Dalian University of Technology, Dalian, Liaoning, China
E-mail: {rangl2, swindle}@uci.edu; mli@dlut.edu.cn

2024 58th Asilomar Conference on Signals, Systems, and Computers | 979-8-3503-5405-8/24/$31.00 ©2024 IEEE | DOI: 10.1109/IEEECONF60004.2024.10942840

***Abstract*—Integrated sensing and communication has emerged as a transformative technology for future wireless communication networks, enabling the simultaneous realization of radar sensing and communication functions by sharing available resources. To fully exploit the available spatial degrees of freedom in monostatic ISAC systems, we propose a dynamic array partitioning architecture that allows the base station to allocate antennas for transmitting dual-functional signals and receiving the corresponding echoes. Based on this architecture, we jointly design the transmit beamforming and array partitioning to minimize the Cramér-Rao bound (CRB) for target direction-of-arrival estimation, while ensuring compliance with signal-to-interference-plus-noise ratio requirements for multiuser communication, power budget constraints, and array partitioning limitations. To address the resulting optimization problem, we develop an alternating algorithm leveraging alternating direction method of multipliers and semi-definite relaxation. Simulation results demonstrate that the proposed joint array partitioning and beamforming design significantly improves the CRB and the resulting DOA estimation performance.**

***Index Terms*—Integrated sensing and communication (ISAC), array partitioning, Cramér-Rao bound, beamforming design**

I. Introduction

In multi-input multi-output (MIMO) integrated sensing and communication (ISAC) systems, beamforming design plays a pivotal role in effectively utilizing the available spatial degrees of freedom (DoFs) to enhance the trade-off between sensing and communication performance. Beamforming designs under various sensing and communication requirements have been extensively studied in the literature [1]-[3]. However, most existing studies assume fixed transmit and receive antenna arrays, which limits the adaptability of the system to dynamic requirements and environments. This fixed array architecture inherently constrains the utilization of available spatial DoFs, potentially leading to performance deterioration.

To address the limitations of fixed antenna arrays, antenna selection has been proposed as an alternative in MIMO-ISAC systems [4]-[7]. The works in [4]-[6] focused on transmit antenna selection and beamforming that minimize the Cramér-Rao bound (CRB) [4], maximize the weighted sum of the communication rate and the Fisher information matrix [5], or maximize the target directional power [6]. The authors of [7] considered joint antenna selection at the dual-functional transmitter and radar receiver. While these approaches have demonstrated the benefits of dynamic antenna selection, they rely on fixed transmit/receive arrays and only activate subsets of antennas, leaving many available antenna elements unutilized. To fully harness the potential of a shared antenna array at the dual-functional base station (BS), a more flexible array partitioning architecture is essential for MIMO-ISAC systems.

Motivated by these issues, in this work we propose a dynamic array partitioning architecture that allows each individual antenna element to function as either a transmit or a receive antenna. Based on this architecture, we formulate a joint optimization framework for array partitioning and transmit beamforming, aiming to minimize the CRB for target direction-of-arrival (DOA) estimation while satisfying communication signal-to-interference-plus-noise ratio (SINR) requirements, the transmit power budget, and inherent constraints on the array partitioning. In order to solve the resulting complicated problem with fractional quadratic terms and binary integer variables, we employ dedicated transformations and typical algorithmic frameworks to convert it into several tractable subproblems leveraging the Schur complement and alternative direction method of multipliers (ADMM). These subproblems are solved iteratively with the aid of semi-definite relaxation (SDR). Simulation results demonstrate that the proposed array partitioning and beamforming design significantly improves performance, achieving notable reductions in root-CRB (RCRB) and root mean squared error (RMSE) compared to conventional fixed antenna configurations.

II. System Model and Problem Formulation

We study a monostatic ISAC system, where a BS is equipped with an N-element uniform linear array. The dual-functional BS simultaneously supports communication with K single-antenna users and performs sensing of a point-like target. To optimize the utilization of the available antennas, the BS dynamically partitions the array into transmit antennas for dual-functional signal transmission and receive antennas for echo signal collection. The array partitioning vector is denoted as $\mathbf{a} \triangleq [a_1, a_2, \ldots, a_N]^T \in \{0,1\}^N$, where $a_n = 1$ indicates that the n-th antenna operates as a transmit antenna and $a_n = 0$ represents a receive antenna. The transmitted dual-functional signal in the l-th time slot is expressed as

$$\mathbf{x}[l] = \mathbf{A}\mathbf{W}_\mathrm{c}\mathbf{s}_\mathrm{c}[l] + \mathbf{A}\mathbf{W}_\mathrm{r}\mathbf{s}_\mathrm{r}[l] = \mathbf{A}\mathbf{W}\mathbf{s}[l], \quad (1)$$

where $\mathbf{A} \triangleq \mathrm{diag}\{\mathbf{a}\}$, $\mathbf{W} \triangleq [\mathbf{W}_\mathrm{c}\ \mathbf{W}_\mathrm{r}] \in \mathbb{C}^{N\times(N+K)}$ with $\mathbf{W}_\mathrm{c} \in \mathbb{C}^{N\times K}$ and $\mathbf{W}_\mathrm{r} \in \mathbb{C}^{N\times N}$ representing the beamformers

This work was supported by the U.S. National Science Foundation under grants CCF-2225575 and Grant CCF-2322191.

for the communication symbols $\mathbf{s}_c \in \mathbb{C}^K$ and the radar probing signals $\mathbf{s}_r \in \mathbb{C}^N$, respectively. We assume that $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_{N+K}$. The received signal at the k-th user is written as

$$y_k[l] = \mathbf{h}_k^T \mathbf{A}\mathbf{W}\mathbf{s}[l] + n_k[l], \tag{2}$$

where $\mathbf{h}_k \in \mathbb{C}^N$ denotes the channel between the BS and the k-th user and $n_k[l] \sim \mathcal{CN}(0, \sigma_k^2)$ denotes additive white Gaussian noise (AWGN). The SINR of the k-th communication user is calculated as

$$\text{SINR}_k = \frac{|\mathbf{h}_k^T \mathbf{A}\mathbf{w}_k|^2}{\sum_{j\neq k}^{K+N} |\mathbf{h}_k^T \mathbf{A}\mathbf{w}_j|^2 + \sigma_k^2}. \tag{3}$$

The signals received at the BS for radar sensing can be expressed as

$$\mathbf{y}_r[l] = \alpha_t(\mathbf{I}_N - \mathbf{A})\mathbf{h}_t\mathbf{h}_t^T\mathbf{A}\mathbf{W}\mathbf{s}[l] + \mathbf{H}_{SI}\mathbf{W}\mathbf{s}[l+\tau] + (\mathbf{I}_N - \mathbf{A})\mathbf{n}_r[l], \tag{4}$$

where α_t is the radar cross section (RCS) of the target, $\mathbf{h}_t \in \mathbb{C}^N$ denotes the line-of-sight (LoS) channel between the N-antenna BS and the target at direction θ_t, $\mathbf{h}_t \triangleq \beta_t[e^{-\jmath\frac{1-N}{2}\pi\sin\theta_t}, e^{-\jmath\frac{3-N}{2}\pi\sin\theta_t}, \ldots, e^{-\jmath\frac{N-1}{2}\pi\sin\theta_t}]^T$, β_t represents the distance-dependent path loss, $\mathbf{H}_{SI} \in \mathbb{C}^{N\times N}$ denotes the self-interference channel, τ is the round-trip delay for the target echoes, and $\mathbf{n}_r \sim \mathcal{CN}(\mathbf{0}, \sigma_r^2\mathbf{I}_N)$ is AWGN. Leveraging knowledge of $\mathbf{s}[l]$ and using advanced interference cancellation techniques, the self-interference between the transmit and receive antennas can be effectively mitigated and is therefore neglected in the subsequent formulation. Consequently, the radar echoes collected over L time slots are expressed as

$$\mathbf{Y}_r = \alpha_t\mathbf{H}_t\mathbf{W}\mathbf{S} + (\mathbf{I}_N - \mathbf{A})\mathbf{N}_r, \tag{5}$$

where $\mathbf{S} \triangleq [\mathbf{s}[1], \ldots, \mathbf{s}[L]]$, $\mathbf{N}_r \triangleq [\mathbf{n}_r[1], \ldots, \mathbf{n}_r[L]]$, and the equivalent channel for the target echoes is defined as

$$\mathbf{H}_t \triangleq (\mathbf{I}_N - \mathbf{A})\mathbf{h}_t\mathbf{h}_t^T\mathbf{A}. \tag{6}$$

Given the echo signals in (5), the BS attempts to estimate the parameters of the target defined as $\boldsymbol{\xi} \triangleq [\theta,\ \boldsymbol{\alpha}^T]^T$ where $\boldsymbol{\alpha} \triangleq [\Re\{\alpha_t\}, \Im\{\alpha_t\}]^T$. In order to derive the CRB for DOA estimation, we vectorize the received signals $\widetilde{\mathbf{y}}_r = \text{vec}\{\mathbf{Y}_r\}$ as

$$\widetilde{\mathbf{y}}_r = \boldsymbol{\eta} + \mathbf{n} \triangleq \alpha_t\text{vec}\{\mathbf{H}_t\mathbf{W}\mathbf{S}\} + \text{vec}\{(\mathbf{I}_N - \mathbf{A})\widetilde{\mathbf{N}}_r\}, \tag{7}$$

which has the distribution $\widetilde{\mathbf{y}}_r \sim \mathcal{CN}(\boldsymbol{\eta}, \sigma_r^2(\mathbf{I}_N - \mathbf{A}))$. Then, according to [8], the (i,j)-th element of the Fisher information matrix $\mathbf{F} \in \mathbb{R}^{3\times 3}$ for estimating $\boldsymbol{\eta}$ is given by

$$\mathbf{F}_{i,j} = \frac{2}{\sigma_r^2}\Re\Big\{\frac{\partial\boldsymbol{\eta}^H}{\partial\xi_i}\frac{\partial\boldsymbol{\eta}}{\partial\xi_j}\Big\}. \tag{8}$$

To derive $\mathbf{F}$, we respectively calculate the partial derivatives of $\boldsymbol{\eta}$ with respect to the DOA θ_t and the RCS $\boldsymbol{\alpha}$ as

$$\frac{\partial\boldsymbol{\eta}}{\partial\theta} = \alpha_t\text{vec}\{\dot{\mathbf{H}}_t\mathbf{W}\mathbf{S}\}, \tag{9a}$$

$$\frac{\partial\boldsymbol{\eta}}{\partial\boldsymbol{\alpha}} = [1\ \jmath]^T \otimes \text{vec}\{\mathbf{H}_t\mathbf{W}\mathbf{S}\}, \tag{9b}$$

where $\dot{\mathbf{H}}_t$ represents the derivative of $\mathbf{H}_t$ with respect to θ_t. It is noted that both $\mathbf{H}_t$ and $\dot{\mathbf{H}}_t$ are functions of the array partitioning vector $\mathbf{a}$. Then, the Fisher information matrix can be obtained as

$$\mathbf{F} = \begin{bmatrix} F_{\theta\theta} & \mathbf{F}_{\theta\boldsymbol{\alpha}^T} \\ \mathbf{F}_{\theta\boldsymbol{\alpha}^T}^T & \mathbf{F}_{\boldsymbol{\alpha}\boldsymbol{\alpha}^T} \end{bmatrix}, \tag{10a}$$

$$\mathbf{F}_{\theta\theta} = \frac{2L|\alpha_t|^2}{\sigma_r^2}\text{Tr}\{\dot{\mathbf{H}}_t\mathbf{W}\mathbf{W}^H\dot{\mathbf{H}}_t^H\}, \tag{10b}$$

$$\mathbf{F}_{\theta\boldsymbol{\alpha}} = \frac{2L}{\sigma_r^2}\Re\{\alpha_t^*\text{Tr}\{\mathbf{H}_t\mathbf{W}\mathbf{W}^H\dot{\mathbf{H}}_t^H[1\ \jmath]^T\}\}, \tag{10c}$$

$$\mathbf{F}_{\boldsymbol{\alpha}\boldsymbol{\alpha}^T} = \frac{2L}{\sigma_r^2}\text{Tr}\{\mathbf{H}_t\mathbf{W}\mathbf{W}^H\mathbf{H}_t^H\}\mathbf{I}_2. \tag{10d}$$

The CRB matrix is found by inverting $\mathbf{F}$, and the diagonal elements of the CRB provide the lower bounds on the variances of the parameters in $\boldsymbol{\xi}$. Thus, the CRB for DOA estimation can be expressed as

$$\begin{aligned}\text{CRB}_\theta &= [\mathbf{F}^{-1}]_{1,1} = [\mathbf{F}_{\theta\theta} - \mathbf{F}_{\theta\boldsymbol{\alpha}^T}^T\mathbf{F}_{\boldsymbol{\alpha}\boldsymbol{\alpha}^T}^{-1}\mathbf{F}_{\theta\boldsymbol{\alpha}^T}]^{-1} \\ &= \frac{\sigma_r^2/(2L|\alpha_t|^2)}{\text{Tr}\{\dot{\mathbf{H}}_t\mathbf{W}\mathbf{W}^H\dot{\mathbf{H}}_t^H\} - \frac{|\text{Tr}\{\mathbf{H}_t\mathbf{W}\mathbf{W}^H\dot{\mathbf{H}}_t^H\}|^2}{\text{Tr}\{\mathbf{H}_t\mathbf{W}\mathbf{W}^H\mathbf{H}_t^H\}}}.\end{aligned} \tag{11}$$

This paper studies minimization of the CRB for DOA estimation, subject to constraints on communication SINR, transmit power, and array partitioning. The optimization problem for the joint design of the array partitioning $\mathbf{a}$ and the transmit beamforming $\mathbf{W}$ is formulated as

$$\max_{\mathbf{a},\mathbf{W}}\ \text{Tr}\{\dot{\mathbf{H}}_t\mathbf{W}\mathbf{W}^H\dot{\mathbf{H}}_t^H\} - \frac{|\text{Tr}\{\mathbf{H}_t\mathbf{W}\mathbf{W}^H\dot{\mathbf{H}}_t^H\}|^2}{\text{Tr}\{\mathbf{H}_t\mathbf{W}\mathbf{W}^H\mathbf{H}_t^H\}} \tag{12a}$$

$$\text{s.t.}\ \frac{|\mathbf{h}_k^T\mathbf{A}\mathbf{w}_k|^2}{\sum_{j\neq k}^{N+K}|\mathbf{h}_k^T\mathbf{A}\mathbf{w}_j|^2 + \sigma_k^2} \geq \Gamma_k,\ \forall k, \tag{12b}$$

$$\|\mathbf{A}\mathbf{W}\|_F^2 \leq P, \tag{12c}$$

$$K \leq \mathbf{1}^T\mathbf{a} \leq N-1, \tag{12d}$$

$$a_n \in \{0,1\},\ \forall n, \tag{12e}$$

where Γ_k is the communication SINR threshold for the k-th user, P is the transmit power budget, and (12d) is imposed to guarantee K different data streams and the desired sensing capability. This optimization problem is inherently complex and highly non-convex, primarily due to the fractional and quadratic terms in the objective and constraints, as well as the binary integer constraints on the array partitioning. In the next section, we propose an efficient alternating optimization algorithm that decomposes the problem into several tractable sub-problems and solves them iteratively.

III. CRB-oriented Joint Array Partitioning and Beamforming Design

A. Problem Reformulation

Since the array partitioning vector $\mathbf{a}$ is implicitly embedded in the objective function (12a), we first derive an equivalent reformulation of (12a) to explicitly express its dependence on $\mathbf{a}$. According to the definition of $\mathbf{H}_t$ in (6), we can re-write the effective channel $\mathbf{H}_t$ and its derivative $\dot{\mathbf{H}}_t$ as

$$\mathbf{H}_t = \text{diag}\{\mathbf{h}_t\}\mathbf{b}\mathbf{a}^T\text{diag}\{\mathbf{h}_t\}, \tag{13a}$$

$$\begin{aligned}\dot{\mathbf{H}}_t &= (\mathbf{I}_N - \mathbf{A})\dot{\mathbf{h}}_t\mathbf{h}_t^T\mathbf{A} + (\mathbf{I}_N - \mathbf{A})\mathbf{h}_t\dot{\mathbf{h}}_t^T\mathbf{A} \\ &= \text{diag}\{\dot{\mathbf{h}}_t\}\mathbf{b}\mathbf{a}^T\text{diag}\{\mathbf{h}_t\} + \text{diag}\{\mathbf{h}_t\}\mathbf{b}\mathbf{a}^T\text{diag}\{\dot{\mathbf{h}}_t\}\end{aligned} \tag{13b}$$

$$= -\jmath\pi\cos\theta_t\big[\text{diag}\{\mathbf{h}_t\}\mathbf{Q}\mathbf{b}\mathbf{a}^T\text{diag}\{\mathbf{h}_t\} + \text{diag}\{\mathbf{h}_t\}\mathbf{b}\mathbf{a}^T\mathbf{Q}\text{diag}\{\mathbf{h}_t\}\big], \tag{13c}$$

where for notational simplicity we define $\mathbf{b} \triangleq \mathbf{1} - \mathbf{a}$, and $\dot{\mathbf{h}}_t \triangleq \partial\mathbf{h}_t/\partial\theta_t = -\jmath\pi\cos\theta_t\mathbf{Q}\mathbf{h}_t$ with $\mathbf{Q} \triangleq \text{diag}\{\mathbf{q}\}$ and $\mathbf{q} \triangleq [\frac{1-N}{2}, \frac{3-N}{2}, \ldots, \frac{N-1}{2}]^T$. Then, each term in (12a) can be rewritten as

$$\text{Tr}\{\dot{\mathbf{H}}_t\mathbf{W}\mathbf{W}^H\dot{\mathbf{H}}_t^H\} = \beta_t^2\pi^2\cos^2\theta_t(\mathbf{a}^T\mathbf{C}\mathbf{a}\mathbf{b}^T\mathbf{Q}^2\mathbf{b} + \mathbf{a}^T\mathbf{C}\mathbf{Q}\mathbf{a}\mathbf{b}^T\mathbf{Q}\mathbf{b} + \mathbf{a}^T\mathbf{Q}\mathbf{C}\mathbf{a}\mathbf{b}^T\mathbf{Q}\mathbf{b} + \mathbf{a}^T\mathbf{Q}\mathbf{C}\mathbf{Q}\mathbf{a}\mathbf{b}^T\mathbf{b}), \tag{14a}$$

$$|\text{Tr}\{\mathbf{H}_t\mathbf{W}\mathbf{W}^H\dot{\mathbf{H}}_t^H\}|^2 = \beta_t^4\pi^2\cos^2\theta_t\big|\mathbf{a}^T\mathbf{C}\mathbf{a}\mathbf{b}^T\mathbf{Q}\mathbf{b} + \mathbf{a}^T\mathbf{C}\mathbf{Q}\mathbf{a}\mathbf{b}^T\mathbf{b}\big|^2, \tag{14b}$$

$$\text{Tr}\{\mathbf{H}_t\mathbf{W}\mathbf{W}^H\mathbf{H}_t^H\} = \beta_t^2\mathbf{a}^T\mathbf{C}\mathbf{a}\mathbf{b}^T\mathbf{b}, \tag{14c}$$

where we define

$$\mathbf{C} \triangleq \text{diag}\{\mathbf{h}_t\}\mathbf{W}\mathbf{W}^H\text{diag}\{\mathbf{h}_t^H\}. \tag{15}$$

Based on the above expressions, we can equivalently transform the objective function as

$$\text{Tr}\{\dot{\mathbf{H}}_t\mathbf{W}\mathbf{W}^H\dot{\mathbf{H}}_t^H\} - \frac{|\text{Tr}\{\mathbf{H}_t\mathbf{W}\mathbf{W}^H\dot{\mathbf{H}}_t^H\}|^2}{\text{Tr}\{\mathbf{H}_t\mathbf{W}\mathbf{W}^H\mathbf{H}_t^H\}} \tag{16a}$$

$$= \beta_t^2\pi^2\cos^2\theta_t\Big(\mathbf{a}^T\mathbf{C}\mathbf{a}\mathbf{b}^T\mathbf{Q}^2\mathbf{b} + \mathbf{a}^T\mathbf{Q}\mathbf{C}\mathbf{Q}\mathbf{a}\mathbf{b}^T\mathbf{b} - \frac{\mathbf{a}^T\mathbf{C}\mathbf{a}\mathbf{b}^T\mathbf{Q}\mathbf{b}\mathbf{b}^T\mathbf{Q}\mathbf{b}}{\mathbf{b}^T\mathbf{b}} - \frac{\mathbf{b}^T\mathbf{b}\mathbf{a}^T\mathbf{C}\mathbf{Q}\mathbf{a}\mathbf{a}^T\mathbf{Q}\mathbf{C}\mathbf{a}}{\mathbf{a}^T\mathbf{C}\mathbf{a}}\Big). \tag{16b}$$

Without loss of generality, we ignore the constant term and define the new objective for the following algorithm development as a function of $\mathbf{a}$ and $\mathbf{W}$ as $f(\mathbf{a},\mathbf{W}) \triangleq \mathbf{a}^T\mathbf{C}\mathbf{a}\mathbf{b}^T\mathbf{Q}^2\mathbf{b} + \mathbf{a}^T\mathbf{Q}\mathbf{C}\mathbf{Q}\mathbf{a}\mathbf{b}^T\mathbf{b} - \frac{\mathbf{a}^T\mathbf{C}\mathbf{a}\mathbf{b}^T\mathbf{Q}\mathbf{b}\mathbf{b}^T\mathbf{Q}\mathbf{b}}{\mathbf{b}^T\mathbf{b}} - \frac{\mathbf{b}^T\mathbf{b}\mathbf{a}^T\mathbf{C}\mathbf{Q}\mathbf{a}\mathbf{a}^T\mathbf{Q}\mathbf{C}\mathbf{a}}{\mathbf{a}^T\mathbf{C}\mathbf{a}}$.

B. Schur Complement Transformation

To address the fractional terms in $f(\mathbf{a},\mathbf{W})$, we introduce two auxiliary variables t_1 and t_2 as

$$t_1 = \mathbf{b}^T\mathbf{Q}^2\mathbf{b} - \frac{\mathbf{b}^T\mathbf{Q}\mathbf{b}\mathbf{b}^T\mathbf{Q}\mathbf{b}}{\mathbf{b}^T\mathbf{b}}, \tag{17a}$$

$$t_2 = \mathbf{a}^T\mathbf{Q}\mathbf{C}\mathbf{Q}\mathbf{a} - \frac{\mathbf{a}^T\mathbf{C}\mathbf{Q}\mathbf{a}\mathbf{a}^T\mathbf{Q}\mathbf{C}\mathbf{a}}{\mathbf{a}^T\mathbf{C}\mathbf{a}}. \tag{17b}$$

Leveraging the Schur complement, the optimization problem can be converted to

$$\max_{\mathbf{a},\mathbf{W},t_1,t_2} \quad \mathbf{a}^T\mathbf{C}\mathbf{a}t_1 + \mathbf{b}^T\mathbf{b}t_2 \tag{18a}$$

$$\text{s.t.} \quad \begin{bmatrix} \mathbf{b}^T\mathbf{Q}^2\mathbf{b} - t_1 & \mathbf{b}^T\mathbf{Q}\mathbf{b} \\ \mathbf{b}^T\mathbf{Q}\mathbf{b} & \mathbf{b}^T\mathbf{b} \end{bmatrix} \succeq \mathbf{0}, \tag{18b}$$

$$\begin{bmatrix} \mathbf{a}^T\mathbf{Q}\mathbf{C}\mathbf{Q}\mathbf{a} - t_2 & \mathbf{a}^T\mathbf{Q}\mathbf{C}\mathbf{a} \\ \mathbf{a}^T\mathbf{C}\mathbf{Q}\mathbf{a} & \mathbf{a}^T\mathbf{C}\mathbf{a} \end{bmatrix} \succeq \mathbf{0}, \tag{18c}$$

$$(12\text{b}) - (12\text{e}).$$

C. Binary Integer Constraint

Since the binary integer constraint (12e) introduces combinatorial complexity that makes the problem challenging to solve directly, we approximate this constraint by reformulating it as a smooth penalty term in the objective function accompanied by a box constraint:

$$\max_{\mathbf{a},\mathbf{W},t_1,t_2} \quad \mathbf{a}^T\mathbf{C}\mathbf{a}t_1 + \mathbf{b}^T\mathbf{b}t_2 - \rho_1\mathbf{a}^T(\mathbf{1}-\mathbf{a}) \tag{19a}$$

$$\text{s.t.} \quad 0 \le a_n \le 1,\ \forall n, \tag{19b}$$

$$(12\text{b}) - (12\text{d}), (18\text{b}), (18\text{c}),$$

where $\rho_1 > 0$ is a preset penalty parameter that regulates the extent to which the binary integer constraint is enforced.

D. ADMM-Based Transformation

We observe that both the objective (19a) and the constraint (18b) involve quadratic terms with respect to $\mathbf{b}$, where $\mathbf{b} \triangleq \mathbf{1} - \mathbf{a}$. To simplify the problem, it is natural to introduce $\mathbf{b} = \mathbf{1} - \mathbf{a}$ as an axillary variable. Then, by employing the ADMM framework, the corresponding augmented Lagrangian formulation of the problem can be expressed as

$$\max_{\mathbf{a},\mathbf{b},\mathbf{W},t_1,t_2} \quad \mathbf{a}^T\mathbf{C}\mathbf{a}t_1 + \mathbf{b}^T\mathbf{b}t_2 - \rho_1\mathbf{a}^T(\mathbf{1}-\mathbf{a}) - \rho_2\|\mathbf{b} - \mathbf{1} + \mathbf{a} + \boldsymbol{\mu}/\rho_2\|^2 \tag{20a}$$

$$\text{s.t.} \quad 0 \le a_n, b_n \le 1,\ \forall n, \tag{20b}$$

$$1 \le \mathbf{1}^T\mathbf{b} \le N - K, \tag{20c}$$

$$(12\text{b}) - (12\text{d}), (18\text{b}), (18\text{c}),$$

where $\rho_2 > 0$ is a penalty parameter and $\boldsymbol{\mu} \in \mathbb{R}^N$ is the dual variable. Then, we employ the block coordinate ascent method to solve this multivariate optimization problem. The update rules for each variable are presented in detail in the following subsection.

E. Block Update

1) Update $\mathbf{W}$*:* Given solutions for the other variables, the sub-problem for finding $\mathbf{W}$ is formulated as

$$\max_{\mathbf{W},t_2} \quad \mathbf{a}^T\mathbf{C}\mathbf{a}t_1 + \mathbf{b}^T\mathbf{b}t_2 \quad \text{s.t.} \ (12\text{b}),\ (12\text{c}), (18\text{c}), \tag{21}$$

where $\mathbf{C}$ is defined in (15). Considering that both the objective and constraints involve quadratic terms with respect to $\mathbf{W}$, we define $\mathbf{R} \triangleq \mathbf{W}\mathbf{W}^H$ and $\mathbf{R}_k \triangleq \mathbf{w}_k\mathbf{w}_k^H$, $\forall k$. Then using the typical SDR approach, we transform problem (21) as

$$\max_{\mathbf{R},\mathbf{R}_k,\forall k,t_2} \quad \mathbf{a}^T\mathbf{C}\mathbf{a}t_1 + \mathbf{b}^T\mathbf{b}t_2 \tag{22a}$$

$$\text{s.t.} \quad \begin{bmatrix} \mathbf{a}^T\mathbf{Q}\mathbf{C}\mathbf{Q}\mathbf{a} - t_2 & \mathbf{a}^T\mathbf{Q}\mathbf{C}\mathbf{a} \\ \mathbf{a}^T\mathbf{C}\mathbf{Q}\mathbf{a} & \mathbf{a}^T\mathbf{C}\mathbf{a} \end{bmatrix} \succeq \mathbf{0}, \tag{22b}$$

$$(1 + \Gamma_k^{-1})\mathbf{a}^T\text{diag}\{\mathbf{h}_k\}\mathbf{R}_k\text{diag}\{\mathbf{h}_k^H\}\mathbf{a} - \mathbf{a}^T\text{diag}\{\mathbf{h}_k\}\mathbf{R}\text{diag}\{\mathbf{h}_k^H\}\mathbf{a} \ge \sigma_k^2,\ \forall k, \tag{22c}$$

$$\text{Tr}\{\text{diag}\{\mathbf{a}\}\mathbf{R}\text{diag}\{\mathbf{a}\}\} \le P, \tag{22d}$$

$$\mathbf{R},\ \mathbf{R}_k,\ \forall k,\ \mathbf{R} - \sum_{k=1}^{K}\mathbf{R}_k \in \mathcal{S}_N^+. \tag{22e}$$

This is a semi-definite programming (SDP) problem, whose solution can be found using standard optimization tools.

After obtaining the optimal solutions $\widetilde{\mathbf{R}}$ and $\widetilde{\mathbf{R}}_k$ to (22), we can construct the optimal solutions $\mathbf{R} = \widetilde{\mathbf{R}}$ and $\mathbf{R}_k$ that satisfy the rank-one constraint $\text{Rank}\{\mathbf{R}_k\} = 1$, $\forall k$ as

$$\mathbf{w}_k = (\mathbf{h}_k^T\mathbf{A}\widetilde{\mathbf{R}}_k\mathbf{A}\mathbf{h}_k^*)^{-1/2}\widetilde{\mathbf{R}}_k\mathbf{A}\mathbf{h}_k^*,\ \forall k, \tag{23a}$$

$$\mathbf{R}_k = \mathbf{w}_k\mathbf{w}_k^H, \tag{23b}$$

where $\mathbf{w}_k$ is the k-th column of the communication beamformer $\mathbf{W}_c$. Then, recalling that $\mathbf{R} = \mathbf{W}\mathbf{W}^H = \mathbf{W}_c\mathbf{W}_c^H + \mathbf{W}_r\mathbf{W}_r^H$, the radar beamformer $\mathbf{W}_r$ should satisfy

$$\mathbf{W}_r\mathbf{W}_r^H = \mathbf{R} - \sum_{k=1}^{K} \mathbf{R}_k, \tag{24}$$

from which we can calculate the optimal radar beamformer $\mathbf{W}_r$ using either a Cholesky or an eigenvalue decomposition.

2) Update $\mathbf{a}$*:* After obtaining the other variables, the subproblem for updating $\mathbf{a}$ is formulated as

$$\max_{\mathbf{a},t_2} \ \mathbf{a}^T\mathbf{C}\mathbf{a}t_1 + \mathbf{b}^T\mathbf{b}t_2 - \rho_1\mathbf{a}^T(\mathbf{1}-\mathbf{a}) - \rho_2\|\mathbf{b}-\mathbf{1}+\mathbf{a}+\boldsymbol{\mu}/\rho_2\|^2 \tag{25a}$$

$$\text{s.t.} \quad 0 \le a_n \le 1, \ \forall n, \tag{25b}$$

$$(12b) - (12d), (18c).$$

This problem involves both quadratic and linear functions of $\mathbf{a}$ along with constraints that include fractional expressions and semidefinite conditions. To address these difficulties, we define the variable $\widetilde{\mathbf{a}} \triangleq [\mathbf{a}^T \ 1]^T$, and the primary variable $\widetilde{\mathbf{A}} \triangleq \widetilde{\mathbf{a}}\widetilde{\mathbf{a}}^T = \begin{bmatrix} \widetilde{\mathbf{A}}_1 & \mathbf{a} \\ \mathbf{a}^T & 1 \end{bmatrix}$ where $\widetilde{\mathbf{A}}_1 = \mathbf{a}\mathbf{a}^T$. After some matrix manipulations and using the SDR approach, problem (25) can be transformed as

$$\max_{\widetilde{\mathbf{A}} \in \mathcal{S}_{N+1}^+, t_2} \ \text{Tr}\{t_1\mathbf{C}\widetilde{\mathbf{A}}_1\} - \text{Tr}\{\widetilde{\mathbf{A}}(\rho_1\mathbf{E}_1 + \rho_2\mathbf{E}_b)\} \tag{26a}$$

$$\text{s.t.} \quad \begin{bmatrix} \text{Tr}\{\mathbf{QCQ}\widetilde{\mathbf{A}}_1\} - t_2 & \text{Tr}\{\mathbf{QC}\widetilde{\mathbf{A}}_1\} \\ \text{Tr}\{\mathbf{CQ}\widetilde{\mathbf{A}}_1\} & \text{Tr}\{\mathbf{C}\widetilde{\mathbf{A}}_1\} \end{bmatrix} \succeq \mathbf{0}, \tag{26b}$$

$$\text{Tr}\{\mathbf{D}_k\widetilde{\mathbf{A}}_1\} \ge \sigma_k^2, \ \forall k, \tag{26c}$$

$$\text{Tr}\Big\{ \sum\nolimits_{j=1}^{K+N} \text{diag}\{|\mathbf{w}_j|^2\}\widetilde{\mathbf{A}}_1 \Big\} \le P, \tag{26d}$$

$$K^2 \le \text{Tr}\{\widetilde{\mathbf{A}}_1\mathbf{1}\} \le (N-1)^2, \tag{26e}$$

$$0 \le \widetilde{\mathbf{A}}_1(n,n) \le 1, \ \forall n, \tag{26f}$$

$$\widetilde{\mathbf{A}}(N+1,N+1) = 1, \tag{26g}$$

where

$$\mathbf{E}_1 = \begin{bmatrix} -\mathbf{I}_N & \mathbf{0.5} \\ \mathbf{0.5}^T & 0 \end{bmatrix}. \tag{27}$$

$$\mathbf{E}_b = \begin{bmatrix} \mathbf{I}_N & \mathbf{b}-\mathbf{1}+\boldsymbol{\mu}/\rho_2 \\ (\mathbf{b}-\mathbf{1}+\boldsymbol{\mu}/\rho_2)^T & 0 \end{bmatrix}. \tag{28}$$

$$\mathbf{D}_k = (1+\Gamma_k^{-1})\text{diag}\{\mathbf{h}_k\}\mathbf{w}_k\mathbf{w}_k^H\text{diag}\{\mathbf{h}_k^H\} - \text{diag}\{\mathbf{h}_k\}\mathbf{W}\mathbf{W}^H\text{diag}\{\mathbf{h}_k^H\}. \tag{29}$$

After obtaining $\widetilde{\mathbf{A}}_1$ by solving problem (26), we can construct the optimal solution as

$$\mathbf{a} = (\mathbf{1}^T\widetilde{\mathbf{A}}_1\mathbf{1})^{-1/2}\widetilde{\mathbf{A}}_1\mathbf{1} \tag{30}$$

if the rank-one constraint is satisfied; otherwise Gaussian randomization is necessary to recover an approximate solution.

3) Update $\mathbf{b}$*:* Fixing the other variables, the update procedure for $\mathbf{b}$ is similar to that for $\mathbf{a}$. In particular, defining $\widetilde{\mathbf{b}} \triangleq [\mathbf{b}^T \ 1]^T$ and $\widetilde{\mathbf{B}} \triangleq \widetilde{\mathbf{b}}\widetilde{\mathbf{b}}^T = \begin{bmatrix} \widetilde{\mathbf{B}}_1 & \mathbf{b} \\ \mathbf{b}^T & 1 \end{bmatrix}$ where $\widetilde{\mathbf{B}}_1 = \mathbf{b}\mathbf{b}^T$,

Algorithm 1 CRB-Oriented Joint Array Partitioning and Beamforming Design Algorithm

Input: $\mathbf{h}_t$, α_t, σ_r^2, $\mathbf{h}_k$, σ_k^2, Γ_k, $\forall k$, P, ρ_1, ρ_2.
Output: $\mathbf{a}$, $\mathbf{W}$.
1: Initialize $a_n = b_n = 0.5$, $\forall n$, $\mathbf{W}$, t_1, t_2, $\boldsymbol{\mu} = \mathbf{0}$.
2: **repeat**
3: Obtain $\widetilde{\mathbf{R}}$, $\widetilde{\mathbf{R}}_k$, $\forall k$ by solving (22).
4: Update $\mathbf{W}$ by (23) and (24).
5: Obtain $\widetilde{\mathbf{A}}_1$ by solving (26).
6: Update $\mathbf{a}$ by (30) or Gaussian randomization.
7: Obtain $\widetilde{\mathbf{B}}_1$ by solving (31).
8: Update $\mathbf{b}$ by (33) or Gaussian randomization.
9: Update $\boldsymbol{\mu}$ by (34).
10: **until** Convergence
11: Return $\mathbf{a}$, $\mathbf{W}$.

the optimization problem is transformed as

$$\max_{\widetilde{\mathbf{B}} \in \mathcal{S}_{N+1}^+, t_1} \ t_1\mathbf{a}^T\mathbf{C}\mathbf{a} + \text{Tr}\{t_2\widetilde{\mathbf{B}}_1\} - \text{Tr}\{\rho_2\widetilde{\mathbf{B}}\mathbf{E}_a\} \tag{31a}$$

$$\text{s.t.} \quad \begin{bmatrix} \text{Tr}\{\mathbf{Q}^2\widetilde{\mathbf{B}}_1\} - t_1 & \text{Tr}\{\mathbf{Q}\widetilde{\mathbf{B}}_1\} \\ \text{Tr}\{\mathbf{Q}\widetilde{\mathbf{B}}_1\} & \text{Tr}\{\widetilde{\mathbf{B}}_1\} \end{bmatrix} \succeq \mathbf{0}, \tag{31b}$$

$$1 \le \text{Tr}\{\widetilde{\mathbf{B}}_1\mathbf{1}\} \le (N-K)^2, \tag{31c}$$

$$0 \le \widetilde{\mathbf{B}}_1(n,n) \le 1, \ \forall n, \tag{31d}$$

$$\widetilde{\mathbf{B}}(N+1,N+1) = 1, \tag{31e}$$

where

$$\mathbf{E}_a = \begin{bmatrix} \mathbf{I}_N & \mathbf{a}-\mathbf{1}+\boldsymbol{\mu}/\rho_2 \\ (\mathbf{a}-\mathbf{1}+\boldsymbol{\mu}/\rho_2)^T & 0 \end{bmatrix}. \tag{32}$$

After solving (31), the optimal solution is obtained as

$$\mathbf{b} = (\mathbf{1}^T\widetilde{\mathbf{B}}_1\mathbf{1})^{-1/2}\widetilde{\mathbf{B}}_1\mathbf{1}, \tag{33}$$

or using Gaussian randomization.

4) Update dual variable $\boldsymbol{\mu}$*:* After updating $\mathbf{W}$, $\mathbf{a}$ and $\mathbf{b}$, the update for the dual variable $\boldsymbol{\mu}$ is given by

$$\boldsymbol{\mu} := \boldsymbol{\mu} + \rho_2(\mathbf{b}-\mathbf{1}+\mathbf{a}). \tag{34}$$

Based on the above derivations, we summarize the proposed CRB-oriented joint array partitioning and beamforming design approach in Algorithm 1. With an appropriate initialization, we alternatingly update the beamformer $\mathbf{W}$, the array partitioning vector $\mathbf{a}$, the auxiliary variable $\mathbf{b}$, and the dual variable $\boldsymbol{\mu}$ until convergence.

IV. Simulation Results

In this section, we evaluate the advantages of the proposed joint array partitioning and beamforming design via simulation. Unless otherwise specified, the following settings are used: $N = 32$, $K = 4$, $\alpha_t \sim \mathcal{CN}(0, \sigma_t^2)$, $\sigma_t^2 = 1$, $\theta_t = \pi/6$, $\sigma_k^2 = \sigma_r^2 = -80$dBm, $P = 10$W, and $\Gamma = \Gamma_k = 10$dB, $\forall k$. We compare the proposed array partitioning scheme (denoted as "**Prop.**") against two benchmarks that assume contiguous array partitions, both of which are commonly used in the ISAC literature. The first (denoted as "**Even**") evenly divides the array into equally-sized contiguous transmit and receive subarrays, i.e. with $N_t = N_r = N/2$. The second (denoted

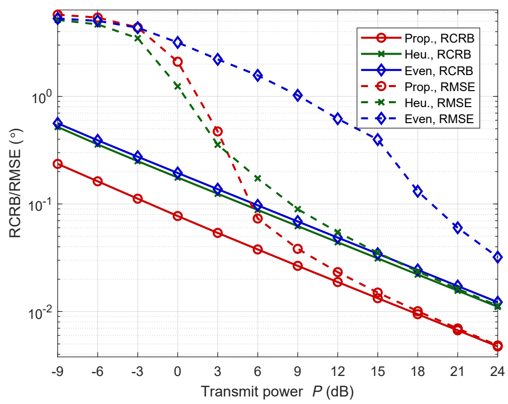

Fig. 1. RCRB & RMSE versus power budget P.

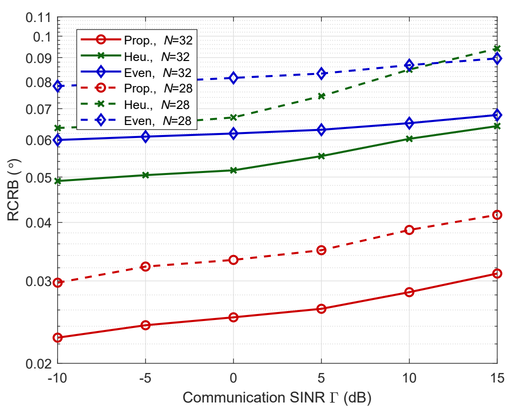

Fig. 2. RCRB versus communication requirement Γ.

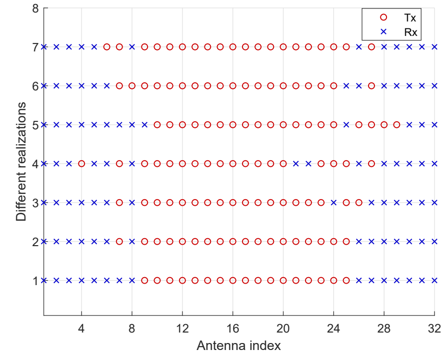

Fig. 3. Representative antenna allocations.

as "**Heu.**") imposes the condition $N_r \geq N_t + K$; for example when $N = 32$, this approach assigns $N_t = 14$ and $N_r = 18$.

We first show the DOA estimation performance versus the transmit power in Fig. 1. The root CRB (RCRB) is the theoretical lower bound determined for the given array partitioning, while the root mean squared error (RMSE) is calculated based on estimation results using the classical MUltiple SIgnal Classification (MUSIC) algorithm [9]. We see that the RMSE approaches the RCRB in the high-power regime. Furthermore, the proposed approach achieves the lowest RCRB, providing approximately a 60% reduction compared to the two contiguous array partitioning benchmarks.

Next, we present the RCRB versus the communication SINR requirement in Fig. 2. A clear trade-off between DOA estimation performance and multiuser communication performance is observed. Additionally, increasing the number of antennas enhances the estimation performance by leveraging the increased spatial DoFs. These results highlight the superiority of the dynamic array partitioning architecture and demonstrate the effectiveness of the proposed joint design algorithm.

Finally, we present several examples of the array partitioning results in Fig. 3 to illustrate typical spatial distribution of the transmit/receive antennas. Notably, the receive antennas are predominantly positioned at the edges of the array, which increases the effective aperture and exploits spatial sparsity to improve DOA estimation accuracy with fewer antennas. This then allows allocation of more antennas to enhance transmit beamforming gains. These findings motivate a heuristic strategy of positioning the receive antennas at the array edges for the considered monostatic ISAC system when sensing a single point-like target.

V. Conclusion

We have proposed an array partitioning architecture for monostatic ISAC systems in which we jointly optimize the array partitioning and transmit beamforming to minimize the CRB for target DOA estimation under constraints on communication SINR, power budget, and the array partitioning. Simulation results demonstrated that the proposed scheme can achieve a significant reduction in RCRB and RMSE compared to conventional contiguous array partitioning benchmarks. Furthermore, our findings revealed that positioning receive antennas at the array edges is favorable for sensing a single point-like target. Future research will extend this work to more complex sensing scenarios.

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