

Promoting Comprehension and Engagement in Introductory Data and Statistics for Blind and Low-Vision Students: A Co-Design Study

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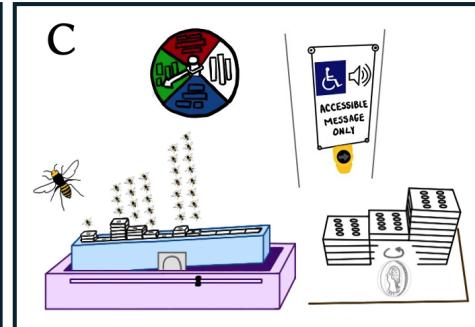
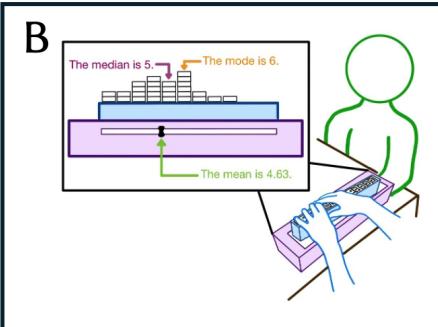
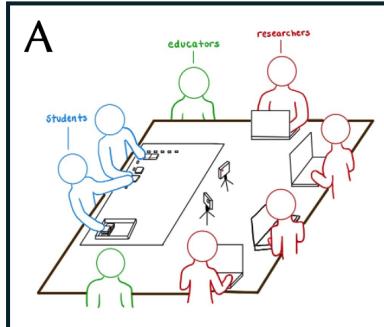


Figure 1: A) Students (blue), educators (green), and researchers (red) participated in a sequence of co-design sessions discussing, designing, and reflecting on practices that build conceptual knowledge and engagement in data and statistics for students who are blind or have low vision (BLV). B) Throughout the co-design process, participants engaged in several inquiry-based activities exploring introductory statistical concepts of distribution and center using both low- and high-tech tools. C) Participants then incorporated knowledge-forming practices synthesized throughout the sessions into the design of their own learning activities.

Abstract

Statistical literacy involves understanding, interpreting, and critically evaluating statistical information in a contextually grounded way. Current instructional practices rely heavily on visual techniques, which renders them inaccessible to students who are blind or have low vision (BLV). To bridge this gap, we formed an extended co-design partnership with a statistics teacher, a teacher for students with visual impairments (TVI), and two BLV students to develop accessibility-first practices for building statistical literacy. Through several months of collaboration that included discussion, exploration, design, and evaluation, we identified specific approaches to promote comprehension and engagement. The enactive approaches we designed, using scaffolding and timely feedback, fostered insights through pattern recognition and analogical reasoning. Additionally, inquiry-based methods promoted contextually situated reasoning and reflection on how statistics can improve students' lives and communities. We present these findings alongside participants' experiences and discuss their implications for inclusive learning frameworks and tools.

CCS Concepts

- Human-centered computing → Accessibility;
- Applied computing → Education.

Keywords

Data, Statistics, Accessibility, Math, Instructional Design, Co-Design, Embodied, Interactive Systems, Education

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1 Introduction

The rapid growth of data and computing during the digital age has marked a paradigm shift in how we consume information. As governments, industries, and individuals increasingly rely on data and statistics for communication and decision-making, understanding

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these concepts is not only a practical skill but also a pathway to greater social inclusion [28].

Although a widely recognized definition is lacking, there is broad agreement that statistical literacy is contextual [61, 63, 128, 144, 147], transnumerative [42, 147], and hierarchical [126, 127, 142], requiring knowledge of the language, procedures, as well as "*higher-order cognitive skills of interpretation, prediction and critical thinking*" [126]. While schools often emphasize calculations and procedures [128, 131], there is a growing acknowledgment of the need to teach data and statistical concepts in more cohesive [36, 129], participatory [135], and contextually situated [61, 63, 128, 135, 142, 147] ways.

Visual representations are widely used for contextualizing and reasoning about statistical measures [110, 136, 140, 144] and are core components of statistical learning curricula [3, 6, 7]. Data visualizations have been shown to promote statistical reasoning [35], particularly for novice learners on problems concerning measures of center [112]. Additionally, there is increasing interest in using dynamic visualizations in exploratory data analysis to shift focus from pure computations towards contextual thinking and inferential reasoning [30, 117]. Several digital learning platforms supporting these features, such as CODAP [1] and TinkerPlots [14], are already being used in classrooms today [105].

However, access to graphics is limited [53, 71, 130], and interactive graphics are virtually nonexistent for blind and low-vision (BLV) students [133], creating significant gaps in the pedagogical tools available to them. Without these tools, reports have found that teaching statistics to BLV students is significantly more challenging [80, 100, 132, 133], particularly when trying to build robust understanding of statistical concepts [102]. Reports have found BLV students lagging in skills to efficiently and accurately interpret graphical information [25].

Prior research in inclusive education has investigated accommodations and adaptations to traditionally inaccessible methods for teaching statistics [51, 65, 68, 69, 100, 102, 133, 134]. However, these efforts often place limited emphasis on fostering conceptual understanding or contextual thinking. In HCI, various systems have been developed to support exploratory and inquiry-driven STEM learning approaches for BLV students [54, 88, 106, 138], but few are designed to directly address the specific knowledge and engagement gaps observed within this population or comprehensively consider the equally critical and interdependent design of the learning activities in which these systems are embedded [20].

Curriculum instructors lack direction in adapting materials for BLV students [24, 51, 68, 102, 133] while Teachers of Visually Impaired Students (TVIs), who specialize in working with BLV students, often do not have the domain knowledge needed to adapt materials to foster robust conceptual understanding [46, 104, 122]. There is a pressing need to identify practices and tools that promote deep conceptual learning, interpretation, and critical evaluation of data and statistics in context from an accessibility-first approach.

In this work, we investigate:

- RQ1: What challenges do educators and BLV students identify in current practices and tools for learning statistics?
- RQ2: How might we design inclusive education practices and tools that promote robust conceptual understanding, interpretation, and critical evaluation of statistical measures in context?

Involving the perspectives of statistics teachers who create curriculum-based activities, TVIs who adapt these activities, and BLV students is essential for designing and evaluating inclusive learning practices [54, 91, 113]. We approach our research questions through a co-design program that solicits the combined expertise of stakeholders directly responsible for BLV students' education experience—a high school statistics teacher, a TVI, and two BLV students—in the design of tools and activities to foster statistical literacy. Spanning multiple half-day sessions over several months, the program leads participants through a complete cycle of the design process—from need-finding and defining learning objectives (Session 1), to exploring educational tools and practices (Session 2), to prototyping (Session 3) and evaluating (Session 4) learning activities.

Conversations and observations from Session 1 highlighted the need for more engaging, exploratory, and contextually relevant practices to help BLV students reason beyond surface-level factual and procedural knowledge. Experimentation with various pedagogies and tools across Sessions 2 through 4 revealed multiple inclusive learning opportunities to strengthen conceptual knowledge in engaging ways, which we synthesize into four engagement and eight learning takeaways. Among these are the use of enactive, embedded, and analogical practices to build intuition and support reasoning about measures in relation to underlying distributions and data; as well as inquiry-based approaches to encourage contextually situated reasoning and reflection. We discuss the implications of our findings for activity and tool design and append the participant-designed activities to demonstrate practical ways to apply these approaches (Appendix A).

2 Prior Work

2.1 Data, Visualization, and Statistical Literacy

While educators have not reached a consensus on a formal definition for statistical literacy, there is widespread agreement on several essential qualities [126].

First, statistical literacy has been conceptualized as interwoven threads of mathematical understanding and contextual engagement [142] in a complex hierarchical construct [127, 142]. Several models have linked levels of statistical understanding with Biggs and Collis' Structure of the Observed Learning Outcome (SOLO) taxonomy [93, 142]. Notably, Watson and Callingham decompose literacy across six levels: from idiosyncratic to critical mathematical [142]. Each level reflects increasing critical engagement with context, uncertainty, and proportional reasoning. While factual and procedural knowledge is traditionally easier to teach, developing higher levels of conceptual knowledge has been reported to be much more challenging [115, 143], especially for BLV students [102].

Second, statistical literacy encompasses an appreciation of context [38, 61, 142], which Gal emphasizes extends beyond simply acknowledging the use of real data; but critically examining *what*

meaning is conveyed by statistical measures and *why* that meaning is relevant to the insights being sought from the data. As society becomes increasingly data-driven, the importance of situating statistical literacy in context has arguably become paramount to social inclusion [28] and modern living [62, 139].

Finally, appreciation and application of statistics in context positions statistical literacy closely with visualization literacy and data literacy [72]. Both visualizations and statistics are powerful, complementary tools for data sense-making and inquiry [18, 136]. While statistical measures often summarize data, they lack the ability to convey the full scope or nuances that visualizations can effectively communicate. Graphical representations have been shown to promote statistical reasoning [35, 112] and are increasingly integrated into core curricula [3, 6, 7, 105].

Our work explores instructional design strategies to foster conceptual understanding of statistics that are both contextually meaningful and grounded in data representations. Specifically, we focus on measures of center, among the first and most widely applied statistical concepts introduced in public education. While many measures exist and are newly constructed to describe data, measures of center—such as the mean or median—have an elevated status given their broad use.

2.2 Accessible Data and Statistics Education

Most BLV students in the United States are enrolled in public schools [13]. In these settings, the curriculum instructor develops and selects activities that TVIs adapt for BLV students [83].

Physical manipulatives, such as tokens and objects, are often used as an introductory tool to teach data and graphical concepts to BLV students [122]. These manipulatives provide concrete experiences that can help students connect abstract concepts to realistic contexts [44, 70, 118]. With increased experience, students transition to more standardized tactile representations of graphs and charts. These representations, called tactile graphics, consist of raised lines that BLV students can explore through touch. Effective use of tactile graphics often takes time to learn [123].

For learning statistics, significant research has highlighted a variety of adaptations for making activities more accessible: which span software tools [69, 134], tactile alternatives to visual graphics [65, 100, 102], and logistical accommodations [100, 102]. However, this approach limits students to the adaptable subset of education activities originally developed for general education purposes.

Contrasting prior work, we take an accessibility-first perspective that begins with identifying the conceptual and engagement challenges of BLV students and then re-imagines tools and instructional practices to address them. This situates comprehension and engagement (rather than adaptation) as the focus. We draw from both visualization and statistical learning practices mentioned in prior work [65, 100, 102, 122], and from more recent audio-tactile systems [54].

2.3 Co-Design for Inclusion

Co-design is a collaborative practice where participants combine their knowledge, skills, and resources to tackle a design task [153]. This approach often results in designs that are better aligned with users' needs [90] and encourages more original ideas [149]. For

inclusive technologies, co-design amplifies the values, experiences, strengths, and ideas of marginalized groups [27, 59]. Prior research highlights the effectiveness of co-design with neurodiverse [26, 27, 81], blind and low vision (BLV) [34, 104, 148], and deaf and hard of hearing (DHH) children [87], contributing to inclusive educational tools such as educational science simulations [148], intelligent tutoring systems [27], learning games [27], collaborative robots [104], smart objects [59], and digital devices [34, 81].

Engaging marginalized students in co-design not only produces better-suited designs but also provides empowering experiences [26, 81, 137] that foster self-efficacy [148]. This is particularly valuable for BLV children, who often face reduced opportunities for collaborative learning and social engagement [22, 60]. Druin et al. outline levels of involvement in design, ranging from users to testers, informants, and, ultimately, design partners [48]. Involving students as equal design partners produces stronger alignment with their needs, abilities, and preferences [76], but requires careful attention to balancing and addressing asymmetries in tools and language [92, 152]. Strategies such as assigning roles [33, 104], using crafts [33], or storytelling can further facilitate equitable and inclusive co-design experiences for BLV participants.

As a research methodology, co-design investigates the plausibility of future realities and demonstrates their existence (though not universality) while providing insights into their impact and relevance [153]. Our work using co-design aims to explore an accessibility-first statistical learning reality, demonstrate its feasibility through student outcomes, and synthesize takeaways and practices that bridge current challenges to such a reality.

3 Positionality Statement

The core research team consists of five faculty members (two BLV and three sighted), four graduate students (all sighted), and two undergraduate students (one blind and one sighted) from schools of information, education, and engineering. Co-design participants who joined the team later helped shape subsequent sessions and provided key insights that informed the research analysis and writing.

4 Methodology

We conducted a series of IRB-approved co-design sessions with a high school statistics teacher, a TVI, and two BLV students in their early high school years. All participants attended four half-day sessions, totaling over 16 hours spread across several months. Each session focused on a different theme, which was: 1) Motivating Data and Statistics, 2) Tools and Practices for Engaging Learning, 3) Design and Prototype, and 4) Evaluation and Reflection (Figure 2). Two additional 90-minute meetings conducted after Sessions 2 and 3 provided educators with the opportunity to reflect and co-plan subsequent sessions.

Six data inquiry activities related to measures of center were interspersed throughout the sessions. These activities provided an opportunity to identify knowledge gaps and experiment with learning approaches. Initial activities reflected standard practices

[4, 10, 11] and provided a shared experience to promote conversations surrounding current practices and needs. Subsequent activities became increasingly embedded in the common values and objectives synthesized by participants.

Activities also provided learning contexts for us to probe a variety of non-visual data and statistical learning technologies. These technologies ranged from low-tech manipulatives often used in classrooms [122] to multimodal learning prototypes that offer tunable instantaneous feedback [54]. They included both participant-submitted tools (Braille notes, calculators, and tactile learning kits), as well as researcher-created artifacts (3D-printed spinners and dice). After using and reflecting on these tools in the early inquiry sessions, participants were given the opportunity to select any combination of these tools or propose their own when designing activities in Session 3.

4.1 Participants

We recruited participants through local mailing lists and our personal network. Before the study sessions, we pre-screened participants and gathered their access preferences during a 30-minute Zoom call. We aimed to include participants with diverse roles and perspectives from typical math learning environments for BLV students. Given that public schools are the most common educational setting for blind children [13], our selection criteria required the general education teacher to have experience designing and teaching statistics in public schools, the TVI to have at least 5 years of experience working with BLV students, and the two BLV students to be in middle or high school with knowledge of basic algebra. We limited the group size to foster engagement, commitment, and a sense of ownership over the process and outcomes. All participants were expected to be open to collaboratively sharing ideas and experiences.

The final group of participants included an AP statistics teacher at a public high school, a TVI with over 48 years of experience who also serves as a university lecturer, and two BLV freshmen from different public high schools. These participants are referred to as the Stats Teacher, the TVI, Student 1, and Student 2, respectively. We also refer to the Stats Teacher and TVI together as "educators." The TVI had worked with one student before, and the two students knew each other but had not met in several years. Both students had prior exposure to concepts of mean, median, and mode in middle school math classes. They also both lost their vision shortly after birth, which is common among blind children [96].

Throughout the co-design process, all participants primarily served as design partners. For the data inquiry activities, the students also acted as users for evaluating activities, testers of new interactions, informants when reflecting on the activities, and design partners in the development of the later activities [48]. Participants were compensated \$45 per hour for their participation in the co-design sessions. Afterward, they contributed to research planning, analysis, and writing in various capacities.

4.2 Session Design

Sessions 1 and 2 focused on exploring *what* to design and *how* to approach the process. The initial group of researchers planned these sessions through multiple discussions. Educators were then

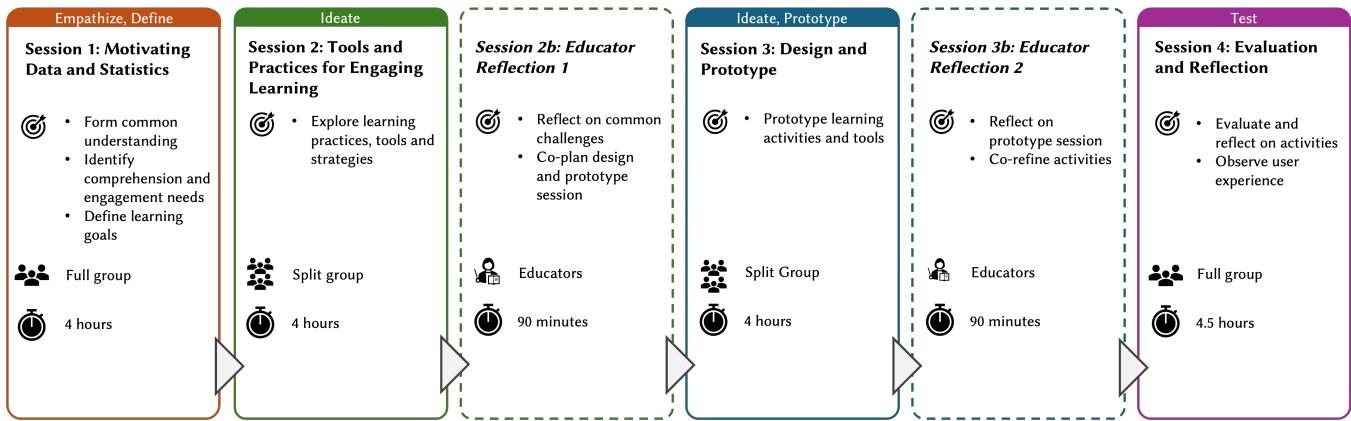


Figure 2: Progression of topics, goals, and logistics across the co-design sessions.

involved in designing Sessions 3 and 4 through follow-up Zoom meetings, which focused on prototyping and evaluation. We used a combination of full-group sessions for collective brainstorming, discussion, and reflection, along with educator-only, student-only, and mixed-pair sessions to encourage open sharing without peer pressure or student-teacher power dynamics.

All but the two educator-only reflection studies were held in person. Each of these sessions lasted approximately four hours and included themed subsections with interspersed breaks. Two to five members of the research team co-led each of the sessions. We asked participants to bring their familiar computing and calculating devices to each session. We videotaped and audio-recorded all sessions with participants' consent.

4.2.1 Session 1: Motivating Data and Statistics. Before engaging in inventive practices, we first needed to establish a shared understanding of learning needs from our participants' diverse perspectives. Thus, Session 1 focused on familiarizing participants with the problem space, identifying needs, and defining learning goals through facilitated discussions and reflective practice grounded in data activities.

Data-centered Icebreaker: Participants and researchers introduced their interests in inclusive STEM practices and shared statistics they found surprising or interesting. Participants and researchers then collaboratively constructed a set of guidelines for healthy and inclusive discussion and collaboration.

Motivating data and statistics: Participants brainstormed ways to motivate data and statistical learning by first imagining themselves as teachers explaining the importance of understanding data and statistics to BLV students, and then as students encountering data and statistics for the first time. Next, students shared types of phenomena they and their peers might find interesting to track. Finally, the group reflected on common themes shared throughout the activity.

Data Inquiry 1: Students participated in an initial inquiry activity, which provided a shared experience to ground future discussions and served as a baseline evaluation of their prior knowledge. We adapted the first activity from common high-school data science curricula [4, 10, 11], which asks students to compute and discuss

measures of center, compare the measures, and reason about them through plots of distributions in context. Students chose one of three possible topics (weather, basketball, or penguins). Plotting was made accessible through a magnetic whiteboard, magnetized acrylic strips to serve as frames, Braille labels, and manipulable magnetic tokens (shown in Figure 3A) [52].

Data Inquiry 2: To further assess understanding, students completed a second activity that asked them to select the highest-rolling die from two weighted Braille dice (shown in Figure 3B). Repeated rolls of the dice would generate two distinct non-uniform probability distributions of different skews. We provided the measures of center for one die, which students could not touch, and gave students the freedom to manipulate and sample from the other die. After choosing a die, students played a game against the researchers in which the highest sum of 20 rolls won.

Reflection on Inquiry Experiences: Participants split into individual student and educator sessions to reflect on their activity experiences. The student group reflected on their engagement and learning across the activities while the educator group reflected on observed strategies and challenges. From those challenges, educators developed an initial set of learning goals focused on center and distribution. The session concluded with a large group sharing of individual group reflections.

4.2.2 Session 2: Tools and Practices for Engaging Learning. Session 2 focused on exploring tools and pedagogical approaches that support engaging learning as a precursor to activity design in Session 3. Facilitated discussions encouraged participants to draw on their experiences and expertise, while data activities provided a concrete foundation for these conversations.

Due to a scheduling conflict, Student 1 participated separately, offering an independent perspective at the cost of collective experience. To keep the student aligned with group discussions, we shared key points from the group session after each reflection period.

Engaging Learning Activities: Participants discussed types of activities that made learning fun by drawing on their prior experiences.

Metaphors and Analogies for Learning: Building on a suggestion from the Stats Teacher to use analogies to explain statistical

measures in Session 1, we discussed the role of metaphors and analogies in teaching data and statistical concepts. The Stats Teacher suggested demonstrating the mean as the "center-of-balance" of a distribution. Students explored this analogy by using their fingers to balance a distribution of magnetic tokens placed atop a magnetic ruler (shown in Figure 3C). Students also explored the median by examining ideas of symmetry using movable magnetic number labels (shown in Figure 3D).

Interactive Learning Systems: While visual graphs leverage interactivity such as layering [86] and highlighting [95] to help sighted students discern data relationships, these benefits have not been extended to BLV students. Addressing the need for more engaging learning tools identified in Session 1, we explored opportunities to incorporate auditory and haptic feedback to enhance statistical learning.

We provided participants with samples of input and feedback methods, including token manipulation, spoken audio, audio effects, kinesthetic feedback, and vibration, which are commonly explored to support non-visual learning [55, 67, 108, 109, 114, 122, 124]. For each, we encouraged participants to think aloud and reflect on ways these methods might support data interaction or demonstrate data concepts.

Data Inquiry 3: To illustrate how metaphors and interactivity can be integrated, participants engaged in a third data inquiry activity using a digital multimodal statistical learning platform developed by Fan et al. (Figure 3E) [54]. This platform provides real-time audio and haptic feedback on statistical measures as students construct and manipulate physical data representations. Teachers can customize feedback to describe statistical measures as data updates, while learners can explore concepts like the mean in embodied and analogical ways. For example, by sliding a physical fulcrum to the distribution's mean and center of mass, students can feel the tilt of the physical representation level. Additionally, students can press on regions of the graph to hear information about those regions.

The inquiry activity focused on classical composers, a topic of interest identified in Session 1. Students created a dataset by measuring their hand sizes on a portable musical keyboard. They then engaged in inquiry activities comparing their hand sizes to classical composers. The activity encouraged them to calculate measures of center, explore how statistical measures changed with individual data updates, and reflect on the sensitivity of those measures. This involved modifying a physical representation based on hypothetical scenarios and observing changes through various feedback mechanisms. Following the inquiry, the group reflected on the platform interactions.

Activities and Strategies for Learning: Having explored both tools and learning methods, participants brainstormed ways to support the learning of measures of center. We introduced participants to Bloom's revised [89] and Fink's [56] taxonomies as a co-design resource and provided a table to organize thoughts across taxonomy categories.

4.2.3 Session 2b: Educator Reflection. We conducted a 90-minute follow-up Zoom meeting with educators to 1) further synthesize and discuss ways to support the learning objectives from Session 1,

2) reflect on students' experiences across the data inquiry activities, and 3) co-plan the design and prototype portion of Session 3.

Researchers synthesized the learning objectives and practices shared by participants at the end of Sessions 1 and 2 and pre-populated a collaborative whiteboard with this information to guide the discussion. We grounded reflections on students' experiences using the CARE methodology proposed by Mouallem et al. [107], which assesses instruction through unproductive struggles, healthy challenges, and rewarding experiences. Based on these reflections, educators and researchers collaboratively defined the structure of Session 3 to support the creation of learning activities, established roles and responsibilities to ensure a productive and enjoyable experience for all participants, and selected materials participants would find useful.

4.2.4 Session 3: Designing and Prototyping. After identifying *what* to design in Session 1 and *how* to approach design in Session 2, participants focused on creating learning activities in Session 3. This open-ended design process allowed them to address observed challenges and translate ideas and themes from earlier sessions into tangible outcomes. The resulting activities served as concrete examples of participants' values, while discussions during their creation revealed the intent behind each component.

Session 3 was split into separate teacher-student subgroups, which created several opportunities. First, students would be able to design activities for each other to evaluate during Session 4, a motivator discussed in an earlier session. Second, designing as student-teacher pairs provides a more active and intimate idea-creation experience that aligns closer to the values of the particular participants. Third, comparing activities across the pairs offers insight into the perspectives each participant brings, such as those of the statistics teacher compared to the TVI.

Recap of Prior Conversations: To promote continuity in activity design based on prior conversations and observations, Session 3 began with a reflection on memorable experiences and conversation topics from the previous sections.

Introduction to Materials: We introduced participants to various materials they could use for prototyping (shown in Figure 3F). These include craft supplies like textured paper, glue sticks, playdough, velcro, tape, scissors, and wooden blocks, and materials commonly used to teach BLV students, such as Braille labels, tactile drawing kits, tactile diagramming materials, Wiki Stix, and a pin board (provided by the TVI). Additionally, we included data exploration materials from previous sessions, such as a magnetic whiteboard, magnetic tokens, and the multimodal data platform. Participants brainstormed potential uses for each type of material as we introduced them.

Prototyping Statistical Learning Activities: Participants prototyped activities following a backward design structure [120]. Participants first selected and refined one of the learning objectives identified in previous sessions, then prototyped activities to help students achieve the objective. While participants led topic selection and activity design, the research team actively contributed by highlighting themes from prior conversations, summarizing participants' ideas, and encouraging the concretization of concepts to move the design process forward.

4.2.5 Session 3b: Educator Reflection. We held a second 90-minute Zoom meeting with educators to reflect on the prototyping process and compare it to their usual activity generation and adaptation practices. During the meeting, educators shared and provided feedback on the activities, allowing us to observe how they integrated their unique perspectives and experiences. The researchers then incorporated this feedback into the activities before Session 4.

4.2.6 Session 4: Evaluation and Reflection. In Session 4, participants engaged in a contextual inquiry (as defined in learning contexts by Druin et al. [48]) to critique and reflect on the effectiveness of the activities designed during Session 3 in addressing the needs and learning goals identified in earlier sessions.

Contextual Inquiry: Participants alternated between experiencing the activities designed for them and observing their peers engaging with the activities they had created. After each activity, a brief reflection focused on what worked well and what could be improved in terms of engagement and learning. In total, participants experienced three activities: the "TVI-Student 1" group developed two shorter activities, while the "Stats Teacher-Student 2" group created a single, longer activity (detailed in Appendix Section A).

Final Activity and Co-Design Reflections: Final reflections followed the contextual inquiry. Educators and students first met in smaller groups to discuss the activities in relation to their prior experiences, learning goals, and the co-design process. The discussion then shifted to a larger group, where participants reflected on designing engaging learning experiences and the importance of feedback and interactivity.

4.3 Data Analysis

We used reflexive thematic analysis [32] to analyze transcripts and video data. Six researchers first reflected on their study notes to identify lenses for coding the data. These lenses include: expectations about learning over time, prior needs and experiences, prior knowledge, challenges, recommendations, interventions, learning outcomes, activity design, and the co-design process. Transcripts were triangulated with artifacts and video observations by multiple investigators to promote consistency and validate findings [40]. Videos were labeled in the following formats:

- "Because (motivation), students (performed action), which made them/ helped them/ led them to believe (reaction)." (i.e. "In order to engage S1 in the construction activity, S2 guided S1's hand to x and y axis locations, which helped S2 understand the structure of the graph.")
- "Students encountered (challenge), but (action) led to (insight)." (i.e. "S1 described not knowing the effect of an added data point on the median, but by re-applying the sticky note to the point that splits the data in half, articulated that the median does not change.")

We then inductively constructed sub-themes and themes from codes and collated data using latent and constructionist approaches, which form the subheadings and headings of Section 5. These were deductively reviewed and refined through several constructivist-aligned learning frameworks, including Keller's ARCS Model of Motivational Design [82], Krathwohl's types of knowledge [89], and Biggs and Collis' Structure of the Observed Learning Outcome

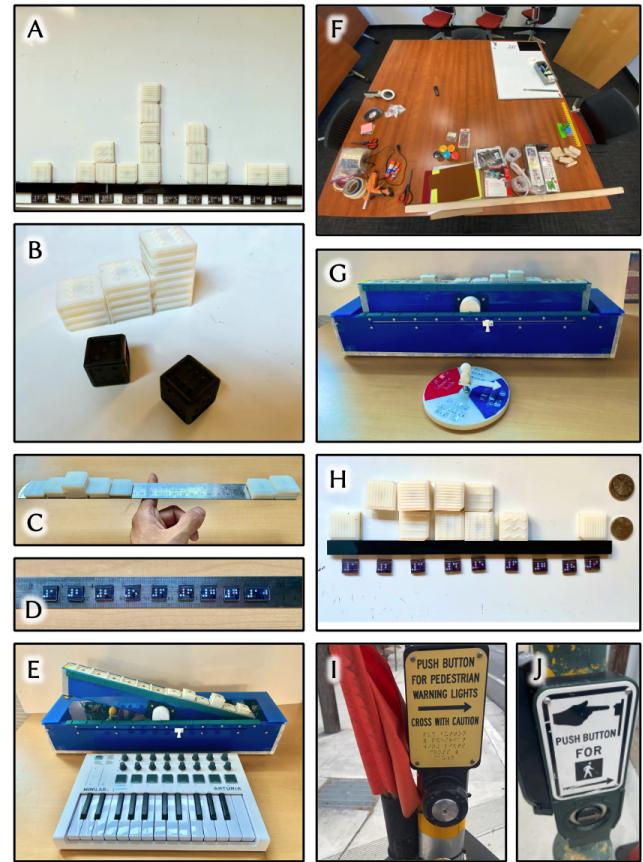


Figure 3: Educational materials used to support inquiries across Sessions 1 (A, B) and 2 (C-E); prototyping in Session 3 (F); and participant-designed activities across Sessions 3 and 4 (G-J). Materials ranged from craft supplies and passive manipulatives to digital tools and learning platforms. Not shown are the educational materials the TVI brought, which include tactile drawing [15], plotting [16], and math kits [17] produced by the American Printing House (APH), and the personal computing devices that the students brought.

(SOLO) taxonomy [31]. Theme generation was an iterative process involving over 4+ meetings with eight researchers, facilitated through affinity diagramming on a collaborative whiteboard. The team held multiple peer debriefing, analysis, and review sessions with the input of the participants to mitigate researcher and confirmation bias. Our codebook is included in the supplementary materials.

5 Results

We identified engagement (Section 5.1), conceptual reasoning (Section 5.2), and contextual reasoning (Section 5.3) as central themes, each encompassing multiple subthemes that structure the subsections. Within each sub-theme, we explore participants' articulated and observed challenges, the strategies they developed in response,

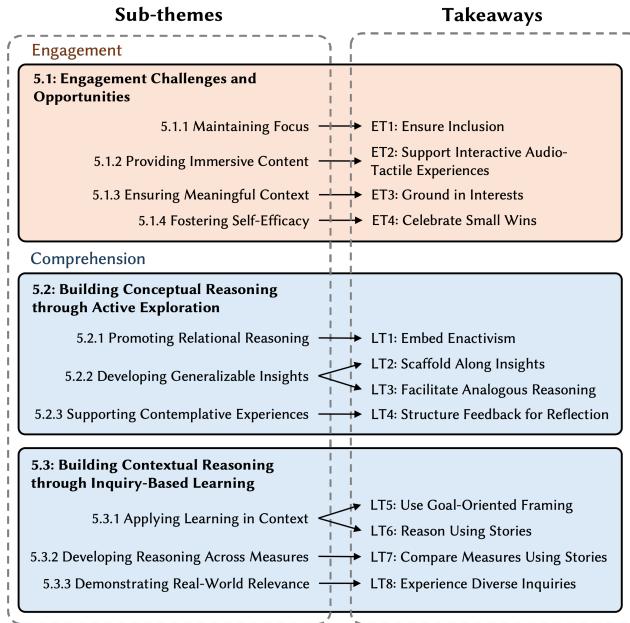


Figure 4: List of sub-themes (left) and takeaways (right) identified for the central themes of engagement (top, beige), conceptual reasoning, and contextual reasoning (bottom, blue). Arrows relate specific sub-themes and corresponding takeaways.

and the interventions that promoted engagement or deepened understanding. These interventions are synthesized into key takeaways, which are woven throughout the subsections (see Figure 4 and Appendix B). Additionally, Appendix A presents participant-created activities that illustrate how they applied these takeaways to design more engaging and knowledge-forming practices.

5.1 Engagement Challenges and Opportunities

Maintaining focus, providing immersive content, ensuring meaningful context, and fostering self-efficacy were identified as important components to sustaining engagement that were often lacking for BLV students. The following subsections describe observed student challenges, corresponding interventions, and key engagement takeaways (labeled as ETs).

5.1.1 Maintaining Focus: During the initial group inquiry sessions, we observed that access barriers produced task and pacing misalignments that disrupted focus. In an early collaborative physicalization activity with magnetic tokens (Data Inquiry 1), Student 2 took the lead in organizing and populating the plot. However, the absence of perceptible cues for Student 1 to track task progress led to their disengagement. The TVI explained that, without vision, “it’s hard [for BLV students] to look at the graph and keep track of where the other students had placed [tokens].” Despite the TVI’s efforts to re-engage Student 1 by placing their hand on the whiteboard, the student continued to struggle with participation.

When Student 2 noticed Student 1’s disengagement, they guided Student 1’s hand through the plot, explained its structure, and offered hints on where to place the data tokens. These efforts to clarify the task and establish joint attention helped Student 1 re-engage with the activity, highlighting the importance of maintaining perceptibility and mutual awareness in all aspects of group work. As a result, we became more intentional about providing audio feedback and clear verbal cues in subsequent activities, which fostered greater vocal and physical engagement from both students.

ET1 (Engagement Takeaway 1): Ensure Inclusion: Perceptibility and mutual awareness in all components of group activities are important for maintaining focus.

5.1.2 Providing Immersive Content: Both students and the TVI corroborated how BLV students often lacked sufficient access to immersive statistical learning materials. The TVI shared that “*A lot of times, there’ll be books or documents with pictures or icons, images, and they eliminate those. They omit them because their rules say they don’t have to include them if it’s not pertinent to the content.*” As a result, students are often inundated with “*pages and pages and pages of Braille (TVI)*,” which Student 2 calls “*pure torture*.”

When designing activities, students and teachers proposed many ideas that used haptic and auditory feedback. Haptically, students particularly enjoyed manipulating tangible tokens during the data inquiry activities, which allowed them to “*actually feel and play around...That’s better than having to do y equals mx plus b*” (Student 2). Auditorily, participants recommended using auditory hooks to motivate inquiry activities (TVI, Stats Teacher) and integrating data-responsive auditory feedback (Student 2). Based on Student 2’s idea, we programmed the digital platform to play buzzing sounds at volumes corresponding to the number of hornets in a participant-designed inquiry about hornets (Appendix A.1). Additionally, pairing token manipulation with sonified auditory feedback led Student 1 to spontaneously create stacks of varying heights to explore pitch-to-data relationships. Through experimentation, students intuitively grasped the correlation between data and pitch without requiring a formal introduction to sonification.

ET2: Support Interactive Audio-Tactile Experiences: Tangible experiences with immersive auditory feedback promote engagement and encourage experimentation.

5.1.3 Ensuring Meaningful Context: Both students shared how their prior statistical learning experiences consisted mainly of calculation-focused tasks rather than context-based reasoning. Student 2 described how their prior learning consisted of “*mostly just numbers. It was just boring.*” The Stats Teacher shared how “*drilling is not necessarily very productive, and it will likely be flushed out of their minds really quickly.*” To the Stats Teacher, drilling can easily be avoided, explaining that “*especially with statistics, it’s nice because you’re working with a specific data set. And if it’s [a] specific data set that is interesting to them, it will be really memorable, and you can refer back to it.*”

As baseline for promoting engagement, Student 2 recommended to “*always make sure to have a story.*” Student 1 further emphasized

how anchoring stories around "subjects that [students] like" improves engagement. Many of the interests students shared involved auditory characteristics of natural and social environments, such as bird sounds, voice impressions, and music; as well as audio-haptic experiences, such as elevators, trains, and blow dryers. The TVI reflected on how "*the rest of us would go in the elevator and wouldn't hear anything. But [Student 1] focuses on it, so [they] pay attention to the sounds that [they're] hearing. And then [they] make a comparison between the sound in this elevator and the sound in that elevator.*" Students were also passionate about accessibility and particularly attentive to accessibility features in pedestrian signals and screen reader interactions. When students tried activities that catered to each other's interests (Appendix A), they naturally raised inquiry questions, shared data-oriented stories about their experiences, and discussed the implications of data findings.

ET3: Ground In Interest: Grounding activities in students' interests, particularly those with interesting audio and haptic features, promotes contextualization, storytelling, and engagement.

5.1.4 Fostering Self-Efficacy: From the first session, Student 2 self-identified as being "*poor at math*" to a point in which the TVI reflected on being "*surprised that [they] continually would say, I don't have good math skills and I can't do this. But then [they] would turn around and demonstrate that [they] could do all of those things and [they] did have those skills, but [they] kept, you know, downgrading [themselves].*"

When reflecting on practices for building confidence, the Stats Teacher shared how "*breaking up open-ended tasks into smaller checkpoints can be pretty effective*," as doing so "*helps keep students accountable and celebrate that they are progressing*." When breaking up tasks, the Stats Teacher recommended "*try[ing] to stick with one main thing... and try and make [activity sections] a little bit more compact. [There might be other] softer skills that they're always working on, but [focus on] one hard skill that is either like new or something that you're trying to emphasize.*"

To celebrate small wins, the TVI shared how explicit praise is particularly important for BLV students because sighted students "*can see the expression on your teacher's face, you can see they are happy with what you've done.*" After receiving verbal praise for completing a sequence of tasks exploring the sensitivity of the median to data values (Session 2), Student 2 responded with "*I'm glad I did [well] because at least I know I still have math skills.*"

During the design session (Session 3), students gleefully proposed sound effects that could be used to validate correct responses, such as "*ding ding ding!*" (Student 1). Taken together, decomposing broader activities into more approachable, goal-oriented portions, and providing opportunities to auditorily obtain positive feedback on the completion of these portions can help students gain confidence in their skills.

ET4: Celebrate Small Wins: Providing opportunities to auditorily celebrate the completion of smaller, goal-oriented objectives can help students confirm their reasoning and build confidence

5.2 Building Conceptual Reasoning through Active Exploration

In the initial inquiry activities, students demonstrated strong factual and procedural knowledge. They clearly defined statistical measures, identified them as types of averages, explained calculation methods, and performed calculations quickly and accurately, both digitally and mentally. However, their conceptual understanding of how statistical measures relate to data values and distributions was limited.

The following subsections examine key aspects of conceptual learning explored during the co-design process: promoting relational reasoning, developing generalizable insights, and supporting contemplative experiences. Each subsection describes observed student challenges, corresponding interventions, and key learning takeaways (labeled as LTs).

5.2.1 Promoting Relational Reasoning: Students initially struggled to reason about the relationships between statistical measures, individual data points, and data distributions. When asked how the addition of a hypothetical datapoint might affect the mean and median, students could only reason using formulas, which either proved difficult or led to errors. For example, Student 2 focused solely on the summation in the mean formula to reason that "*the more [data] you add, the more likely the mean will grow.*" Students also struggled to estimate the mean and median of a distribution without access to the numbers. Student 1 expressed uncertainty about what the shape of the graph revealed about statistical measures. Student 2 reasoned that it "*would be hard [to estimate where the mean is] because there are no data points...You don't know what those measures are.*" The TVI noted that BLV students' unfamiliarity with representation-based inquiry might be due to their limited exposure to data presented in graphical formats. "*When they produce those materials in braille, they don't put them in charts like [sighted] students see them; they just make lists... [because they are] faster and easier to produce.*" As computation alone is a poor way of understanding the underlying concept [115], the Stats Teacher recommended using additional methods of reasoning.

We observed that regardless of whether digital or non-digital tools were used, approaches that allowed students to quickly manipulate and identify patterns in data representations helped them reason about the sensitivity of measures to data and distributions. In one activity (Session 2), students were given a list of manipulable magnetic number labels and shown how to find the median by moving their fingers from the outside labels inward. We then provided additional labels to replace the data. Both students quickly and independently in separate sessions realized that changes in edge values did not affect how their fingers converged toward the center, leading them to conclude that the median was not sensitive to changes in edge values. In another activity (Data Inquiry 3), students received real-time auditory feedback as they adjusted a data physicalization by moving tokens on a digital platform. The physicalization allowed students to enclose their hands around the representation and feel how individual points related to the aggregate. After several brief manipulation-feedback sequences, students independently concluded that higher values increase the mean, while lower values decrease it. The Stats Teacher appreciated how this approach de-emphasized calculations, allowing students

to focus on higher-level relationships. Both students enjoyed the hands-on experimentation and took the initiative to test their own hypotheses about how different values affect statistical measures. As Student 2 shared, *"It's fun how you see different patterns."*

LT1: Embed Enactivism: Embedded and enactive approaches provide engaging ways for students to quickly experiment with and identify patterns in data sensitivity.

5.2.2 Developing Generalizable Insights: Manipulation and feedback alone however did not help students generalize their observations. When asked what types of values would not change the mean, Student 1 could not provide a guess, and Student 2 mistakenly guessed zero, despite recognizing that higher values increase the mean and lower values decrease it. To help students generalize insights, we observed ways the Stats Teacher further scaffolded questions along categories of insights. When trying to help Student 1 generalize types of changes that would affect the median, the Stats Teacher divided the data into a lower (below the median) and higher (above the median) zone, and asked students to explore whether changes within zones or across zones would change the median. This helped Student 1 generalize how cross-zone changes are likely to change the median, while inner-zone changes would likely not unless the change involved a point bordering the median.

LT2: Scaffold Along Insights: Scaffolding exploration along categories of insights can help students develop more generalized understandings.

Enabling students to enact analogies offered additional ways for them to extrapolate from experimentation and reason about how measures of center are influenced by data and distribution shape. We explored incorporating analogies through two mechanisms: having students enact the analogy in a representation-embedded way, and providing analogical feedback.

Enacting analogies in a representation-embedded way provided avenues for students to apply analogical reasoning to validate their understanding. Initially, Student 1 had trouble abandoning their intuition that the median must change with changes in values, even when verbally reminded that the median splits the data in half. When asked how changes in a low boundary value might affect the median, Student 1 reasoned that *"The median [will] decrease a little bit...From, like, slightly above 3 million to slightly below 3 million."* By having students physically place a divider (in our case, a folded up sticky-note) at a point that divides the dataset evenly, they corrected themselves by recognizing through touch that the divider continued splitting the data in half after an edge value was modified. After the activity, the student reflected on how they *"like the divider, [it] really illustrated the median."*

Providing continuous analogical feedback prompted students to reason about data sensitivity while manipulating data. When students interacted with the digital platform, which uses tilt feedback to analogically tie the mean to the center of mass, both students were able to use the concept of weight to reason about and predict changes: *"When we shifted everything to the tall side, it went higher. I have a good feeling that if we do the same thing to the low side because of weight distribution, they might get low"* (Student 2).

However, while students were able to reason about the effect of weight on tilt, students had more difficulty predicting new fulcrum locations to balance distributions, particularly for asymmetrical distributions. We observed this difficulty both through the interface and through passive props, such as balancing tokens on a ruler and wooden blocks on a beam. We hypothesize that this may be because blind students may lack the same exposure to perceiving the fulcrum's position relative to the geometric weight distribution of an object that sighted individuals can holistically see.

LT3: Facilitate Analogical Reasoning: Enacting, physicalizing, and embedding analogical thinking can provide additional methods of reasoning, validation, and problem-solving, but are limited by students' understanding of analogical concepts.

5.2.3 Supporting Contemplative Experiences: When feedback from the digital platform became too readily available, we observed students relying on guess-and-check strategies instead of using the analogical properties of the physical representation to reason about statistical measures. For example, when asked what value would not change the mean using the digital platform, the students tried placing a token at every x-label until they found a location in which they perceived the platform to be balanced. To both students, using the platform felt like *"cheating"* when it produced immediate feedback.

Having observed these challenges, the Stats Teacher reflected on how *"instantaneous feedback should be earned...If it just automatically gives them that information, then they don't have the opportunity to think about it and develop their own answer."* Based on the Stats Teacher's recommendation of having students *"make up their own mind[s] first and then check,"* we modified the digital platform so students must first guess the mean by sliding an emulated fulcrum to it, then press a button to receive feedback and reflect on their performance. In response to this feature, Student 2 described that they *"had to wake [their] brain up...this is a great feature."* After students demonstrate proficiency in distribution-based statistical reasoning, instantaneous feedback can be reintroduced to facilitate quicker pattern-finding.

LT4: Structure Feedback for Reflection: Confirming rather than retrieving solutions through feedback provides space for critical reasoning and reflection.

5.3 Building Contextual Reasoning through Inquiry-Based Learning

Along with a limited conceptual understanding of the relationships between statistical measures, data values, and distributions, students initially struggled to apply and reason about the use of different measures in context.

The following subsections examine key aspects of contextual reasoning explored during the co-design process: applying learning in context, developing reasoning across measures, and demonstrating real-world relevance. Each subsection describes observed student challenges, corresponding interventions, and key learning takeaways (labeled as LTs).

5.3.1 Applying Learning in Context: Students initially had trouble properly applying and validating calculations in context, likely due to the lack of contextual information emphasized in prior learning experiences (Section 5.1.3). For both mean and median, Student 2 performed statistical computations on the labels that make up the x-axis rather than the data values themselves. In another example, Student 2 divided the total number of data values by the data range when asked to compute the mean of a weather and temperature dataset (Data Inquiry 1), resulting in a significant underestimation of the average temperature. When asked what the computed value represents, the student was unable to use context to reason about why such a value was not a likely temperature and could not possibly represent the mean.

The Stats Teacher on several occasions recommended anchoring activities back to the "*underlying goal that reminds us what we're doing. That's important.*" For example, the Stats Teacher recommended framing the exercise through contextually-motivated goals: "*Instead of [asking] what is the average of this data set? You could say: what predictions can we make about the temperature based on this data.*" Throughout the inquiry activities, we observed how framing questions around broader goals can help students apply and refine their use of different statistical measures to accomplish various tasks. Even simple interventions, such as prompting students to recall how the median splits data in half, enabled Student 2 to self-correct and compute the median of data values rather than axis labels.

LT5: Use Goal-Oriented Framing: Framing questions around inquiry goals can encourage reasoning about the proper application of statistics in context.

We learned through participants' reflections and design ideas that story-based instruction can be a powerful way to contextualize and model the application of different statistical measures. When prototyping data activities, participants created characters with goals aligned to different statistical measures, such as individuals who wanted to avoid or seek out hornets in an inquiry about a hornet epidemic (Appendix A.1). They then designed questions where learners help characters make decisions based on their goals. Additionally, they recommended incorporating "random events" to provide more contextually situated opportunities for exploring the sensitivity of various measures. These story components enabled students to use context as a way to reason about why different statistical measures might be applied. For example, after Student 1 completed a story-based activity involving an outbreak of giant hornets across Washington towns (Appendix A.1), they reflected on learning that the median "*is basically the middle number of how, like, severe the town is... [and] allowed us to put [them] into zones, like a [more severe] and [less severe] zone.*"

LT6: Reason Using Stories: Stories offer a powerful and engaging way to motivate statistical measures, encourage contextual reasoning about their significance, and demonstrate their application across a variety of scenarios.

5.3.2 Developing Reasoning Across Measures: Story scenarios also helped students make comparisons across measures, a skill they initially struggled to demonstrate due to a lack of proficiency

in relational reasoning (Section 5.2.1) and applying measures in context (Section 5.3). For example, a story event showed a surge in giant hornets in the highest outbreak cities that left the median unchanged. Students calculated updated measures of center based on the event and were asked to reflect on how citizens of Washington might interpret the data if presented with only one of the measures. Student 1 concluded the mean, unlike the median "*actually [lets citizens] all know if changes were made.*" In another story task, students role-played as characters cherry-picking statistics to sway public opinion. This exercise prompted Student 1 to reflect on how statistics can be used to mislead.

LT7: Compare Measures Using Stories: Using story scenarios can help students compare and reason across different measures of center by contextualizing their advantages and limitations in real-world scenarios.

5.3.3 Demonstrating Real-World Relevance: All participants emphasized the need for students to understand how statistical knowledge can help explain natural phenomena and inform decision-making. The TVI highlighted the importance of "*helping [students] realize that [they] learned this information and [they] can apply it to [their] lives directly.*" Over the course of the co-design, we observed how engaging them in a variety of inquiry-based activities, including case-based learning [85], scientific discovery learning [47], and problem-based learning [23], can help cement different ways statistics might be used to enhance their lives and communities. In one activity that a group of participants designed (Appendix A.2), the learner was tasked with using summary statistics to compare and draw conclusions about a real-world phenomenon. Student 2, who evaluated the activity, reflected on how they learned that they can apply statistics to investigate "*some of the most random everyday things we do!...they're based off of real trials instead of just random. So we actually tried them out and gave them real measures instead of just doing estimates and assumptions.*" In another participant-designed activity (Appendix A.3), students were given a dataset of crosswalks with Accessible Pedestrian Signal (APS) features and asked to identify interesting questions to explore. After the activity, Student 1 reflected on how such a dataset could help BLV individuals "*find a city with the most APS features,*" while Student 2 considered its social impact, noting "*It could show us how to improve our environment and make the world more welcoming for everyone.*" Having performed multiple inquiries, Student 1 summarized, "*when you learn more about the data and how to calculate it, it will help you improve and expand on what you've learned in general.*" The TVI further emphasized the opportunity to promote empathy by including sighted students in accessibility-based inquiries, which would also "*help them develop awareness of their classmates.*"

LT8: Experience Diverse Inquiries: Engaging students in various forms of inquiry can help them understand how to apply statistics to improve their lives and communities.

6 Discussion

In this section, we situate our findings within prior research to draw broader insights, offer recommendations, and propose directions for

future research. We begin by examining how the identified engagement and comprehension needs (RQ1) stem from broader structural barriers, highlighting the need for creative solutions beyond simple adaptations. We then revisit the embodied and inquiry-based approaches identified in the study (RQ2) to discuss their implications for the design of inclusive learning tools and practices.

6.1 The Need to Promote Reasoning and Understanding

From the early stages of our co-design, the experiences and reflections shared by the diverse stakeholders in our participant group quickly directed our focus toward the engagement and comprehension challenges BLV students experience (RQ1). Several, including low self-efficacy (Section 5.1.4) and conceptual understanding (Section 5.2) align with findings from prior research [102, 119, 122]. Study participants highlighted several structural barriers that contribute to these challenges.

6.1.1 Structural Barriers to Inclusive Education. Both the TVI and students noted that BLV students often lack access to engaging and effective learning tools beyond those that are easily adaptable (Section 5.1.2), a finding consistent with prior research [24, 102, 119, 133]. As a result, we observed that students' reasoning tends to be confined to explicit learning experiences that are relatively easier to adapt, such as factual knowledge conveyed by educators and computations supported by Braille and talking calculators. This becomes especially problematic in data and statistical education, which often rely on visual aids to foster engagement and build conceptual knowledge through spatial reasoning [21, 58]. The TVI shared how graphical explanations, which leverage our innate spatial reasoning abilities to understand concepts [141], are often not adapted for BLV students [24] (Section 5.2.1). Furthermore, refreshable graphical tools that can communicate the direct relationship between changes in data and statistical measures non-visually are not available in practice. There is a need for tools that both promote robust conceptual understanding [102, 122] and foster engagement for BLV students (Section 5.1.2).

Additionally, the TVI explained that BLV students often miss out on incidental learning opportunities [24, 102]. These students not only encounter data representations less frequently than their sighted peers, but also have limited exposure to peer work, as observed in our study (Section 5.1.1). Moreover, their ability to benefit from learning analogies is limited by their perceptual experiences (LT3). These gaps highlight the need to recognize how exposure and experiences shape learning and to intentionally broaden the experiences of BLV students.

6.1.2 The Need for Equivalent Learning Experiences. As we began co-developing materials to deepen reasoning, we drew significant inspiration from the Universal Design for Learning Guidelines [121]. Yet, we found the recommendation to *"ensure that key information is equally perceptible to all learners"* insufficient but a good start to promoting more inclusive learning experiences. Our concern was that simply translating key information may not support the same depth of critical thinking and reasoning. Prior research on inclusive STEM education tends to emphasize access and adaptations [51, 65, 68, 69, 100, 102, 133, 134], while reasoning is underexplored.

We propose that, beyond perceptibility, sensorimotor adaptations should strive to support the same quality and depth of reasoning, taking into account prior knowledge and experiences of learners.

Toward this goal, many open research questions remain for teaching statistics non-visually. Among them include: how might we enable the extended cognition benefits [39] that visual diagrams and digital platforms afford with non-visual representations? In our co-design, we explored this question through the interplay of enactivism and analogical thinking.

6.2 Design of Learning Tools

Enactivism (LT1) and analogical thinking (LT3) played key roles in building conceptual knowledge in our study by facilitating experimentation and supporting reasoning about the relationship between statistical measures and data points (RQ2). These features are often associated with educational technologies, particularly those involving tangible user interfaces (TUIs) [19, 20, 74, 75, 77, 97, 111, 151, 154]. After exploring a variety of learning tools with participants, we reflect on how these features can enhance statistical learning for BLV students and propose guidelines and directions for tool development and future research.

6.2.1 Benefits of Enactive Learning. In our work, we observed several benefits of using action and reflection to engage BLV students in data and statistical learning.

First, tightly integrating manipulation and feedback can enable learners to focus on identifying patterns and relationships (LT1), a finding consistent with prior research [97, 111, 151]. We found that this integration can be particularly impactful for BLV students, who often must perform additional steps to access conceptually relevant information. In our study, participants initially accessed statistical measures through manual calculations or sequential screen-reader commands, which made connecting data variations to statistical outcomes difficult. In contrast, tangible tools that directly couple manipulation with feedback promoted experimentation and pattern recognition. The three-dimensional properties of the physicalizations students created also served as cognitive aids, enabling them to reference individual data points in relation to aggregate distributions by physically enclosing specific regions with their hands (Section 5.2.1). While screen reader support is often prioritized in learning tools for BLV students, tools that simplify interactions to focus on conceptually relevant actions can reduce extraneous cognitive load and streamline enactive sense-making.

Second, simple and adaptable components that can be configured in versatile ways can support diverse learning scenarios and foster flexible and robust interpretations, as highlighted in prior research [52, 77, 78, 97, 116, 125, 154]. In our study, tokens functioned as a flexible medium, supporting various data mappings, tasks, and representations. Participants leveraged this versatility to design diverse scenarios (LT6, LT7) and forms of inquiry (LT8) that highlight the wide-ranging insights that data and statistics can provide. These findings emphasize that keeping tools extensible and adaptable allows educators and students to customize them to their specific interests and needs.

Finally, providing students with opportunities to physically enact concepts can make implicit reasoning explicit, revealing inconsistencies, conflicting beliefs, and incorrect assumptions, as noted in

prior research [99, 111, 154]. A notable example from our study occurred when Student 1 used the placement and referencing of a partitioning sticky note to correct their intuition about the types of changes that affect the median (Section 5.2.2). The knowledge "expressed" through students' actions also enabled educators and researchers to identify gaps in reasoning, such as the students' limited intuition about where to place the fulcrum to balance a system. There is potential for intelligent systems to analyze these actions, infer gaps in reasoning, and provide targeted interventions or generate reports for educators to better understand students' thought processes.

6.2.2 Reasoning Using Analogical Thinking. Embedding analogies into exploratory systems can provide additional ways for students to reason about data measures, such as tilting the physical representation to illustrate the mean as the center of mass (LT3). In doing so, we observed students using analogical thinking to deepen their understanding of statistics, such as reasoning that high values shift the mean by redistributing weight. These observations support prior theories suggesting that using conceptual metaphors to structure interaction mappings can facilitate the learning of abstract concepts [20]. However, considering students' prior exposure to analogical concepts is important (LT3). When relating the statistical mean to a distribution's center of mass, the lack of visual experience with how a fulcrum's position affects balance may limit students' ability to estimate the mean's location based on the distribution's shape. These observations underscore the importance of co-designing with BLV students. Gaining a deeper understanding of the naturalistic interactions of BLV students with their everyday environments, as explored by Chundury et al. [43], could help integrate analogical thinking more effectively into the learning process.

We also observed that engaging with conceptual metaphors rooted in image schemas can facilitate the learning of abstract concepts (LT3), consistent with prior work [20, 79]. For instance, having students physically split data with a divider reinforced their understanding of the evolution of data and statistical measures. Other actions performed with tokens during the data inquiries—such as adding, subtracting, arranging, and grouping—also closely mirrored data operations. However, as students transition to more complex datasets that cannot be easily represented with physical tokens, BLV students relying on screen readers often lose access to the schemas associated with these data operations. In contrast, operations like addition, subtraction, arrangement, and splitting remain visibly accessible for sighted learners in interactive visualizations. Exploring non-visual methods, such as haptic or auditory feedback, to evoke action-based image schemas offers a promising avenue for research. Advancing these approaches could enable BLV students to retain the cognitive benefits of embodied interactions while engaging with more complex datasets.

6.2.3 Cues for Supporting Learning. While investigating enactivism and analogies in exploratory systems, we experimented with different forms of cues that supported learning in different ways.

Inclusive cues, drawing from Universal Design for Learning (UDL) [121], ensure that activities are perceptible to all group members (ET1). For example, the sound of a spinning coin, which could be directly heard by everyone (Appendix A.2), noticeably increased engagement compared to the collaborative plotting activity (Data

Inquiry 1), where Student 1 could not perceive the changes made by Student 2.

Immersive cues incorporate sensory and narrative elements that engage learners. This type of cue, supported by multimedia learning principles [101], includes diegetic elements such as story hooks, voice impersonations, audio recordings, and data-responsive audio feedback. Students enjoyed both designing and perceiving them throughout the co-design (ET2, ET3).

Learning cues provide students with specific, constructive guidance that supports reasoning and conceptual understanding. These cues can take multiple forms. *Analogue demonstrations*, such as tilting the distribution to the mean, can support additional modes of reasoning by helping students map familiar concepts to abstract statistical principles [64] (LT3). *Information shortcuts*, such as verbalizing changes in statistical measures as data updates, can streamline complex reasoning and allow students to focus on broader patterns. However, these information shortcuts should be earned only after students have demonstrated proficiency in foundational concepts (LT4). *Confirmation of student accomplishments*, such as a celebratory chime for correct responses, can reinforce positive behavior and boost student motivation [73] (ET4).

Considering a variety of cues in learning tools can provide holistically engaging experiences while fostering deeper understanding.

6.3 Design of Learning Activities

Although we initially adopted a technology-centric approach focused on tool design, the experiences and reflections of participating stakeholders quickly highlighted the equally critical and interdependent need to design the learning activities in which these tools are embedded, an area often underexplored [20]. We observed that the careful design of tasks and exercises is essential not only to provide BLV students with sufficient scaffolds to engage in effective learning (LT2), but also to create opportunities to critically engage with the underlying contexts, a core component of statistical literacy [38, 61, 142] (LT5-LT8) (RQ2). Here, we reflect on our learnings and their implications on the design of inclusive statistical learning activities.

6.3.1 Scaffolding of Critical Problem-Solving. Special education teachers on the research team and in the study shared the importance of explicitly demonstrating concepts first before having students problem-solve on their own, particularly as the lack of incidental learning opportunities commonly requires topics to be explicitly introduced. The "*I do, we do, you do*" mantra, popularized within the Gradual Release of Responsibility (GRR) framework [57], describes a learning process in which responsibility gradually shifts from the teacher to the student. However, modeling from demonstration limits opportunities for BLV students to develop problem-solving skills—experiences that the TVI also identifies as critically lacking. Throughout the co-design process, many instances highlight how with goal-driven framing (LT5), adequate learning tools, and proper scaffolding, BLV students have tremendous problem-solving capabilities.

Co-design participants address these tensions by designing a story with enough structure to guide learners through problem-solving while illustrating how fictional characters might use statistical measures to reason about their environments (Appendix

A.1)(LT6, LT7). The subsequent participant-design activities progressively shift responsibility to students to define, frame, and conduct their own investigations (Appendix A.2, A.3) (LT8).

6.3.2 Structuring Learning Using Stories. When engaging with stories, students naturally reasoned about and applied statistical measures, even with limited prior experience making decisions with these concepts (LT6, LT7). Building on prior work that uses data storytelling to motivate inquiry and enhance communication [50, 94], our findings highlight how specific story elements can actively shape students' statistical reasoning. We identified three key design elements that directly connect statistical learning goals to story constructs:

- (1) story characters with goals that directly embody statistical concepts (i.e. one official wants to split the data in half, another official wants to inflate perceived severity through statistics).
- (2) decision-making opportunities in which learners select actions for characters based on their goals.
- (3) events that encourage ongoing exploration and reflection that reflect realistic and dynamic changes in data.

Embedding statistical reasoning within these narrative structures can make data more concrete, interpretable, and memorable for learners.

6.3.3 Learning as a Way to Care. Participants also emphasized the importance of connecting learned knowledge to the learner's life, a core aspect of Fink's taxonomy of significant learning experiences [56] (Section 5.3.3). The activities they designed reflect this principle on multiple levels—from motivating and providing opportunities to reflect on each statistical concept in context, to demonstrating and facilitating discussions on how statistical inquiry can help inform decision-making and broaden understanding of individuals and communities (LT8). Additionally, our students, like many BLV students across the world, are deeply passionate about topics relating to accessibility and inclusion. Their choice to anchor an activity on Accessible Pedestrian Signals (Appendix A.3) and engagement with the activity highlights opportunities to motivate inquiry through a critical lens [98]. Having students learn data and statistical knowledge that is framed by questions such as *"what data can do to us, what we and others can do with data, and what kind of world we can create with data"* has the potential to position students as capable agents of social change.

6.3.4 Structuring Activities for School Settings: Similar to our co-design, small-group problem-based learning [2, 5, 9, 12, 29, 45] has gained popularity for promoting student involvement, independence, and deeper learning in math classrooms [84, 146]. These methods are particularly effective in inclusive, mixed-ability environments [84, 133], especially when they emphasize social acceptance [84] and provide accessible learning materials that foster meaningful collaboration [133, 145], both elements our participants touched on.

To promote social acceptance, participants suggested framing inquiries around the lived experiences of underrepresented and marginalized students (Section 5.3.3) to amplify their perspectives

and foster empathy among peers. During the co-design, brainstorming meaningful topics, planning data collection, and investigating pedestrian signal accessibility in real-world settings fostered perspective-taking and empathetic communication among sighted and blind members of the research team (Appendix A.3). Engaging mixed-ability students in similar collaborative projects—from inquiry design and hands-on data collection to analysis and discussion—can deepen empathy, enhance perspective-taking, and promote understanding of diverse lived experiences. To ensure these activities are sensitive and effective, educators should consult privately with students beforehand to avoid approaches that might single them out.

With regard to accessible learning materials, enactive, analogical, and embedded tangible experiences explored in this work (LT1–LT4) align well with the emphasis on active experimentation in cooperative problem-based learning. However, as observed in the collaborative construction activities, maintaining joint attention and mutual awareness can be challenging for blind students who may not see the actions of their peers (ET1). Ensuring that group members are attentive to these challenges and explicit about their actions is necessary for promoting an inclusive learning environment. Meaningful ways for enabling the non-visual perception of changes during collaborative activities is an important area of active research [103].

7 Limitations and Future Work

Purposive sampling enabled us to select participants who we felt had sufficient prior experience and expertise to provide rich perspectives on the topic of inclusive math learning. We selected two students who were already acquainted, one of whom was also familiar with the TVI. These prior relationships likely promoted a more comfortable sharing and co-design environment, though at the risk of increasing the representation of experiences shared by familiar individuals. However, the fact that the two students attend different public schools, along with the TVI's extensive experience working with a diverse range of students likely helped broaden insights.

Based on the students' prior knowledge and the needs expressed by participants, our investigation prioritized building conceptual knowledge with a focus on reasoning and application in context, rather than on teaching basic definitions and procedures. This emphasis was expressed as a critical need by our educators and in prior work for both BLV [102] and sighted students [115, 143]. Our takeaways therefore assume that students already have a basic understanding of definitions and procedures.

Both student participants are early-blind and attend public schools, which is common of blind children in the US [13, 66]. However, it is essential to acknowledge the diversity of abilities and prior experiences within the BLV population. Though we do not foresee any of the takeaways and practices to exclude participants with working vision and believe that the pedagogical framings our takeaways derive from may benefit students of diverse abilities, this work does not deeply explore how visual cues and prior visual experiences might alter learning experiences. In line with existing inclusive education guidelines [8, 96], it remains crucial to observe,

recognize, and tailor learning approaches to the individual needs and preferences of students.

We anchored the co-design process around four participants. While this sample size may seem small, it is consistent with prior research involving children in participatory design [34, 37, 41, 49, 148, 150] and with studies that deeply examine the challenges and experiences of BLV students learning math [24, 102, 138]. The close collaboration enabled us to deeply engage with each participant's needs, create space to discuss and address learning challenges, and foster a stronger sense of ownership over their ideas. While generalizability is not guaranteed to broadly extend, thematic alignment between our insights and those identified by prior work [24, 102, 104, 119, 122, 133] suggest that the insights are not idiosyncratic, but represent how broader inclusive learning challenges and strategies manifest in statistical contexts. Adapting and evaluating our takeaways for classroom settings with a broader set of students with mixed abilities remains an important area of future work.

8 Conclusion

Through an extended co-design partnership, we explored instructional practices aimed at fostering robust conceptual understanding, interpretation, and critical evaluation of statistical measures for BLV students. Central to this process was incorporating "data-doing" as a recurring and intentional practice that allowed us to continually uncover, refine, and validate insights directly from participants' experiences. This recurring structure also provided students opportunities to alternate between user and designer roles, which helped them reflect more concretely on prior experiences and apply those insights to their next design.

Through these practices, we found that enactive, embedded, and analogical methods of exploration with deliberate scaffolding and well-timed feedback helped students learn and reason about the relationship between statistical measures to the underlying distribution and data. Inquiry-based approaches encouraged contextually situated reasoning and reflection on ways statistics may be applied to students' lives and communities. Additionally, sustaining inclusion, providing immersive multimodal interactions, grounding inquiry in student interests, and celebrating small wins all promote engagement. Co-designed with a dedicated group of stakeholders, these results lay the initial groundwork for an accessibility-first approach to non-visual statistics education.

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A Participant-Designed Activities: Integrated Examples

We summarize the three learning activities that participants designed for each other in Session 3 to demonstrate how the engagement and learning takeaways can be incorporated in practice. Upon the TVI's recommendation to first provide students with the *"opportunity to take some data and manipulate it into graphs or categorize it... then, to select or to gather their own data... and then do the same activity with things that they've gathered."* we sequenced the activities from a structured to more open-ended approach. The activities begin with a story-based inquiry as a concrete demonstration of ways to reason about and apply statistics in context (LT6, refer to Appendix B for a quick reference of the takeaways). Students then progress to a scientific inquiry, a slightly less structured approach, to demonstrate how statistics can be used to analyze and compare real-world phenomena. Students conclude with an open-ended, self-driven activity in which they choose a real-world topic they are passionate about, plan and conduct data collection, and

explore ways to analyze and communicate the data to deepen their understanding and/or drive social change (LT8). Participating students were able to complete and meet the learning objectives for all activities during Session 4.

A.1 A Story of Giant Hornets in Washington

Learning Objective: *Understand the purpose of measures of center and ways to reason about their sensitivity using a histogram.*

This investigation explores an infestation of giant hornets in the state of Washington, a topic students expressed interest in (ET3). As an engaging hook, the activity begins with a news clip describing the event (ET2). Students are provided with a fictitious dataset of hornet populations across eight cities in Washington. If working in groups, students should agree upon a shared strategy for organizing data and communicating changes before plotting (ET1). As a warm-up exercise, two characters are introduced through voice-acted self-introductions: one who is allergic and wants to avoid hornets and one who is an entomologist tasked with studying these hornets (LT6). Students must recommend cities for the characters to move to (LT6). A small celebration plays when students provide recommendations with reasonable justifications (ET4).

Additional characters are introduced one at a time to represent different statistical measures (LT6). First, a hornet eradicator is tasked with constructing a special trap that must just fit a city's hornet population to be effective. With the goal of eradicating hornets in most cities, students must reason about what size trap (in terms of hornet population) would help them accomplish this goal (LT5). Students then reflect on which measure of center (mode) aligns with their reasoning and what must occur for the mode to change (LT6).

Second, a state official would like to divide cities into equally sized green zones and red zones representing the most and least severe cities respectively (LT5). Students must help the official by placing a physical divider at a point on the distribution that splits the cities in half (LT3) and select a measure of center (median) that meets their objective (LT6). A set of story events causes hornet populations to change, and students must 1) update the representation to demonstrate the change (LT1) and 2) explore changes to the median using the divider (LT3). The questions are scaffolded in a way that reflects categories of observations that might or might not change the median (LT2). Students are asked to generalize on types of changes that would affect the median.

A third character would like a measure that helps them understand changes in outliers, and students must select a measure that meets this objective (mean) (LT5). Students are introduced to the center-of-balance analogy for the mean (LT3), and students must first estimate the mean based on the distribution, and then reflect on their estimation (LT4). Students can either perform the calculation mathematically, or use the digital platform (LT1) (shown in Figure 3G). Like with the median, a series of data changes motivated by story events encourage students to explore mean sensitivity along generalizable categories of scenarios (LT6). Students are celebrated for each correct question (ET4).

To motivate comparisons across the different measures, students select one of two characters who are trying to choose measures that

purposefully inflate or deflate the perceived severity of the hornet outbreak across Washington (LT7). Students flick a spinner (shown in Figure 3G) to trigger a random event that prompts a change they must make to the representation (ET2). Students, depending on their character, select and record a measure that provides the highest or lowest value. A facilitating teacher or peer selects and records for the other character. After several events, students reflect on their strategies for selection, how measures differed across events, and how statistics of the same data might be used to persuade people to feel differently about a topic.

A.2 What factors might affect coin spinning duration?

Learning Objective: Understand ways of applying measures of center to compare factors in daily phenomena.

The activity investigates factors that affect the duration of coin spinning, an activity that students found to be tactiley and auditotily engaging (ET2) and perceptible at a distance (ET1) in that they can hear the coin spinning even as others spin the coin. Students are taught how to spin coins, prompted to consider factors that might affect the spin duration, and asked to choose a factor to anchor the exploration.

To facilitate reasoning about sample size, students perform one to two spins under each factor to compare the factors. Students then reflect on how much they trust their judgment and what action (collecting more samples) might help them trust the data more. Students construct a table collecting additional samples of the data.

After data collection, students are provided with physical tokens to create distributions comparing the factors (shown in Figure 3H). If working in groups, students should agree upon a shared strategy for organizing data and communicating changes before plotting (ET1). Students must use the shape of the distribution to interpret the data (LT1).

Students then develop metrics to help them compare the chosen factors (LT5). Finally, students reflect on how statistics might be used to learn about natural phenomena around them and ways to structure data inquiry to facilitate this learning (LT5).

A.3 Accessible Pedestrian Signals

Learning Objective: Discuss ways of using data and statistics to broaden understanding and enact social change.

This activity investigates city crosswalks for accessible pedestrian signals (APS), a socially significant topic that students were passionate about (ET3). Students first brainstorm aspects of the topic they find interesting, discuss types of data they need to collect, and plan methods of collecting the data (whether in person or through searching databases). If collecting the data in person, students are encouraged to take video clips of interesting crosswalks as a way to contextualize the data points (ET2) (shown in Figure 3H).

After gathering data, students share stories about their experiences and revisit the questions they might be interested in learning having collected data (LT6). Collected video clips can be played back to help students communicate their experiences (ET2). A set of scaffolded tasks helps students structure a data inquiry (LT5), such as: Are there numbers we care about? How might we organize the

data to answer these questions? Can we plot the data in a certain way? What do we learn about data from plotting this way? What types of measures or statistics would help us better understand this information? The instructor prepares several backup questions in case students are unable to formulate inquiry questions at first.

Finally, students reflect on their findings in broader contexts (LT5). Students discuss whether the learnings might be interesting for others in their community and how they might present their findings to interested parties. The activity concludes with a reflection on how knowledge of data and statistics might help them better understand the world.

B Takeaways

Theme	Sub-theme	Takeaway
5.1 Engagement Challenges and Opportunities	5.1.1 Maintaining Focus	ET1: Ensure Inclusion: Perceptibility and mutual awareness in all components of group activities are important for maintaining focus.
5.1 Engagement Challenges and Opportunities	5.1.2 Providing Immersive Content	ET2: Support Interactive Audio-Tactile Experiences: Tangible experiences with immersive auditory feedback promote engagement and encourage experimentation.
5.1 Engagement Challenges and Opportunities	5.1.3 Ensuring Meaningful Context	ET3: Ground in Interests: Grounding activities in students' interests, particularly those with interesting audio and haptic features, promotes contextualization, storytelling, and engagement.
5.1 Engagement Challenges and Opportunities	5.1.4 Fostering Self-Efficacy	ET4: Celebrate Small Wins: Providing opportunities to auditorily celebrate the completion of smaller, goal-oriented objectives can help students confirm their reasoning and build confidence.
5.2 Building Conceptual Reasoning Through Active Exploration	5.2.1 Promoting Relational Reasoning	LT1: Embed Enactivism: Embedded and enactive approaches provide an engaging way for students to quickly experiment with and identify patterns in data sensitivity.
5.2 Building Conceptual Reasoning Through Active Exploration	5.2.2 Developing Generalizable Insights	LT2: Scaffold Along Insights: Scaffolding exploration along categories of insights can help students develop more generalized understandings. LT3: Facilitate Analogical Reasoning: Enacting, physicalizing, and embedding analogical thinking can provide additional methods of reasoning, validation, and problem-solving, but are limited by students' understanding of analogical concepts.
5.2 Building Conceptual Reasoning Through Active Exploration	5.2.3 Supporting Contemplative Experiences	LT4: Structure Feedback for Reflection: Confirming rather than retrieving solutions through feedback provides space for critical reasoning and reflection.
5.3 Building Contextual Reasoning through Inquiry-Based Learning	5.3.1 Applying Learning in Context	LT5: Use Goal-Oriented Framing: Framing questions around inquiry goals can encourage reasoning about the proper application of statistics in context. LT6: Reason Using Stories: Stories offer a powerful and engaging way to motivate statistical measures, encourage contextual reasoning about their significance, and demonstrate their application across a variety of scenarios.
5.3 Building Contextual Reasoning through Inquiry-Based Learning	5.3.2 Developing Reasoning Across Measures	LT7: Compare Measures Using Stories: Using story scenarios can help students compare and reason across different measures of center by contextualizing their advantages and limitations in real-world scenarios.
5.3 Building Contextual Reasoning through Inquiry-Based Learning	5.3.3 Demonstrating Real-World Relevance	LT8: Experience Diverse Inquiries: Engaging students in various forms of inquiry can help them understand how to apply statistics to improve their lives and communities.