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A Comparison of Methods to Calculate Spectral Kurtosis for RFI Detection

Sylvia Llosa(1), Arvind Aradhya(1), Calvin Henggeler(1), Mark Lofquist(1), and Kevin Gifford(1)
(1) University of Colorado Boulder, Boulder, CO, 80309, https://www.colorado.edu/lab/wirg/

Abstract

This research identifies and compares five different ways of calculating spectral kurtosis (sk), discussing their advantages and disadvantages in measuring Radio Frequency Interference (RFI). The traditional way of calculating spectral kurtosis is Fast Fourier Transform (FFT) Based Spectral Kurtosis, which uses an FFT to separate the signal into frequency bins, and kurtosis is then computed for each frequency component. Time-Domain Voltage Kurtosis is calculated directly from the I/Q voltage values of the signal in the time domain. Time-Domain Power Kurtosis (Instantaneous Power) is computed from the I/Q voltage values, and kurtosis is then applied to this power data. Time-Domain Power Kurtosis (Integrated Power) integrates power over time before applying kurtosis. Frequency-Separated Spectral Kurtosis (FSSK) leverages SDR technology to separate frequency directly before calculating kurtosis, eliminating the need for FFT and allowing real-time analysis. Each of these methods could be utilized in different scenarios depending on the researchers needs.

1 Introduction

Radio Frequency Interference (RFI) is an escalating issue in an increasingly digital world. Detecting RFI in real time, with maximum speed and computational efficiency, is therefore crucial. The ability to identify RFI as it occurs allows for immediate mitigation strategies, protecting sensitive radio astronomy observations and communication systems from disruptions caused by unwanted signals. Spectral kurtosis is a powerful technique for detecting and characterizing RFI in the frequency domain, making it a key tool in these efforts. By applying advanced signal processing methods such as spectral kurtosis, this work seeks to strengthen the resilience of radio systems against the growing challenges posed by RFI.

In an idealized scenario, such as at a radio astronomy facility with an exceptionally quiet signal environment, it is expected that spectral kurtosis measurements will show minimal variation between different computational methods [1]. This assumption is based on the lack of significant frequency-specific features that would otherwise lead to disproportionately high kurtosis values in certain frequency channels. Furthermore, in a noise-dominated setting, all frequency channels should closely follow a Gaussian distribution, reducing the influence of the chosen computational approach. As a result, under these conditions, spectral kurtosis should produce consistent results regardless of the specific method used for the calculation.

Spectral kurtosis as a measurement has been around since the 1980s and was developed for the detection of non-Gaussian signals in sonar systems by Dwyer [2]. Since then it has been utilized in many arenas [3] but it is a known effective method for detecting RFI [4] [5]. While multiple papers have shown that sk offers advantages over Power Spectral Density calculations [6] it is possible to achieve measurements similar to FFT based sk through instantaneous and integrated power kurtosis.

Following are the equations for the different ways of calculating spectral kurtosis, although the focus of this work is primarily the traditional FFT based sk and Frequency-Separated Spectral Kurtosis (FSSK):

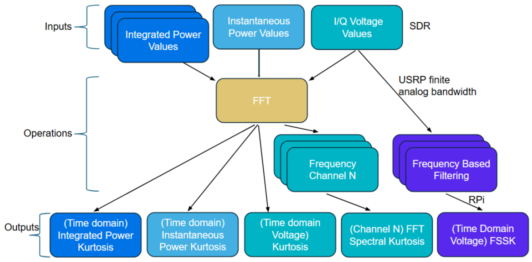

Figure 1. Five Methods of Calculating Spectral Kurtosis

1.1 FFT-Based Spectral Kurtosis (SK)

This method applies a Fast Fourier Transform (FFT) to decompose the signal into frequency bins and then computes kurtosis for each bin:

$$SK(f) = \frac{\langle |X(f)|^4 \rangle}{\langle |X(f)|^2 \rangle^2} - 2$$

where:

- $X(f)$ is the FFT of the time-domain signal $x(t)$.
- $\langle \cdot \rangle$ denotes an expectation (ensemble average) over multiple time segments.

- The subtraction of 2 ensures that SK(f) is zero for Gaussian noise.

1.2 Time-Domain Voltage Kurtosis

This method computes kurtosis directly from the time-domain I/Q voltage samples.

$$K_v = \frac{\mathbb{E}[V^4]}{(\mathbb{E}[V^2])^2}. \tag{1}$$

where:

- V represents the I/Q voltage values of the signal.
- $\mathbb{E}[\cdot]$ represents the expectation (mean) operator.

For Gaussian noise, $K_v \approx 3$. Deviations from this indicate non-Gaussianity, which may suggest the presence of RFI.

1.3 Time-Domain Power Kurtosis (Instantaneous Power)

Here, the power is calculated instantaneously from the I/Q voltage values before applying kurtosis.

$$K_P = \frac{\mathbb{E}[P^4]}{(\mathbb{E}[P^2])^2} \tag{2}$$

For Gaussian noise, K_P should be close to 9, as power follows a chi-square distribution with 2 degrees of freedom.

1.4 Time-Domain Power Kurtosis (Integrated Power)

This method integrates signal power over a time window before computing kurtosis.

$$K_{P_{int}} = \frac{\mathbb{E}[(\sum P)^4]}{(\mathbb{E}[(\sum P)^2])^2}. \tag{3}$$

1.5 Frequency-Separated Spectral Kurtosis (FSSK)

This method separates frequencies using an SDR (Software-Defined Radio) before computing kurtosis, bypassing FFT.

$$FSSK(f) = \frac{\mathbb{E}[P_f^4]}{(\mathbb{E}[P_f^2])^2} - 2. \tag{4}$$

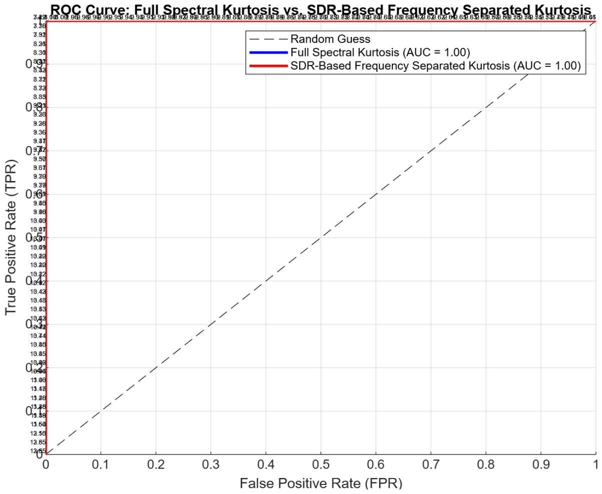

Figure 2. ROC Curve Comparison of FFT-based sk vs. FSSK

For a complex Gaussian random variable $X(f)$, the fourth moment and the second moment are related by

$$\frac{\langle |X(f)|^4 \rangle}{\langle |X(f)|^2 \rangle^2} = 2$$

By subtracting 2 in the spectral kurtosis definition we get $SK(f) = 2 - 2 = 0$ which means that a purely Gaussian noise (which is the reference case) will yield a spectral kurtosis of 0. Any deviation from 0 indicates non-Gaussianity (for example, the presence of radio frequency interference).

2 Analysis

Various RFI noise signals were generated in Matlab and the following measurements were made of the five different methods of kurtosis. Computation time was measured using MATLAB's built-in tic and toc functions with lower computation time means the method is more efficient and can be used in real-time applications. To measure the accuracy of each kurtosis calculation method against the expected theoretical kurtosis of Gaussian noise (which should be 3), we use the absolute deviation formula:

$$Accuracy = |K_{\text{measured}} - K_{\text{theoretical}}|. \tag{5}$$

The statistical variance was calculated by running a hundred trials generating a new RFI signal every trial, computing the kurtosis for each method on those rfi signals and taking the variance across all of the trials.

As shown in the Receiver Operating Characteristic (ROC) in Figure 2 both FFT-based sk and FSSK perform extremely well, as their curves hug the top-left corner and the area under the curve values being close to 1 indicate that both models have excellent discriminative power. Full Spectral Kurtosis is slightly better, but the SDR-based approach is nearly equivalent, which is promising for real-time applications.

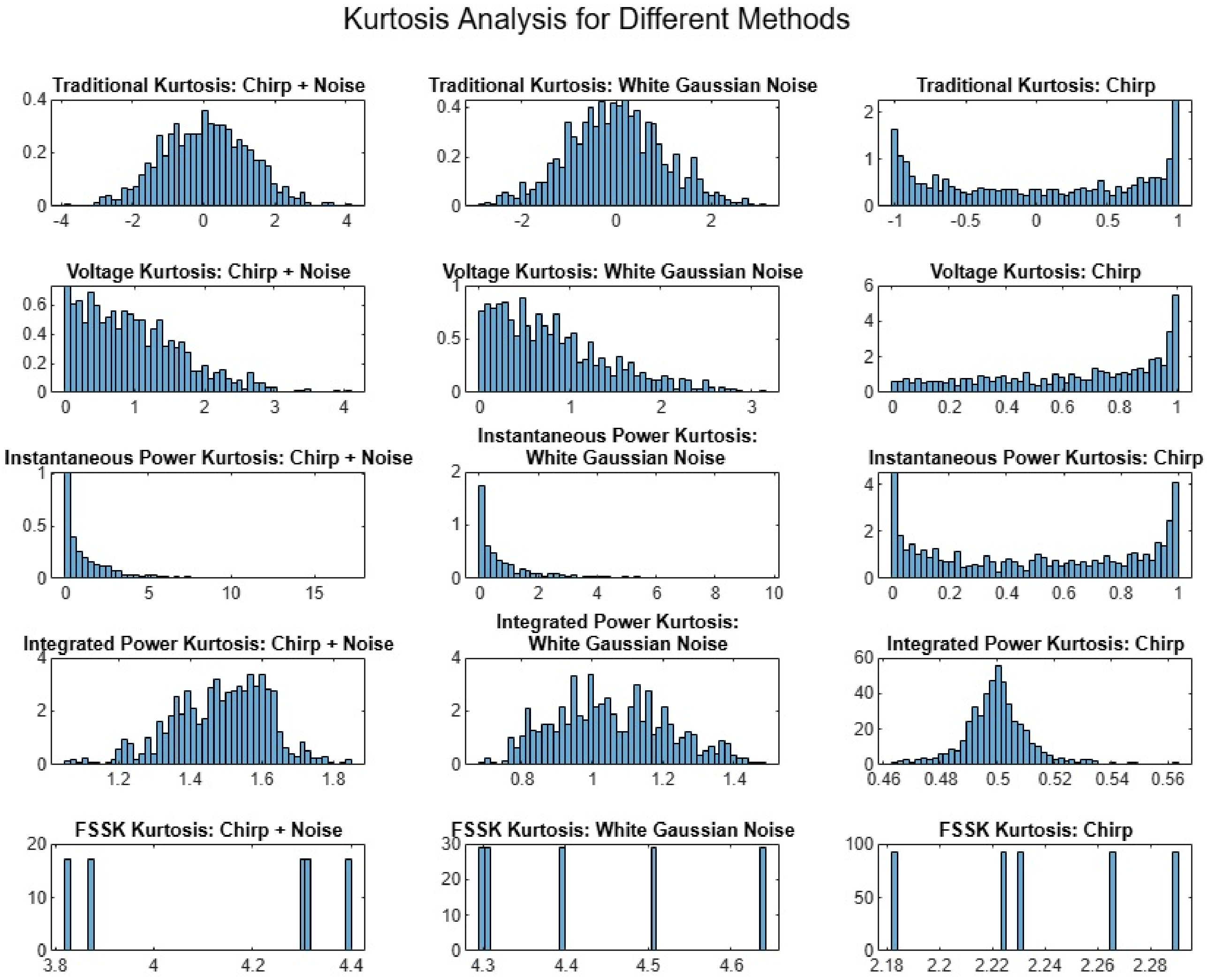

Figure 3. Comparison of five methods of calculating spectral kurtosis with three different chirps.

Method	Kurtosis Value	Computation Time (s)	Accuracy	Statistical Variability
FFT-based	1.01343	0.00574	1.98657	0.00095
Time-Domain Voltage	3.33056	0.00238	0.33056	0.01591
Instantaneous Power	10.58970	0.00061	7.58970	2.26370
Integrated Power	2.42394	0.00052	0.57606	0.15629
FSSK	3.42767	0.00149	0.42767	0.01350

Table 1. Comparison of different spectral kurtosis methods.

3 Results

In Table 1 instantaneous power kurtosis (0.00061 s) and integrated power kurtosis (0.00052 s) are shown to be the fastest methods, making them computationally efficient for real-time applications. Time-Domain voltage kurtosis (0.00238 s) and FSSK (0.00149 s) are relatively fast but slower than the power-based methods. FFT-based spectral kurtosis (0.00574 s) is moderately fast, but has an extreme computational load, which may indicate inefficiency in handling large datasets.

The time-domain voltage kurtosis (3.33056) and FSSK (3.42767) values for Kurtosis are closest to the expected Gaussian kurtosis value of 3, suggesting that they provide the most accurate results in this dataset. Instantaneous power kurtosis (10.58970) and FFT-based spectral kurtosis (1.01343) show significant deviations from 3, meaning that they may not be reliable for estimating kurtosis in this context. Integrated Power Kurtosis (2.42394) is somewhat close to 3 but still has a higher deviation than FSSK.

Instantaneous power kurtosis (2.26370) has the highest statistical variability, making it unstable and less reliable. Integrated power kurtosis (0.15629) and time-domain voltage kurtosis (0.01591) have moderate statistical variability, suggesting that they produce consistent results. FSSK (0.01350) and FFT-based sk (0.00095) are the most stable.

As shown in Figure 3 traditional kurtosis shows clear differences between noise and RFI, but its effectiveness can be reduced when mixed with noise.Voltage Kurtosis exhibits a more skewed distribution, suggesting it reacts strongly to amplitude variations.Instantaneous Power Kurtosis has a highly skewed response, particularly under RFI conditions, making it useful for detecting impulsive signals. Integrated Power Kurtosis provides a more stable distribution with improved separation between noise and RFI. FSSK is the most distinctive, producing discrete peaks that effectively separate RFI from noise, showing its strength in frequency-specific analysis.

4 Conclusion

Out of the five methods of calculating spectral kurtosis examined in this paper, the traditional FFT-based Spectral Kurtosis is the most computationally expensive and the most stable but the alternatives have comparable accuracy. If speed is the highest priority, integrated power kurtosis could be a good alternative. FSSK is computationally efficient and suitable for real-time SDR-based monitoring but may be affected by filtering accuracy, while FFT-based SK offers higher accuracy and robustness to RFI at the cost of increased computation. FSSK seems to be the best overall method for accuracy and stability while keeping computation time reasonable and can be implemented with commercial off the shelf (COTS) components and software.

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