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A Fair Detection Strategy of an Adversary

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***Abstract*— Detecting malicious users or unauthorized activities poses a critical challenge in the realm of dynamic spectrum access. Traditionally, in such a problem, an intrusion detection system (IDS) aims to maximize the detection probability. Meanwhile in the networks or radio spectrum problems with multiple nodes or bands, respectively, protocol maximizing detection probability might lead to focusing on scanning the most plausible nodes or bands for intrusion and neglecting to scan less plausible nodes or bands for intrusion due to restricted scanning resources. To deal with this challenge in this paper we suggest a protocol maximizing fairness of detection probabilities among all the bands in the bandwidth. We consider α-fairness as fairness criteria. Moreover, the proposed detection protocol deals with the adversary, who endorses artificial intelligence (AI) enabling the adversary to not only infiltrate the bandwidth without being detected but also to do so in a less predictable manner for the IDS. The problem is modeled and solved in the framework of a two-player game. An advantage of fairness detection probability protocol in comparison with maximizing detection probability protocol is illustrated.**

***Index Terms*— Detection probability, Equilibrium, Entropy, Fairness**

I. Introduction

The wireless medium's inherent openness, while providing numerous advantages via dynamic spectrum access, also makes cognitive radios a potent instrument for carrying out malicious actions or violating policies by secondary users. That is why, the development of an IDS capable of detecting and identifying illegal or malicious activity in the radio spectrum poses a significant challenge [1]. Game theory has been widely implemented to model security issues and to develop security protocols, as these problems involve different agents with different objectives. For instance, one agent (say, an adversary) aims to infiltrate the radio spectrum, while the other agent (say, the IDS) aims to detect such illegal intrusion. As examples of implementing game theory in such security issues let us refer to [2]–[8]. In all of these papers, the IDS aims to maximize the detection probability of an intruder (adversary). Meanwhile in the networks or radio spectrum problems with multiple nodes or bands, respectively, maximizing detection probability strategies might lead to focusing on scanning the most plausible nodes or bands for intrusion and neglecting to scan less plausible nodes or bands for intrusion. Such neglecting to adequately scan for potential threats could have serious implications for the security of the network or bandwidth since via changing behavior, the adversary might sneak safely within missed nodes or bands by such protocol. Motivated by this observation, in this paper, we suggest a protocol that generalizes the protocol of maximizing detection probability to the protocol maximizing fairness of detection probabilities among all the bands in the bandwidth. We consider α-fairness as fairness criteria. It is worth noting that the model [8] is a boundary case of the considered model in this paper when the fairness coefficient is equal to zero. Moreover, the proposed detection protocol deals with the adversary, who endorses artificial intelligence (AI) enabling the adversary to not only infiltrate the bandwidth without being detected but also to do so in a less predictable manner for the IDS. The problem is modeled and solved in a two-player game-theoretical framework. The equilibrium is designed, and its uniqueness is proven. The scanning protocol's stability is verified by the proven uniqueness of the equilibrium. An advantage of fairness detection probability protocol in comparison with maximizing detection probability protocol is illustrated.

II. Game theoretical model

In this section, we describe the considered game theoretical model of bandwidth scanning to detect an adversary. The bandwidth consists of frequency bands $\mathcal{K} \triangleq \{1, 2, \dots, K\}$. The *adversary* will try to sneak into one of the bands for its illegal usage. Meanwhile, the primary user equipped by the IDS wants to detect the adversary via scanning the bandwidth. That is why we call the primary user by *scanner*. It is assumed that the scanner is capable of scanning only one band at a time to identify any suspicious behavior. Let the scanner scan band k and the adversary sneak into band m. Then, probability $\rho(k, m)$ that the adversary will be detected is equal to γ_k with $\gamma_k \in (0, 1)$, if $k = m$, i.e., if the adversary sneaks into the same band i which is scanned by the scanner. Otherwise, i.e., if $k \neq m$, the detection probability is zero. Thus,

$$\rho(k, m) = \begin{cases} \gamma_k, & k = m, \\ 0, & k \neq m. \end{cases} \quad (1)$$

Note that these detection probabilities are closely related to the associated signal-to-interference and noise ratio

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(SINR) [9].

A. Scanner's strategy and adversary's strategy

Let a scanner's strategy be $\boldsymbol{p} = (p_1, \dots, p_K)$ where p_k is the probability of scanning the band k. Thus,

$$\sum_{k\in\mathcal{K}} p_k = 1 \text{ and } p_k \geq 0 \text{ for } k \in \mathcal{K}. \tag{2}$$

Let an adversary's strategy be $\boldsymbol{q} = (q_1, \dots, q_K)$ where q_k is the probability of sneaking into band k. Thus,

$$\sum_{k\in\mathcal{K}} q_k = 1 \text{ and } q_k \geq 0 \text{ for } k \in \mathcal{K}. \tag{3}$$

So, the set all probability vectors in $\mathbb{R}^K$, which we denote by $\mathcal{P}$, is the set of feasible strategies for both players.

B. Scanner's payoff

The probability for the adversary to be detected at band k, if the scanner uses strategy $\boldsymbol{p} = (p_1, \dots, p_K)$ and adversary uses strategy $\boldsymbol{q} = (q_1, \dots, q_K)$ is given as follows:

$$\mathbf{P}_k(\boldsymbol{p}, \boldsymbol{q}) = \gamma_k p_k q_k, \tag{4}$$

and detection probability of the adversary in all the bandwidth is given as follows:

$$\mathbf{P}(\boldsymbol{p}, \boldsymbol{q}) = \sum_{i\in\mathcal{K}} \mathbf{P}_k(\boldsymbol{p}, \boldsymbol{q}) = \sum_{k\in\mathcal{K}} \gamma_k p_k q_k. \tag{5}$$

Since the detection probability (5) is linear on the scanner's strategy $\boldsymbol{p}$, it might lead to strategy maximizing detection probability which is scanning the only band within the bandwidth and neglecting to scan the others. Such an optimizing strategy might lead to a drastic consequence for all the protected bandwidth, since via changing behavior, the adversary might sneak within missed bands by such scanning protocol. That is why it is important to suggest such a scanning protocol that allows us to support scanning all the bands. Motivated by this observation we suggest a protocol maximizing fairness of detection probability among all the bands in the bandwidth. Specifically, as the scanner's payoff we consider α-fair criteria (please see, for example, [10] and [11] for optimization and game-theoretical scenarios) allocating detection probabilities $\mathbf{P}_k(\boldsymbol{p}, \boldsymbol{q})$, $k \in \mathcal{K}$ among all the bands if the scanner and adversary apply strategies $\boldsymbol{p}$ and $\boldsymbol{q}$, respectively, i.e.,

$$V_{S,\alpha}(\boldsymbol{p}, \boldsymbol{q}) \triangleq \sum_{i\in\mathcal{K}} \varphi_\alpha\left(\mathbf{P}_k(\boldsymbol{p}, \boldsymbol{q})\right), \tag{6}$$

where α-fairness utility $\varphi_\alpha(\cdot)$ is given as follows:

$$\varphi_\alpha(\eta) \triangleq \begin{cases} \dfrac{\eta^{1-\alpha}}{1-\alpha}, & \alpha \neq 1, \\ \ln(\eta), & \alpha = 1. \end{cases} \tag{7}$$

C. Adversary's payoff

The adversary, besides aiming to be undetected, wants to achieve it in the most unpredictable way for the scanner. Such unpredictability in the adversary's strategy for the scanner is measured by the entropy of its strategy (please see also [12], [13] as application of entopy), i.e.,

$$\mathcal{E}(\boldsymbol{q}) = -\sum_{k\in\mathcal{K}} q_k \ln(q_k). \tag{8}$$

For that reason, the payoff to the adversary is taken as a weighted sum of the entropy of its strategy and the negative of its detection probability, i.e.,

$$V_A(\boldsymbol{p}, \boldsymbol{q}) = -(1-w)\mathbf{P}(\boldsymbol{p}, \boldsymbol{q}) + w\mathcal{E}(\boldsymbol{q}), \tag{9}$$

where $1 - w$ and w, with $w \in [0, 1]$, are normalized weighting coefficients.

D. Nash equilibrium and its existence

To complete the definition of the game with scanner and adversary as players we have to define what type of equilibrium we look for. Here each of the players, the scanner and adversary, wants to maximize its payoff, i.e., look for a (Nash) equilibrium [14]. Thus for such strategies $\boldsymbol{p}$ and $\boldsymbol{q}$ of the players that each of them is the best response to the other, i.e., $\boldsymbol{p}$ and $\boldsymbol{q}$ are the solution of the best response equations:

$$\boldsymbol{p} = \text{argmax}\{V_{S,\alpha}(\boldsymbol{p}, \boldsymbol{q}) : \boldsymbol{p} \in \mathcal{P}\}, \tag{10}$$

$$\boldsymbol{q} = \text{argmax}\{V_A(\boldsymbol{p}, \boldsymbol{q}) : \boldsymbol{q} \in \mathcal{P}\}. \tag{11}$$

Proposition 1: This game has is at least one equilibrium.

Proof: Payoff $V_{S,\alpha}(\boldsymbol{p}, \boldsymbol{q})$ is concave in $\boldsymbol{p}$ and payoff $V_A(\boldsymbol{p}, \boldsymbol{q})$ is concave in $\boldsymbol{q}$, since

$$\frac{\partial V^2_{S,\alpha}(\boldsymbol{p}, \boldsymbol{q})}{\partial p_k^2} = -\alpha \frac{(\gamma_k q_k)^{1-\alpha}}{p_k^{\alpha+1}} < 0, \tag{12}$$

$$\frac{\partial V^2_A(\boldsymbol{p}, \boldsymbol{q})}{\partial q_k^2} = -\frac{w}{q_k} < 0 \text{ for } k \in \mathcal{K}. \tag{13}$$

Then, the Nash Theorem [14] implies the result. ∎

III. Equilibrium strategies

In this section, we derive equilibrium strategies and verify their uniqueness using a constructive approach in three steps: (A) we derive the dependence of the solution of best response equations (10) and (11), which are Nonlinear programming (NLP) problems, on their Lagrange multipliers θ and ν (please, see (28) and (30) below), respectively; (B) we establish the auxiliary monotonicity properties of the solution on such Lagrange multipliers and (C) we suggest a modified Gaussian elimination method to find these Lagrange multipliers, and, so, to find the equilibrium, and based on the established monotonicity properties we verify its uniqueness.

A. Solution of the best response equations

In the following proposition, we establish the dependence of the solution of NLP problems (10) and (11), i.e., the best response equations, on their Lagrange multipliers. In other words, we derive the dependence of equilibrium on the corresponding Lagrange multipliers.

PROPOSITION 2: *Let $\alpha \neq 1$. Probabilities vectors $\boldsymbol{p} = (p_1, \dots, p_K)$ and $\boldsymbol{q} = (q_1, \dots, q_K)$ are equilibrium strategies if and only if they are given as follows:*

$$p_k = P_k(\theta, \nu), \tag{14}$$
$$q_k = Q_k(\theta, \nu), \quad k \in \mathcal{K}, \tag{15}$$

where $P_k(\theta, \nu)$ and $Q_k(\theta, \nu)$ are unique roots in (0,1) of Eqn. (16) and Eqn. (17), respectively:

$$\mathcal{F}_k(P_k(\theta, \nu), \theta) = \nu, \tag{16}$$
$$\mathcal{G}_k(Q_k(\theta, \nu), \theta) = \nu, \tag{17}$$

where

$$\mathcal{F}_k(\xi, \theta) \triangleq -(1-w)\gamma_k \xi - w - w \ln\left(\frac{\xi^{\alpha/(1-\alpha)}\theta^{1/(1-\alpha)}}{\gamma_k}\right), \tag{18}$$

$$\mathcal{G}_k(\xi, \theta) \triangleq -(1-w)\gamma_k \frac{\gamma_k^{1/\alpha}\xi^{(1-\alpha)/\alpha}}{\theta^{1/\alpha}} - w - w\ln(\xi). \tag{19}$$

Furthermore, this θ and ν are given as a solution of the following equations:

$$\mathbb{P}(\theta, \nu) = 1, \tag{20}$$
$$\mathbb{Q}(\theta, \nu) = 1 \tag{21}$$

subject to

$$\theta > 0, \tag{22}$$
$$\nu \geq \underline{\nu}, \tag{23}$$

where

$$\mathbb{P}(\theta, \nu) \triangleq \sum_{k \in \mathcal{K}} P_k(\theta, \nu), \tag{24}$$
$$\mathbb{Q}(\theta, \nu) \triangleq \sum_{k \in \mathcal{K}} Q_k(\theta, \nu), \tag{25}$$

$$\underline{\nu} \triangleq -w - (1-w)/\overline{\gamma} - w \sum_{k \in \mathcal{K}} \ln\left(1/(\gamma_k \overline{\gamma})\right)/(\gamma_k \overline{\gamma}), \tag{26}$$

$$\overline{\gamma} \triangleq \sum_{k \in \mathcal{K}} (1/\gamma_k). \tag{27}$$

PROOF: Let $\mathcal{L}_{S,\alpha,\theta}(\boldsymbol{p})$ be Lagrangian of the NLP problem (10) and θ be its Lagrange multiplier, i.e.:

$$\mathcal{L}_{S,\alpha,\theta}(\boldsymbol{p}) = V_{S,\alpha}(\boldsymbol{p}, \boldsymbol{q}) + \theta\left(1 - \sum_{k \in \mathcal{K}} p_k\right). \tag{28}$$

Then, scanner's strategy $\boldsymbol{p} \in \mathcal{P}$ is the best response to $\boldsymbol{q}$ if and only if the following condition holds:

$$\frac{\partial \mathcal{L}_{S,\theta}(\boldsymbol{p})}{\partial p_k} = \frac{(\gamma_k q_k)^{1-\alpha}}{p_k^\alpha} - \theta \begin{cases} = 0, & p_k > 0, \\ \leq 0, & p_k = 0. \end{cases} \tag{29}$$

Similarly, let $\mathcal{L}_{A,\nu}(\boldsymbol{q})$ be Lagrangian of the NLP problem (11) and ν be its Lagrange multiplier, i.e.:

$$\mathcal{L}_{A,\nu}(\boldsymbol{q}) = V_A(\boldsymbol{p}, \boldsymbol{q}) + \nu\left(1 - \sum_{k \in \mathcal{K}} q_k\right). \tag{30}$$

Then, adversary's strategy $\boldsymbol{q} \in \mathcal{P}$ is the best response for $\boldsymbol{p}$ if and only if the following condition holds:

$$\frac{\partial \mathcal{L}_{A,\nu}(\boldsymbol{q})}{\partial q_k} = -(1-w)\gamma_k p_k - w - w\ln(q_k) - \nu \begin{cases} = 0, & q_k > 0, \\ \leq 0, & q_k = 0. \end{cases} \tag{31}$$

Since $\boldsymbol{p} \in \mathcal{P}$ and $\boldsymbol{q} \in \mathcal{P}$, by (29) and (31), we have that

$$0 < p_k < 1 \text{ and } 0 < q_k < 1 \text{ for all } k \in \mathcal{K}. \tag{32}$$

By (32), conditions (29) and (31), respectively, turn into the following equations:

$$(\gamma_k q_k)^{1-\alpha}/p_k^\alpha = \theta, \quad k \in \mathcal{K}, \tag{33}$$
$$-(1-w)\gamma_k p_k - w - w\ln(q_k) = \nu, \quad i \in \mathcal{K}. \tag{34}$$

Then, by (32) and (33), we have that Ineq. (22) holds.

Dividing both sides of Eqn.(34) by γ_k implies

$$-(1-w)p_k - \frac{w}{\gamma_k} - w\frac{\ln(q_k)}{\gamma_k} = \frac{\nu}{\gamma_k}, \quad k \in \mathcal{K}. \tag{35}$$

Summing up (35) over $i \in \mathcal{K}$ and taking into account that $\boldsymbol{p} \in \mathcal{P}$ and notation (27) implies

$$-(1-w) - w\overline{\gamma} - w\sum_{i \in \mathcal{K}} \frac{\ln(q_k)}{\gamma_k} = \nu\overline{\gamma}. \tag{36}$$

Note that $\Phi(\boldsymbol{q}) \triangleq -\sum_{k \in \mathcal{K}} \ln(q_k)/\gamma_k$ is a convex function on $\boldsymbol{q}$ and it achieves its minimum in $\mathcal{P}$ at $\boldsymbol{q} = (1/(\gamma_1\overline{\gamma}), \dots, 1/(\gamma_K\overline{\gamma}))$. This and (36) yield (23).

Solving (33) for p_k implies

$$p_k = (\gamma_k q_k)^{(1-\alpha)/\alpha}/\theta^{1/\alpha}. \tag{37}$$

Substituting p_k given by (37) into (35) implies

$$-(1-w)\gamma_k \frac{\gamma_k^{1/\alpha} q_k^{(1-\alpha)/\alpha}}{\theta^{1/\alpha}} - w - w\ln(q_k) = \nu, k \in \mathcal{K}, \tag{38}$$

and (15) with (17) and (19) follow.

Solving (37) for q_k implies

$$q_k = p_k^{\alpha/(1-\alpha)}\theta^{1/(1-\alpha)}/\gamma_k. \tag{39}$$

Finally, substituting (39) into (34) implies (14) with (16) and (18). ■

B. Monotonicity properties of $\mathbb{P}(\theta,\nu)$ and $\mathbb{Q}(\theta,\nu)$

In this section, we establish auxiliary properties allowing to verify further that the Gaussian elimination method to solve Eqn. (20) and Eqn. (21) leads to the unique equilibrium.

Proposition 3: Let $\alpha < 1$. Then functions $\mathbb{P}(\theta,\nu)$ and $\mathbb{Q}(\theta,\nu)$ have the following properties:

(a) Function $\mathbb{P}(\theta,\nu)$ is continuous and strictly decreasing on both parameters θ and ν;

(b) Function $\mathbb{Q}(\theta,\nu)$ is continuous and strictly increasing on θ and decreasing on ν.

Proof: Note that for $\alpha < 1$, by (18), function $\mathcal{F}_k(\xi,\theta)$, $k\in\mathcal{K}$ is continuous while $\xi\in(0,1)$ and $\theta>0$. Moreover, it has two properties (P-$\mathcal{F}$-1) and (P-$\mathcal{F}$-2) given below:

(P-$\mathcal{F}$-1) *For a fixed $\theta>0$ function $\mathcal{F}_k(\xi,\theta)$ is strictly decreasing on ξ from $\lim_{\xi\downarrow 0}\mathcal{F}_k(\xi,\theta)=\infty$ to*

$$\mathcal{F}_k(1,\theta) = -(1-w)\gamma_k - w + w\ln(\gamma_k) - \ln(\theta)/(1-\alpha); \tag{40}$$

(P-$\mathcal{F}$-2) *For a fixed $\xi\in(0,1)$ function $\mathcal{F}_k(\xi,\theta)$ is strictly decreasing on θ from $\lim_{\theta\downarrow 0}\mathcal{F}_k(\xi,\theta)=\infty$ to $\lim_{\theta\uparrow\infty}\mathcal{F}_k(\xi,\theta)=-\infty$.*

By (P-$\mathcal{F}$-1), we have that for each ν such that

$$\nu \ge -(1-w)\gamma_k - w + w\ln(\gamma_k) - \ln(\theta)/(1-\alpha) \tag{41}$$

there is such $P_k(\theta,\nu)$ that

$$\mathcal{F}_k(P_k(\theta,\nu),\theta) = \nu, \tag{42}$$

and this $P_k(\theta,\nu)$ can be found by the bisection method. Furthermore, this $P_k(\theta,\nu)$, $k\in\mathcal{K}$, has three properties (P-P-1)-(P-P-3) given below:

(P-P-1) *$P_k(\theta,\nu)$ is continuous and strictly decreasing on both parameters θ and ν;*

(P-P-2) *$\lim_{\nu\uparrow\infty}P_k(\theta,\nu)=0$ and $P_k(\theta,\nu)=1$ for $\nu = -(1-w)\gamma_k - w + w\ln(\gamma_k) - \ln(\theta)/(1-\alpha)$;*

(P-P-3) *All these $P_k(\theta,\nu)$, $k\in\mathcal{K}$, are correctly defined for each $\theta>0$ and*

$$\nu + \ln(\theta)/(1-\alpha) \ge \max_{k\in\mathcal{K}}\{-(1-w)\gamma_k + \ln(\gamma_k/e)\}. \tag{43}$$

This and (16) imply that $P_k(\theta,\nu)$ for each $k\in\mathcal{K}$ is continuous and strictly decreasing on both parameters θ and ν. Then $\mathbb{P}(\theta,\nu)$ has the same monotonicity properties as sum of functions $P_k(\theta,\nu)$.

By (19), function $\mathcal{G}_k(\xi,\theta)$, $k\in\mathcal{K}$ is continuous for $(\xi,\theta)\in(0,1)\times(0,\infty)$, and it has two properties (P-$\mathcal{G}$-1) and (P-$\mathcal{G}$-2) given below:

(P-$\mathcal{G}$-1) *For a fixed θ function $\mathcal{G}_k(\xi,\theta)$ is strictly decreasing on ξ from $\lim_{\xi\downarrow 0}\mathcal{G}_k(\xi,\theta)=\infty$ to*

$$\mathcal{G}_k(1,\theta) = -(1-w)\gamma_k^{1/\alpha}/\theta^{1/\alpha} - w; \tag{44}$$

(P-$\mathcal{G}$-2) *For a fixed $\xi\in(0,1)$ function $\mathcal{G}_k(\xi,\theta)$ is strictly increasing on θ from $\lim_{\theta\downarrow 0}\mathcal{G}_k(\xi,\theta)=-\infty$ to $\lim_{\theta\uparrow\infty}\mathcal{G}_k(\xi,\theta)=-w-w\ln(\xi)$.*

By (P-$\mathcal{G}$-1), we have that for each ν such that

$$\nu \ge -(1-w)\gamma_k^{1/\alpha}/\theta^{1/\alpha} - w \tag{45}$$

there is $\xi = Q_k(\theta,\nu)$ such that

$$\mathcal{G}_k(Q_k(\theta,\nu),\theta) = \nu. \tag{46}$$

This $Q_k(\theta,\nu)$ can be found via the bisection method.

Finally, these $Q_k(\theta,\nu)$, $k\in\mathcal{K}$, have three properties (P-Q-1)-(P-Q-3) given below:

(P-Q-1) *$Q_k(\theta,\nu)$ is continuous strictly increasing θ and decreasing on ν;*

(P-Q-2) *$\lim_{\nu\uparrow\infty}Q_k(\theta,\nu)=0$ and $Q_k(\theta,\nu)=1$ for $\nu = -(1-w)\gamma_k^{1/\alpha}/\theta^{1/\alpha} - w$;*

(P-Q-3) *All these $Q_k(\theta,\nu)$, $k\in\mathcal{K}$, are correctly defined for each $\theta>0$ and*

$$\nu + (1-w)\max_{k\in\mathcal{K}}\gamma_k^{1/\alpha}/\theta^{1/\alpha} \ge -w. \tag{47}$$

Then, by (25), $\mathbb{Q}(\theta,\nu)$ also is continuous and strictly increasing on θ and decreasing on ν, and (b) follows. ∎

C. Equilibrium and its uniqueness

In the following theorem, equilibrium is found and its uniqueness is proven.

Theorem 1: Let $\alpha < 1$. Then equilibrium is an unique. Moreover, scanner's and adversary's equilibrium strategies are $(P_1(\theta_*,\nu_*),\dots,P_K(\theta_*,\nu_*))$ and $(Q_1(\theta_*,\nu_*),\dots,Q_K(\theta_*,\nu_*))$, respectively, with their entries given by Proposition 2, and θ_* and ν_* uniquely given by (48) and (52) below.

Specifically, θ_* is an unique positive root of the following equation

$$Z(\theta_*) = 0 \tag{48}$$

with

$$Z(\theta) \triangleq \theta - \Theta(\mathcal{N}(\theta)), \tag{49}$$

where $\Theta(\nu)$ is the unique root for a fixed ν of the equation

$$\mathbb{P}(\Theta(\nu),\nu) = 1, \tag{50}$$

and $\mathcal{N}(\theta)$ is the unique root for a fixed θ of the equation

$$\mathbb{Q}(\theta,\mathcal{N}(\theta)) = 1. \tag{51}$$

Moreover, this unique root θ_* of Eqn. (48) can be found by the bisection method, and ν_* is given as follows:

$$\nu_* = \mathcal{N}(\theta_*). \tag{52}$$

Note that, by Proposition 3, this $\Theta(\nu)$ and $\mathcal{N}(\nu)$ as the unique roots of equations (50) and (51), respectively, can be found via the bisection method.

Proof of Theorem 1: By Proposition 3, we have that scanner's and adversary's equilibrium strategies are $(P_1(\theta,\nu),\dots,P_K(\theta,\nu))$ and $(Q_1(\theta,\nu),\dots,Q_K(\theta,\nu))$, respectively, with their entries given by Proposition 2 where θ and ν are solution of Eqn. (20) and Eqn. (21). By Proposition 3, function $\mathbb{P}(\theta,\nu)$ decreases on both parameters θ and ν. Then for a fixed ν there is the unique root

$$\theta = \Theta(\nu) \tag{53}$$

of Eqn. (21). Moreover, the bisection method converges to this unique root of Eqn. (21). Thus, (50) holds. Moreover, since function $\mathbb{P}(\theta, \nu)$ decreases on parameters θ as well as on ν, such root $\Theta(\nu)$ of Eqn. (21) decreases on ν.

By Proposition 3, function $\mathbb{Q}(\theta, \nu)$ is strictly increasing on θ and decreasing on ν. Then for a fixed θ there is the unique root

$$\nu = \mathcal{N}(\theta) \tag{54}$$

of Eqn. (20), and the bisection method can be applied to find it . Thus, (51) holds. Moreover, since $\mathbb{Q}(\theta, \nu)$ is continuous and strictly increasing on θ and decreasing on ν, such root $\mathcal{N}(\theta)$ of Eqn. (20) is continuous strictly increasing on θ.

Substituting (54) into (53) implies that θ has to be the root of Eqn. (48) with $Z(\theta)$ given by (49). Note that $\Theta(\mathcal{N}(\theta))$ is decreasing function as a superposition of decreasing function $\Theta(\cdot)$ and increasing function $\mathcal{N}(\cdot)$. Thus, $Z(\theta)$ given by (49) increases on θ, and, so, the root of Eqn. (48) is an unique. ■

Theorem 2: Let $\alpha = 1$. Then scanner' equilibrium strategy $\boldsymbol{p} = (p_1, \dots, p_K)$ and adversary's equilibrium strategy $\boldsymbol{q} = (q_1, \dots, q_K)$ are unique and given as follows:

$$p_k = \frac{1}{K}, \tag{55}$$

$$q_k = \frac{\exp\left(-(1-w)\gamma_k/(nw)\right)}{\sum_{m \in \mathcal{K}} \exp\left(-(1-w)\gamma_m/(nw)\right)}, \quad k \in \mathcal{K}. \tag{56}$$

Proof: Let $\boldsymbol{p} = (p_1, \dots, p_K)$ be scanner's equilibrium strategy and $\boldsymbol{q} = (q_1, \dots, q_K)$ be adversary's equilibrium strategy. Then, substituting $\alpha = 1$ into (33) implies

$$p_k = 1/\theta, \quad k \in \mathcal{K}. \tag{57}$$

Since $\sum_{k \in \mathcal{K}} p_k = 1$, summing up (57) implies $1 = K/\theta$. Thus, $\theta = K$, and (57) implies (55).

Substituting (55) into (38) implies

$$-(1-w)\gamma_k/n - w - w\ln(q_k) = \nu, \quad k \in \mathcal{K}. \tag{58}$$

Solving (58) for q_k implies

$$q_k = \exp\left(-\frac{1-w}{w}\frac{\gamma_k}{n} - \frac{w+\nu}{w}\right), \quad k \in \mathcal{K}. \tag{59}$$

Since $\sum_{k \in \mathcal{K}} q_k = 1$, summing up (59) implies

$$1 = \exp\left(-\frac{w+\nu}{w}\right) \sum_{k \in \mathcal{K}} \exp\left(-\frac{1-w}{w}\frac{\gamma_k}{n}\right). \tag{60}$$

Solving this equation for $\exp(-(w+\nu)/w)$ and substituting into (59) implies (56). ■

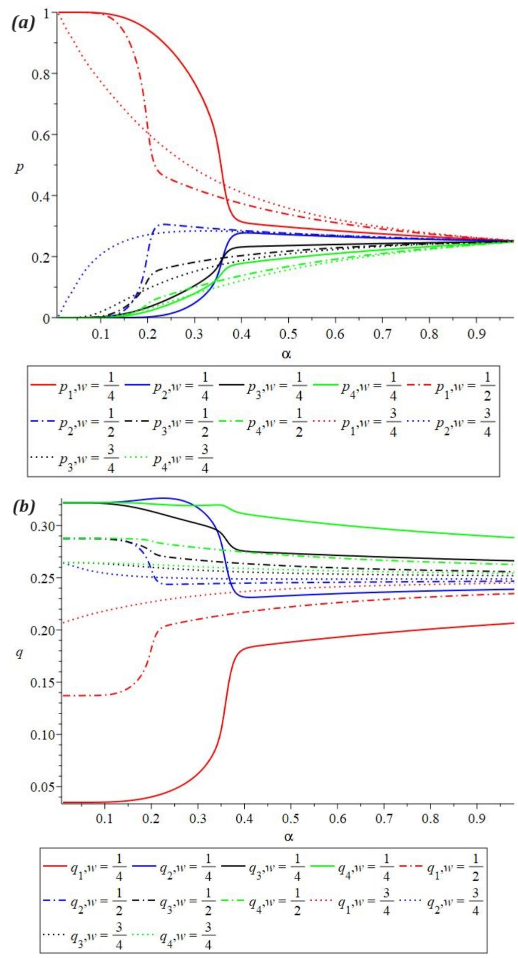

Fig. 1. (a) Scanner's strategy and (b) adversary's strategy as functions on fairness coefficient α.

IV. Numerical evaluation

Let us consider an example of the bandwidth to showcase how the equilibrium strategies in Theorem 1 are influenced by the fairness coefficient of the scanner's payoff and the weighting coefficients of the adversary's payoff. This particular example involves a spectrum comprising of $K = 4$ bands, with the detection probabilities distributed according to an exponential law $\gamma_k = \eta e^{-\delta k}$ with $\eta = 1$ and $\delta = 0.3$, weighting coefficient $w \in \{1/4, 1/2, 3/4\}$ and α-fairness coefficient varying from 0 to 1. Fig. 1 illustrates that tending α to zero makes the scanner scan band 1 with an increasing probability and finally when $\alpha = 0$ the scanner focuses its scanning on the only band 1 that corresponds to the maximizing detection probability protocol (please, see, [8, Theorem 1]]. An increasing α to 1 makes the scanner tend to equiprobability scanning protocol (Theorem 2), i.e., the scanning protocol which does not depend on bandwidth parameters, and, so, designed according to the principle of insufficient reasons. Due to its sophisticated nature, the adversary does not focus on a particular node to sneak into but employs each band from the bandwidth for

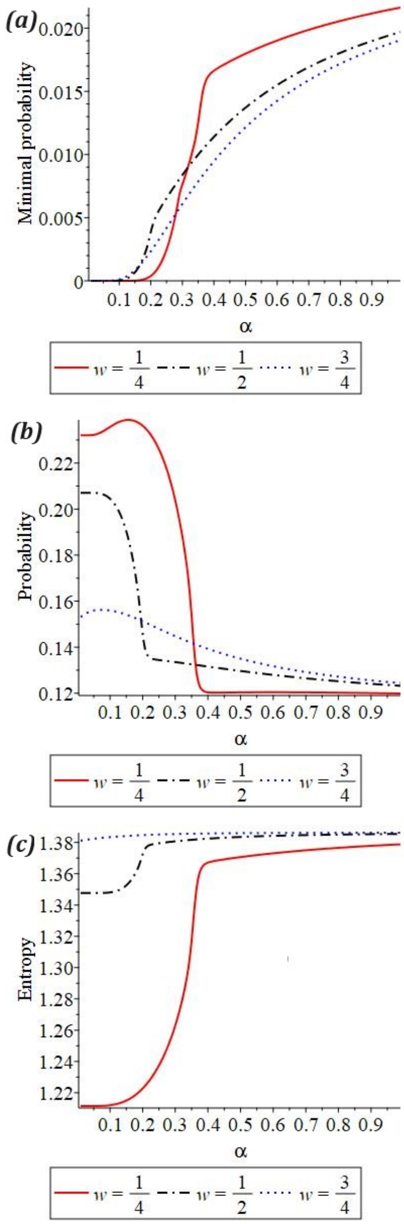

Fig. 2. (a) Minimal detection probability, (b) detection probability and (c) entropy of adversary's strategy as functions on fairness coefficient α.

an attempt to sneak. Since increasing α makes the scanner scan all the bands the minimal detection probability among all the bands also increases (Fig. 2). Furthermore, it increases the adversary's strategy's entropy. Finally, note that due to the difference in the goal of the scanner (to maximize fairness) and the basic goal of the adversary (to minimize detection probability) detection probability as a function on α might achieve its maximum for an inner fairness coefficient (say, in the considered example it is $\alpha = 0.161$ for $w = 1/4$ and $\alpha = 0.08$ for $w = 3/4$). Such fairness coefficient might be considered as the one reflecting a trade-off between protocols designed on the principles of insufficient reasons ($\alpha = 1$) and maximizing detection probability ($\alpha = 0$).

V. Conclusions

A novel approach to identifying an adversary by the scanner has been explored, which focuses on maximizing the fairness of detection probabilities across all bands. An advantage of such protocol, in contrast to maximizing detection probability protocol, is that it maintains scanning through all the bands even in case of restricted resources. α-fairness has been considered as a fairness criterion. The problem has been modeled and solved in the framework of a resource allocation game. An approach has been suggested to find the fairness coefficient reflecting the trade-off between two boundary protocols designed on the principles of insufficient reasons and maximizing detection probability. Finally, note that a goal of our future research is to extend the problem to multi-step scanning based on Bayesian learning.

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