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An ALOHA-type Network Game with Trade-off Between Throughput and Energy Saving

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***Abstract*—The paper considers multi-user ALOHA-type communication in a game-theoretical framework. First, we show that classical communication metric, which reflects the trade-off between throughput and transmission cost could lead to multiple equilibria, and so, instability in communication protocols. The other challenge that might arise is that such users might work with battery-powered communication tools, and, so, efficient energy management might have a crucial role. To deal with these two challenges (stability in communication and energy saving) we introduce an advanced communication metric reflecting the fair trade-off between throughput and energy saving. As fairness criteria, we consider α-fairness utility and model the problem in a game-theoretical framework. We establish such fairness coefficients for which the equilibrium is unique, i.e., such fairness coefficients that allow maintaining stability in such multi-user communication. Finally, to design the optimal protocol among the derived continuum of unique fair communication protocols (a unique protocol per fairness coefficient), the criterion of total throughput maximization is suggested and illustrated.**

***Index Terms*—Energy saving, Fairness, Multi-user**

I. Introduction

The ALOHA protocol, proposed in [1], presents a decentralized approach to MAC protocol in a multi-user setting, operating without carrier sensing. The ALOHA protocol stands as a significant benchmark for the evaluation of other multi-access communication protocols. In [2], the concept of slotted-ALOHA was put forth, incorporating device synchronization. This distributed mechanism has paved the way for numerous extensions and serves as the underlying principle for several cellular network protocols, including the Global System for Mobile Communications. There have been numerous studies conducted on the non-cooperative relation among users in ALOHA-type networks. One such study, referenced as [3], focuses on an ALOHA non-cooperative game-theoretical model, where users determine their transmission probability while keeping their desired throughput confidential. The authors analyzed the presence of equilibrium points that users may potentially achieve based on their throughput requirements. The research conducted by [4] delved into the incorporation of channel state information, which has a direct influence on the transmission policy. The study demonstrated the presence of certain configurations that lead to multiple equilibria.

The existence of these multiple equilibria poses a challenge in the design of communication protocols as it can destabilize users' communication. To deal with possible multiple equilibria, on an example of a homogeneous ALOHA network, an advanced communication metric that effectively balances throughput and proportional transmission fairness was introduced in [5]. Moreover, in [5], is was shown that in contrast to the flat-fading multi-access communication network [6], [7], in an ALOHA network, users' communication might not be stabilized via switching from throughput to latency communication utility.

The other challenge is that since such users might work with battery-powered communication tools, efficient energy management schemes have a crucial role. To deal with this challenge an efficiency function of power efficiency defined as the ratio of the throughput to transmit power is used (please see [8]).

In this paper, to deal with these two challenges (stability in communication and energy saving) we suggest a novel approach. Specifically, on an example of the ALOHA-type network we introduce an advanced communication metric reflecting the fair trade-off between throughput and energy saving. As fairness criteria, we consider α-fairness criteria. We model the problem in a game-theoretical framework. We establish such fairness coefficients for which the equilibrium is unique, i.e., such fairness coefficients that allow maintaining stability in such multi-user communication. Finally, to design the optimal protocol among the derived continuum of unique fair communication protocols (a unique protocol per fairness coefficient), the criterion of total throughput maximization is suggested and illustrated.

II. Basic Communication model

In this section, we describe the basic communication model with a trade-off between throughput and transmission cost and show that in general it leads to multiple equilibria, and, so, instability in communication which is based on such classical protocol. Specifically, we consider an ALOHA-type network with a set of nodes $\mathcal{N} \triangleq \{1, ..., n\}$ transmitting data over a shared collision channel to a base station. Each node i chooses a transmission probability p_i, to transmit a packet (per time slot), which could be regarded as the transmission rate. Let (p_i, p_{-i}) be a strategy profile, i.e., a set consisting of one strategy for each node, where p_{-i} denotes the strategies of all the nodes except node i. A node succeeds in packet transmission at a time slot if it is the only node who transmits a packet at this time slot. Otherwise it fails to transmit. Then the probability of node i to succeed in a packet's transmission, which also reflects the average throughput, is

$$\mathbb{T}_i(p_i, p_{-i}) = p_i \prod_{j\in\mathcal{N}_{-i}} (1 - p_j), \tag{1}$$

where $\mathcal{N}_{-i} = \mathcal{N}\backslash\{i\}$ is the set of all nodes except node i.

Traditionally, difference between throughput and transmission cost is considered as the payoff to the node, i.e.,

$$v_i(p_i, p_{-i}) = \mathbb{T}_i(p_i, p_{-i}) - C_i p_i \text{ for } i \in \mathcal{N} \tag{2}$$

where C_i is the transmission cost of node i per a transmission.

Each node wants to maximize its payoff. Denote this non-zero sum game by Γ_0, and we look for a (Nash) equilibrium [9]. Recall that strategies $p_1, \dots, p_n$ of the nodes $1, \dots, n$ are equilibrium strategies, if and only if each of them is the best response to others, i.e., they are solution of the following n best response equations:

$$p_i = \operatorname{argmax}\{v_i(\tilde{p}_i, p_{-i}) : \tilde{p}_i \in [0, 1]\} \text{ for } i \in \mathcal{N}. \tag{3}$$

Proposition 1: In game Γ_0 there exists at least one equilibrium.

Proof: Since the payoff $v_i(\tilde{p}_i, p_{-i})$ of node i given by (1) and (2) is linear on p_i, the result follows by the Nash theorem [9]. ■

We will find equilibrium by a constructive method solving the best response equations.

Proposition 2: For a fixed set of strategies p_{-i} of nodes $\mathcal{N}_{-i}$ the best response p_i of node i is given as follows:

$$p_i = \begin{cases} 0, & \prod_{j\in\mathcal{N}_{-i}}(1 - p_j) < C_i, \\ \in [0, 1] & \prod_{j\in\mathcal{N}_{-i}}(1 - p_j) = C_i, \\ 1, & \prod_{j\in\mathcal{N}_{-i}}(1 - p_j) > C_i. \end{cases} \tag{4}$$

Proof: By (1) and (2), payoff $v_i(p_i, p_{-i})$ of node i is linear on p_i, and the result follows. ■

To derive equilibrium let us split set of nodes into two set with transmission cost larger and smaller then one, i.e.,

$$\overline{\mathcal{N}} \triangleq \{i \in \mathcal{N} : C_i > 1\}, \tag{5}$$

$$\underline{\mathcal{N}} \triangleq \{i \in \mathcal{N} : C_i < 1\}. \tag{6}$$

Note that to avoid bulkiness in formulas we assume that $C_i \neq 1$ for any i.

Proposition 3: Let none of the nodes have transmission cost smaller than one, i.e., set of nodes $\underline{\mathcal{N}}$ be empty, or, in other words, $\overline{\mathcal{N}} = \mathcal{N}$. Then there is the unique equilibrium $(p_1, \dots, p_n)$ and it is

$$p_i = 0 \text{ for } i \in \mathcal{N} \tag{7}$$

Proof: For feasible strategies $(p_1, \dots, p_n)$ we have that

$$\prod_{j\in\mathcal{N}_{-i}} (1 - p_j) \le 1 \text{ for any } i \in \mathcal{N}. \tag{8}$$

By (4) and (8), we have that

$$\text{if } C_i > 1 \text{ then } p_i = 0. \tag{9}$$

Thus,

$$p_i = 0 \text{ for } i \in \overline{\mathcal{N}}, \tag{10}$$

and the result follows. ■

Proposition 4: Let set of nodes $\underline{\mathcal{N}}$ be not empty. Then multiple equilibria arise. Moreover, each nodes' strategies $(p_1, \dots, p_n)$ given by (a) and (b) bellow is an equilibrium, namely

(a) *for fixed each $j \in \underline{\mathcal{N}}$:*

$$p_i = \begin{cases} 1, & i = j, \\ 0, & i \neq j, \end{cases} \tag{11}$$

(b) *for each fixed subset $\mathcal{L} \subset \underline{\mathcal{N}}$ such that (13) holds below:*

$$p_i = \begin{cases} 1 - \dfrac{\left(\prod_{j\in\mathcal{L}} C_j\right)^{\frac{1}{|\mathcal{L}|-1}}}{C_i}, & i \in \mathcal{L}, \\ 0, & i \notin \mathcal{L}, \end{cases} \tag{12}$$

where $|\mathcal{L}|$ is the number of nodes in subset $\mathcal{L}$ and the following condition has to hold for $\mathcal{L}$:

$$\prod_{j\in\mathcal{L}} C_j \begin{cases} \ge C_i^{|\mathcal{L}|-1}, & i \in \mathcal{L}, \\ < C_i^{|\mathcal{L}|-1}, & i \notin \mathcal{L}. \end{cases} \tag{13}$$

Proof: Let $(p_1, \dots, p_n)$ be an equilibruim, Then these strategies have the following property:

(P) if there exists $i \in \underline{\mathcal{N}}$ such that $p_i = 1$ then $p_j = 0$ for all $j \neq i$.

Property (P) follows from (4), and the fact that if there exists $i \in \underline{\mathcal{N}}$ such that $p_i = 1$, then, $\prod_{j\in\mathcal{N}_{-k}}(1 - p_j) = 0$, for all $j \neq i$, and (a) follows.

(b) Let $(p_1, \dots, p_n)$ be an equilibrium. Then, by (4), (9) and property (P), we have that there is a subset $\mathcal{L}$ of $\underline{\mathcal{N}}$ such that

$$p_i \begin{cases} \in (0, 1), & i \in \mathcal{L}, \\ = 0, & i \notin \mathcal{L}. \end{cases} \tag{14}$$

This and (4) imply that

$$\prod_{j\in\mathcal{N}_{-i}} (1 - p_j) \begin{cases} = C_i, & i \in \mathcal{L}, \\ < C_i, & i \notin \mathcal{L}. \end{cases} \tag{15}$$

Multiplying both sides of (15) on $1-p_i$ implies

$$\prod_{j\in\mathcal{N}}(1-p_j)\begin{cases}= C_i(1-p_i), & i\in\mathcal{L},\\ < C_i(1-p_i)=C_i, & i\notin\mathcal{L}.\end{cases} \quad (16)$$

By (14), we have that

$$\prod_{j\in\mathcal{N}}(1-p_j)=\prod_{j\in\mathcal{L}}(1-p_j). \quad (17)$$

Multiplying up (16) over $i\in\mathcal{L}$, by (17), we have that

$$\left(\prod_{j\in\mathcal{N}}(1-p_j)\right)^{|\mathcal{L}|}=\prod_{j\in\mathcal{N}}(1-p_j)\prod_{i\in\mathcal{L}}C_i. \quad (18)$$

Thus, $\prod_{j\in\mathcal{N}}(1-p_j)=\left(\prod_{i\in\mathcal{L}}C_i\right)^{1/(|\mathcal{L}|-1)}$. This and the first row of (16) solved for p_i imply (12) with (13), and (b) follows. ■

III. Fair throughput and energy saving communication

In this section, we introduce a node's payoff reflecting the trade-off between its throughput and energy saving. Note that, an increase in probability p_i to transmit by node i leads to an increase in its throughput and to a reduction in energy saving. Meanwhile, an increase in probability $1-p_i$, that node i does not transmit, leads to an increase in energy saving and a decrease in throughput. To find a trade-off between these two goals (throughput and energy saving) for the node i we employ fairness criteria, specifically, α-fairness criteria [10] as follows:

$$\varphi_\alpha(\mathbb{P}_i(p_i,p_{-i}),p_i)=\\ =\begin{cases}\dfrac{(\mathbb{T}_i(p_i,p_{-i}))^{1-\alpha}}{1-\alpha}+\dfrac{(C_i(1-p_i))^{1-\alpha}}{1-\alpha}, & \alpha\neq 1,\\ \ln(\mathbb{P}_i(p_i,p_{-i}))+\ln(C_i(1-p_i)), & \alpha=1.\end{cases} \quad (19)$$

This fairness utility is payoff to node i. For convenience let us re-denote this payoff as follows:

$$V_{i,\alpha}(p_i,p_{-i})\triangleq\varphi_\alpha(\mathbb{P}_i(p_i,p_{-i}),p_i). \quad (20)$$

We look for a Nash equilibrium. Thus, for such strategies $p_1,\dots,p_n$ that each of them is the best response to others. In other words, these strategies are given as solution of the following n best response equations:

$$p_i=\mathrm{BR}_i(p_{-i})=\mathrm{argmax}\{V_{i,\alpha}(p_i,p_{-i}):p_i\in[0,1]\}, \quad (21)$$

where for $i\in\mathcal{N}$.

Denote this non-zero sum game by Γ_α.

Note that for $\alpha=0$ equilibrium of this game coincides with the equilibrium of the game with the traditional payoff considered in the previous section since these games' payoffs differ by transmission cost, which is a constant, i.e.,

$$V_{i,0}(p_i,p_{-i})-v_i(p_i,p_{-i})=C_i \text{ for } i\in\mathcal{N}. \quad (22)$$

Proposition 5: In game Γ_α there exists at least one equilibrium.

Proof: By (19) and (20), we have that

$$\frac{\partial^2 V_{i,\alpha}(p_i,p_{-i})}{\partial p_i^2}=-\frac{\alpha\left(\prod_{j\in\mathcal{N}_{-i}}(1-p_j)\right)^{1-\alpha}}{p_i^{\alpha+1}}\\ -\frac{\alpha C_i^{\alpha-1}}{(1-p_i)^{\alpha+1}}<0. \quad (23)$$

Thus, payoff $V_{i,\alpha}(p_i,p_{-i})$ to node i is concave on p_i. This jointly with the fact that set of feasible strategies of node i, $i\in\mathcal{N}$ is compact set $[0,1]$ imply the result by Nash theorem [9]. ■

Proposition 6: The best response p_i of node i to strategies p_{-i} of other nodes is given as follows:

$$p_i=\frac{1}{1+\dfrac{C_i^{(1-\alpha)/\alpha}}{\left(\prod_{j\in\mathcal{N}_{-i}}(1-p_j)\right)^{(1-\alpha)/\alpha}}}. \quad (24)$$

Proof: Note that, by (19), (20) and (21), we have that

$$\frac{\partial V_{i,\alpha}(p_i,p_{-i})}{\partial p_i}=\frac{\left(\prod_{j\in\mathcal{N}_{-i}}(1-p_j)\right)^{1-\alpha}}{p_i^\alpha}-\frac{C_i^{1-\alpha}}{(1-p_i)^\alpha}. \quad (25)$$

Left side of Eqn. (25) is strictly decreasing from infinity for p_i tending to zero to negative infinity for p_i tending to one. Thus, the best response p_i to p_{-i} is given as the unique inner root in $(0,1)$ of the following equation

$$\frac{\left(\prod_{j\in\mathcal{N}_{-i}}(1-p_j)\right)^{1-\alpha}}{p_i^\alpha}-\frac{C_i^{1-\alpha}}{(1-p_i)^\alpha}=0. \quad (26)$$

Solving this equation for p_i implies (24). ■

Corollary 1: (a) If $\alpha=1$ then equilibrium $\boldsymbol{p}=(p_1,\dots,p_n)$ is an unique, and implementing this unique equilibrium strategy each node transmits or does not transmit with equal probabilities, i.e., $(p_1,\dots,p_n)=(1/2,\dots,1/2)$.

(b) If $\alpha>1$ then nodes' strategies assigning them to transmit with certainty, and so, to block transmission of others, i.e., $(p_1,\dots,p_n)=(1,\dots,1)$ compose an equilibrium

Proof: (a) follows by substituting $\alpha=1$ into (24). By (24), if $\alpha>1$ and $p_j=1$ for an $j\in\mathcal{N}_{-i}$ then $p_i=1$, and (b) follows. ■

Corollary 1 implies that for large fairness coefficient ($\alpha>1$) communication protocol might be unstable since for any network parameters $(1,\dots,1)$ is an equilibrium, and this equilibrium completely block communication although each node makes it the best to communicate. For fairness coefficient $\alpha=1$ although equilibrium is unique, it is indifferent to network parameters, namely, all nodes implement a fifty-fifty strategy to communicate or not. This puts forward a question of whether there is such fairness coefficient $0<\alpha<1$ that maintains stability in communication, and whether its equilibrium strategies are flexible to network parameters. In response to this question below we can assume that $0 < \alpha < 1$ and, first, in the following proposition, we derive the relation between the probability of transmitting by a node, and the probability P that none of the nodes transmits, i.e.,

$$P \triangleq \prod_{j\in\mathcal{N}} (1-p_j). \tag{27}$$

Proposition 7: Let $(p_1,\dots,p_n)$ be an equilibrium. Then, (a)

$$p_i > 0 \text{ for } i \in \mathcal{N}; \tag{28}$$

(b) these strategies $(p_1,\dots,p_n)$ are a solution of the equations:

$$F(p_i) = P/C_i \text{ for } i \in \mathcal{N} \tag{29}$$

with P given by (27) and

$$F(\xi) \triangleq \xi^{\alpha/(1-\alpha)}(1-\xi)^{(1-2\alpha)/(1-\alpha)}. \tag{30}$$

Proof: First (24) implies (28). Then, Eqn. (26) is equivalent to

$$\prod_{j\in\mathcal{N}_{-i}} (1-p_j) = \frac{C_i p_i^{\alpha/(1-\alpha)}}{(1-p_i)^{\alpha/(1-\alpha)}}. \tag{31}$$

Multiplying both sides of this equation on $1-p_i$ implies

$$\prod_{j\in\mathcal{N}} (1-p_j) = C_i p_i^{\alpha/(1-\alpha)}(1-p_i)^{(1-2\alpha)/(1-\alpha)}. \tag{32}$$

This and (27) imply (29) with $F(\cdot)$ given by (30). ■

Let us establish auxiliary properties of function F.

Proposition 8: Function $F(\xi)$ given by (32) has the following properties:

(a) Let $0<\alpha<1/2$ then $F(\xi)$ is increasing in $[0,\alpha/(1-\alpha)]$ and decreasing in $[\alpha/(1-\alpha),1]$, and $F(0)=F(1)=0$.

(b) Let $\alpha = 1/2$ then $F(\xi)=\xi$, and it is increasing in $[0,1]$ from $F(0)=0$ to $F(1)=1$.

(c) Let $1/2<\alpha<1$ then $F(\xi)$ is increasing in $[0,1]$ from $F(0)=0$ to $F(1)=\infty$.

Proof: These properties follow from (32) and

$$\frac{dF(\xi)}{d\xi} = \frac{\xi^{(2\alpha-1)/(1-\alpha)}(1-\xi)^{-\alpha/(1-\alpha)}}{1-\alpha}(\alpha-(1-\alpha)\xi). \tag{33}$$

■

Proposition 9(b) and Proposition 8(a) imply that for $0<\alpha<1/2$ multiple equilibria might arise since Eqn. (29) could have two roots. Meanwhile, as it will be proven in the following proposition for $1/2 \le \alpha < 1$ equilibrium is always unique.

Proposition 9: Let $1/2 \le \alpha < 1$. The equilibrium $\boldsymbol{p} = (p_1,\dots,p_n)$ is unique and it is given as follows:

$$p_i = F^{-1}\left(\frac{P_*}{C_i}\right) \text{ for } i \in \mathcal{N}, \tag{34}$$

where $F^{-1}(\cdot)$ is the inverse function to $F(\cdot)$, i.e., $F^{-1}(F(\xi)) = \xi$ for $\xi \in [0,1]$, and P_* is an unique root in $[0,B]$ (with B defined in (37) and detailed in (39) and (i)-(iii) of the proof below) of the following equation

$$\Psi(P_*) \triangleq P_* - \Phi(P_*) = 0, \tag{35}$$

where

$$\Phi(P) \triangleq \prod_{i\in\mathcal{N}} \left(1 - F^{-1}\left(\frac{P}{C_i}\right)\right) \tag{36}$$

and

$$B \triangleq \begin{cases} 1, & 1/2<\alpha<1, \\ \min\{1, \min\{C_i : i \in \mathcal{N}\}\}, & \alpha = 1/2. \end{cases} \tag{37}$$

This unique P_* can be found via the bisection method.

Proof: By Proposition 8, the inverse function $F^{-1}(\cdot)$ exists and it is is increasing function,Thus, by (29), strategy p_i is given uniquely as follows:

$$p_i = F^{-1}(P/C_i) \text{ for } i \in \mathcal{N} \tag{38}$$

and

$$P \in \begin{cases} [0,1], & 1/2<\alpha<1, \\ [0,B], & \alpha = 1/2 \end{cases} \tag{39}$$

with B given by (37).

The first row of (39) follows from (38) and Proposition 8(c). If $\alpha = 1/2$ then by Proposition 8(b) (38) turns into

$$p_i = P/C_i \text{ for } i \in \mathcal{N} \tag{40}$$

Since $p_i \in [0,1]$ for $i \in \mathcal{N}$ (40) implies the second row of (39).

Further, Eqn. (38) is equivalent to

$$1 - p_i = 1 - F^{-1}(P/C_i) \text{ for } i \in \mathcal{N}. \tag{41}$$

Multiplying up (41) over $i \in \mathcal{N}$ and taking into account (27) imply (35) with $\Phi(P)$ given by (36).

By Proposition 8, $F^{-1}(\xi) \in [0,B]$ for $\xi[0,B]$, and it is increasing. Since $0 < B \le 1$, then $\Phi(P)$ is decreasing in $[0,B]$.

Thus, $\Psi(P)$, given by (35), is increasing in $[0,B]$ and $\Psi(0) = 0 - \Phi(0) = -1$ and:

(i) for $1/2<\alpha<1$, i.e., $B=1$, we have that $\Psi(1) = 1 - \prod_{i\in\mathcal{N}}\left(1 - F^{-1}(1/C_i)\right) > 0$.

(ii) for $\alpha = 1/2$ and $B = 1$, i.e., $1 \le \min_i C_i$, we have that $\Psi(1) = 1 - \prod_{i\in\mathcal{N}}(1-1/C_i) > 0$.

(iii) for $\alpha = 1/2$ and $B < 1$, i.e., $B = \min_i C_i < 1$ we have that $\Psi(B) = B - \prod_{i\in\mathcal{N}}(1-B/C_i) = B > 0$.

This implies that Eqn.(35) has the unique root in $[0,B]$, which can be found by the bisection method. ■

IV. Illustration of the results

Let us illustrate the unique equilibrium strategies derived in Proposition 9 by an example of a heterogeneous network where the nodes do not differ essentially from each other by their characteristics. Specifically, let network consist of $n = 3$ nodes with transmission costs $C = (1.1, 1.15, 1.2)$. First note that in contrast to the classical metric reflecting the linear trade-off between throughput and transmission cost, in which, by Proposition 3, none of the nodes transmit due to large transmission cost, the suggested fairness metric allows maintaining transmission for each node. An increase in fairness coefficient leads to an increased probability to transmit, i.e., in nodes' strategies and they turn into fifty-fifty communication strategies, which are indifferent to network parameters when the fairness coefficient becomes equal to 1 (see, Fig. 1(a)). Proposition 9 allows designing the unique communication protocol for each fairness coefficient $\alpha \in [1/2, 1]$. Thus, a question arises which of these protocols might be the best for the network? In response to this question note that the node's throughput is non-monotonous in general and it leads to that the total throughput of all the nodes $\mathbb{T} = \sum_{i \in \mathcal{N}} \mathbb{T}_i(p_i, p_{-i})$ could achieve its maximum for an inner fairness coefficient. Then such a fairness coefficient where the total throughput achieves maximum (in the considered example, it is $\alpha = 0.58$) can be considered as the optimal one for the network as a whole since it allows maintaining the maximal total throughput as well as an individual trade-off for each node between its energy saving and throughput.

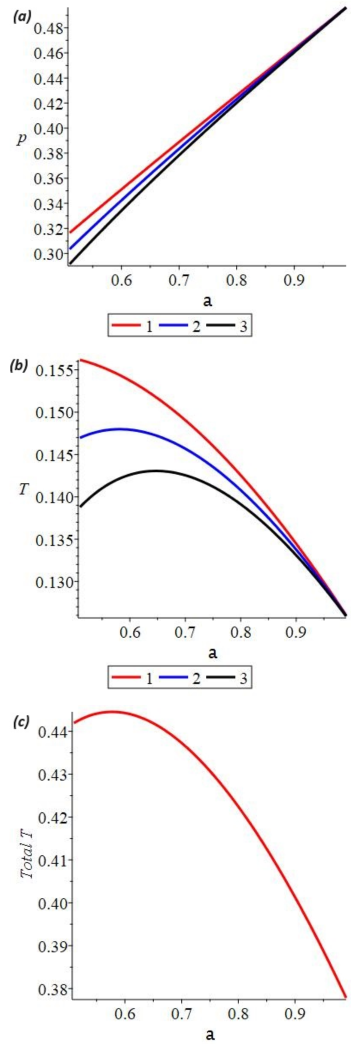

Fig. 1. (a) Nodes' strategies, (b) nodes' individual throughput and (c) total nodes' throughput as functions on α.

V. Conclusions

An advanced communication metric reflecting the fair trade-off between throughput and energy saving has been introduced for multi-user ALOHA-type communication. It has been shown that in contrast to classical communication metric, which reflects the linear trade-off between throughput and transmission cost and leading to multiple equilibria, the suggested advanced metric leads to the unique equilibrium, and, so, to stability in multi-node communication. The other advantages of the suggested metric are that it allows maintaining: (a) uninterrupted communication of all nodes for any network parameters and (b) trade-off between throughput and energy-saving for each (individual) node as well as maintaining the maximal throughput of all the network using control over the fairness coefficient.

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