

An IoT Game with Heterogeneous Communication Nodes

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Abstract—The paper considers a network’s communication, where nodes, communicating with a base station, could differ by the access to information they have on their fading channel gain (channel state). Specifically, some of the nodes know their channel states exactly, while other nodes have only statistical state information. Such difference can be motivated by the fact that the channel gain is a function of the distance to the receiver, and some of the nodes might not know their own location, meanwhile, the others might have complete information about their own location via global positioning system (GPS). This scenario is common in IoT networks, particularly those situated in remote or challenging environments where GPS signals may be unreliable or inaccessible. The problem is modeled by a Bayesian game with latency as a communication metric. A novel approach is developed to solve such a heterogeneous problem by access to information in closed form for any number of channel states even for the continuum of them. The uniqueness of equilibrium is proven which reflects the usability of using latency metric and stability in communication based on the suggested communication protocol even when the nodes might differ by access to information about channel states. The equilibrium strategies are numerically illustrated.

Index Terms—IoT, Latency, Bayesian Equilibrium

I. INTRODUCTION

Wireless networks, such as IoT networks, encompass a variety of mobile devices, nodes, and users. These networks are typically found extensively in the environment and distribute resources in a decentralized manner. Due to their inherent nature, they possess multiple objectives and have been extensively examined within a game theoretic framework [1]. Examples of decentralized communication can be found in various studies such as [2]–[6], where game theory was utilized to analyze a fading multi-access communication scenario. Additionally, in [7]–[9], the focus was on studying an orthogonal frequency division multiplexing (OFDM) scenario. In the works [3]–[10], the communication nodes have complete information on the channel states. In [2], [4], [7]–[9], the communication metric is throughput. In [11], it was shown that depending on the network parameters the throughput metric may return multiple equilibria even in case of complete information on network parameters, meanwhile latency metric leads to

the unique equilibrium, and so, to stability in communication. In [5], an anti-jamming strategy in multiple access communication with complete information and latency metric is developed. Meanwhile, in [10], a network where the users might differ by their communication metrics is studied. In the above works the users have access to complete information about network parameters such as channel states. In other words, these studied networks are homogeneous by users’ access to such information. Meanwhile, in real-world networks such as IoT (Internet of Things) networks, sensor nodes are usually widespread in the environment, and these nodes might differ in access to information about channel gains or states. The variation in channel gain can be attributed to the distance from the receiver, with some nodes lacking knowledge of their location while others have GPS for accurate positioning. This results in a scenario where nodes’ states form an uncountable set in a continuous distribution of locations, significantly increasing network complexity when nodes are spread across a certain area.

Contribution of the paper:

- (i) A Bayesian game-theoretical approach is employed to model an IoT multi-node heterogeneous network by accessing to channel state information. Communication latency is considered as a communication utility. A challenge of such a Bayesian game-theoretical approach is that it might lead to a continuum set of optimization problems associated with channel states if the set of channel states might be a continuum, and a technique to solve such a continuum of optimization problems is not developed in the literature.
- (ii) A novel technique has been introduced to create equilibrium strategies that consider the transmission protocol in a multi-node heterogeneous communication game. This approach is applicable in various scenarios, including cases where a continuum of channel states arises.
- (iii) It has been established that in a heterogeneous network where latency serves as a communication metric, each node consistently adopts a distinct equilibrium strategy that fosters stability in communication. This confirmation underscores the viability of employing the latency metric, even in situations where nodes may differ in their access to information regarding channel states.

II. THE COMMUNICATION MODEL

In this paper we consider scenario of flat-fading multi-access communication in a single cell network consisting of n nodes transmitting data to a base station. To model this scenario in game-theoretical framework we have to define: (a) a set of players, (b) set of feasible strategies for each player and (c) payoff to each player [12]. Moreover, in scenario where player's payoff might depend on knowledge the player has the prior we have to associate a *player type* per such knowledge it has [12]. In our scenario each of the nodes is a player. Thus, the set of the players coincides with the set of nodes $\mathcal{N} \triangleq \{1, \dots, n\}$.

A strategy of node i is its transmit power level P_i , with $P_i \in \mathbb{R}_+$.

The SINR of each node at the base station depends on its transmit power level as well as transmit power levels of others due to mutual signal interference. Specifically, following [4], the SINR of node i at the base station is

$$\text{SINR}_i(P_i, P_{-i}) = \lambda_i h_i P_i / (N + \sum_{j \in \mathcal{N}_{-i}} h_j P_j), \quad (1)$$

with (P_i, P_{-i}) is a *strategy profile* for node i where $P_{-i} \triangleq \{P_j : j \in \mathcal{N}_{-i}\}$ and $\mathcal{N}_{-i} \triangleq \{j \in \mathcal{N}, j \neq i\}$, and λ_i is the spreading gain, h_i is the channel gain, and N is the ambient noise in the network.

According to [11], we will adopt a modeling approach to estimate the latency in communication between node i and the base station as follows

$$\mathcal{L}_i(P_i, P_{-i}) \triangleq 1/\text{SINR}_i(P_i, P_{-i}). \quad (2)$$

It is noteworthy to mention that SINR and negative latency may be integrated into a consistent scale of α -fairness utilities, where α is equal to 0 and 2, respectively [13].

1) *Channel states*: We assume that channel of node i can be in state t_i , $t_i \in D_i$ with *cumulative distribution function* (CDF) $H_i(t_i)$ where D_i is set of feasible states. Also, let $h_i(t_i)$ be the channel gain when channel state t_i occurs (please see an example (32) in section IV "Discussion of the results" below).

Also, let us introduce the following auxiliary notations associated with channel states further used to define nodes' payoffs:

(I) Let (t_i, t_{-i}) be a *channel state profile*, where

$$t_{-i} \triangleq \{t_j : j \in \mathcal{N}_{-i}\}; \quad (3)$$

(II) Let (D_i, D_{-i}) be a *profile for sets of channel states*, where D_{-i} is a Cartesian product of sets of channel states for each node except node i , i.e.,

$$D_{-i} \triangleq \prod_{j \in \mathcal{N}_{-i}} D_j; \quad (4)$$

(III) Let $(dH_i(t_i), dH_{-i}(t_{-i}))$ be the *distribution density profile*, where $dH_{-i}(t_{-i})$ is a product of distribution densities for each node except node i , i.e.,

$$dH_{-i}(t_{-i}) \triangleq \prod_{j \in \mathcal{N}_{-i}} dH_j(t_j). \quad (5)$$

2) *Strategies of the nodes*: Regarding access to (local) information, i.e., information about own channel state, the nodes have the prior, the nodes are split into two sets $\bar{\mathcal{N}}$ and $\underline{\mathcal{N}}$, i.e., $\mathcal{N} = \bar{\mathcal{N}} \cup \underline{\mathcal{N}}$, where

(a) $\bar{\mathcal{N}}$ is the set of nodes who has such access to exact information, i.e., know their channel state. This set consists of \bar{n} nodes;

(b) $\underline{\mathcal{N}}$ is the set of nodes, who only have statistical information on its channel states, i.e., only know the distribution of their channel states. This set consists of $n = n - \bar{n}$ nodes.

Meanwhile regarding channel states of other nodes each node has only statistical information, i.e., the corresponding distributions.

Base on such heterogeneous structure regarding information the nodes have access to, let us associate a node type to a channel state for each node from set $\bar{\mathcal{N}}$. Specifically, a node $i \in \bar{\mathcal{N}}$ is *type- t_i* ($t_i \in D_i$), if channel of node i is in state t_i , i.e., in other words, its fading gain is $h_i(t_i)$. Denote by $P_i(t_i)$ the strategy (power level) of such type- t_i $\bar{\mathcal{N}}$ set node i . Further, each node of set node $\underline{\mathcal{N}}$ does not depend on its channel state, i.e., strategy of such node i , $i \in \underline{\mathcal{N}}$, is power level P_i .

Let \mathbf{P} be the set of strategies of all nodes' types and nodes, i.e.,

$$\mathbf{P} \triangleq \{P_i(t_i) : t_i \in D_i, i \in \bar{\mathcal{N}} \text{ and } P_i, i \in \underline{\mathcal{N}}\}, \quad (6)$$

and let

$$\mathbf{P}_{-i} \triangleq \{P_j(t_j), t_j \in D_j, j \in \bar{\mathcal{N}}_{-i} \text{ and } P_j, j \in \underline{\mathcal{N}}_{-i}\}, \quad (7)$$

where

$$\bar{\mathcal{N}}_{-i} \triangleq \{j \in \bar{\mathcal{N}}, j \neq i\} \text{ and } \underline{\mathcal{N}}_{-i} \triangleq \{j \in \underline{\mathcal{N}}, j \neq i\}. \quad (8)$$

Thus, the problem is heterogeneous by access of the nodes to information on channel states. Note that, in [10], the other type of heterogeneous structure was studied where nodes differ in applied communication metrics.

3) *Payoffs to Nodes*: The node wishes to find the trade-off between a reduction in latency of the signal received by the base station and transmission cost [4]. Thus node's payoff is

$$v_i(P_i, P_{-i}) = -\mathcal{L}_i(P_i, P_{-i}) - C_i P_i, \quad (9)$$

with C_i is a constant reflecting transmission cost per power unit.

Based on such trade-off utility we can define expected payoff to each node depending on the knowledge on the channel state it could have the prior, i.e., depending on which of the set of nodes $\bar{\mathcal{N}}$ or $\underline{\mathcal{N}}$ it belongs to. Specifically,

(a) If node i belongs to set $\bar{\mathcal{N}}$, i.e., it knows its channel state, then the expected payoff to type- t_i node i , with $t_i \in D_i$ is given as follows:

$$V_{i,t_i}(P_i(t_i), \mathbf{P}_{-i}) = -C_i P_i(t_i) - \int_{D_{-i}} \frac{N + \sum_{j \in \bar{\mathcal{N}}_{-i}} h_j(t_j) P_j(t_j) + \sum_{j \in \underline{\mathcal{N}}_{-i}} h_j(t_j) P_j}{\lambda_i h_i(t_i) P_i(t_i)} dH_{-i}(t_{-i}); \quad (10)$$

(b) If node i belongs to set $\underline{\mathcal{N}}$, i.e., it knows only statistical information on its channel states, then the expected payoff to node i is given as follows:

$$V_i(P_i, \mathbf{P}_{-i}) = -C_i P_i - \int_{D_i} \int_{D_{-i}} \frac{N + \sum_{j \in \bar{\mathcal{N}}_{-i}} h_j(t_j) P_j(t_j) + \sum_{j \in \underline{\mathcal{N}}_{-i}} h_j(t_j) P_j}{\lambda_i h_i(t_i) P_i} d_i\{H\}, \quad (11)$$

where $d_i\{H\} = dH_i(t_i) dH_{-i}(t_{-i})$.

The objective of every node is to maximize its payoff. Consequently, we are dealing with a Bayesian game and our aim is to identify its equilibrium. We shall refer to this game as $\Gamma_{\bar{n},n}$.

Recall that set of strategies $\mathbf{P} = \{P_i(t_i) : t_i \in D_i, i \in \bar{\mathcal{N}}\}$ and $P_i, i \in \underline{\mathcal{N}}$ is equilibrium if and only if each of these strategies is the best response to the other, i.e.,

(a) for state t_i with $t_i \in D_i$ and $i \in \bar{\mathcal{N}}$:

$$P_i(t_i) = \text{argmax}\{V_{i,t_i}(P_i(t_i), \mathbf{P}_{-i}) : \tilde{P}_i(t_i) \geq 0\}; \quad (12)$$

(b) for $i \in \underline{\mathcal{N}}$:

$$P_i = \text{argmax}\{V_i(\tilde{P}_i, \mathbf{P}_{-i}) : \tilde{P}_i \geq 0\}. \quad (13)$$

A remarkable feature of game $\Gamma_{\bar{n},n}$ is that it deals with any set of nodes' types including even uncountable sets, i.e., with uncountable sets of best response equations (12) and (13) which are not covered by the classical Nash theorem [12].

III. SOLUTION OF THE GAME

In order to ascertain the equilibrium and validate its uniqueness, we will adopt a constructive approach via solving the best response equations (12) and (13).

First note that payoffs (10) and (11) can be respectively written in the following equivalent form:

(a) For the nodes who know their channel states

$$V_{i,t_i}(P_i(t_i), \mathbf{P}_{-i}) = -C_i P_i(t_i) - \frac{\nu_i(\mathbf{P}_{-i})}{\lambda_i h_i(t_i) P_i(t_i)} \text{ for } t_i \in D_i, i \in \bar{\mathcal{N}}; \quad (14)$$

(b) For the nodes who only have statistical information on their channel states

$$V_i(P_i, \mathbf{P}_{-i}) = -C_i P_i - \frac{\nu_i(\mathbf{P}_{-i})}{\lambda_i P_i} E_i\left(\frac{1}{h_i}\right) \text{ for } i \in \underline{\mathcal{N}}, \quad (15)$$

where $E_i(\xi)$ is the expected value of function $\xi(\cdot)$ defined on D_i according to distribution H_i , i.e.,

$$E_i(\xi) \triangleq \int_{D_i} \xi(t_i) dH_i(t_i) \quad (16)$$

and $\nu_i(\mathbf{P}_{-i})$ is the sum of background noise and the expected interference generated by all nodes except node i , i.e.,

$$\begin{aligned} \nu_i(\mathbf{P}_{-i}) \triangleq & N + \sum_{j \in \bar{\mathcal{N}}_{-i}} \int_{D_j} h_j(t_j) P_j(t_j) dH_j(t_j) \\ & + \sum_{j \in \underline{\mathcal{N}}_{-i}} P_j E_j(h_j). \end{aligned} \quad (17)$$

In the following proposition we find the best response for each node type and corresponding expected latency.

PROPOSITION 1: (a) Let $i \in \bar{\mathcal{N}}$ and \mathbf{P}_{-i} be fixed. Then the best response of type- t_i node i is

$$P_i(t_i) = \sqrt{\frac{N + \sum_{j \in \underline{\mathcal{N}}_{-i}} \mathcal{P}_j}{\lambda_i C_i h_i(t_i)}} \text{ with } t_i \in D_i, \quad (18)$$

where \mathcal{P}_j is the expected fading power gain of node j , $j \in \mathcal{N}$, i.e.,

$$\mathcal{P}_j \triangleq \begin{cases} \int_{D_j} h_j(t_j) P_j(t_j) dH_j(t_j), & j \in \bar{\mathcal{N}}, \\ P_j E_j(h_j), & j \in \underline{\mathcal{N}}. \end{cases} \quad (19)$$

(b) Let $i \in \underline{\mathcal{N}}$ and \mathbf{P}_{-i} be fixed. Then the best response of node i is

$$P_i = \sqrt{\frac{N + \sum_{j \in \bar{\mathcal{N}}_{-i}} \mathcal{P}_j}{\lambda_i C_i}} E_i\left(\frac{1}{h_i}\right). \quad (20)$$

(c) The expected latency L_i of node i , with $i \in \mathcal{N}$, is

$$L_i = (N + \sum_{j \in \mathcal{N}_{-i}} \mathcal{P}_j)^{1/2} B_i, \quad (21)$$

where

$$B_i \triangleq \sqrt{\frac{C_i}{\lambda_i}} \begin{cases} E_i\left(1/\sqrt{h_i}\right), & i \in \bar{\mathcal{N}}, \\ \sqrt{E_i(1/h_i)}, & i \in \underline{\mathcal{N}}. \end{cases} \quad (22)$$

The proof, please, find in Appendix.

In the following proposition we find the total expected fading power gain of all nodes.

PROPOSITION 2: Let \mathbf{P} be an equilibrium. Then the expected fading power gain \mathcal{P}_i of node i is a function of the expected total fading power gain of all nodes \mathcal{P} , i.e.,

$$\mathcal{P}_i = F_i(\mathcal{P}), \quad i \in \mathcal{N}, \quad (23)$$

where

$$\mathcal{P} \triangleq \sum_{j \in \mathcal{N}} \mathcal{P}_j, \quad (24)$$

$$F_i(\mathcal{P}) \triangleq \frac{A_i^2}{2} \left(\sqrt{1 + 4 \frac{N + \mathcal{P}}{A_i^2}} - 1 \right) \quad i \in \mathcal{N}, \quad (25)$$

$$A_i \triangleq \frac{1}{\sqrt{\lambda_i C_i}} \times \begin{cases} E_i(\sqrt{h_i}), & i \in \overline{\mathcal{N}}, \\ E_i(h_i) \sqrt{E_i(1/h_i)}, & i \in \underline{\mathcal{N}}. \end{cases} \quad (26)$$

Moreover, \mathcal{P} is equal to the unique positive root of the fixed point equation

$$\mathcal{P} = F(\mathcal{P}), \quad (27)$$

where

$$F(\mathcal{P}) \triangleq \sum_{i \in \mathcal{N}} F_i(\mathcal{P}). \quad (28)$$

This root can be found via fixed point algorithm

$$\mathcal{P}^{k+1} = F(\mathcal{P}^k), \text{ where } k = 0, 1, \dots \text{ with } \mathcal{P}^0 \triangleq 0. \quad (29)$$

Such sequence \mathcal{P}^k is increasing and converges to the fixed point of (27).

The proof, please, find in Appendix

In the following theorem we establish existence and uniqueness of the equilibrium and derive it in closed form.

THEOREM 1: In game $\Gamma_{\overline{n}, \underline{n}}$, there is a unique equilibrium, and it is equal to $\mathbf{P} = \{P_i(t_i) : t_i \in D_i, i \in \overline{\mathcal{N}} \text{ and } P_i, i \in \underline{\mathcal{N}}\}$, where

(a) for nodes knowing their channel state, i.e., node i with $i \in \overline{\mathcal{N}}$ and its channel in state t_i with $t_i \in D_i$:

$$P_i(t_i) = \sqrt{\frac{N + \mathcal{P} - \frac{A_i^2}{2} \left(\sqrt{1 + 4 \frac{N + \mathcal{P}}{A_i^2}} - 1 \right)}{\lambda_i C_i h_i(t_i)}}; \quad (30)$$

(b) for the nodes having access only to statistical information on its channel states, i.e., for node i with $i \in \underline{\mathcal{N}}$

$$P_i = \sqrt{\frac{N + \mathcal{P} - \frac{A_i^2}{2} \left(\sqrt{1 + 4 \frac{N + \mathcal{P}}{A_i^2}} - 1 \right)}{\lambda_i C_i}} E_i \left(\frac{1}{h_i} \right) \quad (31)$$

with A_i given by (26) and \mathcal{P} is the unique root of Eqn. (27).

The proof, please, find in Appendix.

IV. DISCUSSION OF THE RESULTS

To illustrate our result in the most challenging case, i.e., the case where a continuum of node's types arise, recall that the fading channel gain depends on the distance of the node from the base station. Thus, such distance t_i for node i can be considered as a channel state. Based on this observation let us illustrate this by the following such specific dependence:

$$h_i(t_i) = h_i/t_i^\alpha, \quad t_i \in D_i, \quad (32)$$

where $\alpha > 0$ is the path-loss factor.

Further, to model a continuum of node's types, we consider the scenario where nodes can be present equally likely at any distance t_i , $t_i \in D_i$, from the base station. Thus, $H_i(t_i)$ corresponds to the uniform distribution on $D_i = [\underline{t}_i, \bar{t}_i]$, where

$$H_i(t_i) = \begin{cases} 0, & t_i < \underline{t}_i, \\ (t_i - \underline{t}_i)/(\bar{t}_i - \underline{t}_i), & t_i \in [\underline{t}_i, \bar{t}_i], \\ 1, & t_i = \bar{t}_i. \end{cases} \quad (33)$$

Let the network consist of $n = 10$ nodes with $h = 1$, $N =$

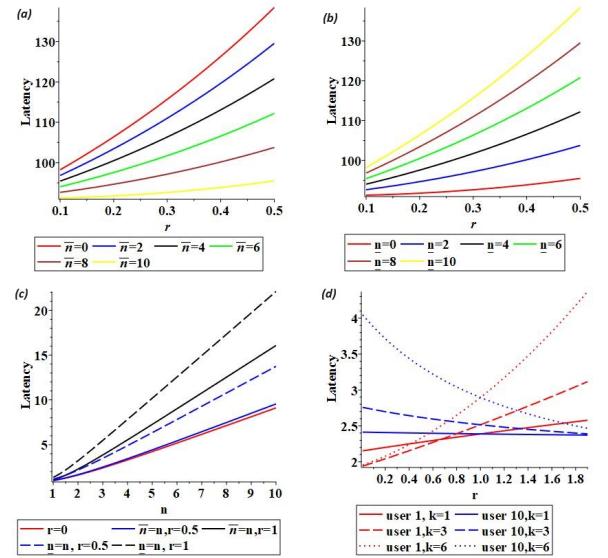


Fig. 1. (a) The expected latency for \overline{N} set node as function on \overline{n} and r , (b) the expected latency for \underline{N} set node as function on \underline{n} and r , (c) the expected latency as function on n , and (d) the expected latency as function on r .

$1, C = 1, \lambda = 1, \alpha = 1$ and $D = [d - r/2, d + r/2]$, where $d = 1$ and $\underline{n} = 10 - \overline{n}$. Note that, the boundary case $\overline{n} = n$ (then, $\underline{n} = 0$) reflects the case where each node has perfect knowledge of its channel state. Meanwhile, the boundary case $\overline{n} = 0$ (then, $\underline{n} = n$) reflects the case where each node has knowledge of distribution of its channel state the prior.

Fig. 1(a) and Fig. 1(b) illustrate that an increase in the number of nodes having complete channel information (i.e., an increase in \overline{n}) leads into a decrease in latency in expected node's communication with the base station. A decrease in uncertainty (i.e., an increase in r) leads to a decrease in latency. Fig. 1(c) illustrates that an increase in the number of nodes, i.e. in n , leads to an increase in latency. Also, an increase in r leads to an increase in latency with the minimal latency at $r = 0$, which corresponds to the complete information scenario. In this case, of course $\overline{N} = \underline{N} = \mathcal{N}$. To investigate how an increase in the set of channel states of one node, or a group of nodes impact latency let us consider a network of the n nodes split into two groups. Each node of the first group, consisting of node 1, ..., node k with $k < n$, has the same set of channel states $D = [d, d + r]$. Similarly, each node

of the second group, consisting of node $k+1, \dots, n$, has the same set of the channel states $D_0 = [d, d+r_0]$. All the other network parameters are the same for the nodes. Let $N = 1$, $h = 1$, $C = 1$, $\lambda = 1$, $d = 1$ and $\alpha = 3$, and $\bar{n} = n = 10$. Also, let $r_0 = 1$. Thus, each node of the first group has the same equilibrium strategy as node 1, while each node of the second group has the same equilibrium strategy as node 10. Fig. 1(d) illustrates that an increase in the set of channel states of the first node group leads to an increase in communication latency of the first group nodes and to a decrease in communication latency of the second group nodes since it leads to an increase in probability that the first group of nodes will be located far from the base station.

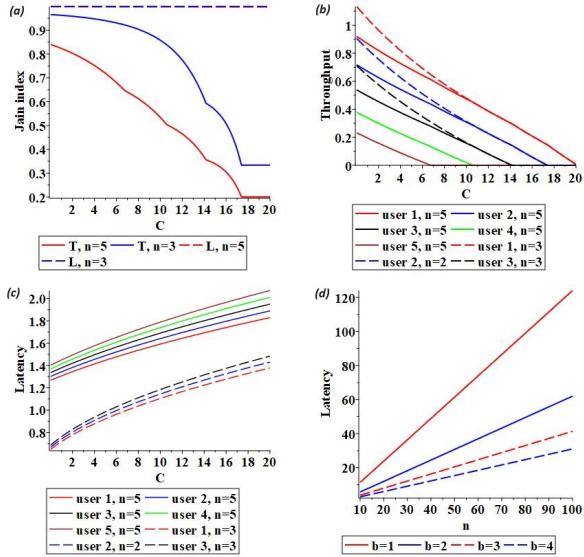


Fig. 2. (a) Jain indexes, (b) throughput, (c) latency as functions on C , and (d) the expected latency as function on n .

Finally, to compare latency and throughput metrics we have to apply common characteristics for these metrics of nodes communication with the base station. For such characteristics it is natural to consider how fair latency and throughput outputs in communication with the base station are, when the nodes implement equilibrium strategies. As fairness criteria we consider Jain's fairness index [14]. Recall that Jain's fairness index $\mathcal{J}(\eta_1, \dots, \eta_n)$ rates the fairness of a set of values (η_1, \dots, η_n) , and it is given as follows:

$$\mathcal{J}(\eta_1, \dots, \eta_n) \triangleq \left(\sum_{i=1}^n \eta_i \right)^2 / \left(n \sum_{i=1}^n \eta_i^2 \right). \quad (34)$$

Jain's fairness achieved maximum 1 (best case) when all values are the same. In our case as the set of values we consider latency and throughput outputs of the nodes implementing the corresponding equilibrium strategies. By [4], the throughput metric might make some nodes to be non-active, while, by Theorem 1, in latency metric, all the nodes are active. Base on this observation we might expect

that the latency metric might give fairer access to the base station compared to the throughput metric. Let us illustrate this observation via an example with $n = 3$ and $n = 5$ nodes, $N = 0.1$, $\lambda_i = 3$, $C_i = C$, $h_i = 20/(3+0.1i)^3$, $t_i = \bar{t}_i = 3 + 0.1i$ for $i \in \mathcal{N}$. We consider here the case $t_i = \bar{t}_i = 3 + 0.1i$ for $i \in \mathcal{N}$ so that the problem corresponds to scenario with complete information on channel state and throughput as communication metric studied in [4] for throughput as a metric. To find the equilibrium with a throughput metric we apply the best response strategy algorithm [4]. Fig. 2 illustrates that the Jain index resulting from the throughput metric is less than that resulting from the latency metric. It is caused by the fact that with an increase in power cost some node in throughput metric becomes non-active, and throughput of such nodes is zero. While in latency metric each node keeps to be active. Also, the Jain index in both metrics decreases with an increase in power cost C , but for the throughput metric such decrease is more essential, while for the latency metric it decreases non-essentially from 0.9990 and 0.9975 for $C = 0.01$ to 0.9989 and 0.9974 for $C = 20$ with $n = 3$ and $n = 5$, respectively.

V. CONCLUSIONS

A multi-node communication with a base station such that the node could differ by the access to information about fading channel gain has been investigated in a Bayesian game formulation. Equilibrium strategies have been determined in closed form using the inverse SINR to measure latency. The uniqueness of equilibrium has been proven which reflects the usability of using latency metric and stability in communication based on the suggested communication protocol even when the nodes might differ by access to information about channel states. Finally, comparing latency and throughput metrics via Jain's fairness index as a common characteristic for these metrics of nodes' communication with the base station, we have shown that the latency metric might give more fair access to the base station compared to the throughput metric.

APPENDIX

1) *Proof of Proposition 1:* Let $i \in \bar{\mathcal{N}}$. By (14), we have that

$$\frac{\partial V_{i,t_i}(x, \mathbf{P}_{-i})}{\partial x} = \frac{\nu_i(\mathbf{P}_{-i})}{\lambda_i h_i(t_i)} \times \frac{1}{x^2} - C_i. \quad (35)$$

Thus, $V_{i,t_i}(x, \mathbf{P}_{-i})$ attains the maximum in $[0, \infty)$ at $x = P_i(t_i)$, where

$$P_i(t_i) = \sqrt{\frac{\nu_i(\mathbf{P}_{-i})}{\lambda_i C_i h_i(t_i)}}. \quad (36)$$

Note that, by (19), the sum of background noise and the expected interference generated by all nodes except node i can be written as follows:

$$\nu_i(\mathbf{P}_{-i}) = N + \sum_{j \in \mathcal{N}_{-i}} \mathcal{P}_j. \quad (37)$$

Then, (37) and (36) imply (18).

Let $i \in \underline{\mathcal{N}}$. By (15), we have that

$$\frac{\partial V_i(x, \mathbf{P}_{-i})}{\partial x} = \frac{\nu_i(\mathbf{P}_{-i})}{\lambda_i} E_i\left(\frac{1}{h_i}\right) \times \frac{1}{x^2} - C_i. \quad (38)$$

So, $V_i(x, \mathbf{P}_{-i})$ attains the maximum in $[0, \infty)$ at $x = P_i$ where

$$P_i = \sqrt{\frac{\nu_i(\mathbf{P}_{-i})}{\lambda_i C_i} E_i\left(\frac{1}{h_i}\right)}. \quad (39)$$

Then, (37) and (39) imply (20). Finally, (18) and (20) imply (21). Q.E.D.

2) *Proof of Proposition 2:* Let $i \in \overline{\mathcal{N}}$. Multiplying both sides of (18) by $h_i(t_i)$ implies

$$h_i(t_i) P_i(t_i) = \sqrt{\frac{h_i(t_i)}{\lambda_i C_i}} \sqrt{N + \sum_{j \in \mathcal{N}_{-i}} \mathcal{P}_j}. \quad (40)$$

Integrating both sides of (40) by $dH_i(t_i)$ and substituting the first row of (19) and the first row of (26) into (40) imply that for $i \in \overline{\mathcal{N}}$ the following relation holds:

$$\mathcal{P}_i = A_i \sqrt{N + \sum_{j \in \mathcal{N}_{-i}} \mathcal{P}_j}. \quad (41)$$

Let $i \in \underline{\mathcal{N}}$. Multiplying both sides of (20) by $E_i(h_i)$ implies

$$P_i E_i(h_i) = E_i(h_i) \sqrt{\frac{E_i(1/h_i)}{\lambda_i C_i}} \sqrt{N + \sum_{j \in \mathcal{N}_{-i}} \mathcal{P}_j}. \quad (42)$$

Substituting the second row of (19) and the second row of (26) into (42) imply that (41) holds also for $i \in \underline{\mathcal{N}}$. Thus, (41) holds for each $i \in \mathcal{N}$.

Since $\sum_{j \in \mathcal{N}_{-i}} \mathcal{P}_j = \mathcal{P} - \mathcal{P}_i$, (41) implies that

$$\mathcal{P}_i = A_i \sqrt{N + \mathcal{P} - \mathcal{P}_i}. \quad (43)$$

We can rewrite this relation as follows:

$$\mathcal{P}_i^2 / A_i^2 + \mathcal{P}_i = N + \mathcal{P}. \quad (44)$$

Solving this quadratic equation on \mathcal{P}_i implies (23) with $F_i(\cdot)$ given by (25). Summing up (23) implies (27) with $F(\mathcal{P})$ given by (28). Finally, since each function $F_i(\mathcal{P})$, $i \in \mathcal{N}$ is concave and increasing such that $F_i(0) > 0$ and $F_i(\mathcal{P})/\mathcal{P} = \infty$ for \mathcal{P} tending to infinity, the result follows. Q.E.D.

3) *Proof of Theorem 1:* Let \mathbf{P} be an equilibrium. Then, by Proposition 1, $P_i(t_i)$ and P_i are given by (18) and (20), respectively, with \mathcal{P}_j given by (19). By Proposition 2, \mathcal{P}_i is given by (23), where \mathcal{P} is the unique root of the fixed point equation (27). By (23) and (24), we have that

$$\sum_{j \in \mathcal{N}_{-i}} \mathcal{P}_j = \mathcal{P} - \mathcal{P}_i = \mathcal{P} - F_i(\mathcal{P}). \quad (45)$$

Substituting this into (18) and (20) implies the result. Q.E.D.

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