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A Bilevel Network Interdiction Problem to Minimize the Number of Active Special Arcs in the Maximum Flow

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Abstract. We consider a bilevel network interdiction problem where the follower aims to maximize the amount of flow from the source node to the sink node, and the leader aims to minimize the number of arcs from a critical set that have positive flow on them, that is, active arcs, in the maximum flow solution obtained by the follower. This problem is motivated by an application in human trafficking disruption. We consider both the optimistic and pessimistic variants of this bilevel optimization problem and develop their respective single-level reformulations. We present a tailored solution method to the pessimistic problem, which solves the problem to optimality for one practically important class of networks. Through computational experiments on randomly generated layered network instances, we show the effectiveness of the proposed methods and demonstrate that the tailored method is orders of magnitude faster than existing approaches in the literature. We also conduct computational experiments on randomly generated test instances inspired by domestic human trafficking networks and draw domain-specific insights.

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Keywords: bilevel optimization • network interdiction • illicit supply chains • computational complexity

1. Introduction

We are interested in a special case of bilevel network interdiction problems where the follower aims to maximize the amount of flow from the source node to the sink node, and the leader aims to minimize the number of arcs from a critical set that have positive flow on them in the maximum flow (max-flow) solution obtained by the follower. This problem setting is motivated by illicit trafficking networks where the trafficker seeks to maximize the quantity of illicit activities (e.g., to achieve maximum profit), whereas the leader aims to minimize the number of components in the system impacted by the illicit operations. For example, in disrupting human trafficking (HT) networks, research has focused on disrupting the control, or, equivalently, exploitation, network of traffickers (Kosmas et al. 2023a, 2024). The network is composed of traffickers, “bottoms” (who are often victims who are forced to traffic other victims), and victims. Kosmas et al. (2023a) model “control” as flow in this network. They focus on a maximum flow problem where there is an arc connecting each victim to the sink node and a unit flow through this arc represents that the victim is being controlled by a trafficker. Clark et al. (2023) use the concept of “flow as control” but focus on the flow representing the number of hours a victim is being forced to participate in illegal activities by the trafficker. In these situations, the leader may wish to limit the number of victims involved in the trafficking operation (which will carry a positive amount of flow on the corresponding arcs if this is the case). We assume the trafficker takes a profit-driven perspective by maximizing the amount of flow sent through the network (Barrick et al. 2023), whereas the leader aims to minimize the number of victims impacted.

We consider a directed network with source node s , sink node t , and capacitated arcs. Each arc has an associated interdiction cost, and there exists a set of special arcs. We say an arc is *active* if there is a positive flow on it.

The leader is given a budget to remove arcs from the network and seeks to minimize the number of active arcs in the special set from the follower’s maximum flow solution, whereas the follower seeks to maximize the flow from s to t in the network after observing the outcome of the interdiction, that is, the set of arcs removed. The bilevel nature of this problem arises from the fact that the objective value of the leader is determined through an optimal solution of the follower’s maximum flow problem. However, because the follower’s problem may have multiple optimal solutions, we investigate both the *optimistic* problem and the *pessimistic* problem, where the follower implements the most favorable and the least favorable maximum flow solution to the leader’s objective, respectively. We provide an illustrative example in Figure 1 to elaborate this problem setting.

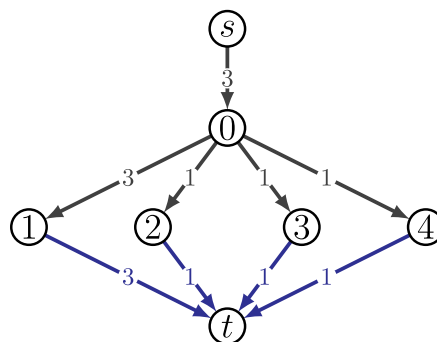
In Figure 1, the arc capacities are specified on each arc, and the interdiction cost is three for arcs $(s, 0)$, $(0, 1)$, $(0, 2)$, $(0, 3)$, and $(0, 4)$, and is one for arcs $(1, t)$, $(2, t)$, $(3, t)$, and $(4, t)$. Let $\{(1, t), (2, t), (3, t), (4, t)\}$ be the set of special arcs. The maximum s - t flow value on the network is three, but there are multiple maximum flow solutions: for example, (i) a flow of 3 units through path $s - 0 - 1 - t$; (ii) a flow of 1.5 units through path $s - 0 - 1 - t$ and a 0.5-unit flow through each of the paths $s - 0 - 2 - t$, $s - 0 - 3 - t$, and $s - 0 - 4 - t$. The choice of a maximum flow solution by the follower may influence the objective value obtained by the leader: (i) would result in an objective value of one whereas (ii) would result in an objective value of four. We consider an interdiction budget of two and the leader’s decision according to the follower’s preference toward the implementation of a maximum flow solution.

In the optimistic variant of this problem, we consider the case where the leader assumes that the follower will implement a maximum flow solution with the smallest number of active special arcs, as in (i). Without interdiction, the maximum flow solution implemented by the follower would have only one active special arc, $(1, t)$, leading to an objective value of one for the leader’s problem. With a budget of two, the leader can remove up to two arcs from $\{(1, t), (2, t), (3, t), (4, t)\}$; in any case, at least one special arc would be active in the maximum flow, and hence one is a lower bound on the objective value of the leader, which can be achieved without interdiction. If the leader chooses to interdict two arcs, one of which is $(1, t)$, then the follower would need to send one unit of flow through each of the remaining special arcs to achieve the maximum flow, resulting in an objective value of two for the leader. Lastly, if $(1, t)$ is the only arc interdicted, then the follower would send flow through each of the remaining special arcs, resulting in an objective value of three for the leader. Therefore, in the optimistic case, any feasible interdiction plan that does not interdict arc $(1, t)$, including the interdiction plan where no arc is interdicted, is an optimal decision by the leader.

In the pessimistic variant of this problem, the leader assumes that the follower would implement a maximum flow solution that has the largest number of active special arcs, as in (ii). In this case, the leader expects that all remaining special arcs in the network after interdiction will be active in the maximum flow implemented by the follower, and therefore the leader would interdict as many special arcs as possible, which leads to the optimal decision of interdicting any subset of two arcs from $\{(1, t), (2, t), (3, t), (4, t)\}$.

In this paper, we propose a bilevel programming formulation for each variant of this problem. For both variants, the leader chooses arcs to interdict and the follower solves a maximum flow problem and chooses a maximum flow solution to implement in the remaining network, and therefore also chooses special arcs to have a positive flow on them, which leads to bilevel formulations with integer (binary) variables on both levels. We discuss a series of theoretical aspects of our problem including the complexity of both variants and properties of optimal flow values for the pessimistic variant. We analyze a class of networks with practically important applications, including human trafficking modeling, and discuss a defining feature of an optimal solution of the pessimistic variant over this class of networks. Based on this feature, we develop a solution method for the pessimistic

Figure 1. (Color online) A Simple Illustrative Example



Note. Special arcs are represented in blue.

problem that is exact for this network class. We then discuss approaches to derive single-level reformulations for both versions of the problem. For the optimistic variant, we apply the “dualize-and-combine” approach (Smith and Song 2020), to derive a single-level reformulation that can be solved by state-of-the-art solvers. For the pessimistic variant, we first introduce a valid relaxation of the integrality of a lower-level variable and then we derive two single-level reformulations: one based on the dualize-and-combine approach, the other based on the level-set relaxation approach by Zeng (2020). To evaluate the performance of our solution approaches for both the optimistic and the pessimistic variants of the problem, we perform computational experiments on a set of randomly generated layered networks for different interdiction budget levels. To draw domain-specific insights, we conduct computational experiments on a set of randomly generated test instances inspired by domestic human trafficking networks. We conclude the paper with a discussion of potential extensions of our work.

Our main contributions can be summarized as follows:

1. We propose a bilevel network interdiction problem where the follower seeks to maximize the flow from source node to sink node, whereas the leader, who interdicts the network, seeks to minimize the number of arcs from a critical set that have positive flow on them in the maximum flow solution obtained by the follower. To the best of our knowledge, this is the first work to consider this problem. Our problem is motivated by applications in disrupting illicit supply chains where the follower aims to maximize the profit obtained from illicit activity, whereas the leader aims to minimize the number of components in the system affected by the operation.

2. We show that both the optimistic and the pessimistic versions of our problem are polynomial-time solvable for the special case of networks with only one layer of nodes between source and sink nodes, but both versions become NP-hard as the networks become generally structured, including when there is only a bipartite network between the source and the sink.

3. We derive single-level reformulations for the optimistic and the pessimistic variants of our problem. For the optimistic variant, we apply the dualize-and-combine approach to achieve a mixed-integer linear single-level reformulation. For the pessimistic variant, we first introduce a valid reformulation based on the relaxation of the integrality of lower-level binary variables, and then we derive two alternative single-level reformulations, one obtained by applying the dualize-and-combine approach and the other obtained by a level-set relaxation scheme by Zeng (2020).

4. Lastly, we introduce a tailored solution method for the pessimistic variant of our problem. This solution method is motivated by the analysis of a practically important class of networks, and we prove that for this class of networks the method is an exact algorithm. Through computational experiments on randomly generated layered networks, we verify that the tailored method is orders of magnitude faster than the benchmark approaches for all instances.

The remainder of this paper is organized as follows. Section 2 presents a brief literature review of bilevel optimization, network interdiction, and applications of these tools in the context of human trafficking disruption. In Section 3 we present our problem and discuss its bilevel optimistic and pessimistic formulations. In Section 4 we discuss the complexity of our problem and other important theoretical considerations related to optimal flow levels of the pessimistic problem as well as a class of networks with important applications; for this class we also develop a specialized solution method. We discuss single-level reformulations for the optimistic and the pessimistic versions of our problem in Section 5 and present computational experiments in Section 6. We close this work with our conclusions and future research directions in Section 7.

2. Literature Review

In this section, we review literature related to our problem and its motivation including the topics of bilevel optimization, interdicting illicit supply chains, and operations research (OR) efforts to counter human trafficking.

2.1. Bilevel Optimization

Bilevel optimization is a popular approach to model the interactions between two decision makers in various domains from economics to engineering and natural sciences (Dempe 2002, Fischetti et al. 2017, Kleinert et al. 2021). The initial development of bilevel optimization originated in the field of economic game theory and focused on market structure and equilibrium problems, particularly duopoly models, now known as Stackelberg games (Dempe and Zemkoho 2020). It consists of an upper-level problem (also referred to as the *outer* or the *leader's* problem) and a lower-level problem (also referred to as the *inner* or the *follower's* problem) where the lower-level problem is embedded as a constraint or objective within the upper-level problem.

The flexibility of bilevel optimization in modeling hierarchical decision-making problems is often counterbalanced by its computational challenges. Bilevel optimization in general is a very challenging class of optimization problems; in fact, it has been shown to be strongly NP-hard (Bard 1991, Hansen et al. 1992) even for linear upper- and lower-level problems. Checking a point's local optimality in a bilevel optimization problem is also NP-hard

(Vicente et al. 1994). Bilevel optimization problems are nonconvex in general, and their feasible points violate the constraint qualifications that are often used to verify optimality and convergence conditions (Dempe et al. 2015).

Moreover, because the lower-level problem may have multiple optimal solutions, bilevel optimization problems are not well-posed in general, which leads to the adoption of two main approaches to handle this ambiguity: the so-called *optimistic* and *pessimistic* problems (Dempe et al. 2014). In the optimistic problem, the leader assumes that the follower will implement the optimal solution that is best according to the leader's interest, whereas in the pessimistic problem the leader assumes that the follower will implement the optimal solution that is worst according to the leader's interest. Pessimistic bilevel problems are the most difficult in the bilevel optimization field; indeed, it is often the case that an optimistic bilevel optimization problem is solvable whereas its corresponding pessimistic version does not have a solution (Dempe and Zemkoho 2020).

Applications of bilevel optimization have been used to model problems in a variety of areas such as energy networks and markets (Dempe et al. 2015, Kleinert and Schmidt 2019), critical infrastructure (Scaparra and Church 2008, Johnson and Dey 2022), pricing and production decisions (Bui et al. 2022, Khorramfar et al. 2022), and location problems (Goyal et al. 2023, Lamontagne et al. 2023).

2.2. Interdiction of Illicit Supply Chains

Network interdiction problems are among the most studied problems in bilevel optimization. Early papers such as Wollmer (1970), McMasters and Mustin (1970), and Golden (1978) set the basis for this research field, which gained much attention with the work by Wood (1993). In that paper, Wood (1993) introduced the maximum flow network interdiction problem (MFNIP), where a leader removes arcs from the network to minimize the maximum flow value obtained by the follower, and derived single-level reformulations for a few variants of the MFNIP. We refer the interested reader to the survey paper by Smith and Song (2020) for a broader discussion of network interdiction models and algorithms, including network interdiction's roots in military and defense applications such as securing borders against smuggling of nuclear material (Morton et al. 2007, Sullivan et al. 2014) and protecting critical infrastructures against attacks (Salmeron et al. 2004, Scaparra and Church 2008). More recently, network interdiction has received increasing attention as an effective tool to model other pressing societal issues. Smith and Song (2020) identify illicit supply chain disruption among the emerging applications of network interdiction. In this section, we discuss recent applications of network interdiction in the context of disrupting illicit supply chain networks.

Griffin et al. (2023) consider a network interdiction problem applied to combating illicit wildlife trading. The authors discuss models where the leader represents law enforcement authorities that allocate limited resources to increase the probability of detecting a wildlife trafficker who aims to smuggle wildlife from an origin to a destination through a network composed of airports and flights between them.

Malaviya et al. (2012) consider a multiperiod network interdiction problem that focuses on scheduling law enforcement operations to remove criminals from an illegal drug supply chain. This network interdiction problem captures some of the characteristics reported by law enforcement agencies in the pursuit of disrupting illegal drug networks, such as the temporal aspect of these interdictions and the need to build a strong case in order to arrest high-ranking traffickers.

Baycik et al. (2018) extend the work of Malaviya et al. (2012) and investigate the problem of interdicting layered physical and information flow networks. Shen et al. (2021) consider the closely related problem of interdicting interdependent contraband smuggling, money, and money laundering networks of a transnational criminal organization, which models the various dependencies among these illegal networks. Kosmas et al. (2023b) consider a maximum flow network interdiction problem to disrupt drug trafficking networks that allows for network restructuring as the trafficker can add new arcs to the network after the antitrafficking agent has taken interdiction actions.

2.3. Countering Human Trafficking

Network interdiction problems have also been used in the context of combating human trafficking networks. Mayorga et al. (2019) present two illustrative robust network interdiction problems to disrupt the flow of human trafficking victims through a network. Xie and Aros-Vera (2022) adapt and extend the work of Baycik et al. (2018) in considering a network interdiction problem for disrupting sex trafficking networks that incorporates mutual interdependencies between the information and physical networks. Tezcan and Maass (2023) propose a multiperiod network interdiction problem where flow through an arc represents the desirability of a trafficker to travel along the respective segment of the network. The work of Mayorga et al. (2019), Xie and Aros-Vera (2022), and Tezcan and Maass (2023) tends to focus on disrupting the *movement* of individuals through a trafficking network but does not focus on how victims are controlled or exploited by traffickers. Kosmas et al. (2023a) address

the issues of how to generate realistic, operational sex trafficking networks and apply the model of Kosmas et al. (2023b) to network models that use flow to model the control that traffickers have over their victims. Kosmas et al. (2024) consider a multiperiod version of the maximum flow network interdiction problem with restructuring and apply it to disrupting human trafficking networks. Clark et al. (2023) extend the concept of “flow as control” to capture that control can be viewed as the number of hours that a victim is forced to participate in illegal activities, which helps to better connect the flow decisions to the revenues generated for the traffickers.

Our work adds to the growing field of network interdiction (and more broadly OR applications) to combat human trafficking (Konrad 2019, Maass et al. 2020, Keskin et al. 2021, Dimas et al. 2022) in response to a call to action made by a variety of researchers (Konrad et al. 2017, Caulkins et al. 2019, Mayorga et al. 2019, Sharkey et al. 2021) that recognized the unique potential of the OR community in providing analytical tools to address this pressing societal problem. The value of these tools, however, is conditional on whether the process that leads to their creation is conducted in a socially responsible manner (Konrad et al. 2023) and on whether it considers the inputs from multiple stakeholders to identify unintended consequences and validate modeling assumptions based on subject-matter experts’ knowledge, which requires a transdisciplinary approach toward research (Lotrecchiano and Misra 2018, Martin et al. 2022).

Similarly to aforementioned problems, the problem we propose in Section 3 has its motivation in combating human trafficking. However, our problem differs from previous works in the literature as the objectives of the leader and the follower are not directly aligned with each other: whereas the follower’s objective is to maximize the flow through the network, the leader’s objective is to minimize the number of “special” arcs with flow on them after the interdiction. We recognize that modeling human trafficking requires a mathematical abstraction of significant trauma and that no model can fully capture the lived experience. However, we believe such modeling can help understand the problem.

From an application perspective, the problem incorporates a profit-driven approach taken by the trafficker (Barrick et al. 2023), and the perspective of a stakeholder who wants to directly minimize the number of victims in the network as opposed to minimizing the maximum profit obtained by the trafficker. However, it is important to note that although this objective is practically motivated, it is not the only potential objective for interventions into human trafficking and that an exploration of the unintended consequences of suggested interventions would need to be done through appropriate partnerships (Konrad et al. 2023). For example, one potential unintended consequence of the objective is that the victims remaining in the network after intervention could have their situation become significantly worse. Through appropriate partnerships and analysis, we believe that our model could help to provide some insights into human trafficking interventions that have a broader and more holistic approach by considering the expectations and contributions of multiple antitrafficking agents such as nongovernmental organizations (NGOs), law enforcement agencies, researchers, social services providers, and survivor-centered groups, among others.

3. Problem Statement

We consider a directed network $G = (N, A)$, where N is the node set with $|N| = n$, and A is the arc set with $|A| = m$. There exists both a source node s and a sink node t . Each arc $(i, j) \in A$ has a positive rational capacity u_{ij} . For the sake of simplicity and without loss of generality, we can assume that all arc capacities are positive integers, because otherwise, we can find a factor, say ρ , such that ρu_{ij} is integer for all $(i, j) \in A$. By multiplying the capacity of each arc by this factor, we obtain a positive integer capacity at each arc. Therefore, we work with integer capacities throughout this paper and address any necessary transformation when needed. There is also an interdiction cost r_{ij} associated with each arc $(i, j) \in A$.

We also introduce a dummy arc from t to s in G with infinite capacity and infinite interdiction cost. Among the set of arcs A , we assume that there is a subset of arcs, $\mathcal{A} \subseteq A$, that are of particular importance to the leader. The leader is given a budget R to remove arcs from the network and seeks to minimize the number of arcs from \mathcal{A} that carry a positive flow on them in the follower’s max-flow solution, which is obtained by the follower who seeks to maximize the flow from s to t in the postinterdiction network. This problem, without loss of generality, can also be used to address situations where the networks have capacitated and/or interdictable nodes, and multiple sources and/or sinks by appropriate network transformations such as node splitting and artificial super-source and/or supersink nodes; see Ahuja et al. (1993) for network transformation details.

Our problem, as stated above, suffers from the ambiguity inherent to bilevel optimization problems as mentioned in Section 2.1. In this specific problem, for a given interdiction decision there may exist multiple maximum flows and each maximum flow can result in a different objective value for the leader’s problem. As defined so far, our problem does not specify which maximum flow should be implemented in the case of multiple optimal

solutions to the follower's problem. We will present two variants of our bilevel optimization problem, an optimistic and a pessimistic version, in Sections 3.1 and 3.2, respectively. In the optimistic version, the follower will implement the maximum flow solution which has the smallest number of special arcs that carry a positive flow, and therefore is the most favorable maximum flow according to the leader's objective. On the other hand, in the pessimistic version, the follower will implement the maximum flow that has the largest number of special arcs that carry a positive flow, which is the least favorable maximum flow from the leader's perspective.

We define binary variables γ_{ij} for all arcs $(i, j) \in A$, where γ_{ij} is equal to one if arc (i, j) is interdicted. Given the interdiction decisions, the follower solves a maximum s - t flow problem, for which we define decision variable x_{ij} to represent the flow value on each arc $(i, j) \in A$. Lastly, we define the active flow indicator variables z_{ij} for $(i, j) \in A$ to indicate whether arc (i, j) in the special set \mathcal{A} has a positive flow on it according to the follower's decision \mathbf{x} . The constraints needed to enforce the relationship between \mathbf{x} and \mathbf{z} differ based on the perspective taken by the leader (i.e., optimistic or pessimistic) with respect to which solution is implemented in case of multiple maximum flows. We discuss how to implement the relationship between these lower-level variables in more detail later; for now we assume that $Z(\mathbf{x})$ is a set containing all $\mathbf{z} \in \{0, 1\}^{|\mathcal{A}|}$ such that

$$z_{ij} = \begin{cases} 1, & \text{if } x_{ij} > 0, (i, j) \in \mathcal{A}, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

We define the set of feasible interdiction vectors, Γ , the set of feasible flows parameterized by the interdiction decision, $X(\boldsymbol{\gamma})$, and the set of maximum flows with respect to the interdiction vector, $S(\boldsymbol{\gamma})$, as follows:

$$\Gamma := \left\{ \boldsymbol{\gamma} \in \{0, 1\}^{|\mathcal{A}|} : \sum_{(i, j) \in \mathcal{A}} r_{ij} \gamma_{ij} \leq R \right\}, \quad (2)$$

$$X(\boldsymbol{\gamma}) := \left\{ \mathbf{x} : \sum_j x_{ij} - \sum_j x_{ji} = 0, \forall i \in N, 0 \leq x_{ij} \leq u_{ij}(1 - \gamma_{ij}), \forall (i, j) \in A \right\}, \quad (3)$$

$$S(\boldsymbol{\gamma}) := \arg \max_{\mathbf{x}} \{x_{ts} : \mathbf{x} \in X(\boldsymbol{\gamma})\}. \quad (4)$$

Given these definitions, we are ready to present the optimistic and the pessimistic versions of our bilevel optimization problem.

3.1. The Optimistic Bilevel Optimization Problem

In the optimistic setting, the leader assumes that the follower will implement the most favorable maximum flow according to the leader's objective. In other words, the leader can *choose* an optimal \mathbf{x} solution from $S(\boldsymbol{\gamma})$ according to their own interests. Because of this choice flexibility for the leader, we must ensure that variable z_{ij} cannot be set to zero when $x_{ij} > 0$. This allows the constraint set $Z(\mathbf{x})$ to be modeled with (5d) and (5e) in Formulation (5).

$$\min_{\boldsymbol{\gamma}} \sum_{(i, j) \in \mathcal{A}} z_{ij} \quad (5a)$$

$$\text{s.t. } \boldsymbol{\gamma} \in \Gamma, \quad (5b)$$

$$\mathbf{x} \in S(\boldsymbol{\gamma}), \quad (5c)$$

$$x_{ij} \leq u_{ij} z_{ij}, \forall (i, j) \in \mathcal{A}, \quad (5d)$$

$$\mathbf{z} \in \{0, 1\}^{|\mathcal{A}|}. \quad (5e)$$

One can observe that Constraint (5d) enforces $z_{ij} = 1$ if $x_{ij} > 0$, for $(i, j) \in \mathcal{A}$, but it does not prevent $z_{ij} = 1$ when $x_{ij} = 0$. This is sufficient in the optimistic version because in an optimal solution, $z_{ij} = 1$ only if $x_{ij} > 0$ for $(i, j) \in \mathcal{A}$.

3.2. The Pessimistic Bilevel Optimization Problem

In the pessimistic setting, the leader assumes that the follower will implement the least favorable maximum flow according to the leader's objective. We model the pessimistic bilevel optimization problem as follows:

$$\min_{\boldsymbol{\gamma}} \max_{\mathbf{x}, \mathbf{z}} \sum_{(i, j) \in \mathcal{A}} z_{ij} \quad (6a)$$

$$\text{s.t. } \boldsymbol{\gamma} \in \Gamma, \quad (6b)$$

$$\mathbf{x} \in S(\boldsymbol{\gamma}), \quad (6c)$$

$$\mathbf{z} \in Z(\mathbf{x}). \quad (6d)$$

The pessimistic bilevel optimization problem has an additional “min-max” structure in the objective, given the fact that the leader is concerned with the worst-case situation for its objective. Notice that in this case, Constraint (5d) would fail to indicate the positive flows according to the \mathbf{x} solutions because of the pessimistic position—the follower would be assumed to set $z_{ij} = 1$ whenever possible regardless of the value of x_{ij} . Therefore, in the pessimistic setting, we must explicitly enforce that $z_{ij} = 1$ only if there is a positive flow through arc $(i, j) \in \mathcal{A}$. We address this issue by replacing Constraint (6d) in Formulation (6) with Constraints (7c) and (7d) for a sufficiently large parameter M . The pessimistic bilevel optimization problem can then be formulated as

$$\min_{\gamma \in \Gamma} \max_{\mathbf{x}, \mathbf{z}} \sum_{(i,j) \in \mathcal{A}} z_{ij} \tag{7a}$$

$$\text{s.t. } \mathbf{x} \in S(\gamma), \tag{7b}$$

$$x_{ij} \geq \frac{1}{M} z_{ij}, \quad \forall (i, j) \in \mathcal{A}, \tag{7c}$$

$$\mathbf{z} \in \{0, 1\}^{|\mathcal{A}|}. \tag{7d}$$

We show that $|\mathcal{A}|$ is a sufficiently large M in Section 4.2.

3.3. Modeling the Motivating Applications

In this section, we discuss how our problem connects to disrupting drug trafficking networks and human trafficking networks. In the work of Malaviya et al. (2012), a problem related to interdicting illegal drug supply chains is considered. Initially, nodes model various people in the illegal drug supply chain, whereas arcs model their connections (note that through standard node expansion techniques, the problems studied focus solely on arc interdiction). Based on realistic features of certain drug trafficking networks, the network data considered by this work are hierarchical in the sense that drugs flow down through different layers of these networks. Further, the bottom layer of the network (the one closest to the sink node) captures people who are using the illegal drug and captures that these nodes are the only ones directly connected to the sink node. The problem we consider in this paper can be directly applied to an illegal drug network similar to the networks considered by Malaviya et al. (2012), where the set of special arcs is considered as those that connect people who use the drugs to the sink node. In this case, the leader is seeking to minimize the number of people who are using the illegal drug, which could help to minimize the harm done by it. This particular application motivates two special considerations in our problem: (i) networks that are “layered” or hierarchical and (ii) problems whose entire set of special arcs is connected to the sink node.

For human trafficking networks, we take an approach similar to the work of Kosmas et al. (2023a) and focus on the control, or exploitation, network of traffickers, bottoms, and victims. This approach focuses efforts on disrupting the exploitation of victims. Further, we use the concept of flow capturing the number of hours victims are forced to participate in illegal activities from Clark et al. (2023). Therefore, each victim has a capacity on this “up-time” (see Clark et al. (2023) for a discussion on the context of capacity for human trafficking victims) in terms of the number of hours they may be forced to participate. Each trafficker has a capacity on the number of hours they choose to have victims underneath them participating in illegal activities, which could be used to capture their desired amount of risk. Flow from a trafficker through a victim represents that the trafficker is forcing the victim to participate in illegal activities for that number of hours. We can connect a source node in the network to all traffickers to represent the fact that they generate control and connect all victims to the sink node. Applying standard network expansion techniques would allow us to model this as a maximum flow problem with only arc capacities and also focus on just arc interdictions. This application motivates two special considerations in our problem: (i) problems where the special arcs are all connected to the source node (e.g., minimize the number of active traffickers in the human trafficking application) and (ii) problems where the special arcs are all connected to the sink node (e.g., minimize the number of active victims).

4. Theoretical Analysis

In this section we develop several theoretical results regarding our problem. We begin by analyzing the complexity of both its optimistic and pessimistic versions. We show that for the special case when there exists a single layer of nodes between the source and sink nodes, both versions are polynomial-time solvable. However, for networks with two or more layers of nodes between source and sink nodes, both versions are NP-hard. Next, we discuss structural properties of the optimal flow in the pessimistic problem, which allows us to show that $M = |\mathcal{A}|$ is sufficiently large in Constraint (7d), and this has further implications in our single-level reformulation of the pessimistic problem. We close this section by discussing a class of networks with practically important

applications, including our motivating application of human trafficking disruption, and a tailored solution method for the pessimistic problem, which is an exact method for this special class of networks.

4.1. Complexity Analysis

We show that both the optimistic and the pessimistic versions of our problem are polynomial-time solvable for the special case of networks with only one layer of nodes between source and sink nodes, but both versions become NP-hard as the networks become generally structured.

4.1.1. Complexity of Single-Layer Networks. The simplest version of our problem is one where there is a single layer of nodes between the source and the sink node; hence, there exist n arc-disjoint paths that move through node i using arcs (s, i) and (i, t) . We refer to this class of networks as *Single-Layer* networks, and we let \mathcal{A} be the set of arcs of the form $(i, t), i \in N$. In this case, we can shift the capacity and interdiction costs to arc (s, i) by setting its capacity to be $\min\{u_{si}, u_{it}\}$ and its interdiction cost to be $\min\{r_{si}, r_{it}\}$.

Throughout this section, we need the assumptions that the source node is capacitated and that the capacity of the source node is strictly smaller than the sum of the capacities of its outgoing arcs to differentiate the optimistic problem from the pessimistic problem. If we did not impose these conditions, both the optimistic and pessimistic problems would have the same optimal interdiction plan: remove as many individual arcs as possible because any arc that remains will have a flow on it equal to its capacity. This interdiction problem reduces to a knapsack problem where each arc is represented through an item, the value of each item is one, its size is equal to its interdiction cost, and the size of the knapsack is equal to the interdiction budget. This problem can be solved by greedily selecting the smallest items until reaching the knapsack size. Finally, to keep our analysis consistent with our problem statement, we apply the node splitting transformation to the source node, where we split the node s into uncapacitated nodes s' and s'' , add arc (s', s'') , and set the capacity of (s', s'') , $u_{s's''}$, equal to the capacity of s .

We have the following results regarding the complexity of Single-Layer networks.

Theorem 1. *The optimistic problem on Single-Layer networks is polynomial-time solvable.*

Theorem 2. *The pessimistic problem on Single-Layer networks is polynomial-time solvable.*

We refer the reader to Online Supplement A.1 for the proofs of Theorems 1 and 2.

4.1.2. Complexity of Two-Layer Networks. We now consider networks where the subnetwork created by removing nodes s and t is bipartite. In other words, the nodes can be split into two sets, the source set which includes all nodes originally connected to the source, and the sink set which includes all the nodes originally connected to the sink. Because there are two layers of nodes between the source and the sink node, we refer to networks in this class as *Two-Layer* networks. For Two-Layer networks, we show that the decision versions of both the optimistic and pessimistic problems are NP-complete.

4.1.2.1. Complexity of the Optimistic Problem on Two-Layer Networks. Consider a Two-Layer network with source set S , with sink set T such that $|T| = 2$, and with special set \mathcal{A} consisting of the two arcs of the form $(j, t), j \in T$. We can prevent a special arc $(j, t) \in \mathcal{A}$ from having flow by interdicting all arcs of the form $(s, i), i \in S$, such that $(i, j) \in \mathcal{A}$. Assuming we are given a target to prevent one arc in \mathcal{A} from having flow, the decision version of the optimistic problem on Two-Layer networks is to determine whether there exists a subset S of arcs $(s, i), i \in S$, with interdiction cost at most R that achieves this target.

Theorem 3. *The optimistic decision problem on Two-Layer networks is NP-complete.*

The proof of Theorem 3 can be found in Online Supplement A.2.

4.1.2.2. Complexity of the Pessimistic Problem on Two-Layer Networks. Consider a Two-Layer network with source set S , with sink set T , and with special set \mathcal{A} consisting of all arcs of the form $(j, t), j \in T$. We prevent a special arc $(j, t) \in \mathcal{A}$ from having flow by interdicting all arcs $(s, i), i \in S$, such that $(i, j) \in \mathcal{A}$. Assuming we are given a target to prevent at least K arcs in \mathcal{A} from having flow on them, the decision version of the pessimistic problem on Two-Layer networks is to determine whether there exists a subset S of arcs of the form $(s, i), i \in S$, with interdiction cost at most R such that by interdicting all arcs in S we achieve the target.

Theorem 4. *The decision version of the pessimistic problem on Two-Layer networks is NP-complete.*

The proof of Theorem 4 can be found in Online Supplement A.2.

One last complexity result of interest is the one with respect to the inner maximization problem in the pessimistic setting. We have the following remark.

Remark 1. The pessimistic inner problem is polynomial-time solvable. In Section 5.2 we show that the pessimistic inner problem becomes a linear programming problem after the relaxation of \mathbf{z} (and the application of the dualize-and-combine approach) and is therefore solvable in polynomial time. Additionally, in the next section we discuss a method that runs in polynomial time (Algorithm 3) to convert any integer maximum flow to a maximum flow with an associated maximal \mathbf{z} , which directly provides a solution to the pessimistic inner problem.

As a summary, we have shown that both the optimistic and pessimistic problems on Single-Layer networks are polynomially solvable, whereas they are NP-hard problems on Two-Layer networks (and more general networks). Hence, we have provided a rather thorough classification of the complexity of both versions of our problem over different classes of networks.

4.2. Properties of Optimal Flow Values to the Pessimistic Problem

In this section we discuss important properties of flows in an optimal solution to the pessimistic problem. Our discussion culminates on the main result of this section, Theorem 5, which supports the development of a single-level reformulation for the pessimistic problem in Section 5.2.

Recall that Constraint (7c) ensures the necessary relationship between decision variables \mathbf{x} and \mathbf{z} in the pessimistic problem, assuming we set an appropriate value to M . In what follows, we show that, assuming integer capacities, for any $M \geq |\mathcal{A}|$, Formulations (6) and (7) have the same set of maximal \mathbf{z} vectors with respect to a maximum flow (the case of rational capacities is addressed with a slight modification and is discussed in Remark 2 at the end of this section). We prove this result constructively by fixing $M \geq |\mathcal{A}|$ in Formulation (7) and presenting an algorithm that takes an integer maximum flow as input and constructs a maximal \mathbf{z} vector with respect to the maximum flow, which implies that, for $M \geq |\mathcal{A}|$, no maximal \mathbf{z} is cut off from the feasible region of Problem (7). The algorithm is based on constructing cycles to send $1/M$ units of flow through a special arc at every iteration, and we show that the algorithm needs no more than $|\mathcal{A}|$ iterations to construct a maximal \mathbf{z} . These cycles are constructed with a special property to ensure that once a special arc (i, j) receives $1/M$ units of flow, its corresponding reverse arc (j, i) is never included in any of these cycles, and hence, the special arc (i, j) will keep at least $1/M$ units of flow on the arc as we construct the maximal \mathbf{z} .

The first step in our development is a simple procedure, which we refer to as UpdateFlow. At any iteration k , procedure UpdateFlow takes a flow vector $\hat{\mathbf{x}}^k$ and a directed cycle \mathcal{C} as input, sends $1/M$ units of flow through \mathcal{C} , and updates the flow vector $\hat{\mathbf{x}}^k$. For completeness, the pseudocode corresponding to the UpdateFlow procedure is presented in Algorithm 1. We observe that, even though the UpdateFlow procedure redistributes flow over arcs of the network, it does not change the maximum flow value from s to t .

Algorithm 1 (UpdateFlow)

- 1: **procedure** UpdateFlow($\hat{\mathbf{x}}^k, \mathcal{C}$)
- 2: **for** $(i, j) \in \mathcal{C}$ **do**
- 3: $\hat{x}_{ij}^k \leftarrow \hat{x}_{ij} + 1/M$
- 4: $u_{ij}^r \leftarrow u_{ij}^r - 1/M; u_{ji}^r \leftarrow u_{ji}^r + 1/M$ ▷ u_{ij}^r is the residual capacity of arc (i, j)

The UpdateFlow procedure plays a central role in our proof, and its use motivates the following definitions. When we use the UpdateFlow procedure to send $1/M$ units of flow through a special arc $(i, j) \in \mathcal{A}$ which has a flow value of zero, thus ensuring that the flow on (i, j) after the update satisfies Constraint (7c) with the corresponding z_{ij} variable set to be one, we say that the special arc (i, j) has been *activated*. Consequently, every activated arc is associated with a cycle, which we refer to as its *activating cycle*. Note that there may be special arcs that are used in the original maximum flow, but those are not activated according to our definition. An important remark is that a single activating cycle may be associated with multiple activated arcs, that is, the activating cycle may contain more than one special arc. Furthermore, given an activated arc $(i, j) \in \mathcal{A}$, we refer to arc (j, i) as a *reverse activated arc*. It is important in our construction to ensure that no activated arc is *deactivated* by sending flow through its reverse activated arc. Lemma 1 provides a mechanism (codified by Algorithm 2) to avoid this issue.

Lemma 1. For any special arc $(i, j) \in \mathcal{A}$ such that $\hat{x}_{ij}^k = 0$ and which is contained in a directed cycle in the residual network $G(\hat{\mathbf{x}}^k)$, there exists a directed path from j to i in $G(\hat{\mathbf{x}}^k)$ that does not contain a reverse activated arc.

Proof. Let (i, j) be a special arc such that $\hat{x}_{ij}^k = 0$. Suppose that $(i, j) \in \mathcal{C}$, where \mathcal{C} is a directed cycle in $G(\hat{\mathbf{x}}^k)$ that contains a reverse activated arc. For any reverse activated arc $e' = (w, v)$ in \mathcal{C} , there exists an activating cycle \mathcal{C}^e

with respect to the activated arc $e = (v, w)$ in $G(\hat{x}^k)$. We replace e' with $C^e \setminus \{(v, w)\}$ in \mathcal{C} and by doing so we create a directed walk \mathcal{W} from j to i . This scheme, however, may introduce cycles in \mathcal{W} when there exist multiple reverse activated arcs in \mathcal{C} and their corresponding activating cycles intersect with each other. By eliminating any cycles (as described in Algorithm 2 and illustrated in Example 2 in Online Supplement B) in \mathcal{W} , we obtain a directed path \mathcal{P} from j to i with no reverse activated arcs. \square

Algorithm 2 implements Lemma 1 and constructs an activating cycle with the special property that it does not contain any reverse activated arcs. Cycles with this property can be used to activate special arcs without deactivating any special arc already activated, which implies that we can activate all the special arcs that can be activated without reducing the maximum flow with at most $|\mathcal{A}|$ calls to Algorithm 2.

Algorithm 2 (ActivatingCycle)

```

1: function ActivatingCycle( $e, \mathcal{C}, C, \mathcal{Z}_r$ )
2:    $\mathcal{W} \leftarrow \mathcal{C} \setminus \{e\}$  ▷ Construction of walk  $\mathcal{W}$ 
3:    $\mathcal{P} \leftarrow \mathcal{W}$ 
4:   if  $\mathcal{C} \cap \mathcal{Z}_r \neq \emptyset$  then
5:     for  $e' = (w, v) \in \mathcal{C} \cap \mathcal{Z}_r$  do
6:       Replace  $e'$  with  $C^{e'} \setminus \{(v, w)\}$  in  $\mathcal{W}$ 
7:       for  $i \in \mathcal{W}$  do ▷ Cycle Elimination
8:         if  $i \in \mathcal{P}$  then
9:           Let  $p$  be the position of  $i$  in  $\mathcal{P}$ 
10:          for  $k \geq p$  do
11:            Delete node at position  $k$  in  $\mathcal{P}$ 
12:            Add  $i$  to the first available position in  $\mathcal{P}$ 
13:    $C \leftarrow \mathcal{P} \cup \{e\}$ 
14: return  $C$ 

```

With these definitions, we are in a position to discuss a simple algorithm (Algorithm 3) that takes an initial integer maximum flow \hat{x} as its input and constructs a maximal z with respect to the maximum flow. Note that the integrality of the initial maximum flow \hat{x} is not a restriction, because the assumption of integrality on the capacities of all arcs leads to an integral maximum flow value and the existence of an integral maximum flow is guaranteed. The case of rational arc capacities is addressed in Remark 2 at the end of this section.

Algorithm 3 (maximal z)

```

Input:  $\hat{x}, G(\hat{x})$ 
1:  $C \leftarrow \emptyset, \mathcal{Z}_r \leftarrow \emptyset, A^0 \leftarrow \emptyset$ 
2: for  $(i, j) \in \mathcal{A}$  do ▷ Mark special arcs used in the initial max-flow
3:   if  $\hat{x}_{ij} > 0$  then
4:      $A^0 \leftarrow A^0 \cup \{(i, j)\}$ 
5:  $k \leftarrow 0; \hat{x}^k \leftarrow \hat{x}$ 
6: for  $(i, j) \in \mathcal{A} \setminus A^0$  do ▷ Skip special arcs already used
7:   if  $\hat{x}_{ij}^k < 1/M$  then
8:     if  $(i, j) \in \mathcal{C}$ , where  $\mathcal{C}$  is a directed cycle in  $G(\hat{x}^k)$  then
9:        $k \leftarrow k + 1; \hat{x}^k \leftarrow \hat{x}^{k-1}$ 
10:       $C^k \leftarrow \text{ActivatingCycle}((i, j), \mathcal{C}, C, \mathcal{Z}_r)$ 
11:      UpdateFlow( $\hat{x}^k, C^k$ )
12:      for  $(v, w) \in C^k \cap \mathcal{A}$  do
13:         $\text{arcs}^k \leftarrow \text{arcs}^k \cup \{(v, w)\}$  ▷ Set of arcs activated by cycle  $C^k$ 
14:         $\mathcal{Z}_r \leftarrow \mathcal{Z}_r \cup \{(w, v)\}$ 
15:       $C \leftarrow C \cup C^k$ 

```

Algorithm 3 scans the flow on each special arc and marks the ones that have a positive flow (hence, because of the integrality assumption, larger than or equal to $1/M$) in the initial maximum flow. For any other special arc $e = (i, j)$, the algorithm verifies whether e can be activated without changing the overall value of the maximum flow. This verification is done by checking whether e is in a directed cycle in the residual network $G(\hat{x}^k)$; if it is, then the algorithm calls Algorithm 2 to construct an activating cycle C^k associated with e that does not contain any reverse special arc. Lemma 1 ensures that we can always construct such a cycle. Algorithm 3 then calls procedure UpdateFlow to activate e as well as any other special arcs in C^k , stores the corresponding reverse activated

arcs, and adds C^k into a set containing activating cycles. Observe that $\mathcal{P} \cup \{(i, j)\}$ is a directed cycle in $G(\hat{x}^k)$ and it can be used to activate (i, j) . Example 2 in Online Supplement B illustrates the procedure described above.

We next discuss two supporting results regarding Algorithm 3 that follows from Lemma 1.

Corollary 1. *No activated arc is deactivated during an execution of Algorithm 3.*

Proof. It follows directly from Lemma 1 and from Algorithm 3; Lemma 1 and Algorithm 2 ensure that we can always construct an activating cycle without any reverse activated arcs for each special arc contained in a directed cycle in the residual network and that has not been activated. From Algorithm 3, because only the cycles without reverse activated arcs are used with procedure UpdateFlow, the result follows. \square

Corollary 2. *No activating cycle is eliminated before all the special arcs have been scanned during an execution of Algorithm 3.*

Proof. Recall that all arc capacities are integer values and Algorithm 3 is initialized with an integral maximum flow. Therefore, the residual capacity of each arc in the activating cycle corresponding to the first activated arc is at least one. Algorithm 3 creates a directed, elementary cycle corresponding to at least one activated arc through Algorithm 2, ensuring that no arc is traversed more than once when UpdateFlow is called. Every time UpdateFlow is deployed, the residual capacity of each arc in the corresponding activating cycle is decreased by $1/M$. Because UpdateFlow is called at most once for each special arc and at most M times throughout the algorithm execution, the residual capacity of any arc in an activating cycle cannot drop to zero before the last special arc has been scanned. \square

The discussion from this section culminates in the following theorem.

Theorem 5. *Given a feasible interdiction vector γ , for $M \geq |\mathcal{A}|$ Problems (6) and (7) have the same set of feasible \mathbf{z} vectors.*

Remark 2. For problems with rational arc capacities, we need a slight modification: let ρ be a factor such that ρu_{ij} is integer for all $(i, j) \in \mathcal{A}$, then Theorem 5 holds for $M \geq \rho|\mathcal{A}|$.

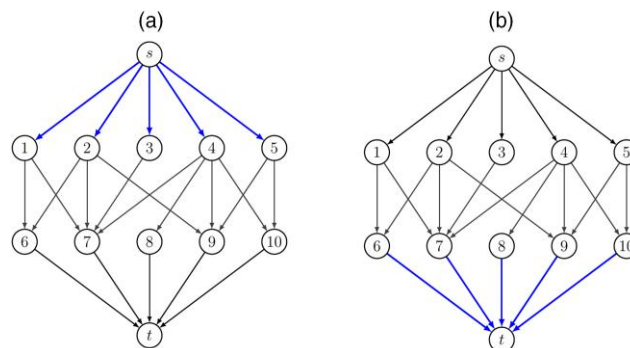
Remark 3. The lower bound $|\mathcal{A}|$ for integer capacities ($\rho|\mathcal{A}|$ in the case of rational capacities) is the smallest valid value for the big-M parameter M , as we can find counterexamples (see Online Supplement C) showing Theorem 5 does not hold for smaller values.

4.3. Analysis of a Practically Important Class of Networks

In this section, we present a class of networks with important motivating applications, we prove a property of the optimal solution of the pessimistic problem on this class of networks, and based on this property, we derive a tailored solution method to the pessimistic problem which is an exact method for this network class.

We consider networks where all the special arcs are connected to the same terminal node, that is, all the special arcs are either connected to the source node or to the sink node, and we make the standard assumption of maximum flow problems that there is no arc coming into the source node or leaving the sink node in the original network. We refer to networks where all the special arcs are connected to the source as “all-source” networks, and we refer to networks where all special arcs are connected to the sink node as “all-sink” networks. An illustration of networks in this class can be seen in Figure 2. This class of networks is particularly important in the context of human trafficking operations, where the arcs connected to the source node may represent traffickers and the arcs

Figure 2. (Color online) Special Class of Networks Where All Special Arcs Are Connected to One of the Terminal Nodes



Notes. (a) All-source network (special arcs in blue). (b) All-sink network (special arcs in blue).

connected to the sink node may represent victims. We have the following results for all-source and all-sink networks.

Theorem 6. Consider a network G such that all special arcs are of the form (i, t) , for $i \in N$. Let $(i', t) \in \mathcal{A}$, $i' \in N$. If there exists a direct path from s to i' in G , then $z_{i't} = 1$ for any optimal solution of the pessimistic problem.

Proof. Let the network G be such that all special arcs are of the form (i, t) , for $i \in N$, and let (i', t) be a special arc of G . Suppose that there exists a direct path $p_{si'}$ from s to i' in G , but $z_{i't}^* = 0$ for an optimal solution (γ^*, x^*, z^*) of the pessimistic problem. From the maximality of x^* , it follows that at least one arc in $p_{si'}$ must be at full capacity, for otherwise a positive flow could be sent through $p_{si'}$ increasing the value of $x_{is'}^*$. Let (v, w) be the arc at full capacity that is closest to i' in $p_{si'}$, that is, the arc at full capacity in $p_{si'}$ with the subpath along $p_{si'}$ with the lowest number of arcs to reach i' . By the flow decomposition theorem applied to x^* , arc (v, w) can appear in a set of cycles and a set of s - t paths, but it follows from the maximality of x^* that (v, w) does not appear in a cycle, because if it did, some positive flow from the cycle could be rerouted through the path $w - \dots - i' - t$, increasing the value of $x_{is'}^*$. Hence, (v, w) must appear only in a set of s - t paths in decomposing x^* . Choose one of these s - t paths of x^* , denote it by p' , and observe that the subpath $w - \dots - t$ of p' together with the subpath $w - \dots - i' - t$ of $p_{si'}$ form a cycle in G , which gives rise to a directed cycle in the residual network $G(x^*)$. This directed cycle in $G(x^*)$ can be used to send a positive flow along the subpath $w - \dots - i' - t$ of $p_{si'}$, activating the special arc (i', t) without changing the value of $x_{is'}^*$, and thus contradicting the optimality of z^* . \square

Corollary 3. Consider a network G such that all special arcs are of the form (s, i) , for $i \in N$. Let $(s, i') \in \mathcal{A}$, $i' \in N$. If there exists a direct path from i' to t in G , then $z_{si'} = 1$ for any optimal solution of the pessimistic problem.

It follows from Theorem 6 that for all-sink networks, any special arc to which there exists a directed path from s after interdiction will have a positive flow and hence the corresponding z_{ij} variable will take one as its value in any optimal solution of the pessimistic problem. Similarly, Corollary 3 implies that for all-source networks, any special arc from which there exists a directed path to t after interdiction will have a positive flow and its corresponding indicator variable will take one as its value in any optimal solution of the pessimistic problem. Therefore, for these networks, we can solve the pessimistic problem by solving an MFNIP (Wood 1993) in a modified network to identify the special arcs that remain in a directed s - t path. Henceforth, this approach is referred to as the MFNIP method, and it is described below.

MFNIP Method

Given a network $G = (N, A)$, construct a new network $\bar{G} = (\bar{N}, \bar{A})$ with $\bar{N} = N$ and $\bar{A} = A$, and let the arc capacities of \bar{G} be defined as follows:

$$u_{ij} = \begin{cases} 1, & \text{if } (i, j) \in \mathcal{A}, \\ \infty, & \text{if } (i, j) \in A \setminus \mathcal{A}. \end{cases}$$

Solve the MFNIP on \bar{G} to obtain an optimal interdiction decision $\hat{\gamma}^*$.

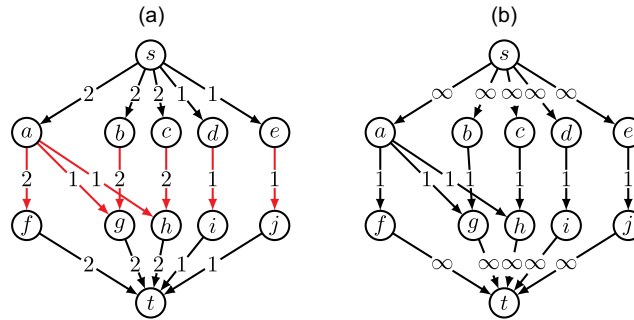
Solve the inner maximization problem for the fixed interdiction decision $\hat{\gamma}^*$ to obtain the corresponding values for the remaining variables of interest.

In our computational experiments, the MFNIP method will be demonstrated to be orders of magnitude faster than the single-level reformulation approach that we propose to solve the pessimistic problem on all-source and all-sink network instances. We point out, however, that for networks without these special structures, the MFNIP method applied to the resulting transformed network may not obtain the same number of arcs in its pessimistic flow calculation. We illustrate these cases with the following example.

Example 1. Consider the network shown in Figure 3(a), where the special arcs are represented in red, and the arc capacities are shown in each corresponding arc. Assume the interdiction budget is $R = 2$ and the interdiction costs are as follows: $r_{sa} = r_{sb} = r_{sc} = 2$, $r_{sd} = r_{se} = 1$, and for $i \neq s$, $r_{ij} = 2 + \Delta$, with $\Delta > 0$. Therefore, the leader can only interdict arcs connected to the source and the optimal interdiction decision for the pessimistic problem is to interdict arcs (s, d) and (s, e) . The maximum flow value in the remaining network is six and can only be achieved if the special arcs (a, g) and (a, h) are left without flow, which results in an objective value of three because only the special arcs (a, f) , (b, g) , and (c, h) are active in the maximum flow. Solving the MFNIP in the modified network of Figure 3(b), however, would result in the leader interdicting arc (s, a) and consequently an optimal objective value of four.

Example 1 demonstrates that the MFNIP method may fail to produce an optimal solution for networks where special arcs are not all connected to the source node or all connected to the sink node. Therefore, for these cases

Figure 3. (Color online) Example of a Network Where the Interdiction Obtained from the MFNIP Method Is Not Optimal to the Pessimistic Problem



Notes. (a) Original network with special arcs depicted in red. Arc capacities are shown in the network. (b) Modified network for the MFNIP method. Arc capacities are shown in the network.

the MFNIP method is a heuristic approach. We observe, however, that the MFNIP produced an upper bound on the optimal objective value for the pessimistic problem as well as a high-quality initial interdiction decision, which may help to speed up solution processes when solving the pessimistic problem. The special feature of this example is that the network is layered and the special arcs are all located in the same layer. For networks that do not possess these features, the MFNIP method would still produce a feasible interdiction solution, but this solution may not correspond to a high-quality upper bound.

5. Single-Level Reformulations

In this section we develop single-level reformulations for both the optimistic and pessimistic versions of our problem. For the optimistic version, we rely on the dualize-and-combine approach, that is, the strong duality-based approach that is typically applied to bilevel optimization problems. For the pessimistic version, first we derive a valid relaxation and then we discuss two approaches to achieve a single-level formulation: the first one, based on dualize-and-combine, is discussed in Section 5.2, and the second one, based on a level-set relaxation approach proposed in Zeng (2020), is discussed in Online Supplement D.

5.1. A Single-Level Reformulation for the Optimistic Problem

Our optimistic bilevel optimization problem was given in (5). In developing a single-level reformulation of the optimistic problem, recall that the leader can select the most advantageous optimal solution of the lower-level problem given in (5c) according to their own objective. Therefore, the reformulation mainly needs to ensure that the optimality conditions of the follower's maximum flow decisions are satisfied.

First, we introduce the dual of the maximum s - t flow problem, the minimum s - t cut Problem (8), for a fixed interdiction decision γ . In this problem, α_i indicates whether node i shares the same side of the s - t cut with the source s ($\alpha_i = 0$) or the sink ($\alpha_i = 1$), and θ_{ij} indicates whether arc (i, j) is a forward arc in the minimum s - t cut.

$$\min_{\alpha, \theta} \sum_{(i,j) \in A} u_{ij}(1 - \gamma_{ij})\theta_{ij} \quad (8a)$$

$$\text{s.t. } \alpha_i - \alpha_j + \theta_{ij} \geq 0, \quad \forall (i, j) \in A, \quad (8b)$$

$$\alpha_t - \alpha_s \geq 1, \quad (8c)$$

$$\alpha \in \{0, 1\}^{|N|}, \quad (8d)$$

$$\theta \in \{0, 1\}^{|A|}. \quad (8e)$$

The following results will help in our reformulation as they will allow us to remove the binary restrictions on variables α and θ .

Lemma 2 (Wood 1993). *The dual of the maximum flow problem has an optimal solution in which all variables are zero or one.*

Corollary 4. *The dual of the maximum flow problem has an optimal solution in which $\theta_{ij} \leq 1$ for all $(i, j) \in A$.*

The proof of Corollary 4 follows directly from Lemma 2 by observing that the optimal solution in the lemma satisfies $\theta_{ij} \leq 1, \forall (i, j) \in A$.

Next, we impose the strong duality condition enforcing that the value of the maximum flow is equal to the capacity of the minimum cut, thereby ensuring the optimality of \mathbf{x} . This is done by Constraint (9d) together with the weak duality. The single-level reformulation is presented as

$$\min_{\gamma, \mathbf{x}, \mathbf{z}, \alpha, \theta} \sum_{(i,j) \in A} z_{ij} \quad (9a)$$

$$\text{s.t.} \quad (5b), (5d), (8b), (8c), \quad (9b)$$

$$\mathbf{x} \in X(\boldsymbol{\gamma}), \quad (9c)$$

$$x_{ts} \geq \sum_{(i,j) \in A} u_{ij}(1 - \gamma_{ij})\theta_{ij}, \quad (9d)$$

$$0 \leq \theta_{ij} \leq 1, \quad \forall (i,j) \in A, \quad (9e)$$

$$0 \leq \alpha_i \leq 1, \quad \forall i \in N, \quad (9f)$$

$$\mathbf{z} \in \{0, 1\}^{|A|}. \quad (9g)$$

We observe that Constraint (9d) involves bilinear terms. This can be addressed by introducing variables $\beta_{ij} := (1 - \gamma_{ij})\theta_{ij}$ and replacing the bilinear terms with these variables. We will then need to express this bilinear relationship using linear constraints. Recall that there exists an optimal solution to (8) in which all $\alpha_i, \forall i \in N$ and $\theta_{ij}, (i,j) \in A$, are zero or one by Lemma 2, which allows us to linearize this bilinear relationship by $\beta_{ij} \geq \theta_{ij} - \gamma_{ij}$ and arrive at the Reformulation (10).

Using an argument similar to the one developed in Wood (1993) for the MFNIP, we show that (10) is a valid reformulation of (9). We state this result in Proposition 1.

$$\min_{\gamma, \mathbf{x}, \mathbf{z}, \alpha, \theta, \beta} \sum_{(i,j) \in A} z_{ij} \quad (10a)$$

$$\text{s.t.} \quad (5b), (5d), (8b), (8c), \quad (10b)$$

$$\mathbf{x} \in X(\boldsymbol{\gamma}), \quad (10c)$$

$$x_{ts} \geq \sum_{(i,j) \in A} u_{ij}\beta_{ij}, \quad (10d)$$

$$\beta_{ij} + \gamma_{ij} - \theta_{ij} \geq 0, \quad \forall (i,j) \in A, \quad (10e)$$

$$0 \leq \theta_{ij} \leq 1, \quad \forall (i,j) \in A, \quad (10f)$$

$$0 \leq \alpha_i \leq 1, \quad \forall i \in N, \quad (10g)$$

$$\beta_{ij} \geq 0, \quad \forall (i,j) \in A, \quad (10h)$$

$$\mathbf{z} \in \{0, 1\}^{|A|}. \quad (10i)$$

Proposition 1. *Formulation (10) is a valid reformulation of (9).*

Proof. It follows from the preceding discussion that we just need to show the validity of modeling the relationship $\beta_{ij} = (1 - \gamma_{ij})\theta_{ij}$ via Constraints (10e) and (10h). We first observe that enforcing β_{ij} to take its minimum allowed value according to Constraints (10e) and (10h) will not eliminate all optimal solutions because $u_{ij} \geq 0, \forall (i,j) \in A$ in Constraint (10d). Now for each $(i,j) \in A$: (i) if $\gamma_{ij} = 1$, then Constraint (10e) becomes $\beta_{ij} \geq \theta_{ij} - 1$, and according to Corollary 4, β_{ij} will take its lower bound of zero here; (ii) if $\gamma_{ij} = 0$, then Constraint (10e) becomes $\beta_{ij} \geq \theta_{ij}$, and β_{ij} will take its lower bound of θ_{ij} here. In both cases, we see that the relationship $\beta_{ij} = (1 - \gamma_{ij})\theta_{ij}$ holds. \square

5.2. Single-Level Reformulations for the Pessimistic Problem

The pessimistic problem in general is considerably harder than its optimistic counterpart, and it is not guaranteed to be solvable even when the optimistic version has a solution (Dempe and Zemkoho 2020). In what follows, we discuss an approach, referred to as the direct reformulation approach, to derive a single-level reformulation for the pessimistic problem that can be solved using state-of-the-art solvers. A second approach, referred to as the alternative approach, to derive a single-level reformulation for the pessimistic problem is discussed in Online Supplement D.

5.2.1. The Direct Reformulation Approach. Recall that in our pessimistic bilevel optimization problem stated in (7), the relationship between the active flow indicator vector, \mathbf{z} , and the flow vector, \mathbf{x} , is unambiguously

characterized by Constraints (7c) with an appropriate value of M . Furthermore, we have shown through Theorem 5 that for a given feasible interdiction γ and a corresponding maximum flow \hat{x} , setting $M \geq |\mathcal{A}|$ does not cut off any feasible z (maximal w.r.t. \hat{x}). Hence, by fixing $M = |\mathcal{A}|$, for any special arc $(i, j) \in \mathcal{A}$ such that $\hat{x}_{ij} > 0$, we have that $\hat{x}_{ij} \geq 1/M$, from which it follows that $z_{ij} = 1$. This allows us to maintain the validity of the formulation with the integrality restriction on z being relaxed. We fix $M = |\mathcal{A}|$ and present the following result:

Lemma 3. *Formulation (11) is a valid reformulation of Formulation (7).*

$$\min_{\gamma \in \Gamma} \max_{x, z} \sum_{(i,j) \in \mathcal{A}} z_{ij} \tag{11a}$$

$$\text{s.t. } x \in S(\gamma), \tag{11b}$$

$$x_{ij} \geq \frac{1}{|\mathcal{A}|} z_{ij}, \quad \forall (i, j) \in \mathcal{A}, \tag{11c}$$

$$0 \leq z_{ij} \leq 1, \quad \forall (i, j) \in \mathcal{A}. \tag{11d}$$

Proof. It follows directly from Theorem 5 and the above discussion. \square

We now consider the inner problem of Formulation (11), presented as (12) for a fixed $\gamma' \in \Gamma$ below:

$$\max_{x, z} \sum_{(i,j) \in \mathcal{A}} z_{ij} \tag{12a}$$

$$\text{s.t. } x \in S(\gamma'), \tag{12b}$$

$$x_{ij} \geq \frac{1}{|\mathcal{A}|} z_{ij}, \quad \forall (i, j) \in \mathcal{A}, \tag{12c}$$

$$0 \leq z_{ij} \leq 1, \quad \forall (i, j) \in \mathcal{A}. \tag{12d}$$

Notice that Constraint (12b) can be replaced using linear programming strong duality, similar to what we have shown for the optimistic version in Section 5.1. This is presented in Formulation (13), where Constraint (13b) is the strong duality condition, Constraints (13c)–(13d) are primal feasibility conditions, and (13e)–(13f) are dual feasibility conditions, and the variables defined in the parentheses are the dual variables associated with each constraint.

$$\max_{x, z, \alpha, \theta} \sum_{(i,j) \in \mathcal{A}} z_{ij} \tag{13a}$$

$$\text{s.t. } x_{ts} - \sum_{(i,j) \in \mathcal{A}} u_{ij}(1 - \gamma'_{ij})\theta_{ij} \geq 0, \quad (\eta) \tag{13b}$$

$$\sum_j x_{ij} - \sum_j x_{ji} = 0, \quad \forall i \in N, \quad (\chi_i) \tag{13c}$$

$$x_{ij} \leq u_{ij}(1 - \gamma'_{ij}), \quad \forall (i, j) \in \mathcal{A}, \quad (\omega_{ij}) \tag{13d}$$

$$\alpha_i - \alpha_j + \theta_{ij} \geq 0, \quad \forall (i, j) \in \mathcal{A}, \quad (\delta_{ij}) \tag{13e}$$

$$\alpha_t - \alpha_s \geq 1, \quad (\pi) \tag{13f}$$

$$\theta_{ij} \leq 1, \quad \forall (i, j) \in \mathcal{A}, \quad (\nu_{ij}) \tag{13g}$$

$$\alpha_i \leq 1, \quad \forall i \in N, \quad (\epsilon_i) \tag{13h}$$

$$x_{ij} - \frac{1}{|\mathcal{A}|} z_{ij} \geq 0, \quad \forall (i, j) \in \mathcal{A}, \quad (\phi_{ij}) \tag{13i}$$

$$z_{ij} \leq 1, \quad \forall (i, j) \in \mathcal{A} \quad (\kappa_{ij}) \tag{13j}$$

$$x_{ij} \geq 0, \quad \forall (i, j) \in \mathcal{A}, \tag{13k}$$

$$z_{ij} \geq 0, \quad \forall (i, j) \in \mathcal{A}, \tag{13l}$$

$$\alpha_i \geq 0, \quad \forall i \in N, \tag{13m}$$

$$\theta_{ij} \geq 0, \quad \forall (i, j) \in \mathcal{A}. \tag{13n}$$

For a given γ' , the inner problem (13) is a linear program, and we can apply the dualize-and-combine approach using strong duality here again to arrive at the single-level Reformulation (14) for the pessimistic problem overall. We note that even though Reformulation (14) is a single-level formulation, it involves bilinear products of a binary

and a continuous variable $(1 - \gamma_{ij})\omega_{ij}$ in the objective function and $(1 - \gamma_{ij})\eta$ in Constraints (14j). We have tried to derive and prove the validity of tight bounds on the continuous variables in order to linearize these bilinear terms, beginning with the most probable values, for example, $|\mathcal{A}|$ and $|A|$, but we could not prove their validity, and in some preliminary tests, using $|\mathcal{A}|$ as a bound cut off optimal solutions. We point out, however, that state-of-the-art solvers such as Gurobi (2023) can directly and efficiently handle this type of bilinear term.

$$\min_{\gamma, \chi, \omega, \phi, \eta, \kappa, \delta, \epsilon, \pi, \nu} \sum_{(i,j) \in A} u_{ij}(1 - \gamma_{ij})\omega_{ij} + \pi + \sum_{i \in N} \epsilon_i + \sum_{(i,j) \in A} v_{ij} + \sum_{(i,j) \in A} \kappa_{ij} \quad (14a)$$

$$\text{s.t. } \gamma \in \Gamma, \quad (14b)$$

$$\chi_i - \chi_j + \omega_{ij} \geq 0, (i, j) \in A \setminus \{\mathcal{A} \cup (t, s)\}, \quad (14c)$$

$$\chi_i - \chi_j + \omega_{ij} + \phi_{ij} \geq 0, (i, j) \in \mathcal{A}, \quad (14d)$$

$$\chi_t - \chi_s + \omega_{ts} + \eta \geq 0, \quad (14e)$$

$$\kappa_{ij} - \frac{1}{|\mathcal{A}|} \phi_{ij} \geq 1, (i, j) \in \mathcal{A}, \quad (14f)$$

$$\sum_j \delta_{ij} - \sum_j \delta_{ji} + \epsilon_i \geq 0, i \in N \setminus \{s, t\}, \quad (14g)$$

$$\sum_j \delta_{tj} - \sum_j \delta_{jt} + \epsilon_t + \pi \geq 0, \quad (14h)$$

$$\sum_j \delta_{js} - \sum_j \delta_{sj} + \epsilon_s - \pi \geq 0, \quad (14i)$$

$$\delta_{ij} - u_{ij}(1 - \gamma_{ij})\eta + v_{ij} \geq 0, (i, j) \in A, \quad (14j)$$

$$\chi \text{ unrestricted}, \omega \geq 0, \phi \leq 0, \eta \leq 0, \kappa \geq 0, \quad (14k)$$

$$\delta \leq 0, \epsilon \geq 0, \pi \leq 0, \nu \geq 0. \quad (14l)$$

6. Computational Experiments

We evaluate our proposed approaches through a series of computational experiments divided into two sections based on the network instances: (i) general layered networks and (ii) synthetic but realistic domestic human trafficking networks. We present the computational performances of both versions of the bilevel optimization problem, evaluate how optimal interdiction plans differ in these two different versions, and describe what insights can be obtained from the application perspective.

We implemented all the proposed approaches in Python using Gurobi version 10.0.1 (Gurobi 2023) as the optimization solver. We set a time limit of 7,200 seconds (two hours) for solving each instance. All the computational experiments were performed on the Palmetto Computing Cluster at Clemson University, using an Intel® Xeon® Gold 6148 CPU at 2.40 GHz, with 40 physical cores, 40 logical processors, and up to 32 threads. The algorithms, results, and statistics discussed in this section are available in the GitHub repository Lopes da Silva et al. (2025).

6.1. Implementation Details

In this section, we present the implementation details of the following four approaches in our computational experiments: (i) `Opt`, which solves Formulation (10); (ii) `MFNIP`, which implements the MFNIP method as discussed in Section 4.3; (iii) `Pessimistic Direct`, which solves Formulation (14); and (iv) `Pessimistic Alternative`, which solves Formulation (8) discussed in Online Supplement D.

We note that although `MFNIP` is only a heuristic approach for networks that are neither all-source nor all-sink networks, we can feed the obtained interdiction decision to the inner problem of the pessimistic formulation (13) to obtain a feasible solution. In addition, we leverage the `Opt` and the `MFNIP` to accelerate the solution approaches for the pessimistic problem as follows. First, the optimal objective value of the optimistic problem serves as a valid lower bound for the pessimistic problem. Second, we use the optimal interdiction decisions obtained from both `Opt` and `MFNIP` as warm-starting initial feasible solutions for the pessimistic problem. More details about the implementation of `Pessimistic Alternative` are discussed in Online Supplement D.1.

It is worth noting that we have also developed a penalty-based Benders decomposition approach (similar to the one presented in Cormican et al. (1998)) where the master problem finds a feasible interdiction vector to minimize the sum over the z_{ij} values whereas the subproblem finds a maximum flow that maximizes the sum over z_{ij} adjusted by a penalty term associated with violations of the interdiction decision. However, this approach was

found to not be competitive in our preliminary experiments, from which we observe that the information retained at the master problem level is not enough to produce good interdiction decisions in a reasonable amount of time. Please see Online Supplement E for details on the Benders decomposition approach and preliminary results with computational experiments.

6.2. Experiment Results on Layered Networks

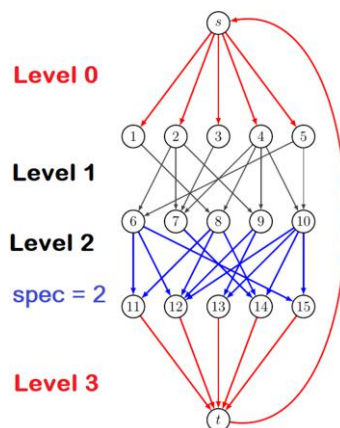
In this section, we present the performances of our proposed approaches on randomly generated layered networks where all the special arcs are located in a single level. These networks are generated based on user-provided input vectors including nodes and density, as well as input parameters cap_{LB} , cap_{UB} , and $spec$. Here nodes specifies the number of nodes in each layer of the network, density specifies the arc density between consecutive layers of nodes, cap_{LB} and cap_{UB} denote, respectively, the lower and upper bounds of the discrete uniform distribution used to generate the capacities of the arcs between the first and second layers, and $spec$ specifies the level in which the special arcs are located. Next, we provide a brief description of the network generation process.

We add a source node, connect the source node to every node in the first layer, and assign infinite capacity and infinite interdiction cost to each of the arcs in this level (level 0). We add a sink node, connect every node in the last layer to the sink node, and assign infinite capacity and infinite interdiction cost to each of the arcs in this level. We also add an arc from the sink node to the source node with infinite capacity and infinite interdiction cost. Arcs between layers are generated according to the density vector entry corresponding to that particular level. The capacities of arcs in level 1 (between the first and second layers of nodes) are randomly generated according to a discrete uniform distribution between cap_{LB} and cap_{UB} . The total capacity of level 1 (the sum of the capacities of all the arcs in that level) is calculated and replicated in the other levels so all the interdictable levels have approximately the same total capacity. Once we have obtained the number of arcs in each level and the total capacity of the level, we distribute the total capacity of the level by randomly assigning integer capacities to the arcs in the level subject to the constraint that each arc has a capacity of at least one. The interdiction cost of each arc in the bottom-most interdictable level is set to one, and the total cost of interdicting all the arcs in that level is computed (i.e., the number of arcs in that level). For the interdictable levels above the bottom-most interdictable level, we randomly assign an interdiction cost to each arc according to a discrete uniform distribution between the floor and the ceiling of the number of arcs in that level divided by the total cost of interdicting all the arcs in the bottom-most interdictable level, so that the total cost of interdicting all arcs in a level is approximately equal.

Figure 4 shows the topology of a layered network generated with parameters $nodes = [5 \ 5 \ 5]$, $density = [0.4 \ 0.6]$, $spec = 2$, where we have hidden the values of capacities and interdiction cost to prevent the picture from being too cluttered. In this network, the arcs in red have infinite capacity and infinite interdiction costs and the special arcs are represented in blue.

We conducted computational experiments with two types of layered networks, referred to as “Type 1” and “Type 2,” which are described below. For each type, we generated 10 networks, and for each network, we run all the solution approaches for budget levels 3, 6, 9, and 12. We report the statistical summary of the computational results and discuss our findings. The full tables with the results are available in the GitHub repository Lopes da Silva et al. (2025).

Figure 4. (Color online) An Example of a Layered Network



6.2.1. Network Type 1. We generate 10 networks according to the parameters $\text{nodes} = [5 \ 5 \ 5 \ 5]$, $\text{density} = [0.6 \ 0.75 \ 0.9]$, $\text{spec} = 1$, $\text{capLB} = 15$, and $\text{capUB} = 30$, that is, networks with four layers of nodes, each layer containing five nodes, and special arcs in the top-most interdictable level. These networks have 22 nodes, and their number of arcs varies between 61 and 71. The expected number of special arcs is 15. Note that these networks are all-source networks because the special arcs are all in level 1 and the arcs connecting the source node to the nodes in the first layer have infinite capacity and infinite interdiction cost. We have shown in Section 4.3 that for these networks, MFNIP is guaranteed to produce an optimal solution. The following table shows the statistics summary of the running time in seconds for each approach (“—” indicates that the time limit was reached).

Table 1 shows that the pessimistic problem is significantly more challenging to solve than the optimistic problem, with both the *Direct* and the *Alternative* reaching the two-hour time limit in several instances. However, exploiting the special structure of this type of network instance, MFNIP solves to optimality each instance in less than 0.1 second, outperforming the other two approaches by orders of magnitude for all budget levels. Comparing the two general approaches for solving the pessimistic problem, we see that *Direct* outperforms *Alternative* in all budget levels, but by a substantial margin for budget levels 6 and 9. Both approaches take significantly more time as the budget level increases from 3 to 9, but interestingly take less time when the budget level increases to 12. Upon further inspection, we noticed that for a budget of 12, in many networks, the optimal objective value of the pessimistic problem coincides with the optimistic optimal objective value. Because we feed the optimistic objective value as a lower bound and the optimal interdiction decision from the optimistic problem (as well as from the MFNIP) as an initial solution to both *Direct* and *Alternative*, they are able to solve these instances almost instantaneously. In terms of optimality gap, *Alternative* is unable to close the gap in 11 of the 40 instances, with an optimality gap varying between 17% and 50%, whereas *Direct* is unable to solve 9 of the 40 instances to optimality, with the optimality gap varying between 17% and 50%. More details about the optimality gap for these instances can be found in Online Supplement F.

6.2.2. Network Type 2. We generate 10 networks according to the parameters $\text{nodes} = [5 \ 5 \ 5 \ 5]$, $\text{density} = [0.6 \ 0.75 \ 0.9]$, $\text{capLB} = 15$, $\text{capUB} = 30$, and $\text{spec} = 2$ (special arcs in the middle interdictable level). These networks have 22 nodes, and their number of arcs varies between 58 and 75. The expected number of special arcs is 18.75. Observe that the special arcs for these networks are located in an intermediate level. Therefore, for Type 2 networks, MFNIP is only a heuristic. Here again “—” indicates that the time limit was reached.

Table 2 shows similar results for Type 2 networks as compared with Type 1 networks. The *Direct* outperforms the *Alternative* for all budget levels. The computational time for both approaches increases significantly as the budget level increases to 9 and 12, even though for budget level 12 we observed the same phenomenon as for Type 1 networks where the optimal solutions of the optimistic and of the pessimistic problems coincide for many instances. MFNIP is again very efficient, but for Type 2 networks, MFNIP is only a heuristic, so we were also interested in evaluating this method in terms of the solution quality.

Direct and *Alternative* were able to solve (the same) 29 out of the 40 instances to optimality within the time limit. In 26 of these 29 instances, MFNIP produced an optimal solution. For the remaining three instances where an optimal solution was obtained by *Direct* and *Alternative*, the solutions produced by MFNIP achieved an objective value that was greater than the optimal objective value by one unit, representing an optimization gap between 7% and 10%. For the 11 instances that neither *Direct* nor *Alternative* was able to solve to optimality within the time limit, MFNIP produced a solution that achieved the same objective value of the best known solutions produced by *Direct* and *Alternative*. For these 11 instances, the optimality gap for the best-known solution produced by *Direct* and *Alternative* was between 9% and 33%. Please see Online Supplement F for more details on the optimality gaps from the pessimistic approaches with Type 2 networks.

Table 1. Statistics Summary of the Running Times of Opt, MFNIP, Pessimistic Direct, and Pessimistic Alternative in Each Budget Level for Type 1 Networks

Budget	Opt Avg	MFNIP Avg	Pessimistic Direct				Pessimistic Alternative			
			Avg	StDev	Max	Min	Avg	StDev	Max	Min
3	0.04	0.02	0.95	0.58	2.33	0.10	2.20	1.09	3.93	0.11
6	0.05	0.03	124.79	106.67	298.61	0.09	443.99	395.50	1,118.26	0.10
9	0.07	0.03	3,848.70	3,023.20	—	0.11	5,002.60	3,276.70	—	0.14
12	0.05	0.03	3,600.08	3,599.92	—	0.11	3,600.10	3,599.90	—	0.14

Note. All the entries are in seconds.

Table 2. Statistics Summary of the Running Times of Opt, MFNIP, Pessimistic Direct, and Pessimistic Alternative by Budget Level for Type 2 Networks

Budget	Opt Avg	MFNIP Avg	Pessimistic Direct				Pessimistic Alternative			
			Avg	StDev	Max	Min	Avg	StDev	Max	Min
3	0.06	0.03	1.35	0.82	2.98	0.33	2.82	1.23	4.67	0.51
6	0.09	0.02	199.24	159.28	515.32	0.11	645.46	486.91	1,430.75	0.11
9	0.08	0.03	4,851.48	3,031.65	—	0.09	5,377.51	2,917.07	—	0.09
12	0.09	0.03	3,600.08	3,599.92	—	0.10	3,600.11	3,599.89	—	0.13

Note. All the entries are in seconds.

6.3. Other Performance Metrics for the Pessimistic Approaches

To provide a more complete evaluation of our pessimistic approaches, we briefly discuss complementary metrics to the running time averages.

In Online Supplement G we report the number of constraints, number of variables, number of nonzero elements, and the initial gap at the root node for both *Direct* and *Alternative* for Type 1 and Type 2 networks. The comparison between the two approaches shows that *Direct* has a much smaller average number of constraints and average number of nonzero elements, whereas *Alternative* has a much smaller average number of variables. We also observe the phenomenon that for some instances (and mostly for a budget of 12), building off the warm start solution provided, both algorithms are able to solve the problem to optimality at the *presolve* stage and have a 0% gap at the root node.

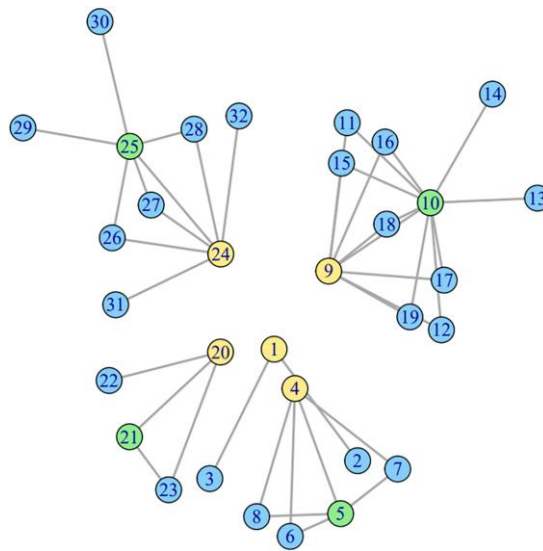
In Online Supplement H, we report the running times for *Direct* and *Alternative* with and without mixed-integer programming (MIP) warm start solutions for Type 2 networks. These results demonstrate the substantial effect of the warm start solutions in speeding up the solution process, especially considering that it takes less than 0.1 second to evaluate and process the MIP start solutions for any of the instances used, as reported in their log files. This effect is more pronounced for *Alternative*, where several of the instances that were not solved to optimality within the two-hour time limit without warm start could be solved to optimality with warm start.

6.4. Experiment Results on Human Trafficking Networks

Kosmas et al. (2023a) proposed a transdisciplinary approach to generate a domestic sex trafficking network through the integration of operations research concepts and qualitative research methods. Based on multiple sources including interviews with domain experts, analysis of law enforcement case files, and a survivor-centered advisory group with first-hand knowledge of sex trafficking operations, the authors created a network generator whose input is the number of single-trafficker operations and whose output models the relationships between traffickers, so-called “bottoms,” and victims. Bottom is a term found in the literature and in some operations to designate a victim in whom the trafficker trusts and to whom the trafficker assigns additional responsibilities, including helping to traffic other victims (Belles 2018).

This generator provides synthetic but realistic domestic sex trafficking networks which can be used as test beds for evaluating interdiction plans and other interventions. An example of such a network can be seen in Figure 5, where the yellow nodes represent traffickers, the green nodes represent bottoms, and the blue nodes represent victims. We use the methodology of Kosmas et al. (2023a) to generate 10 networks each composed of five single-trafficker operations. We adapt these networks by adding a source node and a sink node, connecting the source node to each trafficker node, connecting each bottom and victim node to the sink node, and adding a returning arc from the sink node to the source node. Observe that the adapted networks are more general than layered networks, with trafficker nodes in the first layer, bottom nodes in the second layer, and victim nodes in the third layer, but with the additional feature that there are arcs between the first and third layer. The special arcs are the ones connecting bottoms and victims to the sink, so these are all-sink networks. A summary of each network characteristics is shown in Table 3.

An arc between the source node and a trafficker node has an interdiction cost of eight, an arc between a trafficker node and a bottom node as well as an arc from a bottom node to the sink node has an interdiction cost of four, and an arc from a victim node to the sink node has an interdiction cost of two. This interdiction cost assignment scheme is intended to reflect the relative difficulty of removing each of the different types of people in the network. The cost of every other arc is infinite. The capacity of the arcs between the victim nodes and the sink node as well as between a bottom node and the sink node is randomly generated according to a discrete uniform distribution between 1 and 10. In the baseline setting, the capacity of the arc between the source node and a trafficker node equals the sum of the capacities of the arcs between the victim nodes (and bottom node if present) in

Figure 5. (Color online) A Synthetic Sex Trafficking Network Generated by the Method of Kosmas et al. (2023a)

Note. Yellow nodes represent traffickers, green nodes represent bottoms, and blue nodes represent victims.

that trafficker operation and the sink node, which in practical terms means that the trafficker is uncapacitated. In subsequent settings, we modify the capacity of the trafficker arcs and evaluate the effect on the optimal interdiction plans. We refer to these setting choices as the “capacity rate,” and we perform computational experiments for rate values 1, 0.75, and 0.50, denoting, respectively, the baseline model where traffickers have capacity equal to the sum of the capacities of all victims in the operation, a reduction to 75% of the initial capacity, and a further reduction to 50% of the baseline capacity.

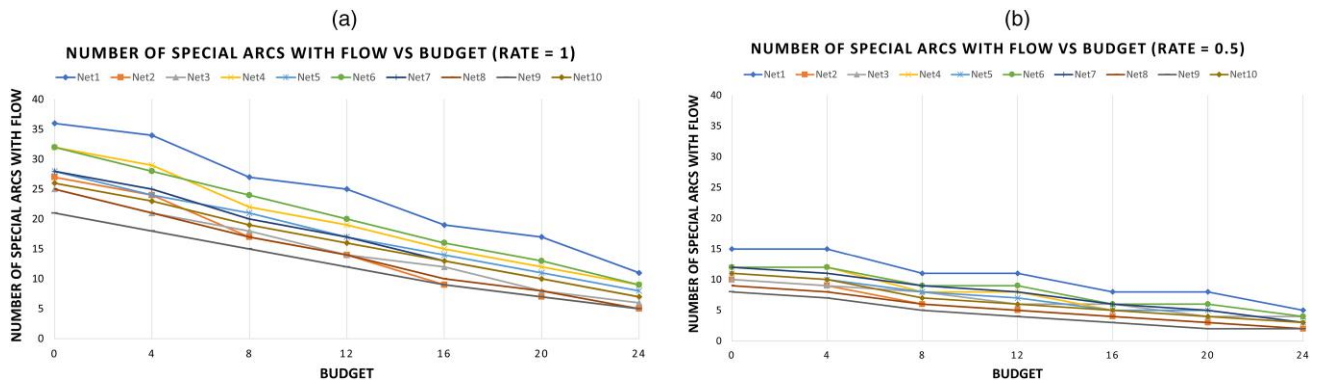
In this set of computational experiments, we consider the budget levels 4, 8, 12, 16, 20, and 24, and we solve each instance with `Opt` and `MFNIP`. We consider neither the `Pessimistic Direct` nor the `Pessimistic Alternative` because `MFNIP` is an exact algorithm to solve the pessimistic problem on these networks and was shown to dominate the other pessimistic algorithms by orders of magnitude. In this set of experiments, each instance was solved by `Opt` and `MFNIP` in under one second.

Figure 6, (a) and (b), illustrates the variation of the optimal objective value (which denotes the number of active victim arcs and bottom arcs in the network) of the optimistic problem for different budget levels and for capacity rates 1 and 0.50, respectively. These plots show that the optimistic problem is highly sensitive to the trafficker’s capacity. In the optimistic problem, the trafficker implements the maximum flow that uses the smallest number of special arcs, that is, the trafficker concentrates flow on the special arcs with the largest capacities until the maximum flow is reached. One can observe that, even without interdiction (budget of zero), the optimal objective value drops significantly for all networks as the capacity rate decreases. One can also observe that, for rate 0.50, in some instances, the optimal objective value of the optimistic problem does not change when the budget is increased to the next budget level (e.g., Network 1, budget level changes from 0 to 4 while the objective value

Table 3. Characteristics of HT Networks Used in Our Computational Experiments

Network	Characteristics of the HT networks				
	Nodes	Arcs	Traffickers	Bottoms	Victims
1	41	101	5	5	31
2	32	74	5	4	23
3	30	64	5	3	22
4	37	86	5	5	27
5	33	79	5	5	23
6	37	83	5	5	27
7	33	70	5	4	24
8	30	64	5	3	22
9	26	56	5	3	18
10	31	73	5	4	22

Figure 6. (Color online) Optimal Objective Value Variation with Capacity Rate



Notes. (a) Optimal objective value of optimistic problem for various budget levels and fixed capacity rate 1. (b) Optimal objective value of optimistic problem for various budget levels and fixed capacity rate 0.50.

remains equal to 15). In these cases, the trafficker’s capacity can be matched by the capacity of a few victim arcs and the increase in budget is not enough to implement an interdiction plan that achieves a better objective value, so a leader’s optimal solution is to not interdict any additional arc. In fact, it can be shown that if the leader is forced to always use up all the budget, its optimal objective value can be hurt. However, when the budget is a multiple of eight, then generally there is a corresponding significant drop in the optimal objective value. This is because the leader has enough budget to remove another trafficker.

Recall that for a capacity rate equal to one, the capacity of the trafficker’s arcs equals the sum of the capacities of its victims (and the bottom if present in the operation), which effectively means that the trafficker has an infinite capacity. For these cases, the optimistic and pessimistic problems are equivalent in the sense that the trafficker in the optimistic problem cannot concentrate flow in fewer arcs because there is enough capacity in the trafficker arc to saturate all special arcs. Therefore, for rate = 1, the pessimistic problem presents the same variation in the number of victims connected to the network as shown in Figure 6(a) for the same budget levels. Furthermore, because in the pessimistic problem the trafficker spreads the flow over as many special arcs as possible, the optimal objective value of the pessimistic problem is unaffected by variations in the capacity rate. Indeed, the pessimistic problem presents the same variation in the number of victims connected to the network as shown in Figure 6(a) for the same budget levels for all capacity rates.

A general interdiction pattern emerges from the results of both the optimistic and pessimistic problems: as the budget increases, interdictions are concentrated on the trafficker arcs connected to the largest number of victim arcs. In other words, the leader removes as many traffickers as possible. There are, however, nuances and exceptions to the general pattern, which require a responsible evaluation by a transdisciplinary team, and are out of the scope of this paper.

7. Conclusions and Future Directions

In this work, we introduce a novel bilevel network interdiction problem where the follower wants to maximize the flow through the network whereas the leader wants to minimize the number of arcs from a critical set that have a positive flow on them in the maximum flow solved by the follower. Our problem is motivated by applications in illegal network disruption where the follower seeks to maximize profit, whereas the leader seeks to minimize the number of components affected by the operation. We develop a single-level reformulation for the optimistic problem and two closely related single-level reformulations for the pessimistic problem. We also develop a specialized solution method (referred to as MFNIP method) to the pessimistic problem that is an exact algorithm for a class of networks with practically important applications including human trafficking disruption, and can be an attractive heuristic approach to other classes of networks.

Through computational experiments on general networks, we show that whereas the optimistic problem can be solved almost instantaneously, the pessimistic problem is much harder and may take an unpractical amount of time to be solved to optimality. The specialized method, however, outperforms the two benchmark pessimistic approaches by orders of magnitude in all settings, and is shown to be robust against variations in parameters of the problem. For the special class of networks, the MFNIP method solves each instance to optimality in under one second, whereas for a class of networks where it is a heuristic, it solves the majority of the instances to

optimality in under one second, and for the instances for which it failed to produce an optimal solution, its solution provides an upper bound on the objective value within a 10% gap from the optimal solution.

From computational experiments conducted on domestic human trafficking networks, we have obtained several interesting insights. The optimistic optimal objective value is highly sensitive to variations in the trafficker capacity because an optimistic trafficker will concentrate flow on arcs with the largest capacities. For this reason, there are instances where the optimal interdiction plan for the optimistic problem is null (i.e., to not interdict any arc). On the other hand, the optimal objective value in the pessimistic problem is unaffected by variations in the trafficker capacity. For a trafficker with infinite capacity, both problems are equivalent. There is a general pattern of interdicting as many traffickers as possible in both problems.

We reiterate that our problem is meant to provide a deeper understanding of the dynamics of human trafficking networks from the perspective of a stakeholder whose objective is to directly minimize the number of victims in the operation rather than to provide policy decisions on interventions. We acknowledge that our work takes an incomplete view of the real problem because we do not consider metrics of interest other than the number of victims in the network and the profit obtained by the trafficker and these may not be the most important metrics for interdiction decisions. We highlight that despite being in accordance with the leader's objective, minimizing the number of victims in the network may result in greater suffering for the victims who remain in the network, a metric that is not fully captured by our model, and which deserves further consideration by human trafficking domain experts and those with lived experience.

Future research can consider the closely related problem setting where the leader represents a consortium of antitrafficking agents composed of (at least) two stakeholders who contribute to the interdiction budget: one wants to minimize the number of victims, whereas the other wants to minimize the maximum profit obtained by the traffickers. Another future research direction could rely on collaboration with social scientists, social service providers, and other subject-matter experts who can help to formulate other important metrics (e.g., related to victims' suffering, vulnerabilities, time in the operation), which can be considered when elaborating interdiction models. Finally, because illicit supply chains are hidden systems supported by covert operations, in a problem setting close to the one we discuss in this paper, the leader makes decisions under incomplete information. Furthermore, the literature in this field points out that illicit supply chains possess a restructuring feature, which adds to the complexity of the problem. Therefore, future works can consider uncertainty in different parameters of the problem, as well as restructuring networks.

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