Optimizing Police Response with the Multiple-Type Demand & Multiple-Type Facility Maximal Covering Location Problems

Ashleigh N. Price and Kevin M. Curtin

Abstract This work seeks to build on the previous literature regarding coverage models as a means of optimizing location planning under varying demand and facility type conditions. The existing police response and patrol literature has not yet recognized that not all police units are identical. There are categories of police units (e.g., bomb squad, canine, SWAT, detectives) that differ in their numbers, in their equipment, in their training, and in their response behavior. This work will demonstrate how these variations can be modeled in order to more realistically represent police response and location scenarios and to optimize those operations. The literature in the areas of police optimization, related covering models, and the integration of multiple types of facilities in location optimization is reviewed. The motivating application for this work is the optimal covering of multiple different types of calls for police services in the city of Chicago, Il, and the optimal stationing of multiple types of police units throughout the city. Optimization formulations that cast the multiple-type demand maximal covering problem (MTD-MCLP) and the multiple-type facility maximal covering problem (MTF-MCLP) in the context of police response and unit location are provided. Demonstrative results are included along with the descriptions of the models. The chapter concludes with perspectives on the potential benefits of these models for police operations planning, the limitations of these models, and extensions in the context of police operations and additional application areas.

Keywords Police optimization, covering location models, multiple type covering, police location, police geography

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1. Introduction

There is a long tradition of employing spatial optimization in the context of emergency response and systems planning (Toregas et al. 1971), including police response and planning (Mitchell 1972). Covering models have been a prominent and persistent component of this work (Church and Murray 2018). Novel variants of the Maximal Covering Location Problem recently appeared in the literature, designed for an application in the area of drug interdiction at a hemispheric scale (Price et al. 2022). The novelty of these models lay primarily in their ability to maximize coverage while recognizing that there are multiple different types of demands to be covered, and multiple different types of facilities that can – or cannot – cover those demands. In this chapter the concepts of multiple-type demand and multiple-type facilities are applied to the motivating example of police response and police unit location.

This work seeks to build on the previous literature regarding coverage models as a means of optimizing location planning under varying demand conditions (Curtin et al. 2010). While this previous work recognized that there were different priorities associated with different types of calls, it did not distinguish between categories of calls that may necessitate entirely different types of response. For example, the urgency, the level of response, and the needed equipment for a call of an "active shooter" is far different from the response necessary for a report of vandalism or a minor car accident. Moreover, the existing police response and patrol literature has not yet recognized that not all police units are identical. There are categories of police units (e.g., bomb squad, canine, SWAT, detectives) that differ in their numbers, in their equipment, in their training, and in their response behavior. This work will demonstrate how these variations can be modeled in order to more realistically represent police response and location scenarios and to optimize those operations.

The following section reviews the literature in the areas of police optimization, related covering models, and the integration of multiple types of facilities in location optimization. This is followed by a presentation of the motivating application for this work, the optimal covering of multiple different types of calls for police services in the city of Chicago, Il, and the optimal stationing of multiple types of police units throughout the city. This is followed by optimization formulations that cast the multiple-type demand maximal covering problem (MTD-MCLP) and the multiple-type facility maximal covering problem (MTF-MCLP) in the context of police response and unit location. Demonstrative results are included along with the descriptions of the models. The chapter concludes with perspectives on the potential benefits of these models for police operations planning, the limitations of these models, and extensions in the context of police operations and additional application areas.

2. Background and Literature Review

The literature pertinent to this research lies in three broad areas, 1) the use of optimization techniques in police operations planning, 2) maximal and backup covering modeling, and 3) multiple-type covering models. Each of these areas is briefly reviewed.

2.1. *Police Optimization*

Organizational strategies that focus on police presence, such as community-oriented, problem-oriented, or other place-based policing methods and targeted intervention efforts, rely on the proper spatial allocation of police resources and can readily be modeled with spatial optimization techniques. Allocating police assets optimally relies not only on the identification of criminogenic areas, but on differentiating among the types of calls for service and the number and type of assets available. In the context of crime prevention and deterrence, the objective is to optimally allocate police resources (e.g., patrol units) among known criminogenic locations to maximize the demand for police services that is covered by police presence, which in turn can contribute to improved public safety.

Spatial optimization models can be used to represent various aspects of law enforcement operations, including routine patrol and reactive policing, via the spatial allocation of police presence. The literature on police patrolling problems can be broadly (Samanta et al. 2021) grouped as focusing on 1) designing patrol areas (e.g., (Yang et al. 2021)), 2) allocating patrol units or other resources within those areas (e.g., (Adler et al. 2014; Liberatore et al. 2020)), 3) designing patrol routes on a street network (e.g., (Chen et al. 2018; Dewinter et al. 2020; Samanta et al. 2021)), and to a lesser extent, some combination of the three (e.g., (Curtin et al. 2010)).

Collectively, spatial optimization problems for police patrol most commonly have the objective of minimizing response time given historical spatial and temporal patterns of crime, the type and capacity of each unit, and the frequency and duration (dosage) of police presence needed at demand locations (Samanta et al. 2021). The focus on response time minimization persists (Zhu et al. 2022) even though reduced response time is not consistently correlated with lower crime rates. Models applied to routing or allocating other emergency services such as fire or EMS, are of less utility as several aspects of police operations create unique objectives and constraints. For example, police patrols do not include return to a base station (unless there is an arrest or administrative duties), meaning the dispatch location is dynamic. Police patrols are also sensitive to dosage (the time spent at a location) and timing quotas, which controls the frequency of visits to locations, and reactive policing can in turn influence timing and dosage at those locations (Dewinter et al. 2022; Dau et al. 2023). Given that there is some stochasticity in the timing and location of calls for service, other considerations include probabilistic elements, such as random patrols during idle time with maximum dosage and frequency at those locations, and random reactive policing to locations within the patrol area. In those cases, the objective is not necessarily to minimize response time, but to maximize the calls responded to, balance the time spent at patrol locations, or minimize waiting or idle time (Dewinter et al. 2020).

With regard to patrol area and police district design, the most common objectives are to minimize response time or idle time in order to minimize the deviations in the level of police activity between jurisdictions (Liberatore et al. 2020; Samanta et al. 2021). District design problems can be situated in a network space (Adler et al., 2014) or rely on areal units, and typical constraints include assigning all demand locations to a district and balancing the level of police activity/resources needed across districts. Chevaleyre (2004) and Portugal and Rocha (2010) developed multi-agent patrolling models to minimize idle time and presented heuristic solution procedures using synthetic networks. Santana et al. (2004) used a Markov decision process to design patrol districts by minimizing idle time on a weighted graph. D'Amico et al. (2002) presented a constrained-graph partitioning model and simulated annealing heuristic solution procedure to minimize response and idle times among reporting districts in Buffalo, NY. Liberatore and Camacho-Collados (2016) used a similar graph partitioning model to minimize response times while constraining the deviations among the size of patrol areas and allowing for backup coverage from adjacent districts in Madrid. The same problem, dataset, and constraints were used in Camacho-Collados et al., (2015), albeit on a rasterized version of the street network. Chen et al., (2019) formulated the street-network police districting problem (SNPDP) to balance police activity among patrol districts using crime risk and the travel distance within each district. Similar adjacency constraints appear in the redistricting formulation in Kong et al., (2019), which used a clustering algorithm to assign smaller area units to larger districts. Bucarey et al., (2015) extends the p-median problem to balance policework among districts by minimizing the sum of distances from each block to its assigned patrol district and constraining the allowable travel distance that can be patrolled during a single shift.

Spatial optimization problems for the allocation of police resources typically rely on existing district geography and can have the objective(s) of maximizing police coverage/visibility, or minimizing average response time, operating costs, or variation in police activity among districts. Node routing problems have been formulated, but may be of less utility in representing police patrols, as the demand for police presence is more intuitively located along street segments (Dewinter et al., 2020). Patrol routing along street networks can be represented by directed arc routing problems, with required edges (patrol areas), where the demand along edges is a dosage and frequency. There are also multi-period models that can accommodate flexible assignments at each time step (Haghani et al. 2004) which minimizes response time based on the vehicles' current locations. Liberatore, Camacho-Collados, and Quijano-Sánchez (2021) used dual objectives to maximize the minimum time spent patrolling an area and the ratio of police contact to the size of population groups so that patrol time is allocated proportional to the expected crime risk and to the population.

Routine police presence influences criminal outcomes at patrol locations by deterring crime, reducing call volumes, and preventing traffic and other minor violations, especially when directed at spatial and temporal hotspots (Telep and Weisburd 2012; Dau et al. 2023). Law enforcement resources are limited in comparison to the volume and frequency of criminal activity that occurs, the location and timing of individual incidents is subject to many dynamic influences, and there are numerous and diverse types of police interventions that may be required at specific locations or of individual law enforcement agencies. In this way, the nature of the objectives and constraints on actual police operations is readily accommodated by location allocation problems.

2.2. Maximal Covering and Backup Covering

The focus on minimizing response time in the literature reviewed above highlights a contrast in the perception of the quality of police service by policymakers, elected officials, and the population at large on one hand, and by police officers and officials on the other. While police are often judged on the time (or average time) to respond to calls for service, it has been noted that fast responses do not equate to safe neighborhoods. There are examples of neighborhoods where there are high incidences of crime, and significant police presence in close proximity, but the pervasive nature of the crime does not permit policing to be effective. Police themselves describe the need to cover areas where their services are needed in order to provide thorough public service. For this reason, location covering problems have become one of the primary approaches relevant to police operations. These problems optimally locate a resource or set of resources that can serve or "cover" spatially distributed demand within a given distance or time period. In police operations, these resources are located to be available to respond to, or "cover," spatially distributed criminal or emergency response incidents.

Research in facility location has created a family of variant location covering problems. There are those with the objective of complete coverage using a minimum number of facilities as formulated in the Location Set Covering Problem (LSCP); those that maximize coverage based on cost, a limited number of facilities, or other resource constraints as addressed by the Maximal Covering Location Problem (MCLP) (Church and Revelle 1974) and problems that avoid coverage to the largest extent possible as addressed in the Minimum Impact Location Problem (MILP) (Church and Murray 2018). It should be noted that these models are also related to a wider range of location problems (Church and Weaver 1986).

The LCSP and the MCLP share a common limitation; once a facility is located, all demand points under its coverage are considered completely covered. In many instances, this is not the case. This is addressed with multiple extensions and variations of the LSCP and the MCLP that allow for multiple coverage. Daskin and Stern (1981) proposed the hierarchical objective set covering model (HOSC) to maximize secondary coverage to the LSCP model (Murray et al. 2010). Later, Hogan and Revelle (1986) addressed problems of the HOSC formulation by presenting a multi-objective problem rather than a hierarchical formulation in the back-up covering problem BACOP and subsequent variations (BACOP or BACK UP 2-4) (Daskin et al. 1988). Others yet have the objective of maximizing the demands that are covered multiple times (Gendreau et al. 1997; Li et al. 2011). There are also many more variants that can accommodate multiple coverage (multiple facilities at a single location), gradual coverage (the quality of coverage decreases with distance), and cooperative coverage, in which multiple facilities can contribute to fully covering a demand (e.g., (Price and Curtin 2024)). In the specific context of police operations, Curtin, Hayslett-McCall, and Qiu (2010) traded off backup coverage with optimal single coverage to allow a comparison of allocations where high-priority calls for service could be covered by multiple police patrols.

2.3. Multiple-Type Covering

The spatial allocation of law enforcement resources relies not only on the location of calls for service, but on differentiating among potential targeted intervention efforts, the types of police assets available, and the consideration of districts or jurisdictions that can respond to specific locations. In the context of law enforcement operations, the objective is to optimally allocate police assets (e.g., patrol, K9, or marine units) among police districts or incident locations. The demand associated with incident locations has a priority based on the nature of the incident reported to police (e.g., robbery, narcotics, fraud), and there can be multiple units and types of assets available to respond to the incident locations. Location covering models in particular can be suited to modeling and solving these spatial allocation problems given the objectives and constraints on law enforcement operations.

There is an established set of location allocation models cast in the context of police operations, and there is a range of spatial optimization models that have considered locating multiple types of facilities and others that can accommodate covering multiple types of demands. There are those that have considered multiple facility types (Wilt and Sharkey 2019) albeit in the context of task forces for illicit trafficking rather than policing, those that have addressed co-locating multiple types of facilities at the same location (Magliocca et al. 2022; Price et al. 2022) and others yet that focus on maximizing the spatial dispersion of multiple types of facilities (Curtin and Church 2006; Church and Drezner 2022). Multiple-type facility location models exist in numerous derivations and extensions applied to healthcare facilities, but typical constraints avoid multiple coverage of the same demand location (Farahani et al. 2019) or do not permit multiple facilities of the same type at the same location. Similarly, of those models that can accommodate multiple types of demands, many are concerned with locating a single facility type (Mirzaei et al. 2021), with a system of hierarchal facilities, or with maintaining existing service locations (Paul et al. 2017; Stanimirović et al. 2017). There are multi-objective formulations to model covering multiple types of flows (Jabarzare et al. 2020), but the typical objectives aim to maximize disruption over the entire study area and do not account for isolating multiple types of demands (e.g., types of crime) at a single location. Others have examined locating multiple types of facilities across multiple time periods, albeit with the objective of maximizing coverage over the entire planning horizon (Zarandi et al. 2013; Porras et al. 2019). Addressing multiple types of crimes as a component of policing operations has only recently been integrated into the operations research literature (Brandt et al. 2022).

In summary, we know from the literature that police planners have been increasingly accepting of optimization modeling to assess the effectiveness of a range of operational decisions. The maximal covering and related models have proven particularly useful given the nature of policing, the constraints on resources, and the spatially distributed need for police services. Finally, we know that considering multiple types of demands is of increasing interest in spatial optimization across domain areas but is only recently being considered in the context of policing, and the consideration of how to locate multiple types of facilities or police units is entirely absent from the literature. The research presented here seeks to address this gap in the literature and provide a useful model for police decision-makers. Given that police specialization is ongoing, with more and varied types of police units designed for and trained to respond to different types of calls for service (Reaves 2015), and given that police resources are under nearly constant threat of reallocation (Piza and Connealy 2022; Lum et al. 2022) models that capture this changing nature of police staffing and response and can optimize the use of those scarce resources would appear to have some potential practical application. Therefore, this research uses an example dataset and formulations in the following sections to demonstrate how location covering models can be used to allocate multiple types of law enforcement units throughout an urban area and cover multiple types of demands.

3. Motivating Example and Data

The purpose of this chapter is not to provide a particular police patrol scheme, but rather to demonstrate that using multiple-type demand and multiple-type facility covering models can inform the process of allocating police units should practitioners choose to do so. Instances of the multiple-type demand and facility problems will be solved using the police geography and incident-level crime data from the Chicago Police Department (Figure 1). Shapefiles containing district and beat boundaries and police station locations were obtained from the Chicago Data Portal (2023). Incident locations and details were available via a spreadsheet containing latitude and longitude coordinates, which were subsequently geocoded in ArcGIS Pro, and transformed to a spatial reference that is appropriate for the Chicago area. A sample subset of the available incident data can be seen in the inset map of Figure 1. The example data consists of 870 geocoded locations for calls for service in the city of Chicago on August 2, 2019.

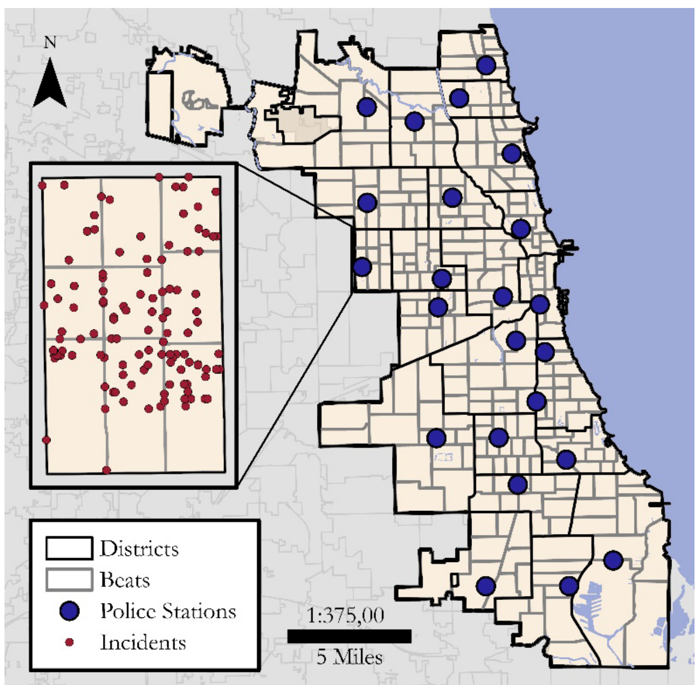

Figure 1: Chicago Police Geography. Districts *(n = 22)*, beats *(n = 274)*, and the locations of police stations and incidents

3.1. The Multiple-Type Demand Maximal Covering Location Problem

The spatial optimization formulations presented here are inspired by the multiple-type formulations that appeared recently in Price et. al, (2022) in the context of hemispheric drug trafficking and interdiction. The rationale for the Multiple-Type Demand Maximal Covering Location Problem (MTD-MCLP) is that, in general, there may be more than one kind of demand that needs to be covered to some extent, and covering more of one type is not equivalent to covering the required amount of each type of demand. In terms of police response consider that police may need to exert pressure on certain groups (e.g., different street gangs) in such a way that all of the groups are influenced rather than targeting a single group while the others flourish with no police pressure. The same could be true of calls for police to patrol locations for crime suppression, or allocating police for outreach, community engagement, traffic control, or other targeted interventions. The demand locations are areas where the police units are allocated for a specific type of intervention, but where that presence may also provide a protective effect or provide the ability for a unit to respond to nearby incidents. The MTD-MCLP can be cast in the context of police response with the following formulation and notation.

Maximize

$$\sum_{i \epsilon I} \sum_{c \epsilon C} a_{ic} y_{ic} \tag{1}$$

Subject To:

$$\sum_{j \in N_{ic}} x_{jc} \geq y_{ic} \qquad \forall \qquad i,\ c \tag{2}$$

$$\sum_{j \in J_c} x_{jc} = P_c \qquad \forall \qquad c \tag{3}$$

$$x_{jc} = \{0,1\} \qquad \forall \qquad \begin{matrix} j \in J_c \\ c \in C \end{matrix} \tag{4}$$

$$y_{ic} = \{0,1\} \qquad \forall \qquad i \in I \tag{5}$$

Where:

I, i = the set and index of incident locations
C, c = the set and index of types of targeted interventions
i, j = the indices of demand incident locations to which police may respond
J_c = the set of locations j where police units can respond to call for intervention type c
x_{jc} = 1 if a police unit responds to call type c at location j, and 0 otherwise
y_{ic} = 1 if demand incident i of call type c is covered by a police unit, and 0 otherwise
a_{ic} = priority of covering a call of type c at demand incident location i
P_c= number of police units allocated to respond to call type c
N_{ic}= the set of locations j where police units can respond to demand incident i of type c

In the MTD-MCLP, the objective (1) is to cover as many demands for the targeted intervention of a number of criminal organizations as possible. The demands additionally have priorities for coverage for each organization. Constraints (2) are the covering constraints and serve to ensure that a demand incident location i can only be considered covered if a police unit responds to an incident location j that is within the neighborhood set with a targeted intervention of type c. Constraints (3) serve as the cardinality constraints and ensure that only the available number of police units P_c allocated to respond to organization type c are located. Constraints (4) and (5) require only integer values in the solution, meaning a police unit cannot be partially assigned to a call at incident location j and similarly, incidents cannot be partially covered. The location decision variables indicate which locations/organizations receive a police intervention. If a minimum amount of police units are available to target organizations, the model will enforce the minimum required allocation of police units. As written, constraint (3) serves as the cardinality constraint by defining the exact number of available units allocated to each intervention type *c*. The MTD-MCLP can be made to encourage a minimum number of police units available for a particular intervention type by replacing constraints (3) with a constraint on the total number of police units to allocate (6) and a set of constraints setting the minimum number of units (7) available for each for each intervention.

$$\sum_{j \in J_c} \sum_{c \epsilon C} x_{jc} = P \qquad (6) \qquad\qquad \sum_{j \in J_c} x_{jc} \geq P_{cmin} \qquad \forall \quad c \qquad (7)$$

3.2. *The Multiple-Type Facility Maximal Covering Location Problem*

Similarly, the Multiple-Type Facility Maximal Covering Location Problem (MTF-MCLP) can model the different types of law enforcement units that may be located at each police station:

Maximize

$$\sum_{i \epsilon I} \sum_{t \epsilon T} a_{it} y_{it} \qquad (8)$$

$$\sum_{j \in N_{it}} x_{jt} \geq y_{it} \qquad \forall \quad i \in I,\ t \in T \qquad (9)$$

$$\sum_{j \in J} x_{jt} = P_t \qquad \forall \quad t \qquad (10)$$

$$x_{jt} \leq Q_{jt} \qquad \forall \quad t \in T,\ j \in J \qquad (11)$$

$$x_{jt} \geq K_{jt} \qquad \forall \quad t \in T,\ j \in J \qquad (12)$$

$$x_{jt} = \{0,1, \dots P_t\} \qquad \forall \quad j \in J \qquad (13)$$

$$y_{it} = \{0,1, \dots P_t\} \qquad \forall \quad i \in I \qquad (14)$$

Where:

T, t	= the set and index of law enforcement unit types
x_{jt}	= 1 if a police unit of type *t* is located at police station *j*, and 0 otherwise
y_{it}	= 1 if incident *i* is covered by law enforcement unit of type *t*, and 0 otherwise
a_{it}	= priority for call at incident *i* that can be covered by police unit type *t*
P_t	= number of law enforcement units of type *t*
Q_{jt}	= the capacity of district *j* for law enforcement units of type *t*

K_{jt} = the minimum number of units of type t to located in district j

The MTF-MCLP demonstrates the case where it is not the interventions that are of a different kind, but rather the law enforcement units themselves differ. These differences may limit the sets of police station locations where those units can be located, or where their location would have a covering effect. In the context of police response consider that different available law enforcement units may have different assets, equipment, or training. In the MTF-MCLP, each police station location j is assigned a capacity Q_{jt} and a minimum allocation K_{jt} defined as the maximum and minimum, respectively, number of law enforcement units of type t that can be located at police station j. For those types that can be located at a given location, one or more may be located there if this leads to maximal coverage. Only those incidents that can be served by a given law enforcement unit type ($a_{it} > 0$) will be covered by the law enforcement unit of that of that type. The decision variables x_{jt} and y_{it} now indicate which types t of police units are located at location j and which demand i is covered by a unit of type t. In the MTF-MCLP the objective (8) is to maximize the number of priority-weighted calls for service that are covered by a police unit of the appropriate type. Constraints (9) are the covering constraints and serve to ensure that an incident of type t in police district j can only be considered covered if a unit of the appropriate type is located within that district. Constraints (10) are the cardinality constraints and indicate the total number of units available. Constraints (11) require that the number of facilities located at each location j do not exceed the capacity of that location. Constraints (12) similarly require that a minimum number of facilities in each district are met for each particular type. In the context of police operations, capacity may be influenced by police geography (e.g., number of patrol officers per beat) or by the available equipment, such as the number of patrol cars. The decision variables are restricted to integer values, (13,14) and now indicate the number of law enforcement units of type t that are allocated to district j, and the demand locations i that are covered by a facility.

In practice, the demand for, and availability of, law enforcement fluctuates over time. The MTF-MCLP can accommodate multiple variants of constraints on the law enforcement response by assigning the different sets of facility types to locations, by altering the capacity of each potential facility location, or by adjusting the constraints on the total number of available facilities. For example, major events, emergency response, budget shortages, or other routine changes in personnel or equipment can all necessitate a redirection or reallocation of law enforcement resources.

4. Results

The MTD-MCLP and the MTF-MCLP were written as Python (3.1) programs in Jupyter Notebook within ArcGIS Pro 2.9 and solved using the Gurobi 10.0.1 module.

4.1. The Multiple-Type Demand Maximal Covering Location Problem (MTD-MCLP)

The MTD-MCLP was tested on the sample dataset with the goal of covering three different types of demands for targeted intervention. Each demand location in the dataset was randomly assigned to at least two of three hypothetical types of demands, with each demand allocated to a similar number of demand locations. The objective for the MTD-MCLP is cover as many demands for different types of targeted interventions as possible. These problem instances of the

MTD-MCLP employed beat-level neighborhood sets, meaning a potential facility sited within a beat is assumed to provide a protective or deterrent effect to nearby incidents. Figure 2 shows the results of the MTD-MCLP results using the equality constraint in (3), where P_c was constrained to 8, 11, and 9 for demands of types 1, 2, and 3, respectively. The MTD-MCLP was then tested with the goal of targeting each demand with at least 1 police unit using the total P_c constraint in (6) and the inequality constraint in (7), and the results are shown in Figure 3. Both instances of the MTD-MCLP resulted in a targeted 38% of the total demand locations, although the use of the alternative constraints ($P_c = 28, x_{jc} \geq 1$) resulted in 12 facilities of types 1 and 3, while type 2 was allocated four interventions.

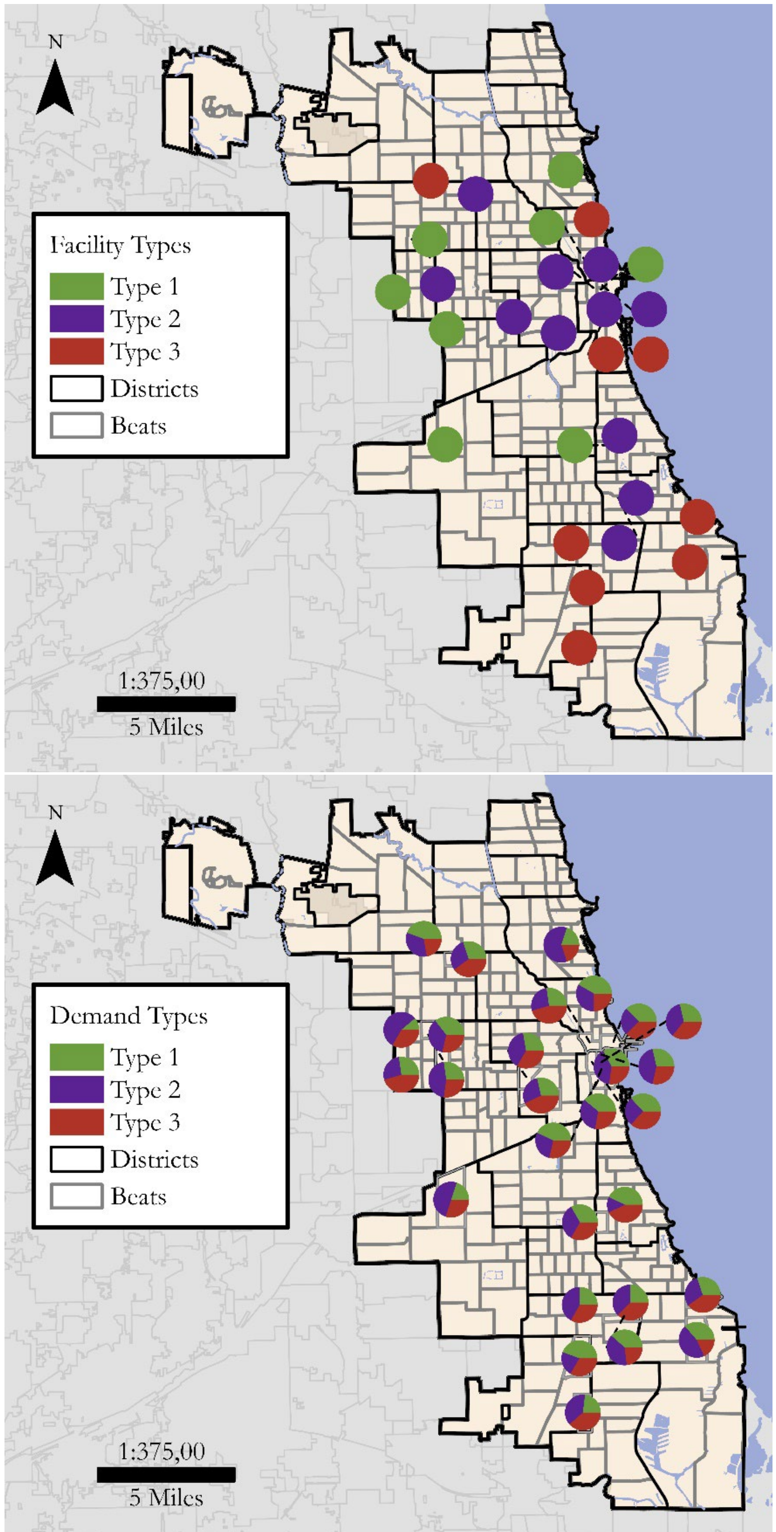

Figure 2: MTD-MCLP (a) intervention locations and (b) demands (beats) covered using the equality constraint in (3).

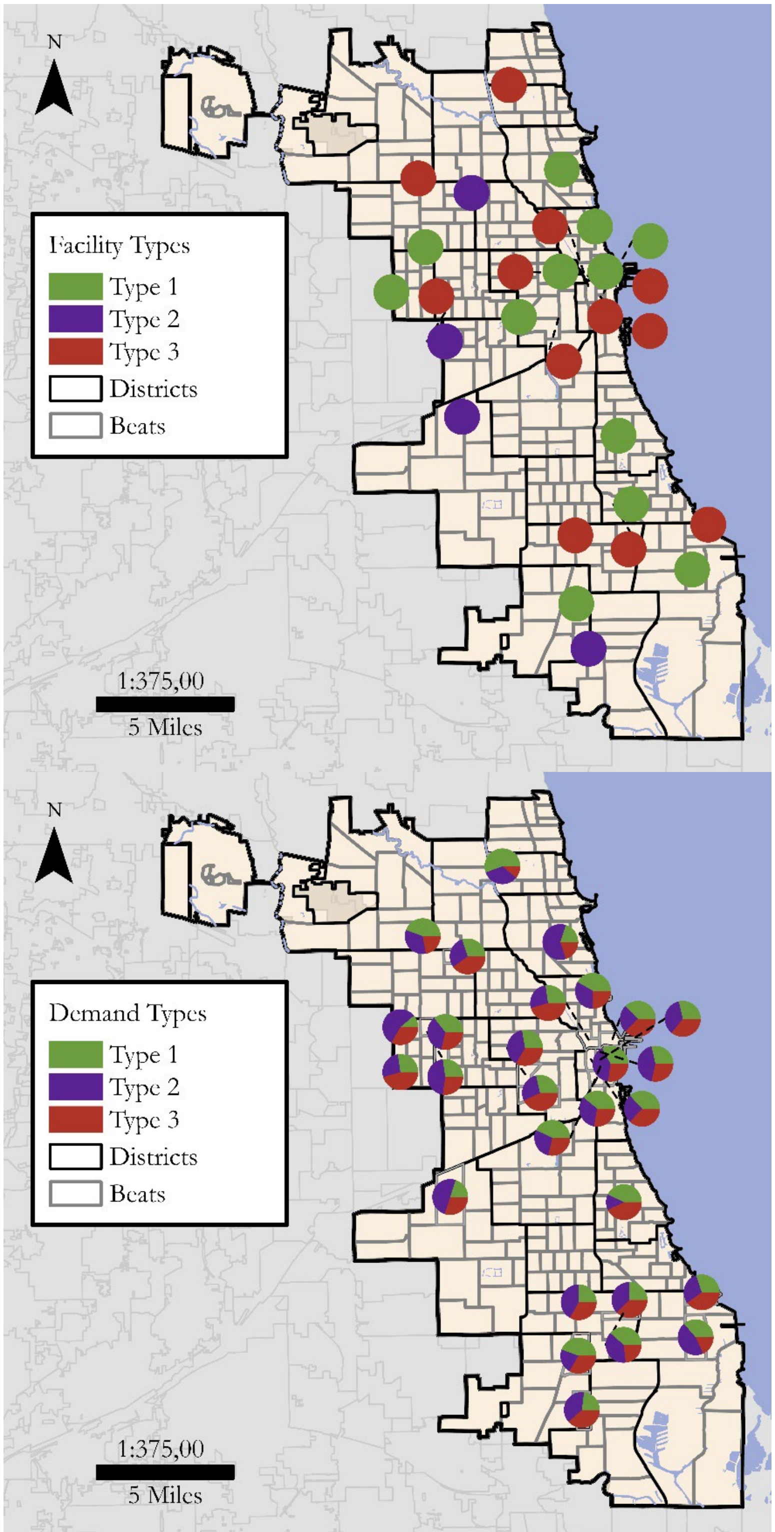

Figure 3:MTD-MCLP (a) intervention locations and (b) demands covered (beats) using the inequality and total constraints in (6) and (7).

4.2. The Multiple-Type Facility Maximal Covering Location Problem (MTF-MCLP)

In this instance of the MTF-MCLP, the objective is to allocate a number of multiple types of police units among 22 police districts in the city of Chicago. Each incident in the sample dataset was assigned a priority (a_{it}) based on the nature of the call reported (e.g., murder, robbery, fraud) and the location of the incident. Incidents were assigned priority values for patrol units ranging from 2-5 based on FBI Uniform Crime Reporting (UCR) offense definitions (FBI 2019). The a_{it} values were assigned as follows: Criminal homicide, sexual assault ($a_{it} = 5$); robbery, aggravated assault, and burglary/breaking & entering ($a_{it} = 4$); larceny, motor vehicle theft, and arson ($a_{it} = 3$); *Part 2* offenses ($a_{it} = 2$). The total number of patrol officers to allocate ($P_t = 2192$) assumes an average of 8 patrol officers per beat ($n = 274$), and the capacity for each district similarly assumes a maximum of 9 patrol officers per beat (Chicago Police Department, 2023a). This problem instance employed the minimum allocation constraint in (12) to ensure each beat was allocated at least one patrol officer ($K_{jt} \geq 1$). No other unit types were constrained to a minimum number of facilities. The constrained values for each type is shown in Table 1.

All incidents at transit stations were assigned priority values of $a_{it} = 5$ for transit units. In this example it is assumed that law enforcement units assigned to transit stations may also be able to respond to nearby incidents. As such, transit units were assigned to the neighborhood set belonging to those incidents in the beat surrounding the transit station. Assuming nearby incidents will be of lower priority to transit units, all other incidents in the beat were assigned a_{it} values of one less than those values assigned to patrol units. Location-based selection was used to identify 84 beats within 18 districts that contained a transit station. The minimum allocation was constrained to 1 per beat and 65 transit units were located.

The mounted patrol units, as part of CPD’s Special Functions division, are both limited in number, and are assigned to a small scope of specialized duties in specific police districts. For example, these assignments may include crowd control in downtown Chicago, patrolling major festivals/events, or maintaining community relations as ‘Ambassadors of Good Will (Chicago Police Department 2023b).’ Mounted units were assigned to the neighborhood sets of incidents in four police districts that include downtown, the waterfront, and Grant Park. All incidents in which the location description indicated waterfront or park property were assigned priority values of $a_{it} = 5$ to mounted units. As with the priority values assigned to transit incidents, all other a_{it} values were assigned as one less those values assigned to patrol units.

Police canine (K9) units are tasked with tracking individuals, attending community outreach events, and detecting narcotics and explosives, among many other duties (Chicago Police Department 2023c) . For this demonstration, incidents a K9 unit can cover were assigned the highest priority value ($a_{it} = 5$) for incidents involving narcotics and other incidents were similarly assigned a_{it} values of one less that those values assigned to patrol units.

This example instance of the MTF-MCLP also considered two other specialized units: Marine and Helicopter. Marine units consist of both maritime and land-based operations and among many other specialized duties, are present at major waterfront events (Chicago Police

Department 2023d). Incidents on park or waterfront property were assigned the highest priority for marine type facilities(a_{it} = 5). Other incidents were assigned a_{it} values of one less that those values assigned to patrol units, excluding all Part two offenses, which were given no priority. Marine units appear in the neighborhood set of incidents in districts adjacent to water features including Lake Michigan, the Chicago River, Wolf Lake, Lake Calumet, and the Sanitary and Ship Canal. Helicopter units can cover incidents in any district. Incidents were assigned a_{it} for helicopter units equal to those for patrol, and similar to marine units, all part two offenses were given no priority for helicopter units.

Table 1: Constraint values by police unit type

Type	P_t	K_{jt}	Q_{jt}
Patrol	**2192**	**1 per beat**	**9/beat**
Transit	**65**	**0**	**1/beat**
Mounted	**27**	**0**	**8/district**
K9	**13**	**0**	**1/district**
Marine	**7**	**0**	**1/district**
Helicopter	**2**	**0**	**1/district**

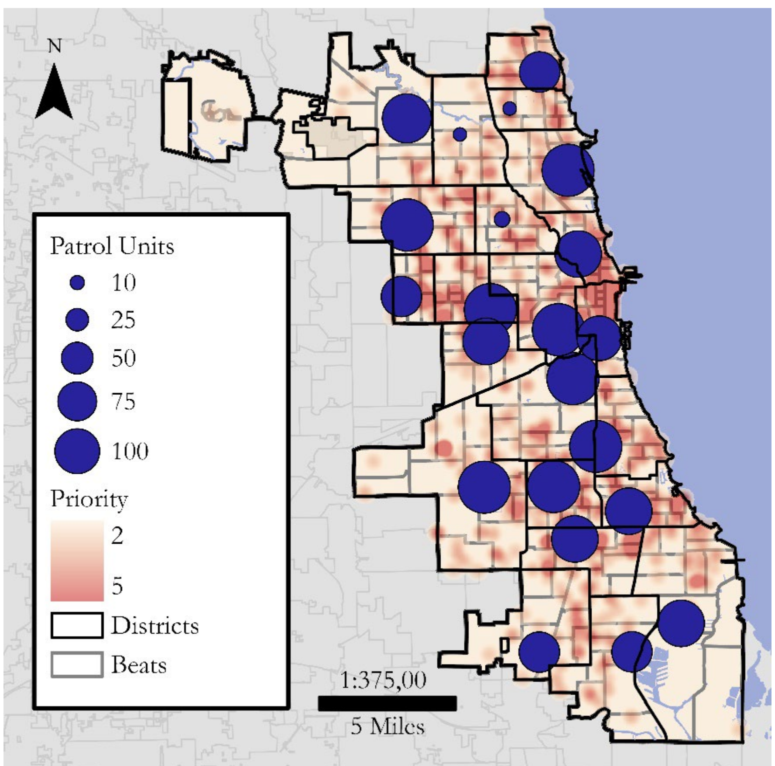

Figure 4: Patrol units allocated among 22 police districts in Chicago. Incident priority displayed as heat map.

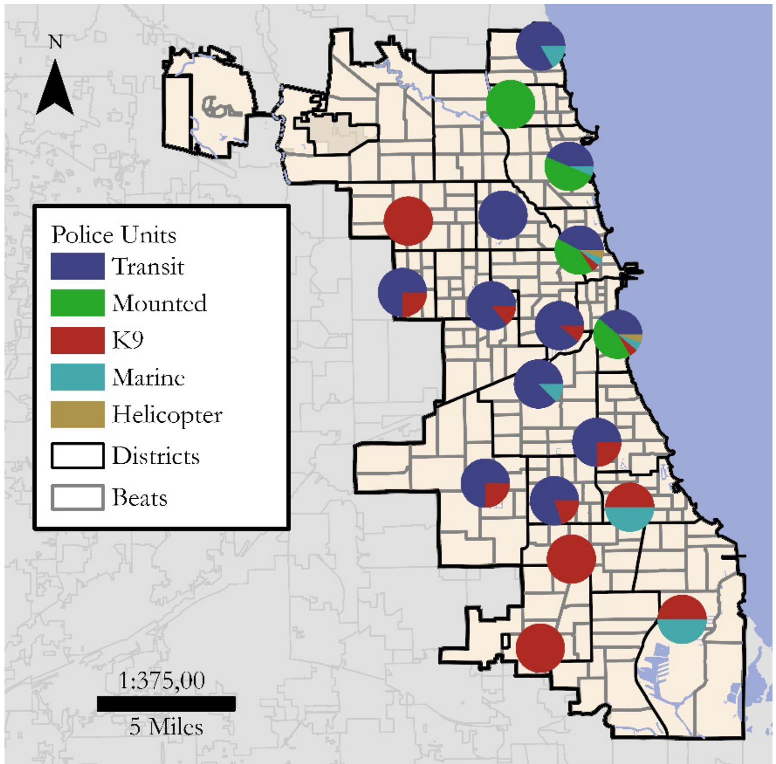

Figure 5: The allocation of different types of police units among districts. Pie symbols show the proportion of total facilities (excluding patrol) located in each district.

5. Discussion, Limitations, and Potential Extensions

Fundamentally, the purpose of generating models of these types is to permit practitioners – police decision-makers or staff – to explore alternative deployment designs. These models allow them to experiment with different data (e.g., alternative numbers of police units of different types, alternative station capacities, or alternative demands for police services). Police can experiment with different numbers of units (increasing or decreasing units by type) to determine if additional coverage can be achieved, or the extent to which coverage will be foregone if units are removed/unfunded. These experiments can reflect the uncertainties that are inherent in police operations. Police planners know that they are subject to changes in the police operations environment due to changing budget restrictions, changing availability of police resources such as during police union actions or serious illness among police officers, and changing demand circumstances (e.g., during special events or natural or man-made disasters), including the seasonal changes in demand that have been well documented in the criminological literature. Along those lines, the models presented here seek to increase that planning flexibility.

The models presented here are intended as a general framework to illustrate the potential for spatial optimization to inform the allocation of law enforcement resources. The MTD-MCLP accommodates targeting multiple types of police interventions at specific locations. This is important, as a primary constraint on police operations is the need to prioritize how, and to what degree, different types of calls or incidents are responded to. The objective for the MTD-MCLP was to maximize coverage of incident locations associated with three types of police interventions. The alternative covering constraints in (7) can be used to prioritize the level of police activity assigned to specific types of crimes, criminal organizations, routine duties, or during temporary changes in police workloads.

The MTD-MCLP could also accommodate many other modeling considerations in the context of police operations. The problem instances of the MTD-MCLP presented above employed the use of beat-level neighborhood sets, meaning a potential facility location sited in a particular beat is assumed to provide a protective or deterrent effect to nearby incidents. This method differs slightly from the classic MCLP in that neighborhood sets are usually designed using a user-defined service distance, wherein all facilities within the service distance of a demand may provide coverage of that demand. Work has already begun on spatial optimization formulations that can accommodate allocating multiple facilities, at a single location, while considering spatial deterrence using distance-decay. In the context of police operations, the number and types of available units can be highly variable, and targeting specific locations or types of incidents may be of greater priority than maximizing coverage over the study area. In that case, the neighborhood set of any demand i would be user-defined based on the specific operational context, and the objective function would aim to maximize coverage of specific types of interventions.

The MTF-MCLP can be used to allocate different types of units, resources, or assets among police districts. The MTF-MCLP aims to maximize coverage of incidents within districts, while maintaining minimum levels of police presence throughout the study area. The demand for, and availability of, police resources can be highly dynamic. For example, major events or emergency situations can necessitate the redirection of first response resources to the affected districts, while routine maintenance or other budget or equipment shortages can reduce the number of available units. The MTF-MCLP can be made to encourage new or alternative spatial allocations of units among districts by simply updating the capacity and minimum allocation parameters. Further, the objective of the MTF-MCLP aims to maximize coverage of incidents with the most appropriate type of facility. In the problem instance presented above, each district was assigned a minimum number of each type of unit, although not all types of units can be located in all districts. For example, patrol, mounted, and transit units can provide coverage to the same set of incident types, although those incidents will be of lower priority than the value assigned for more specialized units. Similarly, while patrol units can be located in any district, mounted and transit units are limited to those districts where the specialized function takes place (e.g., near transit stations or the waterfront). In this way, the allocation of a more specialized unit in a particular district can influence the number of patrol units available to other areas.

Moreover, models such as these reflect the changing nature of policing. There is increased specialization within police forces, with specific training limited to a relatively small percentage of the entire force. In the context of large urban policing, these specialized police services need to be positioned so that their effectiveness in responding to appropriate calls for police services is maximized. The costs associated with establishing these units at stations with the equipment necessary to efficiently operate further argues for the use of planning tools such as these models in the decision-making process.

This effort to increase flexibility in operations planning is incremental, however. Police operations – particularly in large urban areas – represent a complex system that is unlikely to be captured by any single model. While this work builds on previous models there are still many elements of police operations that have not yet been captured. In one sense these current deficiencies represent limitations of the current body of work, in another sense they represent opportunities for future research and application. As examples, the current models do not consider equity among police responders. That is, neither the number or the severity of calls that may need response for a particular station have been considered in the current literature, even though such equity is a serious concern for police officers and their labor representatives. Moreover, the current modeling standards do not accurately capture the level of flexibility that police planners have in altering their operations, including the location of facilities and personnel.

Even though there is much work to be done in this area, the research presented here attempted to move the research frontier forward. The ultimate goal is to provide an increasing diversity of quantitative planning tools that can generate results that can be integrated with insights from police and policy makers that cannot be readily quantified in order to produce the best holistic outcomes for the police and the population they serve.

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Index Terms

- Backup Covering
- Chicago
- Chicago Police Department (CPD)
- Crime
    - Mapping
    - Modeling
    - Type
- Geographic Information System (GIS)
    - ArcGIS Pro
- Location Covering Problems
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