Exact solution for a charged particle in an inductive time-dependent increasing magnetic field

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We study the classical motion of a charged particle in presence of an inductively increasing time-dependent magnetic field as the one created inside a resistor-inductor series circuit driven by a voltage source. The inductor is treated as an infinite solenoid. In such a scenario, the expression for the time-dependent magnetic field generated when the circuit is turned on can be easily derived. We consider the case study of two-dimensional motion since the generalization to three-dimensions is elementary. The resulting differential equations for the two-dimensional motion of the charged particle are solved by using a particular method which relies in deployment of complex variables. The ensuing motion has interesting features that highlight the challenges faced in studies of charged particles in a time-dependent magnetic field. This study has applications in magnetic plasma confinement, where understanding charged particle dynamics in time-varying magnetic fields helps optimize stability and energy retention in fusion devices.

Keywords: Magnetic field; Cyclotron motion; Charged particle; Equations of motion; Differential equations.

I. INTRODUCTION

Circular motion in a magnetic field is a phenomenon that occurs when charged particles, such as electrons or ions, move with velocity perpendicular to a uniform constant magnetic field. For such a situation, the motion results in a circular trajectory confined to a two-dimensional (2D) plane [1]. This behavior is a consequence of the Lorentz force, which describes the force experienced by a charged particle moving in an electromagnetic field. The radius of the circular trajectory depends on the mass, velocity, charge of the particle, and the strength of the magnetic field. This principle has numerous applications in physics and engineering and is well described in the literature [2–11]. Its quantum counterpart is the backbone of many important phenomena in condensed matter physics [12–26]. Any different situation, for instance, a magnetic field that is non-uniform but constant with time will lead to a mathematical problem that is very difficult to solve [27]. Same difficulties are encountered when the magnetic field is uniform but varies as a function of time. A time-dependent magnetic field might be seen as a minor inconvenience, but its presence drastically modifies the equation of motion in such a way as making it impossible to solve analytically. As a result, one must use special numerical methods of integration in order to understand the resulting motion.

A realistic example in which a time-dependent magnetic field can be generated is that of a resistor (R) and an inductor (L) connected in series with a direct current (DC) voltage source. This type of circuit is commonly studied as it exhibits a simple transient and steady-state behavior. When the circuit is first connected to the DC voltage source, the current does not immediately reach its maximum value. Instead, it increases gradually over time. The current, $I(t)$ at any time, t can be expressed as: $I(t) = I_{max}\left[1 - \exp\left(-\frac{t}{\tau}\right)\right]$, where $I_{max} = \frac{V}{R}$ is the maximum current value, V is the DC voltage of the source and $\tau = \frac{L}{R}$ is known as the time constant. This means that only after a long time (theoretically, in the $t \to \infty$ limit), the current reaches a steady state maximum value, I_{max}. Within the framework of this study, we treat the inductor as an infinite solenoid. An infinite solenoid is a theoretical model of a solenoid that is considered to be infinitely long and uniformly wound with wire. In an infinite solenoid, the magnetic field inside the solenoid is uniform and parallel to the axis of the solenoid. The field outside the solenoid is considered to be negligible (effectively zero). This is a result of the symmetry of the solenoid and the assumption of infinite length. The magnetic field strength inside an infinite solenoid is directly proportional to the current. Thus, a time-dependent current, $I(t)$ will create a time-dependent magnetic field, $B(t)$ which can be expressed as: $B(t) = \mu_0\, n\, I(t)$ where μ_0 is the permeability of free space and n is the number of turns per unit length. The key feature of the infinite solenoid is that the magnetic field inside does not depend on the length of the solenoid. In this special case the magnetic field inside the infinite solenoid is uniform but time-dependent. While treating an inductor as an infinitely long solenoid is a valid idealization for theoretical analysis, practical implementations involve solenoids of finite length. To justify the "infinite" approximation, it is common to consider solenoids where the length is much greater than the radius, typically with a ratio length/radius $\gtrsim 10$. This ensures the magnetic field inside remains approximately uniform and axial, minimizing edge effects that deviate from the ideal model [28]. Such an approximate dimensional estimate for this ratio is a useful practical benchmark for the physical relevance of the model and helps clarify the conditions under which the infinite solenoid approximation

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holds [29].

The objective of this work is to study the effect of this inductive time-dependent magnetic field on the motion of a charged particle. We consider a scenario in which the charged particle is injected or happens to be inside an infinite solenoid when magnetic field is turned on. We assume that the charged particle is initially either at rest or has a nonzero initial velocity that is perpendicular to the magnetic field. This guarantees that the charged particle will move in a 2D path. The solution to this problem is challenging since one must not forget that a time-dependent magnetic field induces an electric field which in turn generates an electric force acting on the charged particle. This means that equation of motion of the charged particle is governed by magnetic and electric forces acting simultaneously. Our solution method relies on a particular mathematical approach that employs complex variables. This approach is elegant and considerably simplifies the final form of the resulting differential equations of motion.

Furthermore, in addition to its physical applications, the approach used in this study offers significant pedagogical value, as it provides a clear and exact example of charged particle motion in a non-uniform, time-dependent magnetic field. Such an example can greatly aid students in visualizing and understanding complex electromagnetic phenomena beyond static or uniform cases typically covered in textbooks. The numerical and analytical treatment presented here bridges the gap between abstract theory and practical computation, enhancing conceptual clarity. This makes the work a valuable resource for undergraduate electromagnetism courses or computational physics modules. Highlighting this educational benefit would further demonstrate the broader impact and versatility of the study.

The article is organized as follows. In Section II we introduce the relevant theory, model and describe the key results obtained. In Section III we present the main conclusions.

II. THEORY AND RESULTS

An inductive increasing magnetic field refers to a magnetic field that is increasing in strength over time, typically generated by a changing electric current in a solenoid. This concept is fundamental in electromagnetism and is related to Faraday's law leading to the creation of an induced electric field, too. The motion of a charged particle in a magnetic and electric field can be modeled using the principles of electromagnetism, particularly the Lorentz force law. In a Cartesian coordinate system, unit vectors are typically denoted by $\vec{i}$, $\vec{j}$ and $\vec{k}$ for the x, y and z axes, respectively. These unit vectors have a magnitude of 1 and point in the direction of their respective axes: For the present case scenario, we assume that the uniform time-dependent magnetic field points along the positive z-direction and can be written

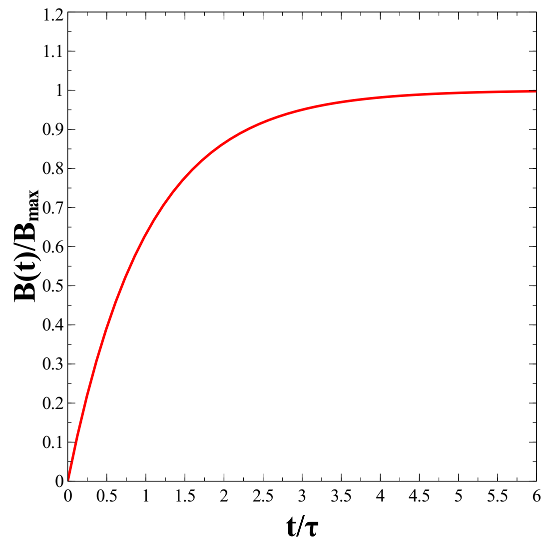

FIG. 1. Plot of $B(t)/B_{max}$ as a function of t/τ for $0 \leq t/\tau \leq 6$. The uniform time-dependent magnetic field is applied along the z-direction.

as:

$$\vec{B}(t) = B(t)\,\vec{k}\ . \tag{1}$$

The expression for $B(t)$ can be represented in the following way:

$$B(t) = B_{max}\left[1 - \exp\left(-\frac{t}{\tau}\right)\right], \tag{2}$$

where $B_{max} = \mu_0\, n\, I_{max}$. The relationship between $B(t)/B_{max}$ and t/τ is illustrated in Fig. 1, demonstrating how the inductive increasing magnetic field changes as a function of time.

In electromagnetism, electric and magnetic fields can be expressed in terms of scalar (V) and vector potentials ($\vec{A}$). This formulation is particularly useful because it simplifies the analysis of electromagnetic fields and helps in solving problems involving potentials. In this particular case, there is no scalar potential ($V = 0$). By following the same procedure as for the case of a uniform time-dependent magnetic field that is linearly increasing with time [30], one eventually obtains:

$$\vec{E}(\vec{r}, t) = \frac{\dot{B}(t)}{2}\, y\, \vec{i} - \frac{\dot{B}(t)}{2}\, x\, \vec{j}\ , \tag{3}$$

where we use a concise notation for the derivative, $\dot{B}(t) = dB(t)/dt$.

The classical motion of a charged particle with charge, $q > 0$ and mass, $m > 0$ can be described using Newton's second law of motion, modified to include the effects of electric and magnetic forces. The equation of motion for a charged particle in an electric and magnetic field is derived from the Lorentz force law:

$$m\,\frac{d\vec{v}}{dt} = q\,\vec{E}(\vec{r}, t) + q\,\vec{v} \times \vec{B}(t)\ , \tag{4}$$

where $\vec{v}$ is the velocity, $\vec{E}(\vec{r},t)$ is the induced electric field given from Eq.(3) and $\vec{B}(t)$ is the time-dependent magnetic field given from Eq.(1) and Eq.(2). In classical mechanics, the initial conditions for Newtonian motion are crucial for determining the future behavior of a particle or object under the influence of forces. These initial conditions include the initial position and initial velocity. For this model, we assume that the initial position vector and the initial velocity vector lie in the $x-y$ plane (perpendicular to the magnetic field). One can verify that, for such initial conditions, there is no dynamics in the z-direction. Thus, the resulting motion is 2D and only the x and y components of the initial position and initial velocity matter:

$$x(t=0) = x_0 \quad ; \quad y(t=0) = y_0 \tag{5}$$

and

$$v_x(t=0) = v_{0x} \quad ; \quad v_y(t=0) = v_{0y} \ . \tag{6}$$

We now equate the components of the vectors on both sides of Eq.(4) to form a system of scalar equations:

$$\begin{cases} \dot{v}_x = +\frac{\dot{\omega}(t)}{2}\, y + \omega(t)\, v_y \ , \\ \dot{v}_y = -\frac{\dot{\omega}(t)}{2}\, x - \omega(t)\, v_x \ , \end{cases} \tag{7}$$

where

$$\omega(t) = \frac{q}{m}\, B(t) \ , \tag{8}$$

can be viewed as a time-dependent angular frequency. Note that the definition of $\omega(t)$ in Eq.(8) mirrors that of the familiar cyclotron angular frequency for the case of a uniform constant magnetic field. Obviously, $\dot{\omega}(t) = \frac{q}{m}\, \dot{B}(t)$ is the time derivative of $\omega(t)$.

Solving coupled differential equations of this nature is very difficult by using conventional techniques [31]. The mathematical solution method that we introduce has significant advantages because it allows one to simplify considerably the final differential equation to solve. At the core of the method is the idea of using complex variables to rewrite Eq.(7) in a more compact and convenient form.

The uncoupling of the two differential equations in Eq.(7) cannot be accomplished by differentiating (with respect to time) one of the two equations. The most elegant and effective approach that we have found to achieve this goal relies on using complex variables. To start with, we can combine the two sides of Eq.(7) to obtain:

$$\dot{v}_x + i\, \dot{v}_y = \frac{\dot{\omega}(t)}{2}\, (y - i\, x) + \omega(t)\, (v_y - i\, v_x) \ , \tag{9}$$

where $i = \sqrt{-1}$ represents the imaginary unit. At this juncture, we introduce complex variables which are a fundamental concept in mathematics, particularly in the field of complex analysis. Complex numbers are numbers that have both a real part and an imaginary part. In this particular case, the 2D position can be written as a complex variable of the form, $z = x + i\, y$. This approach leads to $\dot{z} = v_x + i\, v_y$ and $\ddot{z} = \dot{v}_x + i\, \dot{v}_y$. By using these results, one obtains a complex second-order linear homogeneous ordinary differential equation with variable coefficients for the complex 2D position:

$$\ddot{z} + i\, \omega(t)\, \dot{z} + \frac{i}{2}\, \dot{\omega}(t)\, z = 0 \ . \tag{10}$$

Based on Eq.(2) and Eq.(8), one can rewrite $\omega(t)$ as:

$$\omega(t) = \omega_{max} \left[1 - \exp\left(-\frac{t}{\tau} \right) \right] , \tag{11}$$

where

$$\omega_{max} = \frac{q\, B_{max}}{m} \ . \tag{12}$$

The value, ω_{max} represents the value the cyclotron frequency for a uniform constant magnetic field, B_{max}. By substituting $\omega(t)$ from Eq.(8) into Eq.(10) one obtains:

$$\frac{d^2 z}{dt^2} + i\, \omega_{max} \left[1 - \exp\left(-\frac{t}{\tau} \right) \right] \frac{dz}{dt} + \frac{i}{2}\, \frac{\omega_{max}}{\tau}\, \exp\left(-\frac{t}{\tau} \right) z = 0 \ . \tag{13}$$

The two linearly independent solutions of this differential equation are:

$$f_1(t) = \exp[-i\, \omega_{max}\, \tau\, e^{-t/\tau}]\, \exp[-i\, \omega_{max}\, t]\, U\left(\frac{1}{2} \, , \, 1 + i\, \omega_{max}\, \tau \, , \, i\, \omega_{max}\, \tau\, e^{-t/\tau} \right) \ , \tag{14}$$

and

$$f_2(t) = \exp[-i\, \omega_{max}\, \tau\, e^{-t/\tau}]\, \exp[-i\, \omega_{max}\, t]\, L_{-1/2}^{(i\, \omega_{max}\, \tau)} \left(i\, \omega_{max}\, \tau\, e^{-t/\tau} \right) \ . \tag{15}$$

In the above expressions, $U(a,b,z)$ is a confluent hypergeometric function of the second kind and $L_n^{(\alpha)}(z)$ is a generalized Laguerre polynomial. Obviously, a, b, α and n are parameters while z is just a dummy complex argument of the function. We are sure that our educated readers will not confuse the argument of these two special functions with the 2D complex position.

For practical purposes, it is convenient to rewrite Eq.(13) in terms of dimensionless variables:

$$\frac{d^2 z}{d(\omega_{max}\, t)^2} + i\left[1 - \exp\left(-\frac{t}{\tau}\right)\right]\frac{dz}{d(\omega_{max}\, t)} + \frac{i}{2}\,\frac{\exp\left(-\frac{t}{\tau}\right)}{\omega_{max}\,\tau}\, z = 0\ . \quad (16)$$

At this juncture, one can define a dimensionless time variable of the form:

$$t\,' = \omega_{max}\, t = 2\,\pi\,\frac{t}{T}\ , \quad (17)$$

where T would represent the period of the cyclotron motion should the magnetic field had been a uniform constant magnetic field with value, B_{max}. An important quantity that determines the 2D trajectory of the charged particle is:

$$\frac{t}{\tau} = \frac{\omega_{max}\, t}{\omega_{max}\,\tau} = \frac{t\,'}{\tau\,'}\ , \quad (18)$$

where

$$\tau\,' = \omega_{max}\,\tau = 2\,\pi\,\frac{\tau}{T}\ . \quad (19)$$

Thus, one can write Eq.(16) in terms of dimensionless variables as:

$$\frac{d^2 z}{dt\,'^2} + i\left[1 - \exp\left(-\frac{t\,'}{\tau\,'}\right)\right]\frac{dz}{dt\,'} + \frac{i}{2}\,\frac{\exp\left(-\frac{t\,'}{\tau\,'}\right)}{\tau\,'}\, z = 0\ . \quad (20)$$

The idea is to solve Eq.(20) for different values for parameter, $\tau\,'$. For example, $\tau\,' = 2\,\pi$ corresponds to $\tau = T$ while $\tau\,' = 20\,\pi$ corresponds to $\tau = 10\,T$ which would be quite a large value of the time constant. The most general situation to study would involve arbitrary initial conditions of the form $z(t=0) = z_0 = x_0 + i\,y_0 \neq 0$ and $v(t=0) = v_0 = v_{0x} + i\,v_{0y} \neq 0$ where the trajectory would be determined in parametric form from the knowledge of $x(t) = Re[z]$ and $y(t) = Im[z]$.

For simplicity, we choose two values for parameter, $\tau\,' = \omega_{max}\,\tau = 2\,\pi\tau/T$ to study in detail:

$$\tau\,' = 2\,\pi \quad ; \quad \tau = T \quad (21)$$

and

$$\tau\,' = 20\,\pi \quad ; \quad \tau = 10\,T\ . \quad (22)$$

In order to understand the motion of the charged particle as the magnetic field is increasing from zero towards its maximum value, we solve Eq.(20) for values of parameter $\tau\,' = \omega_{max}\,\tau$ that were chosen and different initial positions and velocities.

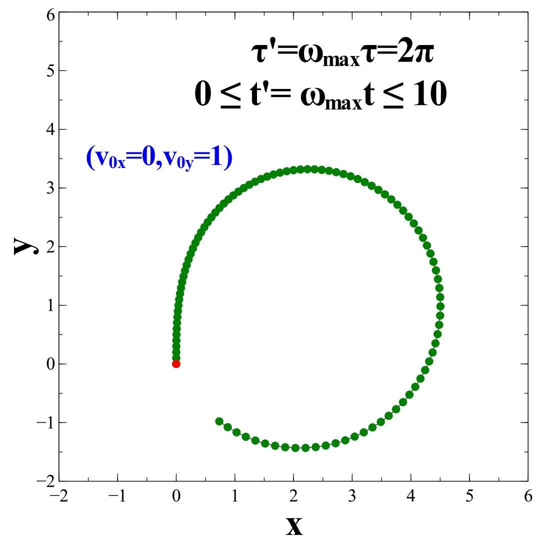

FIG. 2. Trajectory of the particle for $\tau\,' = \omega_{max}\,\tau = 2\,\pi$ and a relatively short time range, $0 \leq t\,' = \omega_{max}\, t \leq 10$. The initial position is taken at the origin, $(x_0 = 0, y_0 = 0)$ and is represented by a solid circle. The initial velocity is along the y-direction, $(v_{0x} = 0, v_{0y} = 1)$. The uniform inductive time-dependent magnetic field is applied along the z-direction.

The first choice considered, $\tau\,' = 2\,\pi$ corresponds to a time constant $\tau = T$. This means that should the charged particle had been exposed to a uniform constant magnetic field with value, B_{max}, the particle should have been able to complete only one revolution on a circular path for a time $t = T$. Instead, for the current inductively increasing time-dependent magnetic field, the charged particle experiences a magnetic field far from the saturation value of B_{max} value for a time interval of $t = T$ (since $\tau = T$). For simplicity, we may consider $\omega_{max} = 1$ wherever necessary when calculating velocities $(\frac{dz}{dt\,'} = \frac{1}{\omega_{max}}\frac{dz}{dt} = \frac{\dot{z}}{\omega_{max}})$. For the first scenario that we consider, we assume that motion starts at $z_0 = 0$ and the initial velocity is taken along the y-direction, $v_0 = i$ which means $(v_{0x} = 0, v_{0y} = 1)$. The result found for the 2D trajectory of the charged particle during a relatively short time interval, $0 \leq t\,' = \omega_{max}\, t \leq 10$ is shown in Fig. 2. One can see that the charged particle is not able to come close to completing a full loop. The 2D trajectory followed by the charged particle for a longer time range, $0 \leq t\,' = \omega_{max}\, t \leq 100$ is shown in Fig. 3. Note that the particle's motion has quickly stabilized to a circular trajectory. The effect of changing the initial position to $z_0 = 1 + i$ $(x_0 = 1, y_0 = 1)$ and changing the initial velocity to $v_0 = 2 + 3\,i$ $(v_{0x} = 2, v_{0y} = 3)$ is shown in Fig. 4 where one can see that the trajectory is more far-reaching.

Let us now consider a much larger value of $\tau\,' = \omega_{max}\,\tau = 20\,\pi$ which is ten times larger than the previously considered one. This means that the inductively increasing time-dependent magnetic field with need much

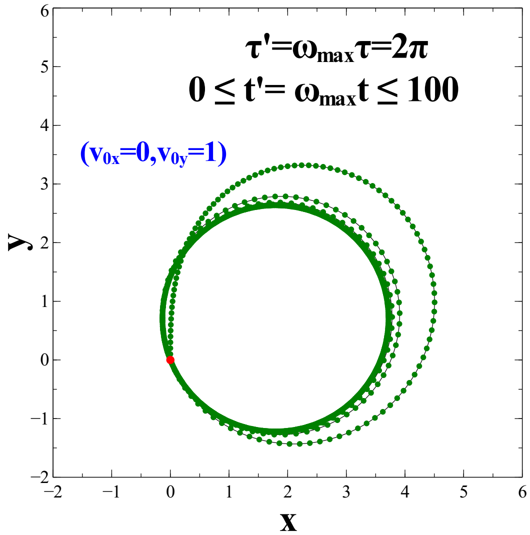

FIG. 3. Same as in Fig. 2 but for a longer time range, $0 \leq t' = \omega_{max}\, t \leq 100$.

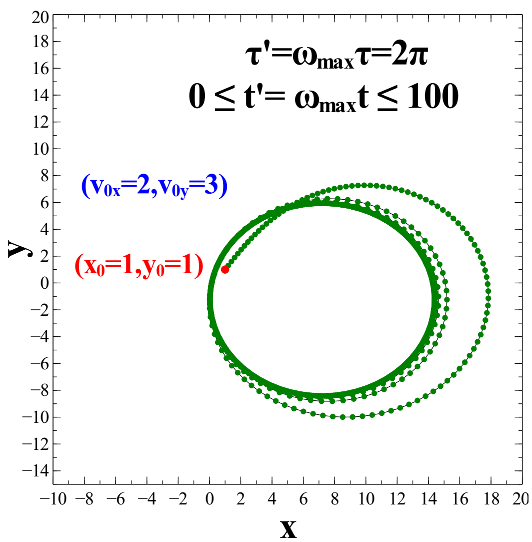

FIG. 4. Trajectory of the particle for $\tau' = \omega_{max}\, \tau = 2\,\pi$ and a long time range, $0 \leq t' = \omega_{max}\, t \leq 100$. The initial position is taken at $(x_0 = 1, y_0 = 1)$ and is represented by a solid circle. The initial velocity is $(v_{0x} = 2, v_{0y} = 3)$. The uniform inductive time-dependent magnetic field is applied along the z-direction.

more time to stabilize close to its maximum value of B_{max}. For this second scenario, we assume that motion starts at $z_0 = 0$ and the initial velocity is taken along the y-direction, $v_0 = i$. The result found for the 2D trajectory of the charged particle for the same relatively short time interval as before, $0 \leq t' = \omega_{max}\, t \leq 10$ is shown in Fig. 2. One can see that the charged particle is not able to come close at all to completing a full loop since the magnetic field during this short time interval is

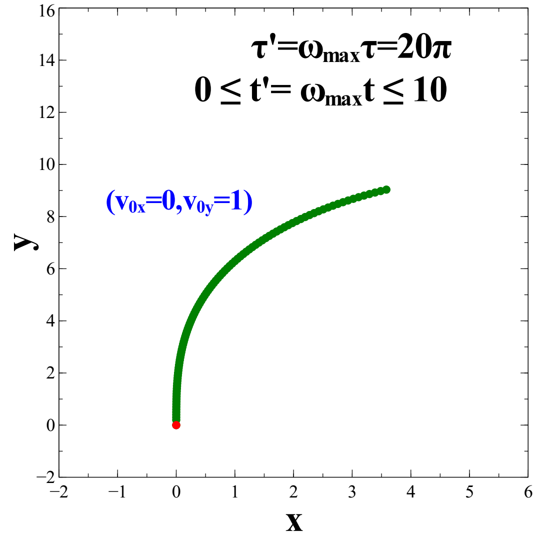

FIG. 5. Trajectory of the particle for $\tau' = \omega_{max}\, \tau = 20\,\pi$ and a time range, $0 \leq t' = \omega_{max}\, t \leq 10$. The initial position is taken at the origin, $(x_0 = 0, y_0 = 0)$ and is represented by a solid circle. The initial velocity is along the y-direction, $(v_{0x} = 0, v_{0y} = 1)$. The uniform inductive time-dependent magnetic field is applied along the z-direction.

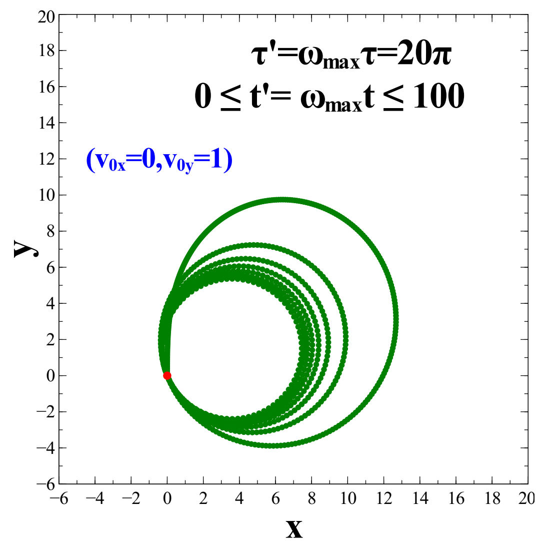

FIG. 6. Same as in Fig. 5 but for a longer time range, $0 \leq t' = \omega_{max}\, t \leq 100$.

very weak give the large value of the time constant. The path followed by the charged particle for a longer time range as studied earlier, $0 \leq t' = \omega_{max}\, t \leq 100$ is shown in Fig. 6. Changing the initial position to $z_0 = 1 + i$ $(x_0 = 1, y_0 = 1)$ and changing the initial velocity to $v_0 = 2 + 3\,i$ $(v_{0x} = 2, v_{0y} = 3)$ has an important effect by expanding further away the traveling paths as shown in Fig. 7. Overall, the path patterns observed are not as elaborate as those for the case of a uniform time-

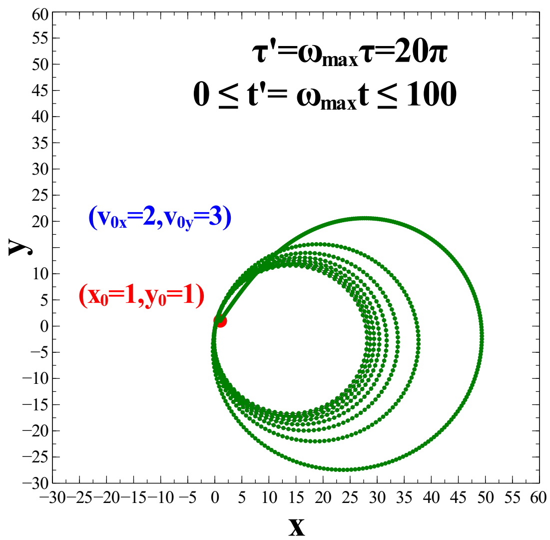

FIG. 7. Trajectory of the particle for $\tau' = \omega_{max}\tau = 20\pi$ and a long time range, $0 \leq t' = \omega_{max}t \leq 100$. The initial position is taken at $(x_0 = 1, y_0 = 1)$ and is represented by a solid circle. The initial velocity is $(v_{0x} = 2, v_{0y} = 3)$. The uniform inductive time-dependent magnetic field is applied along the z-direction.

dependent magnetic field that is linearly increasing with time.

III. CONCLUSIONS

Current research on plasma confinement indicates the need for additional studies on edge effects in order to limit container wall damage. A plasma is typically a system at high temperature containing high energy charged particles. Confinement of charged particles is achieved by using a sufficient magnetic field, or magnetic field combinations. To support such investigations, various magnetic field arrangements may be employed. In many instances, a charged particle experiences a varying magnetic field, thus exact theoretical solutions to the path of a charged particle are essential parts of theoretical investigations. In this paper, we furnish an exact solution from Maxwell's equations for a charged particle in an inductively increasing magnetic field, because when a current is turned on to produce a magnetic field, the current commonly experiences induction in the circuitry. For the current study we assume that the inductive magnetic field is applied perpendicular to the plane of motion of the charged particle. Initially, the magnetic field is zero and gradually increases as the inductor allows more current to pass, reaching a steady-state value (B_{max}) after long time (much longer than the time constand, τ). The great difficulty on understanding the motion of a charged particle in a time-varying magnetic field of this nature stems from the fact that a changing magnetic field produces an induced electric field which alters the commonly expected circular path of the particle.

The differential equation resulting from the Lorentz force acting on the charged particle is highly non-trivial giving rise to coupled differential equations for respective x and y components. Instead of using standard conventional methods, we solve this problem by using a method that relies on complex variables for the 2D position and counterpart velocity. Complex variables are known to simplify problems by providing a unified framework for handling oscillatory and exponential behaviors, especially in engineering and physics. They make easier to solve integrals and differential equations that are otherwise cumbersome in purely real-number form. In this particular case, the formalism of complex variables allows us to obtain a single second-order complex linear homogeneous differential equation with variable coefficients for the complex 2D position instead of the two uncoupled original equations. We solve this resulting differential equation via specialized software [32]. This allows us to obtain the position coordinates as a function of time and, thus, we can identify the path followed by the charged particle.

We observe interesting features in the 2D trajectory of the charged particle as a function of the chosen initial conditions. Non-trivial combinations of initial position and initial velocity (when they are both nonzero) lead to complicated paths. Overall, the ensuing paths are very sensitive to the initial conditons. While we illustrated our findings only for simple choices, a brief summary of most pertinent patterns observed for more elaborate choices of the initial conditions is provided in the following: (i) For right-of-center nonzero initial positions, circular convergence occurs sooner to the right of the origin with a converging circle center in a right quadrant. (ii) For a nonzero initial position, all curving motion is clockwise such that the positively charged particle's path is perpendicular to an outward B field and the orbital motion moves closer to the origin and away from its initial position. (iii) The size of the convergent curved path varies with the initial position of the particle, initial velocity direction, as well as initial velocity magnitude of the charged particle. (iv) For a nonzero initial posotion, a charged particle apparently has greater difficulty in establishing final circular motion when the initial position is above and to the right of the axis origin while its velocity vector is pointing nearly westward (left.) The particle has opposing propensities to turn clockwise while at the same time seeking to move toward the origin southwest of its initial position. The resulting velocity is smaller and the circular path diameter is subsequently smaller than in other formations. (v) From its nonzero initial position, a charged particle follows a path that curves and drifts toward the origin until circular motion is established.

Because of our choice of initial conditions for the velocity (being perpendicular to the magnetic field), the resulting motion is confined to a 2D plane. If the initial velocity of the charged particle has a component parallel to the solenoid's axial magnetic field, the parallel component results in a uniform motion along the axis of the solenoid. In a constant, uniform magnetic field the trajectory would have been helical. When the magnetic field is time-dependent, the standard picture of helical motion must be reconsidered since the trajectory becomes a much more complicated three-dimensional (3D) one. In particular, if the particle's initial velocity has a component parallel to the axial (solenoidal) magnetic field, the motion still consists of a helical-like path, but both the transverse and longitudinal components evolve with time due to the changing field strength. In a time-dependent increasing magnetic field, the charged particle tends to spiral along the magnetic field lines, but with time-varying characteristics, resulting in a non-uniform helical-like trajectory that reflects both the magnetic and the induced electric field influences. Overall, the 3D motion is still helical-like in character but becomes more elaborate and must be described numerically or analytically with care.

As already hinted, the insights gained from this work may be useful to plasma confinement studies [33–39]. By adjusting the strength of the magnetic field in different regions of the confinement device, researchers can study how it affects plasma stability and particle loss mechanisms. This can be of interest to investigating the use of rapidly changing magnetic fields (pulses) to actively control plasma instabilities and optimize confinement. By carefully tailoring the magnetic field, researchers can potentially achieve better plasma confinement, leading to longer fusion reaction times and higher energy output. Furthermore, time-dependent magnetic fields like the inductively increasing one that we studied in this work can help suppress certain types of plasma instabilities that can lead to energy loss. This is also important to numerical modeling studies which are used to predict plasma behavior under different magnetic field configurations, helping to optimize designs before building physical experiments.

At this juncture, we would like to clarify that the primary goal of this study is not to provide a direct quantitative tool for controlling plasma physics in experimental fusion devices, but rather to offer a basic theoretical framework that captures essential qualitative features of charged particle motion in time-dependent magnetic fields. The model presented is deliberately simplified to allow for exact or numerically precise solutions, which can serve as benchmarks or reference cases for more complex numerical studies. While the model does not include full plasma self-consistency, collisions, or boundary effects, which are features essential in realistic plasma simulations or experimental studies, it does highlight fundamental mechanisms, such as the role of inductive electric fields and evolving magnetic fields, which are present in real confinement systems. By analyzing particle behavior in a controlled, idealized setting, the study helps build intuition and provides conceptual insight into how time-varying magnetic fields affect particle dynamics, insight that can inform more advanced modeling efforts. Moreover, such theoretical approaches are valuable for interpreting trends, testing computational algorithms, and even guiding diagnostic expectations in experimental setups. Future work could benefit from comparison with more complete models or experimental data. However, from this point of view, we see this study as a stepping stone that captures essential qualitative features toward those directions rather than a replacement for them. Therefore, the results of this work should be welcomed by a broad audience of both researchers and students working in various scientific disciplines.

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