Monochromatic mass spectrum of primordial black holes

Matthew Kleban and Cameron Norton

Center for Cosmology and Particle Physics, Department of Physics, New York University, New York, New York 10003, USA

(Received 21 December 2023; accepted 8 January 2025; published 28 January 2025)

During slow-roll inflation, nonperturbative transitions can produce bubbles of metastable vacuum. These bubbles expand exponentially during inflation to superhorizon size, and later collapse into black holes when the expansion of the Universe is decelerating. Estimating the rate for these transitions during a time-dependent slow-roll phase requires the development of new techniques. Our results show that in a broad class of models, the inflationary fine-tuning that gives rise to small density fluctuations causes these bubbles to appear only during a time interval that is short compared to the inflationary Hubble time. As a result, despite the fact that the final mass of the black hole is exponentially sensitive to the moment bubbles form during inflation, the resulting primordial black hole mass spectrum can be nearly monochromatic. If the transition occurs near the middle of inflation, the mass can fall in the "asteroid" range 10^{17}–10^{22} g in which all known observations are compatible with black holes comprising 100% of dark matter.

DOI: 10.1103/PhysRevD.111.023538

I. INTRODUCTION

One of the greatest mysteries in modern physics is the nature of dark matter. Decades of searches for weakly interacting massive particles have so far failed to find any conclusive signal [1]. Axion dark matter is another interesting possibility, as these are well motivated beyond the Standard Model particles [2] and can simultaneously account for other features of our Universe [3]. A different possibility is that dark matter is composed of primordial black holes (PBHs) that formed in the early Universe [4,5]. PBHs could be formed from Standard Model matter and radiation without any exotic particle that survives until today, although the primordial mechanism that produced them in sufficient abundance likely requires new physics. Current observational constraints leave a 5 order of magnitude window of "asteroid" mass black holes in which a monochromatic spectrum of PBHs could account for all of dark matter [4,6].

Various PBH production mechanisms have been proposed in literature (see Ref. [6] for a review). These include a peak in the spectrum of primordial density fluctuations [7], first-order phase transitions [8], second-order phase transitions [9], crossovers [10,11], and collapse of cosmic strings [12].

Here we present a variation of the mechanism proposed in [13] and followed up in [14] and [15], in which the quantum nucleation and expansion of vacuum bubbles or domain walls during inflation creates regions that collapse later in the evolution of the universe, forming black holes. Because the vacuum bubbles form *during* inflation, their size and abundance at the end of inflation—and the masses and quantity of black holes that eventually form—is exponentially sensitive to when during inflation the transition occurred. These previous works assumed that the rate of production of these objects was approximately constant during inflation, and hence predicted a very broad, power-law spectrum of PBH masses [16].

By contrast, according to our analysis the transition takes place over a fraction of an inflationary e-fold, so the resulting PBH mass spectrum is nearly a delta function. If the peak of the mass distribution lies in the "asteroid" mass range, the abundance can be such that PBHs constituting all of dark matter is consistent with observational constraints.

A delta functionlike mass distribution was also found in [17], though this paper focused on domain walls with time-varying tension instead of vacuum bubbles.

Another variation was studied in [18], where a qualitatively different potential led to an approximately constant tunneling rate and a broad PBH mass spectrum. An interesting alternative mechanism to produce PBHs from single-field inflation that gives a fairly narrow mass distribution was studied in [19].

II. BUBBLE NUCLEATION DURING INFLATION

The basic mechanism we are interested in is the production of defects during inflation. If the defect expands to the inflationary horizon size, it will be caught in the

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

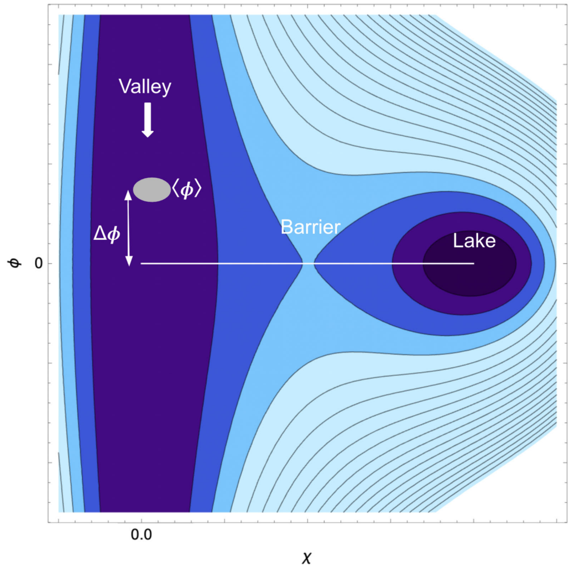

FIG. 1. Schematic of the potential for a two-field inflationary model. The inflaton ϕ is the vertical direction and slow-roll inflation can occur as ϕ evolves downward along a gently sloping "valley." The valley is separated from a local minimum (a "lake") by an interval in the second scalar field χ. The line indicates the (unique) instanton trajectory that connects the lake to the valley. This instanton describes the formation of a bubble of radius R, inside of which the fields take values corresponding to the endpoint in the lake, and outside of which take values corresponding to the endpoint in the valley. At a time during inflation when the vacuum expectation value $\langle\phi\rangle$ of the inflaton is displaced from the valley endpoint of the instanton trajectory by a distance $\Delta\phi$, the bubble can still appear but with probability exponentially suppressed in $(\Delta\phi)^2$.

pseudo-de Sitter expansion and grow exponentially for the remainder of the inflating phase. After reheating it will eventually reenter the horizon, after which it can collapse into a black hole. Vacuum bubbles or membranes are two examples of such defects that can be produced by nonperturbative quantum effects. If less than one such defect is produced per Hubble volume per Hubble time, the transition will not percolate because the space expands fast enough to dilute the number density exponentially.

To be definite, we will assume that during inflation two scalar fields have a potential similar to the one shown in Fig. 1, containing a "valley" with a small slope (vertical direction), separated from a "lake" by a barrier

$$V(\chi,\phi) = V_{\rm tun}(\chi) + V_{\rm infl}(\chi,\phi), \tag{1}$$

$$V_{\rm tun}(\chi) = \alpha\chi^2(\chi-\chi_0)^2 + M_V^2, \tag{2}$$

$$V_{\rm infl}(\chi,\phi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{\beta}{2}\chi^2(\phi-\phi_0)^2. \tag{3}$$

where $\alpha, \chi_0, M_V, m_\phi, \beta$, and ϕ_0 are parameters [20]. Slow-roll inflation is driven by the field ϕ rolling vertically down the valley in the figure, with a second scalar field χ acting as a spectator. We further assume that the vacuum energy in the lake ρ_b is lower than the inflationary energy density at any time during inflation, but higher than the energy density in the radiation dominated phase well after inflation ends when the vacuum bubbles reenter the horizon [21].

Potentials of this form will generally admit a unique instanton; a trajectory in field space that solves the field and gravity equations in Euclidean signature, connecting a point near the lake minimum to a point on the other side of the barrier in the valley via a domain wall of radius R [22]. Tunneling between the valley and lake corresponds to the formation of a bubble. If the bubble has walls thin compared to its radius, the fields inside and outside the wall will take values approximating the two end points of the trajectory [23].

Two-field models like this constitute effective descriptions for a wide variety of microphysical models [24–27] and provide a definite mechanism for a nonperturbative transition during inflation. However as we will see, our conclusion that the mass spectrum is monochromatic goes beyond this large class of models. It applies whenever there is a nonperturbative transition during inflation that produces a defect that grows beyond horizon size, and for which the transition rate is non-negligible only for a range of inflaton field $\Delta\phi$ that is not too large.

A. Approximating the tunneling rate

Tunneling between two local minima in the presence of gravity occurs via the Colemann-DeLuccia instanton [28]. In our case the initial state is an inflating universe, so tunneling connects slow-roll down the valley into the lake. This presents an interesting complication that has not been previously studied (to our knowledge), where the initial state is time-dependent and the field is not near a minimum. Nevertheless, there is generally a unique instanton connecting a specific point in the valley to the lake. If the potential were symmetric around $\phi = 0$ in Fig. 1, the tunneling trajectory would lie along the line $\phi = 0$. Slow roll breaks this symmetry slightly, but—absent special features or other symmetries—there is still only a single instanton trajectory. The instanton solution for a two-field potential similar to this was found numerically in [29], where the authors were interested in tunneling from lake to valley.

We expect the tunneling rate to be maximized at the time during inflation when the vacuum expectation value (vev) of the inflaton coincides with the end point of the instanton trajectory in the valley. Away from this time, when the field vev differs by $\Delta\phi$ from the end point of the trajectory, the rate should be suppressed. It is essential for our analysis to understand how quickly the tunneling rate goes to zero away from this maximum. To our knowledge this question has not been considered previously. We develop two approaches to this question, described below.

Numerically estimating the rate: One way to estimate the tunneling rate from a point with $\Delta\phi \neq 0$ is to deform the potential slightly to create an infinitesimal potential minimum at that point. This deformation creates another instanton connecting the new minimum to the lake [30]. We can calculate the new instanton's action and trajectory numerically (for instance with the "anybubble" package [31]). Because the deformation can be made arbitrarily small we expect this method to give a good approximation to the actual tunneling rate.

Analytically estimating the rate: To understand the dependence on $\Delta\phi$ more generally, consider a "two step" analytic estimate. We approximate the actual tunneling trajectory by a first step where the field fluctuates vertically the distance $\Delta\phi$ to the end point of the instanton trajectory, and a second step where it tunnels across the barrier via the standard instanton. The action for the full transition can be approximated as the sum of the actions for these two steps.

In order to create the initial conditions for the Colemann-DeLuccia (CdL) instanton, the first fluctuation must occur in a region that is at least of size R, the radius of the CdL bubble. The Euclidean action for such a fluctuation can be estimated by dimensional analysis

$$S \sim \int d^4x(\partial\phi)^2 \sim \int d^4x(\Delta\phi/R)^2 \sim c(\Delta\phi)^2R^2 \quad (4)$$

with associated probability $\sim e^{-S}$. An estimation for this c is shown in appendix, which gives $c \gtrsim 16\pi^2/9$.

We compare this approximation to numerical results for a specific potential using the deformation technique mentioned above (Fig. 2). We find that the quadratic scaling of $(\Delta\phi)^2$ in the exponent provides an excellent fit to the numerical estimates of the action made using the deformation technique, and a best-fit coefficient $c > \frac{16\pi^2}{9}$.

B. Slow roll

During inflation, an interval in the inflaton field $\Delta\phi$ is related to an interval in the number of inflationary e-folds ΔN by

$$\Delta N = H_i\Delta t = \frac{H_i\Delta\phi}{\dot{\phi}} = \frac{H_i^2}{\dot{\phi}}\frac{\Delta\phi}{H_i} = 2\pi\Delta_{\mathcal{R}}\frac{\Delta\phi}{H_i}. \quad (5)$$

Here $\Delta_{\mathcal{R}}^2$ is the power spectrum of the gauge-invariant curvature perturbation, with $\Delta_{\mathcal{R}}^2 \approx 10^{-9}$ during the observable period of inflation, and we approximate the Hubble rate during inflation H_i as constant. Our analysis in the previous subsection shows that the transition rate is unsuppressed relative to the maximum rate when

$$|\Delta\phi| \lesssim \frac{1}{\sqrt{c}R} \approx \frac{1}{10R}. \quad (6)$$

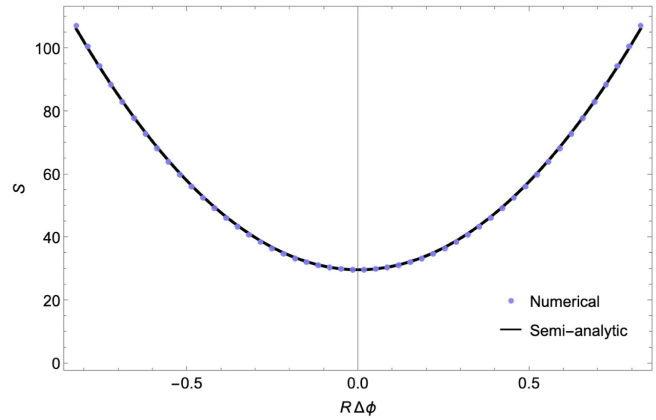

FIG. 2. Action for tunneling from a point displaced from the valley endpoint of the instanton by a distance $\Delta\phi$. Blue points: numerical approximation $S_{\text{num}}(\Delta\phi)$ calculated using [31] from the potential given in (1) (with parameters $\alpha = 800, \beta = 700, \chi_0 = 2, M_V = 0.1, m_\phi = 1$ and $\phi_0 = 0$), with a small deformation added to create a local minimum when $\Delta\phi \neq 0$. (The asymmetry in $\Delta\phi$ due to the slope of the valley is too small to be visible.) Black line: Semianalytic approximation explained in the text, $S \approx S_0 + c(\Delta\phi)^2R^2$, where $S_0 = S_{\text{num}}(\Delta\phi = 0)$, R is the radius of the bubble at $\Delta\phi = 0$, and $c \approx 94 > 16\pi^2/9$ is the best fit to the data points shown in blue.

where the last approximation uses our numerical estimate $c \approx 94$ (Fig. 2). This gives

$$\Delta N \lesssim \frac{\pi}{5}\frac{\Delta_{\mathcal{R}}}{H_iR}. \quad (7)$$

During or shortly after the observable part of inflation, the numerator $\Delta_{\mathcal{R}} \approx 10^{-4.5}$ and the spectral tilt is small and red (so that $\Delta_{\mathcal{R}}$ decreases slowly with time). However, we will see that for PBHs in the asteroid mass range the transitions must take place after this phase of inflation, where we do not have a direct measurement of (or strong constraints on) $\Delta_{\mathcal{R}}$. In the thin wall approximation the bubble radius R is determined by the potential energy in the valley and lake, and by the potential barrier separating the two. For high-scale inflation $H_iR \gtrsim 10^{-5}$ since the radius R must be larger than the Planck length. For lower-scale inflation H_iR could be smaller, but (for $\Delta_{\mathcal{R}} \ll 1$) there is a large class of potentials for which $\Delta N \ll 1$.

It is apparent from this analysis that $\Delta N \propto \Delta_{\mathcal{R}} \ll 1$ is a generic feature of any model in which the transition takes place over a range of inflaton field $\Delta\phi$ that is not too large. The two-field model we considered here is just one example. In the opposite extreme where the decay rate is close to constant during inflation, $\Delta N \gg 1$ and the results of [13,14] would be recovered. This can occur in a two-field model with an approximate symmetry, for instance the one considered in [18].

Following the "step" $\Delta\phi$ that creates the initial conditions for the instanton, the field must tunnel through the potential barrier. The tunneling rate is $\lambda \sim e^{-B}$, where $B = S_I - S_V$ is the action of the standard instanton minus the action for the inflaton to stay in the valley. In the next section we will calculate how large λ should be to give the observed dark matter abundance.

III. VACUUM BUBBLES AND BLACK HOLES

After a vacuum bubble nucleates, pressure due to the lower energy state on the inside causes it to expand to horizon size, after which de Sitter expansion inflates it exponentially to superhorizon scales. After inflation it continues to grow, comoving with the expansion of the universe, until eventually reentering the horizon. We are assuming that at this horizon-crossing time the vacuum energy inside the bubble is higher than the energy density of the radiation-dominated universe around it. In that case the bubble begins to collapse once it reenters the horizon. The resulting black hole has a mass that is exponentially sensitive to the time during inflation that the bubble appeared [13,15].

The bubble's radius grows exponentially during inflation, and at reheating is approximately

$$R_i \approx H_i^{-1} \exp\{N_n\}, \tag{8}$$

where $N_n = H_i(t_i - t_n)$ be the number of e-folds before reheating that the bubble nucleates. After reheating, any initial velocity of the bubble walls rapidly decreases due to the pressure of the fluid around the bubble, so that it expands at rest with respect to the cosmic comoving frame until it reenters the horizon at time t_H and subsequently collapses. The mass of the resulting black hole is can be approximated as $GM \sim t_H$, where t_H is the horizon crossing time of the comoving scale corresponding to R_i [32]. We can find this by setting the Hubble radius equal to the radius of the bubble after inflation, $H(t_H) = \frac{a(t_i)}{a(t_H)}$. Assuming radiation domination $a(t) \sim \sqrt{t}$, we have $t_H \sim \frac{R_i^2}{t_i}$ and the mass of the black hole as a function of N_n is

$$M \sim \frac{1}{GH_i} \exp\{2N_n\}. \tag{9}$$

(Had we considered domain walls instead [13], the mass would depend as $M \sim e^{4N_n}$.) Once the wall reenters the horizon it will rapidly collapse into a black hole due to its wall tension and the fact that the vacuum inside has higher energy than the universe outside. A black hole of mass M has a Schwarzschild radius (with $c = 1$)

$$R = 2GM = 1.5 \times 10^{-10}\ \mathrm{m}\left(\frac{M}{10^{20}\ \mathrm{g}}\right) \tag{10}$$

and horizon crossing time

$$2t_H = R = 2.5 \times 10^{-19}\ \mathrm{s}\left(\frac{M}{10^{20}\ \mathrm{g}}\right). \tag{11}$$

The number of e-folds before the end of inflation when the bubble nucleated is

$$N_n \approx 24 + \frac{1}{2}\ln\left(\frac{M}{10^{20}\ \mathrm{g}}\right) + \frac{1}{2}\ln\left(\frac{H_i}{10^{15}\ \mathrm{GeV}}\right). \tag{12}$$

If the bubble expands for a time longer than its internal inflationary Hubble time before it collapses, it will form a baby universe connected to ours through a (nontraversible) wormhole. There is a critical mass M_{cr}, above which a baby universe is formed and below which an ordinary black hole is formed. Following [13], $GM_{\mathrm{cr}} \sim \mathrm{Min}\{t_\sigma, t_b\}$, where t_σ, t_b are the gravitational times associated with the wall tension and vacuum energy inside the bubble. We assume $GM_{\mathrm{cr}} \sim t_b = H_b^{-1} \equiv \sqrt{\frac{3}{8\pi\rho_b}}$ where ρ_b is the vacuum energy in the lake, so that for a bubble to be supercritical it is sufficient that

$$\rho_b > \frac{3}{8\pi G^3 M^2} = (3.3 \times 10^6\ \mathrm{GeV})^4 \left(\frac{M}{10^{20}\ \mathrm{g}}\right)^{-2}, \tag{13}$$

well below the energy density in typical inflation models. Hence, these hydrogen-atom sized PBHs contain baby universes that undergo their own internal exponential expansion and some form of decay or reheating, since the lake is at best metastable to further transitions.

The mass distribution has a width due to the uncertainty in the nucleation time, $H\Delta t = \Delta N$ (7). From (9) we have

$$\frac{\Delta M}{M} \approx 2\Delta N. \tag{14}$$

It is natural for $\Delta N \ll 1$, so the mass distribution can be very close to monochromatic. (Later accretion roughly doubles the mass [14], but given the homogeneity of the early universe, we do not expect this to increase the width of the black hole mass distribution significantly.)

A. Tunneling rate

The number density of vacuum bubbles at the time they were produced is $\lambda \Delta t H_i^3$. This dilutes like the volume, so at reheating the number density of bubbles is $\lambda \Delta t H_i^3 e^{-3N_n}$. Hence the mass density of PBH dark matter today is

$$\rho_{\mathrm{PBH}} \approx \lambda \Delta t M H_i^3 e^{-3N_n} (T_0/T_{\mathrm{rh}})^3. \tag{15}$$

Equating this to the measured density of dark matter today and using Eq. (9) gives

$$\lambda \Delta t \approx 1.3 \times 10^{-16} \left(\frac{T_{\mathrm{rh}}}{\sqrt{H_i M_{\mathrm{Pl}}}}\right)^3 \left(\frac{M}{10^{20}\ \mathrm{g}}\right)^{1/2}, \tag{16}$$

where $M_{\rm Pl}$ is the Planck mass. This is the fraction of Hubble volumes in which a bubble nucleates during the transition. The maximum possible reheat temperature is $T_{\rm rh,max} \approx \sqrt{H_i M_{\rm Pl}}$, so this is small and collisions between bubbles are rare. Because transitions are so rare, we expect the effects on standard inflationary observables would be negligible (even if it were possible to measure them on these length scales).

IV. CONSTRAINTS AND DETECTION

Currently, there are no observations constraining PBHs in the "asteroid" range 10^{17} g $< M < 10^{23}$ g from constituting 100% of dark matter [4,6]. Possible approaches to detecting this form of dark matter include lensing, accumulation of one or more PBHs inside stars that affect stellar evolution over a long period of time, and stellar explosions triggered by a transit of the PBH though a star.

The lower bound arises from Hawking radiation, which for lighter PBHs produces gamma rays and energetic electron/positron pairs [33] [35–38]. These bounds could potentially be improved with future MeV telescopes or 21 cm observations [39–41]. A study of microlensing of stars in M31 provides the upper bound on the mass range [42]. The microscopic size of the PBHs in this range relative to optical wavelengths, combined with finite-source size effects, makes it very difficult to push theses constraints to lower PBH mass. Lensing of gamma ray bursts is of interest because their cosmological distance and much shorter wavelength of electromagnetic radiation makes lensing by PBHs in this mass range stronger, but there are no current constraints from this effect [43].

If a PBH passes through a star, gravitational friction heats the star and reduces the kinetic energy of the PBH. This can lead to a bound orbit where the PBH repeatedly passes through the star, eventually settling into the center. The PBH will gradually accrete matter, eventually growing to the point that it strongly affects stellar evolution.

The analysis in Ref. [43] shows that survival of stars does not provide constraints on PBHs in the allowed mass window because captures in galaxies are rare. A constraint would arise only if globular clusters have high dark matter densities and low PBH velocity dispersion. Observational signatures from rare stellar destruction events present a more promising avenue for future constraints. More modeling is needed in order to better understand the evolution and destruction of the star after the PBH is captured and accretes a substantial amount of mass (see Ref. [44] for some recent work on neutron stars).

Observations of white dwarfs in certain mass ranges might have implications for PBH dark matter, as PBHs could trigger an explosion via heating even in the case that they are not dynamically captured by the white dwarf, and most white dwarfs will experience at least one such transit. While an initial analysis indicated this might occur for a certain range of PBHs [45], a more detailed treatment shows that this process does not provide any constraints in this mass range [43].

V. CONCLUSION

It is remarkable that dark matter could be composed of microscopic black holes produced in the earliest phase of the universe [46]. The scenario considered here requires physics not far removed from what is already needed to drive inflation, without any new forces or particle species at accessible energies.

There are a number of ways our analysis could be extended or generalized. One is to study potentials in which the transitions occur not at one time during inflation, but at a discrete series of times. This can be natural in inflationary models involving a pseudo-periodic potential such as unwinding inflation [27,47,48].

We assumed that the vacuum energy in the "lake" was well below the energy density at the end of inflation. It would be interesting to analyze the situation where the energy density instead falls below that of the lake before inflation ends.

We focused on the "asteroid" mass range because of the lack of constraints on PBHs in this range. There is another range where the constraints are weak—the so-called stupendously large BHs [49,50]. These black holes are larger than galactic halos and cannot constitute all of dark matter, but evidently current constraints allow them to form an $\mathcal{O}(1)$ fraction. It would also be of interest to extend our treatment of tunneling from slow roll to a more general analysis of tunneling from time-dependent initial states.

ACKNOWLEDGMENTS

We would like to thank Yacine Ali-Haimoud, Heling Deng, Sergei Dubovsky, Oliver Janssen, Mehrdad Mirbabayi, and Giovanni Villadoro for useful discussions. Our work is supported by NSF Grants No. PHY-1820814 and No. PHY-2112839.

APPENDIX: ANALYTICAL APPROXIMATION

To approximate the probability for the field to fluctuate down the valley, we calculate the variance of the field ϕ averaged over a sphere of radius R

$$\phi_R(\vec{x}, t) \equiv \frac{1}{V_3} \int_R d^3 y \phi(\vec{x} + \vec{y}, t), \tag{A1}$$

where $V_3 = \frac{4}{3}\pi R^3$. The averaged field is approximately a Gaussian random variable because the inflaton is a nearly free field. The probability therefore scales as

$$P \sim \exp\left\{-\frac{(\Delta\phi_R)^2}{2\sigma^2}\right\}, \tag{A2}$$

where σ is the variance of the field.

Setting $\langle\phi_R\rangle = 0$, σ^2 is given by the two point function of ϕ_R,

$$\sigma^2 = \langle\phi_R(\vec{x},t)\phi_R(\vec{x},t)\rangle = \frac{1}{V_3^2}\int_{R_x} d^3y \int_{R_{x'}} d^3y' \langle\phi(\vec{x}+\vec{y},t)\phi(\vec{x}+\vec{y}',t)\rangle. \quad \text{(A3)}$$

Evaluating the propagator for a massless field with spatially separated vectors gives

$$\langle\phi_R(\vec{x},t)\phi_R(\vec{x},t)\rangle = \frac{1}{V_3^2}\int_{R_x} d^3y \int_{R_x} d^3y' \frac{1}{4\pi^2(\vec{y}-\vec{y}')^2}. \quad \text{(A4)}$$

Integrating this expression,

$$\langle\phi_R(\vec{x},t)\phi_R(\vec{x},t)\rangle = \frac{1}{V_3^2}\frac{R^4}{2} = \frac{9}{32\pi^2}\frac{1}{R^2}. \quad \text{(A5)}$$

This gives that the dependence of the probability on $\Delta\phi$ is

$$P \sim \exp\left\{-\frac{16\pi^2}{9}(\Delta\phi)^2 R^2\right\}. \quad \text{(A6)}$$

Indeed, this is the same dependence on R^2 and $(\Delta\phi)^2$ that we obtained from dimensional analysis in the main text.

[1] G. Arcadi, M. Dutra, P. Ghosh, M. Lindner, Y. Mambrini, M. Pierre, S. Profumo, and F. S. Queiroz, Eur. Phys. J. C **78**, 203 (2018).
[2] D. J. E. Marsh, Phys. Rep. **643**, 1 (2016).
[3] T. C. Bachlechner, K. Eckerle, O. Janssen, and M. Kleban, arXiv:1902.05952.
[4] A. M. Green and B. J. Kavanagh, J. Phys. G **48**, 043001 (2021).
[5] A. Escrivà, F. Kuhnel, and Y. Tada, arXiv:2211.05767.
[6] B. J. Carr, K. Kohri, Y. Sendouda, and J. Yokoyama, Rep. Prog. Phys. **84**, 116902 (2021).
[7] B. J. Carr and S. W. Hawking, Mon. Not. R. Astron. Soc. **168**, 399 (1974).
[8] M. Crawford and D. N. Schramm, Nature (London) **298**, 538 (1982).
[9] S. G. Rubin, M. Y. Khlopov, and A. S. Sakharov, Gravitation Cosmol. **6**, 51 (2000).
[10] A. Escrivà and J. G. Subils, Phys. Rev. D **107**, L041301 (2023).
[11] A. Escrivà, E. Bagui, and S. Clesse, J. Cosmol. Astropart. Phys. 05 (2023) 004.
[12] S. W. Hawking, Phys. Lett. B **231**, 237 (1989).
[13] J. Garriga, A. Vilenkin, and J. Zhang, J. Cosmol. Astropart. Phys. 02 (2016) 064.
[14] H. Deng, J. Garriga, and A. Vilenkin, J. Cosmol. Astropart. Phys. 04 (2017) 050.
[15] H. Deng and A. Vilenkin, J. Cosmol. Astropart. Phys. 12 (2017) 044.
[16] J. He, H. Deng, Y.-S. Piao, and J. Zhang, Phys. Rev. D **109**, 044035 (2024).
[17] J. Liu, Z.-K. Guo, and R.-G. Cai, Phys. Rev. D **101**, 023513 (2020).
[18] A. Kusenko, M. Sasaki, S. Sugiyama, M. Takada, V. Takhistov, and E. Vitagliano, Phys. Rev. Lett. **125**, 181304 (2020).
[19] A. Escrivà, V. Atal, and J. Garriga, J. Cosmol. Astropart. Phys. 10 (2023) 035.
[20] Our main conclusions are not sensitive to the details of the potential. For instance, a model that produces domain walls rather than vacuum bubbles would change our results quantitatively, but not qualitatively.
[21] Again, these assumptions are not necessary and could be relaxed without changing the qualitative results.
[22] It is possible for multiple discrete instantons to exist, for instance if there are several local minima, or for a continuum of transitions to exist when there is a symmetry or the potential for the second field is independent of that of the first.
[23] While the conditions for thin wall tunneling and slow roll inflation are not compatible in a single field model, in this case the tunneling takes place in a field space direction transverse to the slow-roll.
[24] A. D. Linde, Phys. Lett. **158B**, 375 (1985).
[25] A. D. Linde, Phys. Rev. D **49**, 748 (1994).
[26] M. Berg, E. Pajer, and S. Sjörs, Phys. Rev. D **81**, 103535 (2010).
[27] G. D'Amico, R. Gobbetti, M. Kleban, and M. Schillo, J. Cosmol. Astropart. Phys. 11 (2013) 013.
[28] S. R. Coleman and F. De Luccia, Phys. Rev. D **21**, 3305 (1980).
[29] K. Sugimura, D. Yamauchi, and M. Sasaki, J. Cosmol. Astropart. Phys. (2012) 027.
[30] We thank Giovanni Villadoro for suggesting this approach.
[31] A. Masoumi, K. D. Olum, and B. Shlaer, J. Cosmol. Astropart. Phys. 01 (2017) 051.
[32] This applies for the case of a supercritical bubble, which we explain a bit later in this discussion.
[33] Reference [34] points out that if PBHs were close to extremal, the lower bound on the mass would be reduced.
[34] J. A. d. F. Pacheco, E. Kiritsis, M. Lucca, and J. Silk, Phys. Rev. D **107**, 123525 (2023).
[35] B. Carr, K. Kohri, Y. Sendouda, and J. Yokoyama, Phys. Rev. D **94**, 044029 (2016).
[36] M. Boudaud and M. Cirelli, Phys. Rev. Lett. **122**, 041104 (2019).

[37] R. Laha, Phys. Rev. Lett. **123**, 251101 (2019).
[38] B. Dasgupta, R. Laha, and A. Ray, Phys. Rev. Lett. **125**, 101101 (2020).
[39] A. Ray, R. Laha, J. B. Muñoz, and R. Caputo, Phys. Rev. D **104**, 023516 (2021).
[40] A. K. Saha and R. Laha, Phys. Rev. D **105**, 103026 (2022).
[41] S. Mittal, A. Ray, G. Kulkarni, and B. Dasgupta, J. Cosmol. Astropart. Phys. (2022) 030.
[42] H. Niikura, M. Takada, N. Yasuda, R. H. Lupton, T. Sumi, S. More, T. Kurita, S. Sugiyama, A. More, M. Oguri, and M. Chiba, Nat. Astron. **3**, 524 (2019).
[43] P. Montero-Camacho, X. Fang, G. Vasquez, M. Silva, and C. M. Hirata, J. Cosmol. Astropart. Phys. (2019) 031.
[44] Y. Génolini, P. D. Serpico, and P. Tinyakov, Phys. Rev. D **102**, 083004 (2020).
[45] P. W. Graham, S. Rajendran, and J. Varela, Phys. Rev. D **92**, 063007 (2015).
[46] It is perhaps even more remarkable that each such atom-sized black hole contains a large universe that underwent its own period of inflationary expansion and potentially reheating and further evolution.
[47] G. D'Amico, R. Gobbetti, M. Kleban, and M. L. Schillo, Phys. Lett. B **725**, 218 (2013).
[48] G. D'Amico, R. Gobbetti, M. Kleban, and M. Schillo, J. Cosmol. Astropart. Phys. (2013) 004.
[49] B. Carr, F. Kuhnel, and L. Visinelli, Mon. Not. R. Astron. Soc. **501**, 2029 (2021).
[50] H. Deng, J. Cosmol. Astropart. Phys. 11 (2021) 054.