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Optimal Interventions in Coupled-Activity Network Games: Application to Sustainable Forestry

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***Abstract*— We consider the problem of promoting sustainability in production forests wherein a given number of strategic entities are authorized to own or manage concession regions. These entities harvest agricultural commodities and sell them in a market. We study optimal price-shaping in a coupled-activity network game model in which the concession owners (agents) engage in two activities: (a) the *sustainable activity* of producing a commodity that does not interfere with protected forest resources, and (b) the *unsustainable activity* of infringing into protected regions to expand their agricultural footprint. We characterize the policy that maximally suppresses the aggregate unsustainable activity under budget constraints. Our analysis provides novel insights on the agents' influence on each other due to intra-activity and cross-activity network effects. We also identify a measure of node centrality that resembles the Bonacich-Katz centrality and helps us determine pricing incentives that minimize the aggregate unsustainable activity over the set of all feasible policies.**

I. INTRODUCTION

As our planet loses 23 million hectares of tree cover annually, the global impact of deforestation remains significant. In particular, more than 1.47 gigatons of CO_2 are emitted per year as a result of the conversion of tropical forests for large-scale commercial agriculture [1].

These observed effects of commercially driven deforestation have led researchers to explore the connection between its negative societal impacts and the commercial interests of the entities involved in deforestation. A survey of these works can be found in [2]. Besides, there also exist studies that reveal the effectiveness of tools from network science, control [3], and game theory in advancing our understanding of forest management and the economics of deforestation. An example of the use of game theory in this context comes from [4], which studies two-person games played by two landowners who own neighboring plots of land and need to compute their rewards to decide between two possible actions in every time period: forest conservation and deforestation.

Our present work is a contribution to the above stream of works in that we use a blend of tools from game theory and network analysis to design economic interventions that incentivize sustainable practices in forest concession networks. Specifically, we focus on the problem of restraining the expansion of agricultural plantations into forested lands. This is an issue of growing global importance as is evident from the European Union (EU) Deforestation Regulation's latest rules that will, starting in 2025, prohibit the trade of goods produced using deforested lands [5]. One approach that has been used to address this issue is sustainability certification, which identifies commercial entities whose harvesting and manufacturing practices are environmentally sustainable in that they do not contribute to the deforestation of areas designated as protected forests, primary forests, High Conservation Value (HCV) areas, etc. However, a significant fraction (over 50%) of palm oil cultivators are non-certified [6], and as [7] notes, increasing this fraction is essential for improved forest protection.

The limited success of current sustainability standards can be primarily attributed to the following shortcomings: (a) even though sustainable goods (i.e., goods produced by sustainability-certified cultivators) can be sold at prices higher than those of their unsustainable counterparts, the current monetary costs of certification are prohibitively high for cultivators to have sufficient incentive to adopt sustainable agricultural practices, and (b) current standards are sub-optimal because they ignore the geospatial distributions of palm oil concessions, thereby failing to incorporate the network topologies implicit in the ownership and strategic interaction structures of the cultivators, especially those of small-holders.

We overcome this challenge by studying a network game in which every agent is a concession owner who is free to choose her individual effort levels in sustainable and unsustainable production practices separately. We use this setup to devise agent-dependent[1] pricing policies aimed at minimizing the concession owners' aggregate equilibrium effort in the conversion of protected forests into palm oil plantations. In the process, we obtain several insights into how our optimal policy depends on the structure of the strategic interactions network, the pre-intervention prices of sustainable and unsustainable goods, and the pre-defined price limits that the planner may require the post-intervention prices to satisfy. In particular, we show that in most cases of practical interest, the optimal post-intervention prices depend on a hitherto-unexplored measure of node centrality that is similar to, but not the same as, Bonacich-Katz centrality. In fact, this centrality measure is defined by the difference between two scaled *Leontief* matrices – matrices that are

[1]As we clarify below, our pricing policies only determine the *effective* per-unit prices (i.e., the differences between per-unit selling prices and per-unit costs) of sustainable goods. Therefore, they also apply to the case of uniform pricing, wherein they can be interpreted as agent-dependent cost adjustment policies.

pivotal in the analysis of shocks in economic networks [8]–[10].

Related Works: Our work is a contribution to the literature on intervention design in network games. However, unlike this paper, most of this literature is only concerned with single-activity games, e.g., [11], [12]. A landmark paper in this category is [13], which studies a linear best-response network game and identifies the "key player" – the agent whose deletion from the network causes the maximum decrease in the aggregate effort level. [14] consider a related model of a linear-quadratic network game with the goal of maximizing the social welfare over the space of all possible network topologies. The main result therein is that the optimal network necessarily belongs to a class of graphs called *nested split graphs*. The recent work [15] extends these results to the case of networks modeled as directed graphs and shows that, under mild assumptions, the optimal networks in this case are hierarchical.

The policies proposed in all of the above works lie in the category of network interventions, i.e., the interventions are aimed at achieving the planner's objective by changing the structure of the strategic interactions network. On the other hand, the policies that we design lie in the category of *characteristic interventions* [16] or interventions that seek to achieve the planner's objective by altering agent characteristics (such as marginal utilities) rather than the network structure. This set of works includes [17], which designs interventions that maximize the aggregate action in a single-activity network game by enhancing the availability of resources to the agents under budget constraints. A recent paper in this line of works is [18], one of whose key findings is that the desired (optimal) interventions are given by the eigenvectors of the graph adjacency matrix. Given that our work considers the problem of maximizing social welfare, it is related to [18] but differs from the latter in the crucial respect that we borrow a *coupled*-activity network game model from [19] to incorporate cross-activity and intra-activity network effects. Moreover, our intervention design problem also incorporates optimization constraints that vary across the two activities.

Notation: In this paper $\mathbb{N} := \{1, 2, \ldots\}$, $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the set of n-dimensional real-valued column vectors, and $\mathbb{R}^{n\times n}$ denotes the set of $n \times n$ real-valued matrices. For $n \in \mathbb{N}$, we let $[n] := \{1, 2, \ldots, n\}$. For a vector $v \in \mathbb{R}^n$, v_i denotes its ith entry, and b_{ij} denotes the (i, j)th entry of a matrix $B \in \mathbb{R}^{n\times n}$. All matrix inequalities hold entry-wise.

We denote the column vectors with all zero entries and all one entries in $\mathbb{R}^n$ by $\mathbf{0}$ and $\mathbf{1}$, respectively.

II. PROBLEM SETUP AND DESIGN OBJECTIVES

Our results are founded on the coupled-activity network game model proposed recently in [19, Section 6.2]. A salient feature of this model is that it enables us to model network effects as well as agent participation in a pair of interdependent activities. We use this model to analyze strategic interactions between forest concession owners involved in one or both of the following: (a) the *unsustainable activity* or activity B, defined as the set of all harvesting and manufacturing activities that entail the conversion of protected forests into crop plantations, and (b) the *sustainable activity* or activity A, which encompasses all the harvesting and manufacturing activities that do not require clearing protected forests. All the agents in our setup are simultaneously engaged in the production of goods and compete in a market to sell these goods. Under this setup, we design characteristic interventions (i.e. changes in the effective per-unit prices of sustainable goods, or equivalently, goods resulting from activity A) to either (a) maximize what we call social welfare (defined below) while keeping the aggregate unsustainable activity in the network acceptably low, or (b) maximally suppress the aggregate unsustainable activity without compromising on sustainable production levels. We formally state and define these problems in the next section.

A. Model Definition

The model, in which a network of agents participate in a pair of activities A and B, is defined by [19, Equation (9)], which we reproduce below.

$$
\begin{aligned}
&u_i\left(x^A, x^B\right) \\
&= p_i^A x_i^A + p_i^B x_i^B - \left\{\frac{1}{2}\left(x_i^A\right)^2 + \frac{1}{2}\left(x_i^B\right)^2 + \beta x_i^A x_i^B\right\} \\
&\quad + \delta \sum_{j=1}^{n} g_{ij} x_i^A x_j^A + \delta \sum_{j=1}^{n} g_{ij} x_i^B x_j^B \\
&\quad + \mu \sum_{j=1}^{n} g_{ij} x_i^A x_j^B + \mu \sum_{j=1}^{n} g_{ij} x_i^B x_j^A, \qquad (1)
\end{aligned}
$$

Here n is the number of agents in the network, x_i^A (respectively, x_i^B) denotes the effort level of agent $i \in [n]$ in activity A (respectively, activity B), u_i denotes the utility or the net payoff of agent $i \in \{1, 2, \ldots, n\}$ as a function of all the agents' effort levels, p_i^A (respectively, p_i^B) denotes the marginal utility of activity A (respectively, activity B) to agent i, the *intra-concession substitutability* $\beta \in (-1, 1)$ denotes the extent of complementarity or substitutability between A and B for agent i, δ is the *intra-activity network effects parameter* and quantifies the effect of agent-to-agent strategic interactions associated with either activity A or activity B on the net payoff of agent i, μ is the *cross-activity network effects parameter* and quantifies the effect of agent-to-agent strategic interactions across the two activities on the net payoff of agent i, and $g_{ij} \geq 0$ quantifies the extent of substitutability or complementarity between the activities of agents $i \in [n]$ and $j \in [n] \setminus \{i\}$. It is also convenient to define the strategic interactions matrix of the network as the $n \times n$ non-negative matrix $G = (g_{ij})$.

In our setup (illustrated below in Figure 1), x_i^A (respectively, x_i^B) denotes the quantity of goods produced as a result of the i-th agent's participation in activity A (respectively, activity B). In addition, p_i^A (respectively, p_i^B) denotes the *effective* price per unit good produced as a result of the i-th agent's participation in activity A (respectively, activity

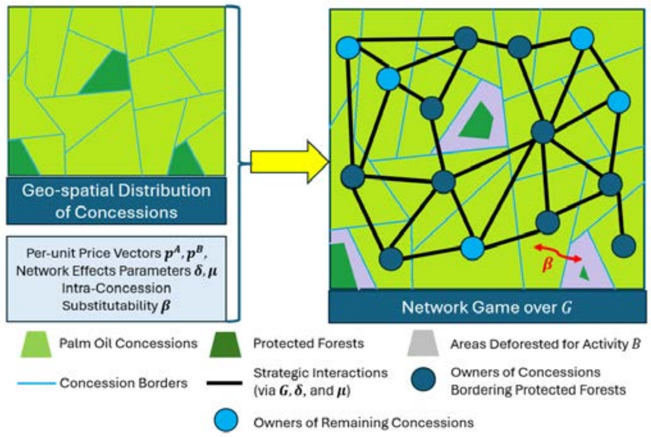

Fig. 1. Illustration of our game-theoretic setup.

B), where *effective* price means the difference between the price in question and the total of all associated costs of participation in the concerned activity. In the case of activity A, one of these costs is the cost of acquiring sustainability certificates. This implies that it is possible to have $p_i^A < p_i^B$ in scenarios in which the market prices of both sustainable and unsustainable goods produced by agent i are equal. Henceforth, we drop the word *effective* whenever we refer to effective per-unit prices, and we define two price vectors p^A and p^B such that p_i^A (respectively, p_i^B) is the i-th entry of p^A (respectively, p^B) for every $i \in \{1, 2, \ldots, n\}$.

On the basis of (1), our intervention design problems P can be stated precisely as follows, where the subscript $_0$ refers to the pre-intervention scenario.

P: Minimize the aggregate unsustainable effort (i.e., the sum of all the agents' effort levels in activity B) subject to the following constraints: (i) no agent's sustainable effort (i.e., effort in activity A) drops below her pre-intervention effort level, and (ii) the post-intervention per-unit prices of sustainable goods are no less than their pre-intervention counterparts and no greater than a pre-defined maximum $p_{\max}$, where $p_{\max} > p_{i0}^A$ for all $i \in [n]$. P can be expressed compactly as

$$\underset{p^A}{\text{Minimize}} \ \sum_{i=1}^{n} x_i^B$$

$$\text{s.t.} \quad x_i^A \geq x_{i0}^A \quad \text{for all } i \in [n], \tag{2}$$

$$p_i^A \geq p_{i0}^A \quad \text{for all } i \in [n], \tag{3}$$

$$p_i^A \leq p_{\max} \quad \text{for all } i \in [n]. \tag{4}$$

The problem P seeks to maximally suppress the prevalence of unsustainable harvesting and manufacturing activities. Note that this problem has as its decision variables the agent-dependent per-unit prices $\{p_i^A : i \in [n]\}$ of goods produced as a result of activity A. Therefore, $p_i^B = p_{i0}^B$ for all $i \in [n]$.

B. *Modeling Assumptions and Parameter Ranges*

We first define the value ranges of the game-theoretic modeling parameters δ, μ, and β.

Since the interactions between the agents are collaborative and complement each other's efforts, both μ and δ are positive in our model.

On the other hand, the contributions of intra-concession cross-activity effects are non-positive and non-increasing, because every agent has a finite capital and limited resources that she must divide between A and B. This makes the agent's individual effort levels substitutable across the two activities, i.e., $\beta > 0$. In addition, we scale the utility function to ensure that $\beta < 1$. Thus, $\beta \in (0, 1)$.

Next, we assume that the network effects parameters δ and μ are small enough for [19, Assumption 3] to hold. We state this assumption formally below.

Assumption 1: We have $\max\left(\frac{|\delta+\mu|}{1+\beta}, \frac{|\delta-\mu|}{1-\beta}\right)\lambda_1(G) < 1$, where $\lambda_1(G)$ denotes the spectral radius of G.

Next, we assume the following.

Assumption 2: We have $\mu < \delta$.

This means that cross-activity network effects are weaker than intra-activity network effects, which models the real-world scenario in which agents whose harvesting and manufacturing practices are largely sustainable try to avoid cooperating with those whose practices are largely unsustainable.

Finally, we make a standard connectivity assumption.

Assumption 3: The network is connected and undirected. Equivalently, G is irreducible and symmetric.

III. MAIN RESULTS

In this section, we solve the intervention design problem P, interpret the resulting optimal policy, and explain its significance for sustainable forestry.

As our goal is to find a pricing policy that achieves a positive reduction in the aggregate unsustainable equilibrium effort, there should exist at least one policy that is feasible and strictly outperforms the current policy. This motivates the following definition.

*Definition 1 (**Essential Feasibility**):* For $i \in \{1, 2\}$, we say that P_i is *essentially feasible* if the feasible set of P_i contains at least one policy p^A for which $\sum_{i=1}^n x_i^B < \sum_{i=1}^n x_{i0}^B$. If P_i is not essentially feasible, we say that it is *essentially infeasible*.

We now define the notation that we will use to characterize P and its optimal solution. We reproduce the definitions of the following scaled Leontief matrices from [19]:

$$M^+ := ((1+\beta)I - (\delta+\mu)G)^{-1}$$
$$M^- := ((1-\beta)I - (\delta-\mu)G)^{-1}.$$

The existence of M^+ and M^- is implied by Assumption 1. In addition, we let $b^+ := M^+\mathbf{1}$, $b^- := M^-\mathbf{1}$, and $b_\Delta = b^- - b^+$. We also let $d_{\min} := \min_{i\in[n]} \sum_{j=1}^n g_{ij}$ denote the minimum weighted node degree of the network.

We now state our main result: the optimal solution of P. The proof of this result can be found in the extended version of this paper [20].

Proposition 1: The following statements hold true for P.

1) For all G, β, δ, and μ satisfying Assumptions 1 - 4, P is essentially feasible if $\mu < \beta\delta$ and essentially infeasible if $\mu > \max\{\frac{2\beta}{1+\beta^2}\delta, \frac{\beta}{d_{\min}}\}$.
2) Suppose $\mu < \beta\delta$. Then there exists a threshold

$$p^*_{\max} := \frac{(b^- + b^+)^\top p^B}{(b^- - b^+)^\top \mathbf{1}} = \frac{(b^- + b^+)^\top p^B}{b_\Delta^\top \mathbf{1}} \quad (5)$$

such that the solution to P satisfies the following:

(i) If $p_{\max} \le p^*_{\max}$, then we have $p_i^A = p_{\max}$ for all $i \in \{1, 2, \ldots, n\}$.

(ii) If $p_{\max} > p^*_{\max}$, then every p^A satisfying $p^*_{\max} \le p_i^A \le p_{\max}$ for all $i \in \{1, 2, \ldots, n\}$ is optimal. In addition, $x_i^B = 0$ for all $i \in \{1, 2, \ldots, n\}$.

Proof:

1) We first observe from [19, Theorem 4] that P can be expressed as follows.

$$P\textbf{:}\quad \text{Minimize } (b^+ - b^-)^\top z$$
$$\text{s.t. } \left(M^+ + M^-\right) z \ge 2x_0^A - (M^+ - M^-)p^B, \quad (6)$$
$$z \ge p_0^A \quad (7)$$
$$z \le p_{\max}\mathbf{1}, \quad (8)$$

where $z = p^A$ is the vector of optimization variables (the post-intervention effective prices for activity A), and constraints (6), (7), and (8) are equivalent to (2), (3) and (4), respectively.

To establish the essential feasibility of P for $\mu < \beta\delta$, we first claim that $B_\Delta := M^- - M^+$ is entry-wise positive. To prove this claim, we observe that

$$\begin{aligned} &B_\Delta \\ &= (1-\beta)^{-1}(I - (1-\beta)^{-1}(\delta - \mu)G)^{-1} \\ &\quad - (1+\beta)^{-1}(I - (1+\beta)^{-1}(\delta+\mu)G)^{-1} \\ &\overset{(a)}{\ge} (1-\beta)^{-1}(I - (1-\beta)^{-1}(\delta - \mu)G)^{-1} \\ &\quad - (1-\beta)^{-1}(I - (1+\beta)^{-1}(\delta+\mu)G)^{-1} \\ &\overset{(b)}{=} (1-\beta)^{-1}\sum_{k=0}^{\infty}\left(\left(\frac{\delta-\mu}{1-\beta}\right)^k - \left(\frac{\delta+\mu}{1+\beta}\right)^k\right)G^k, \end{aligned}$$

where (a) holds because $\beta > 0$ and (b) follows from Neumann series expansion [21, 5.6.P26], which converges due to Assumption 1. Now, we know from Assumption 3 that G is the non-negative adjacency matrix of a graph in which there exists a path from every node $i \in [n]$ to every other node $j \in [n] \setminus \{i\}$. Equivalently, for every pair $(i, j) \in [n]\times[n]$, there exists a $k_{ij} \in \mathbb{N}$ such that $(G^k)_{ij} > 0$ for $k = k_{ij}$. Therefore, it suffices to show that the coefficient of G^k in the above series expansion is positive for each k. To this end, we observe upon some simplification that $\mu < \beta\delta$ implies $(1-\beta)^{-1}(\delta - \mu) > (1+\beta)^{-1}(\delta+\mu)$. As a result, $(1-\beta)^{-k}(\delta-\mu)^k > (1+\beta)^{-k}(\delta+\mu)^k$ for all k. This completes the proof of the claim that B_Δ is positive.

Therefore, the objective function $(b^+ - b^-)^\top z = -\mathbf{1}^\top B_\Delta^\top z$ is decreasing in every entry of z. It now follows from [19, Theorem 4] that $2\sum_{i=1}^n x_i^B = (b^+ - b^-)^\top p^A + \mathbf{1}^\top(M^+ + M^-)p^B$ is decreasing in each entry of $p^A = z$. Moreover, as all Leontief matrice are positive, $M^+ + M^-$, which is the sum of two scaled Leontief matrices with positive scaling factors, is also positive. Therefore, any price vector z that satisfies $p_0\mathbf{1} < z < p_{\max}\mathbf{1}$ also satisfies (6) - (8). Such a z ensures that $2\sum_{i=1}^n x_i^B < (b^+ - b^-)^\top p_0^A + \mathbf{1}^\top(M^+ + M^-)p^B = 2\sum_{i=1}^n x_{0i}^B$. Hence, P is essentially feasible. Suppose now that $\mu > \max\{\frac{2\beta}{1+\beta^2}\delta, \frac{\beta}{d_{\min}}\}$. As $\beta < 1$, we can use $\mu > \frac{2\beta\delta}{1+\beta^2}$ to verify that the inequality $\frac{(\delta+\mu)^k}{(1+\beta)^{k+1}} - \frac{(\delta-\mu)^k}{(1-\beta)^{k+1}} > 0$ holds for all $k \in \mathbb{N}$. On the other hand, we know from the definition of $d_{\min}$ that $G\mathbf{1} \ge d_{\min}\mathbf{1}$. Using induction, this can be generalized to $G^k\mathbf{1} \ge d_{\min}^k\mathbf{1}$ for all $k \ge 1$. Consequently, using Neumann series expansions of M^+ and M^- yields

$$\begin{aligned} &B_\Delta\mathbf{1} \\ &= \frac{2\beta}{1-\beta^2}\mathbf{1} - \sum_{k=1}^{\infty}\left(\frac{(\delta+\mu)^k}{(1+\beta)^{k+1}} - \frac{(\delta-\mu)^k}{(1-\beta)^{k+1}}\right)G^k\mathbf{1} \\ &\le \frac{2\beta}{1-\beta^2}\mathbf{1} - \sum_{k=1}^{\infty}\left(\frac{(\delta+\mu)^k}{(1+\beta)^{k+1}} - \frac{(\delta-\mu)^k}{(1-\beta)^{k+1}}\right)d_{\min}^k\mathbf{1} \\ &\overset{(a)}{=} \frac{2\beta}{1-\beta^2}\mathbf{1} - \frac{(\delta+\mu)d_{\min}}{(1+\beta)^2(1-(1+\beta)^{-1}(\delta+\mu)d_{\min})}\mathbf{1} \\ &\qquad + \frac{(\delta-\mu)d_{\min}}{(1-\beta)^2(1-(1-\beta)^{-1}(\delta-\mu)d_{\min})}\mathbf{1}, \end{aligned}$$

where (a) follows from Geometric series expansion. On simplification, the above inequality reduces to $B_\Delta\mathbf{1} \le -c_0(2\beta d_{\min}^2(\delta^2 - \mu^2) + (1-\beta^2)(d_{\min}\mu - \beta))\mathbf{1}$ for some $c_0 > 0$. It now follows from Assumption 2 and $\mu \ge \frac{\beta}{d_{\min}}$ that $B_\Delta\mathbf{1} \le 0$, i.e., $b^+ - b^- \ge 0$. Therefore, the value of the objective function is non-decreasing in every entry of z, and hence, (7) implies that the objective function is minimized at p_0^A. In other words, P is essentially infeasible.

2) We know from [19, Theorem 4] that $z = p^*_{\max}\mathbf{1}$ implies $2\sum_{i=1}^n x_i^B = -(b^- - b^+)^\top\mathbf{1}p^*_{\max} - (b^- + b^+)^\top p^B = 0$. Thus, if $p_{\max} > p^*_{\max}$, then every value of z satisfying $p^*_{\max}\mathbf{1} \le z \le p_{\max}\mathbf{1}$ is optimal because it drives the aggregate unsustainable equilibrium effort to 0. On the other hand, if $p_{\max} < p^*_{\max}$, then the optimal solution is attained at the upper bound on z specified by (8) because the objective function is decreasing in z by virtue of B_Δ being positive (as proved above).

■

Assertion 1 provides a set of necessary conditions and also a set of sufficient conditions for the essential feasibility of P. Taken together, these conditions mean the following: for us to be able to reduce the aggregate unsustainable activity $\sum_i x_i^B$ in the network without decreasing the agents' sustainable effort levels with respect to their pre-intervention values $\{x_{0i}^A\}_{i=1}^n$ and without decreasing the per-unit prices of sustainable goods w.r.t. their pre-intervention counterparts $\{p_{0i}^A\}_{i=1}^n$, the cross-activity network effects (quantified by μ) must not exceed the combined effect of the intra-activity network effects (quantified by δ) and the intra-concession substitutability (quantified by β and the function $\max\{\frac{2\beta}{1+\beta^2}\delta, \frac{\beta}{d_{\min}}\}$, which is increasing in β on $(-1,1)$) across the two activities.

To explain these conditions intuitively, we first note that increasing the per-unit price of sustainable goods for any agent increases the agent's incentive to produce sustainably, which further increases the incentive of the agent's neighbors to produce sustainably (via the network G and the intra-activity network effects parameter δ). This increase has two conflicting effects on the agents' incentives to engage in unsustainable production: (a) via β (intra-concession substitutability), it decreases the incentive to produce unsustainably (i.e., as the agents increase their participation in sustainable production, their cost of dividing effort and resources between the two activities compels them to decrease their participation in unsustainable production), and (b) via cross-activity agent-to-agent complementarities, it increases the incentive to produce unsustainably. To minimize the aggregate unsustainable activity, we need to ensure that the former effect (a) dominates over the latter effect (b), and this is made precise by the conditions in Assertion 1.

Note that the necessary and sufficient conditions provided by Assertion 1 are not tight. This is because for $\mu \in \left(\beta\delta, \max\{\frac{2\beta}{1+\beta^2}\delta, \frac{\beta}{d_{\min}}\}\right)$, the feasibility of P depends on the structure of the network (as captured by G) and the values of δ and β.

Next, observe that Assertion 2 requires all the post-intervention per-unit prices of sustainable goods to equal the maximum allowable price $p_{\max}$ when this price is below the threshold $p^*_{\max}$. This is because the assumption $\mu < \beta\delta$ guarantees that the post-intervention unsustainable effort levels $\{x_i^B\}_{i=1}^n$ are monotonically non-increasing in every per-unit price p_i^A, and in particular, $\{x_i^B\}_{i=1}^n$ are monotonically decreasing in $\{p_i^A\}_{i=1}^n$ when $p_{\max} < p^*_{\max}$. However, as $p_{\max}$ crosses the threshold $p^*_{\max}$, the aggregate unsustainable activity $\sum_{i=1}^n x_i^B$ vanishes in the post-intervention equilibrium, which means that increasing the per-unit prices beyond $p^*_{\max}$ does not yield any additional benefit over setting all of them to $p^*_{\max}$. Nevertheless, as there is no undesirable effect of increasing p_i^A beyond $p_{\max}$, there exists a range of optimal prices for the case $p_{\max} > p^*_{\max}$.

Remark 1: Observe that the aggregate unsustainable activity $\sum_{i=1}^n x_i^B$ is more sensitive to increases in p_i^A for higher values of $b_{\Delta i} = b_i^- - b_i^+$. This leads to an interpretation of b_Δ as a vector of node centralities: agents with higher values of $b_{\Delta i}$ are more central in the network in the sense that they are better-positioned than other agents for the purpose of transferring the unsustainability-suppressing effects of their individual price interventions to the rest of the network. Furthermore, the entry-wise positivity of B_Δ implies the entry-wise positivity of b_Δ, which means that the centralities of interest are all positive.

IV. Conclusion

We have designed a pricing policy by formulating and solving a budget-constrained optimization problem that seeks to maximally reduce the aggregate unsustainable activity in the post-intervention equilibrium. We have shown that this problem is essentially feasible (i.e., it has an optimal solution that achieves a positive reduction in the aggregate unsustainable activity) provided cross-activity network effects are weaker than intra-activity network effects as quantified by Assertion 1 of Proposition 1. Conversely, the problem becomes essentially infeasible if cross-activity network effects are significant relative to intra-activity network effects. Furthermore, we observe that when the maximum allowable price exceeds the threshold $p^*_{\max}$, the price adjustment budget is sufficiently large in the sense that it is possible to eliminate the aggregate unsustainable activity entirely.

In future, we will investigate the role of network structure in the context of our setup. Another important direction is to use the framework of supermodular games to analyze coupled-activity network games in which the agents' equilibrium effort levels are bounded by heterogeneous thresholds.

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