

Overstatement-Net-Equivalent Risk-Limiting Audit: ONEAudit

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Abstract. A procedure is a risk-limiting audit (RLA) with risk limit α if it has probability at least $1 - \alpha$ of correcting each wrong reported outcome and never alters correct outcomes. One efficient RLA method, card-level comparison (CLCA), compares human interpretation of individual ballot cards randomly selected from a trustworthy paper trail to the voting system’s interpretation of the same cards (cast vote records, CVRs). CLCAs heretofore required a CVR for each cast card and a “link” identifying which CVR is for which card—which many voting systems cannot provide. This paper shows that every set of CVRs that produces the same aggregate results overstates contest margins by the same amount: they are *overstatement-net-equivalent* (ONE). CLCA can therefore use CVRs from the voting system for any number of cards and ONE CVRs created *ad lib* for the rest. In particular:

- Ballot-polling RLA is algebraically equivalent to CLCA using ONE CVRs derived from the overall contest results.
- CLCA can be based on batch-level results (e.g., precinct subtotals) by constructing ONE CVRs for each batch. In contrast to batch-level comparison auditing (BLCA), this avoids manually tabulating entire batches and works even when reporting batches do not correspond to physically identifiable batches of cards, when BLCA is impractical.
- If the voting system can export linked CVRs for only some ballot cards, auditors can still use CLCA by constructing ONE CVRs for the rest of the cards from contest results or batch subtotals.

This works for every social choice function for which there is a known RLA method, including IRV. Sample sizes for BPA and CLCA using ONE CVRs based on contest totals are comparable. With ONE CVRs from batch subtotals, sample sizes are smaller than for BPA when batches are homogeneous, approaching those of CLCA using CVRs from the voting system, and much smaller than for BLCA: A CLCA of the 2022 presidential election in California at risk limit 5% using ONE CVRs for precinct-level results would sample approximately 70 ballots statewide, if the reported results are accurate, compared to about 26,700 for BLCA. The 2022 Georgia audit tabulated more than 231,000 cards (the expected BLCA sample size was $\approx 103,000$ cards); ONEAudit would have audited $\approx 1,300$ cards. For data from a pilot hybrid RLA in Kalamazoo, MI, in 2018, ONEAudit gives a risk of 2%, substantially lower than the 3.7% measured risk for SUITE, the “hybrid” method the pilot used.

Keywords: Risk-limiting audit, BPA, card-level comparison audit, batch-level comparison audit

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1 Introduction: Efficient Risk-Limiting Audits

A procedure is a risk-limiting audit (RLA) with risk limit α if it guarantees that if the reported outcome is right, the procedure will not change it; but if the reported outcome is wrong, the chance the procedure will not correct it—the “risk”—is at most α . “Outcome” means who or what won, not the precise vote tallies. RLA methods have been developed for many sampling designs [20,16,10,11,18,22], and to use the audit data in different ways to measure “risk” [20,16,21,17,9,8,18,22], to accommodate legal and logistical constraints and heterogeneous equipment within and across jurisdictions.

“Card” or “ballot card” means a sheet of paper; a ballot comprises one or more cards. A “cast-vote record” (CVR) is the voting system’s interpretation of the votes on a particular card. A “manual-vote record” (MVR) is the auditors’ reading of the votes on a card. The main approaches to RLAs are *ballot-polling RLAs* (BPA), which examine individual randomly selected cards but do not use data from the voting system other than the totals; *batch-level comparison RLAs* (BLCA), which compare reported vote subtotals for randomly selected batches of cards (e.g., all cards cast in a precinct) to manual tabulations of the same batches; *card-level comparison RLAs* (CLCA), which compare individual CVRs to the corresponding MVRs for a random sample of cards; and *hybrid* audits, which combine two or more of the approaches above.

BLCA is closest to historical statutory audits, but requires larger sample sizes than other methods when outcomes are correct. BPA requires almost no data from the voting system. It is generally more efficient than BLCA, but its sample size grows approximately quadratically as the margin shrinks. The most efficient approach is CLCA, for which the sample size grows approximately linearly as the margin shrinks. But it requires the most information from the voting system: an exported CVR for every card and a way to link exported CVRs to the corresponding physical cards, without compromising voter privacy.

This paper shows that applying CLCA to any combination of CVRs provided by the voting system and CVRs created by the auditors to match batch subtotals or contest totals gives a valid RLA. When the CVRs are derived entirely from contest totals, the method is algebraically equivalent to BPA. When the CVRs are derived from batch subtotals, the method is far more efficient than BLCA and approaches the efficiency of ‘pure’ CLCA when the batches are sufficiently homogeneous.

Many modern voting systems can provide linked CVRs for some ballot cards (e.g., vote-by-mail) but not others (e.g., in-precinct voters). This has led to a variety of strategies:

- give up the efficiency of CLCA and use BPA
- *hybrid* RLAs that use different audit strategies in different strata [10,18,22,13]

- BLCA using weighted random samples [17,7,10], with batches of size 1 for cards with linked CVRs
- CLCA that rescans some or all of the cards to create linked CVRs the voting system did not originally provide [12,5]
- CLCA using cryptographic nonces to link CVRs to cards without compromising voter privacy [19].

Section 4 develops a simpler approach that in examples is more efficient than a hybrid audit or BLCA, works even when a BLCA is impracticable, avoids the expense of re-scanning any ballots, and does not require new or additional equipment. When reporting batches are sufficiently homogeneous, the sample size for the method approaches that of CLCA.

2 Testing net overstatement does not require CVRs linked to ballot cards

2.1 Warmup: 2-candidate plurality contest

Consider a two-candidate plurality contest, Alice v. Bob, with Alice the reported winner. We encode votes and reported votes as follows. There are N cards. Let $b_i = 1$ if card i has a vote for Alice, -1 if it has a vote for Bob, and 0 otherwise. Let $c_i = 1$ if card i was counted by the voting system as a vote for Alice, $c_i = -1$ if it was counted as a vote for Bob, and $c_i = 0$ otherwise. The true margin is $\sum_{i=1}^N b_i$ and the reported margin is $\sum_{i=1}^N c_i$. The *overstatement* of the margin on the i th card is $c_i - b_i \in \{-2, -1, 0, 1, 2\}$. It is the number of votes by which the voting system exaggerated the number of votes for Alice. Alice really won if the *net overstatement of the margin*, $E(\{c_i\}) := \sum_i (c_i - b_i)$, is less than the reported margin $\sum_i c_i$. Because addition is commutative and associative, if $\{c_i\}$ and $\{c'_i\}$ are any two sets of CVRs for which $\sum_i c_i = \sum_i c'_i$, then $E(\{c_i\}) = E(\{c'_i\})$: they are *overstatement net equivalent* (ONE).

$$E(\{c_i\}) := \sum_i (c_i - b_i) = \sum_i c_i - \sum_i b_i = \sum_i c'_i - \sum_i b_i = \sum_i (c'_i - b_i) = E(\{c'_i\}). \quad (1)$$

Hence, if we have an RLA procedure to test whether $E(\{c_i\}) < \sum_i c_i$ using the “real” CVRs produced by the voting system, the same procedure can test whether the outcome is correct if it is applied to other CVRs $\{c'_i\}$ provided $\sum_i c_i = \sum_i c'_i$, *even if the CVRs $\{c'_i\}$ did not come from the voting system.* (Audit sample sizes might be quite different.)

Thus, we can conduct a CLCA using *any* set $\{c'_i\}$ of CVRs that reproduces the contest-level results: the net overstatement of every such set of CVRs is the same. If the system reports batch-level results, we can require that the CVRs reproduce the batch-level results as well., which might reduce audit sample sizes, especially when the batches have different political preferences. If the voting system reports CVRs for some individual ballot cards, we can conduct a CLCA that uses those CVRs, augmented by ONE CVRs for the remaining ballot cards

(derived from batch subtotals or from contest totals, by subtraction). Better agreement between the MVRs and VCRs will generally allow the audit to stop after inspecting fewer cards.

2.2 Numerical example

Consider a contest in which 20,000 cards were cast in all, of which 10,000 were cast by mail and have linked CVRs, with 5,000 votes for Alice, 4,000 for Bob, and 1,000 undervotes. The other 10,000 cards were cast in 10 precincts, 1,000 cards in each. Net across those 10 precincts, Alice and Bob each got 5,000 votes. In 5 precincts, Alice showed more votes than Bob; in the other 5, Bob showed more than Alice. The reported results are thus 10,000 votes for Alice, 9,000 for Bob, and 1,000 undervotes. The margin is 1,000 votes; the *diluted margin* (margin in votes, divided by cards cast) is $1000/20000 = 5\%$. Consider two sets of precinct subtotals:

- 5 precincts show 900 votes for Alice and 100 for Bob; the other 5 show 900 votes for Bob and 100 for Alice.
- 5 precincts show 990 votes for Alice and 10 for Bob; the other 5 show 990 votes for Bob and 10 for Alice.

Construct ONE CVRs for the 10,000 cards cast in the 10 precincts as follows: if the precinct reports a votes for Alice and $1000 - a$ for Bob, the net vote for Alice is $a - (1000 - a) = 2a - 1000$. The “average” CVR for the precinct has $(2a - 1000)/1000 = 2a/1000 - 1$ votes for Alice; that is the ONEAudit CVR for every card in the precinct. For instance, a precinct that reported 900 votes for Alice and 100 for Bob has a net margin of $900 \times 1 + 100 \times -1 = 800$ for Alice, so that precinct contributes 1,000 ONE CVRs, each with $c_i = (0.9) \times 1 + (0.1) \times (-1) = 0.8$ votes for Alice.

Table 1. Overstatement-net equivalent CVRs that match batch subtotals. If a precinct of 1000 voters reported 990 votes for Alice and 10 for Bob, the net overstatement is the same as if there had been 1,000 CVRs, one for each card, each showing 0.98 votes for Alice.

batch total	ONE CVR
990 Alice, 10 Bob	0.98 Alice
900 Alice, 100 Bob	0.8 Alice
100 Alice, 900 Bob	-0.8 Alice (0.8 Bob)
10 Alice, 990 Bob	-0.98 Alice (0.98 Bob)

To audit, draw ballot cards uniformly at random, without replacement. To find the overstatement for each audited card, subtract the MVR for the card (-1, 0, or 1) from the CVR (-1, 0, or 1) if the system provided one, or from the ONE CVR for its precinct (a number in $[-1, 1]$) if the system did not provide a CVR. Apply a “risk-measuring function” (see, e.g., [18,22]) to the overstatements to measure the risk that the outcome is wrong based on the data collected so far;

the audit can stop without a full hand count if and when the measured risk is less than or equal to the risk limit.

The random selection can be conducted in many ways, for instance, conceptually numbering the cards from 1 to 20,000, where cards 1–10,000 are the ballots with CVRs, ordered in some canonical way; cards 10,001–11,000 are the cards cast in precinct 1, starting with the top card in the stack; cards 11,001–12,000 are the cards cast in precinct 2, starting with the top card in the stack; etc. Auditors draw random numbers between 1 and 20,000, and retrieve the corresponding card. Alternatively, if the resulting number is between 1 and 10,000, retrieve the corresponding card; but if the number is larger, draw a ballot at random from the precinct that numbered card belongs to, for instance, using the k -cut method [14]. That approach avoids counting into large stacks of ballots.

Table 2 summarizes expected audit sample sizes. If there had been a CVR for every card and the results were exactly correct, the sample size for a standard CLCA with risk limit 5% would be about 125 cards. A BPA at risk limit 5% would examine about 2,300 cards. A BLCA (treating individual cards as batches for those with CVRs) using sampling with probability proportional to an error bound and the Kaplan-Markov test [21] would examine about 7250 cards on average in the 900/100 scenario and 5300 in the 990/10 scenario. For ONEAudit, the expected sample size is about 800 cards in the 900/100 scenario and 170 in the 990/10 scenario. As preferences within precincts become more homogeneous, ONEAudit approaches the efficiency of CLCA.

2.3 The general case

We use the SHANGRLA audit framework [18] because it can be works with every social choice function for which an RLA method is known; however, ONE CVRs can be used with every extant RLA method for comparison audits. SHANGRLA reduces auditing election outcomes to multiple instances of a single problem: testing whether the mean of a finite list of bounded numbers is less than or equal to $1/2$. Each list results from applying a function A called an *assorter* to the votes on the ballots; each assorter maps votes to the interval $[0, u]$, where the upper bound u depends on the particular assorter. Different social choice functions involve different assorters and, in general, different numbers of assorters. An *assertion* is the claim that the average of the values the assorter takes on the true votes is greater than $1/2$. The contest outcome is correct if the all the SHANGRLA assertions for the contest are true.

A *reported assorter margin* is the amount by which that assorter applied to the reported votes exceeds $1/2$. For scoring rules (plurality, supermajority, multi-winner plurality, approval voting, Borda count, etc.), reported assorter margins can be computed from contest-level tallies, batch tallies, or CVRs. For auditing IRV using RAIRE [4], CVRs are generally required to construct an appropriate set of assorters and to find their margins—but the CVRs do not have to be linked to individual ballot cards.

Let b_i denote the true votes on the i th ballot card; there are N cards in all. Let c_i denote the voting system’s interpretation of the i th card. Suppose we

		expected		
scenario	method	cards	vs BPA	vs CLCA
both	BPA	2,300	1.00	18.40
	CLCA	125	0.05	1.00
900/100	BLCA	7,250	3.15	58.00
	ONE CLCA	800	0.35	6.40
990/10	BLCA	5,300	2.30	42.40
	ONE CLCA	170	0.07	1.36

Table 2. Expected RLA workloads for a two-candidate plurality contest with a ‘diluted margin’ of 5% at risk limit 5%. In all, 20,000 cards were cast, of which 10,000 have linked CVRs; the other 10,000 cards are divided into 10 precincts with 1,000 cards each. Among cards with CVRs, 5,000 show votes for the winner, 4,000 show votes for the loser, and 1,000 have invalid votes. In the 900/100 scenario, 5 precincts have 900 votes for the winner and 100 for the loser; the other 5 have 900 for the loser and 100 for the winner. In the 990/10 scenario, 5 precincts have 990 votes for the winner and 10 for the loser; the other 5 have 990 for the loser and 10 for the winner. BPA: ballot-polling audit. CLCA: card-level comparison audit (which would require re-scanning the 10,000 cards cast in precincts). BLCA: batch-level comparison audit. ONE CLCA: card-level comparison audit using the original 10,000 CVRs, augmented by 10,000 ONE CVRs derived from precinct subtotals. Column 4: sample size divided by BPA sample size. Column 5: sample size divided by CLCA sample size. Expected workload for BPA and CLCA is the same in both scenarios. Sample sizes for CLCA use the “super-simple” method [17], computed using <https://www.stat.berkeley.edu/~stark/Vote/auditTools.htm> (last visited 19 March 2023). Sample sizes for BPA use ALPHA [22].

have a CVR c_i for every ballot card whose index i is in \mathcal{C} . The cardinality of \mathcal{C} is $|\mathcal{C}|$. Ballot cards not in \mathcal{C} are partitioned into $G \geq 1$ disjoint groups $\{\mathcal{G}_g\}_{g=1}^G$ for which reported assorter subtotals are available. For instance \mathcal{G}_g might comprise all ballots for which no CVR is available or all ballots cast in a particular precinct. Unadorned overbars denote the average of a quantity across all N ballot cards; overbars subscripted by a set (e.g., \mathcal{G}_g) denote the average of a quantity across cards in that set, for instance:

$$\begin{aligned}\bar{A}^c &:= \frac{1}{N} \sum_{i=1}^N A(c_i), & \bar{A}_{\mathcal{C}}^c &:= \frac{1}{|\mathcal{C}|} \sum_{i \in \mathcal{C}} A(c_i), & \bar{A}_{\mathcal{G}_g}^c &:= \frac{1}{|\mathcal{G}_g|} \sum_{i \in \mathcal{G}_g} A(c_i) \\ \bar{A}^b &:= \frac{1}{N} \sum_{i=1}^N A(b_i), & \bar{A}_{\mathcal{C}}^b &:= \frac{1}{|\mathcal{C}|} \sum_{i \in \mathcal{C}} A(b_i), & \bar{A}_{\mathcal{G}_g}^b &:= \frac{1}{|\mathcal{G}_g|} \sum_{i \in \mathcal{G}_g} A(b_i).\end{aligned}$$

The *assertion* is the claim $\bar{A}^b > 1/2$. The reported assorter mean $\bar{A}^c > 1/2$: otherwise, according to the voting system’s data, the reported outcome is wrong. Now $\bar{A}^b > 1/2$ iff

$$\bar{A}^c - \bar{A}^b < \bar{A}^c - 1/2. \quad (2)$$

The right hand side is known before the audit starts; it is half the “reported assorter margin” $v := 2\bar{A}^c - 1$ [18]. We assume we have a *reported assorter total*

$\sum_{i \in \mathcal{G}_g} A(c_i)$ from the voting system for the cards in the group \mathcal{G}_g (e.g., reported precinct subtotals) and define the *reported assorter mean* for \mathcal{G}_g :

$$\hat{A}_{\mathcal{G}_g}^c := \frac{1}{|\mathcal{G}_g|} \sum_{i \in \mathcal{G}_g} A(c_i). \quad (3)$$

We have

$$\bar{A}^c = \frac{\sum_{g=1}^G |\mathcal{G}_g| \hat{A}_{\mathcal{G}_g}^c + \sum_{i \in \mathcal{C}} A(c_i)}{N} = \frac{\sum_{g=1}^G \sum_{i \in \mathcal{G}_g} \hat{A}_{\mathcal{G}_g}^c + \sum_{i \in \mathcal{C}} A(c_i)}{N}. \quad (4)$$

Thus if we *declare* $A(c_i) := \hat{A}_{\mathcal{G}_g}^c$ for $i \in \mathcal{G}_g$, the reported assorter mean for the cards in group \mathcal{G}_g , the mean of the assorter applied to the CVRs—including these faux CVRs—will equal its reported value: using a “mean CVR” for the batch is overstatement-net-equivalent to any CVRs that give the same assorter batch subtotals. Condition 2 then can be written

$$\frac{1}{N} \sum_i (A(c_i) - A(b_i)) < v/2. \quad (5)$$

Following SHANGRLA [18, section 3.2], define

$$B(b_i) := \frac{u + A(b_i) - A(c_i)}{2u - v} \in [0, 2u/(2u - v)]. \quad (6)$$

Then $\bar{A}^b > 1/2 \iff \bar{B}^b > 1/2$, which can be shown as follows, using the fact that $v := 2\bar{A}^c - 1 \leq 2u - 1 < 2u$:

$$\begin{aligned} \bar{B}^b &:= \frac{1}{N} \sum_i \frac{u + A(b_i) - A(c_i)}{2u - v} \\ &= \frac{u + \bar{A}^b - \bar{A}^c}{2u - v} \\ &= \frac{u + \bar{A}^b - \bar{A}^c}{2u - 2\bar{A}^c + 1}. \end{aligned} \quad (7)$$

Thus if $\bar{B}^b > 1/2$,

$$\begin{aligned} \frac{u + \bar{A}^b - \bar{A}^c}{2u - 2\bar{A}^c + 1} &> 1/2 \\ u + \bar{A}^b - \bar{A}^c &> u - \bar{A}^c + 1/2 \\ \bar{A}^b &> 1/2. \end{aligned} \quad (8)$$

If the reported tallies are correct, i.e., if $\bar{A}^c = \bar{A}^b = (v + 1)/2$, then

$$\bar{B}^b = \frac{u}{2u - v}. \quad (9)$$

3 Auditing using batch subtotals

The oldest approach to RLAs is batch-level comparison, which involves exporting batch subtotals from the voting system (e.g., for precincts or tabulators), verifying that those batch subtotals yield the reported contest results, drawing some number of batches at random (with equal probability or with probability proportional to an error bound), manually tabulating all the votes in each selected batch, comparing the manual tabulation to the reported batch subtotals, assessing whether the data give sufficiently strong evidence that the reported results are right, and expanding the sample if not [20,16,15,21,6].

BLCAs have two logistical hurdles: (i) They require manually tabulating the votes on *every* ballot card in the batches selected for audit. (ii) When the batches of cards for which the voting system reports subtotals do not correspond to identifiable physical batches (common for vote-by-mail and vote centers), the audit has to find and retrieve every card in the audited reporting batches. Those cards may be spread across any number of physical batches, which can also make recounts prohibitively expensive [1].

Both can be avoided using CLCA with ONE CVRs. The following algorithm gives a valid RLA, but selects and compares the manual interpretation of *individual* cards to the implied “average” CVR of the reporting batch each card belongs to. We assume that the canvass and a *compliance audit* [2] have determined that the ballot manifest and physical cards are complete and trustworthy.

Algorithm for a CLCA using ONE CVRs from batch subtotals.

1. Pick the risk limit for each contest under audit.
2. Export batch subtotals from the voting system.
3. Verify that every physical card is accounted for,^a that the physical accounting is consistent with the reported votes, and that the reported batch subtotals produce the reported winners.
4. Construct SHANGRLA assorters for every contest under audit; select a risk-measuring function for each assertion (e.g., one in [22]); set the measured risk for each assertion to 1.
5. Calculate the reported mean assorter values for each reporting batch; these are the ONE CVRs.
6. While any measured risk is greater than its risk limit and not every card has been audited:
 - Select a card at random, e.g., by selecting a batch at random with probability proportional to the size of the batch, then selecting a card uniformly at random from the batch using the *k*-cut method [14], or by selecting at random from the entire collection of cards.
 - Calculate the overstatement for the selected card using the ONE CVR for the reporting batch the card belongs to.
 - Update the measured risk of any assertion whose measured risk is still greater than its risk limit.

- If the measured risk for every assertion is less than or equal to its risk limit, stop and confirm the reported outcomes.
- 7. Report the correct contest outcomes: every card has been manually interpreted.

^a For techniques to deal with missing cards, see [3,18].

This algorithm be made more efficient statistically and logistically in a variety of ways, for instance, by making an affine translation of the data so that the minimum possible value is 0 (by subtracting the minimum of the possible overstatement asserters across batches and re-scaling so that the null mean is still 1/2) and by starting with a sample size that is expected to be large enough to confirm the contest outcome if the reported results are correct.

3.1 Numerical case studies

Table 3 compares expected sample sizes for BLCA to CLCA using ONE CVRs derived from the same batch subtotals, and to BPA, for two contests: the 2022 midterm Georgia Secretary of State’s contest, which had a diluted margin (margin in votes divided by cards cast) of about 9.2%¹ and the 2020 presidential election in California, which had a diluted margin of about 28.7%. The Georgia contest was audited using batch-level comparisons. The Georgia SoS claims that audit was a BLCA with a risk limit of 5%, but in fact the audit was not an RLA, for a variety of reasons.² BPA and CLCA using ONE CVRs are generally much more efficient than BLCA when batches are large. CLCA with ONE CVRs is more efficient than BPA when batches are more homogenous than the contest votes as a whole, i.e., when precincts are polarized in different directions.

4 Auditing heterogenous voting systems

When the voting system can report linked CVRs for some but not all cards, we can augment the voting system’s linked CVRs with ONE CVRs for the remaining cards, then use CLCA. The ONE CVRs can be derived from overall contest results or from reported subtotals, e.g., precinct subtotals. Finer-grained subtotals generally give smaller audit sample sizes (when the reported outcome is correct) if the smaller groups are more homogeneous than the overall election.

SUITE [10], a hybrid RLA designed for this situation, was first fielded in a pilot audit of the gubernatorial primary in Kalamazoo, MI, in 2018. The stratum with linked CVRs comprised 5,294 ballots with 5,218 reported votes in the contest; the “no-CVR” stratum comprised 22,372 ballots with 22,082 reported votes. The sample included 32 cards were drawn from the no-CVR stratum and

¹ <https://sos.ga.gov/news/georgias-2022-statewide-risk-limiting-audit-confirms-results>, last visited 26 February 2023.

² <https://www.stat.berkeley.edu/~stark/Preprints/cgg-rept-10.pdf>, last visited 15 December 2022.

Contest	total turnout	total batches	actual sample size	K-M BLCA	ONE CLCA	Wald BP	BLCA/ CLCA
2020 CA U.S. Pres	17,785,667	21,346	$\approx 178,000$	26,700	70	72	381
2022 GA SoS	3,909,983	12,968	$> 231,000$	103,300	1,380	700	75

Table 3. Actual and estimated expected sample sizes for various RLA methods for the 2020 U.S. presidential election in California and the 2022 Georgia Secretary of State contest, at risk limit 5%. Columns 2, 3: turnout per state records. CA data from <https://statewidedatabase.org/d10/g20.html> (last visited 2 March 2020); the CA Statement of Vote gives slightly smaller turnout 17,785,151 (<https://elections.cdn.sos.ca.gov/sov/2020-general/sov/complete-sov.pdf>, last visited 2 March 2023). GA data from (<https://sos.ga.gov/news/georgias-2022-statewide-risk-limiting-audit-confirms-results>, last visited 26 February 2023). Column 4: approximate number of cards examined in the actual batch-level audits (which were not RLAs). Column 5: expected sample size for BLCA using the Kaplan-Markov risk function. Column 6: expected sample size for CLCA using ONE CVRs based on batch subtotals, for the ALPHA risk-measuring function with the truncated shrinkage estimator with parameters $c = 1/2$, $d = 10$, estimated from 100 Monte Carlo replications. Column 7: expected sample size for BPA using Wald’s SPRT. Column 8: column 5 divided by column 6. A different risk function should reduce the GA ONE CLCA sample size to at most the BPA sample size: see section 5.1. Estimates assume that reported batch subtotals are correct.

8 from the CVR stratum.³ Table 4 summarizes the contest and audit results; auditors found no errors in the 8 CVRs, each of which yields the overstatement assorter value $u/(2u-v)$. The new method compares the 32 cards without CVRs to ONE CVRs derived from all the votes without CVRs (not from subtotals for smaller batches) by subtracting the votes on the linked CVRs from the reported contest totals. Ignoring the fact that the sample was stratified and the difference in sampling fractions in the two strata, in 100,000 random permutations of the data, the ALPHA martingale test using a fixed alternative $0.99(2u)/(2u-v)$ had a mean P -value of 0.0201 (90th percentile 0.0321), about 54% of the SUITE P -value of 0.0374 [10]. The ONEAudit P -value is larger than the P -value of the best product supermartingale test in [13], but comparable to or smaller than the P -values for the other supermartingale tests. See table 5. If precinct subtotals were available to construct the ONE CVRs, the measured risk might have been lower, depending on precinct heterogeneity.

5 Sample sizes for contest-level ONE CLCA vs. BPA

5.1 Theory

Moving from tests about raw assorter values to tests about overstatements relative to ONE CVRs derived from overall contest totals is just an affine transformation: no information is gained or lost. Thus, if we audited using an affine

³ See https://github.com/kellieotto/mirla18/blob/master/code/kalamazoo_SUITE.ipynb.

Candidate	CVR	no-CVR	polling sample
Butkovich	6	66	0
Gelineau	56	462	1
Kurland	23	284	0
Schleiger	19	116	0
Schuette	1,349	4,220	8
Whitmer	3,765	16,934	23
Non-vote	76	290	0
Total	5,294	22,372	32

Table 4. Reported votes in the stratum with CVRs and the stratum without CVRs, and the audited votes in the random sample of 32 cards from the stratum without CVRs in the 2018 RLA pilot in Kalamazoo, MI.

Method	P -value	SD	90th percentile
SUITE	0.037	n/a	n/a
ALPHA P_F^*	0.018	0.002	0.019
ALPHA P_M^*	0.003	0.000	0.003
Empirical Bernstein P_F^*	0.348	0.042	0.390
Empirical Bernstein P_M^*	0.420	0.134	0.561
ALPHA ONEAudit	0.020	0.010	0.032

Table 5. P -values for the 2018 RLA pilot in Kalamazoo, MI, for different risk-measuring functions. SUITE is a hybrid stratified approach [10]. Rows 2–5 are from [13, Table 3]; the ALPHA and Empirical Bernstein stratumwise supermartingales combined using either Fisher’s combining function (P_F^*) or multiplication (P_M^*). The 6th row is for ONEAudit, using the ALPHA supermartingale with fixed alternative $\eta = 0.99$, a non-adaptive choice. Technically, ONEAudit should not be applied to this sample because the sample was stratified, while the risk calculation assumes the sample was a simple random sample of ballot cards: this is just a numerical illustration.

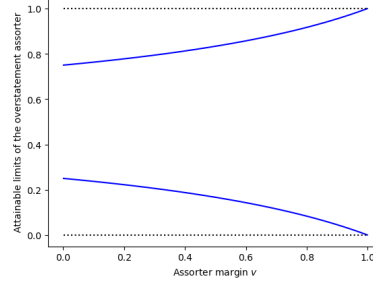
equivariant statistical test, the sample size should be the same whether the data are the original assorter values (i.e., BPA) or overstatements from ONE CVRs.

However, the statistical tests used in RLAs are not affine equivariant because they rely on *a priori* bounds on the assorter values. The original assorter values will generally be closer to the endpoints of $[0, u]$ than the transformed values are to the endpoints of $[0, 2u/(2u - v)]$. To see why, suppose that there are no reported CVRs ($\mathcal{C} = \emptyset$) and that only contest totals are reported from the system—so every cast ballot card is in \mathcal{G}_1 . For a BPA, the population values from which the sample is drawn are the original assorter values $\{A(b_i)\}$, which for many social choice functions can take only the values 0, $1/2$, and u . For instance, consider a two-candidate plurality contest, Alice v. Bob, where Alice is the reported winner. This can be audited using a single assorter that assigns the value 0 to a card with a vote for Bob, the value $u = 1$ to a card with a vote for Alice, and the value $1/2$ to other cards. In contrast, for a comparison audit, the possible population values $\{B(b_i)\}$ are

$$\left\{ \frac{1 - (v + 1)/2}{2 - v}, 1/2, \frac{2 - (v + 1)/2}{2 - v} \right\}.$$

Unless $v = 1$ —i.e., unless every card was reported to have a vote for Alice—the minimum value of the overstatement assorter is greater than 0 and the maximum is less than u . Figure 1 plots the minimum and maximum value of the overstatement assorter as a function of v for $u = 1$.

Fig. 1. Upper and lower bounds on the overstatement assorter as a function of the diluted margin v , for $u = 1$.



A test that uses the prior information $x_j \in [0, u]$ may not be as efficient for populations for which $x_j \in [a, b]$ with $a > 0$ and $b < u$ as it is for populations where the values 0 and u actually occur. An affine transformation of the overstatement assorter values can move them back to the endpoints of the support constraint by subtracting the minimum possible value then re-scaling so that the null mean is $1/2$ once again, which reproduces the original assorter, A :

$$\begin{aligned}
 C(b_i) &:= \frac{1/2}{1/2 - \frac{u - (v+1)/2}{2u-v}} \cdot \left(B(b_i) - \frac{u - (v+1)/2}{2u-v} \right) \\
 &= \frac{2u-v}{2u-v - (2u - (v+1))} \cdot \left(\frac{u + A(b_i) - (v+1)/2}{2u-v} - \frac{u - (v+1)/2}{2u-v} \right) \\
 &= (2u-v) \cdot \frac{A(b_i)}{2u-v} = A(b_i). \tag{10}
 \end{aligned}$$

5.2 Numerical comparison

While CLCA with ONE CVRs is algebraically equivalent to BPA, the performance of a given statistical test will be different for the two formulations. We now compare expected audit sample sizes for some common risk-measuring functions applied to CLCA with ONE CVRs derived from contest-level results and applied to the original assorter data. This is assessing the particular *statistical tests*, not any intrinsic difference between BPA and ONE CLCA.

Tables 7–11 in the appendix report expected sample sizes when the reported winner received a share $\theta \in \{0.505, 0.51, 0.52, 0.55, 0.6\}$ of the reported votes, for percentages of cards that do not contain a valid vote for either candidate ranging from 10% to 75%, and various values of tuning parameters in the risk-measuring functions. Table 6 gives the geometric mean of the ratios of the mean sample size for each condition to the smallest mean sample size for that condition (across

risk-measuring functions). Transforming the assorter into an overstatement assorter using the ONEAudit transformation, then testing whether the mean of the resulting population is $\leq 1/2$ using the ALPHA test martingale with the truncated shrinkage estimator of [22] with $d = 10$ and η between 0.505 and 0.55 performed comparably to—but slightly worse than—using ALPHA on the raw assorter values for the same d and η , and within 4.8% of the overall performance of the best-performing method.

Method	Parameters	Score	Method	Score
ALPHA	$\eta = 0.505 \ d = 10$	1.51	ONEAudit	1.52
	$\eta = 0.505 \ d = 100$	1.54		1.55
	$\eta = 0.505 \ d = 1000$	1.79		1.83
	$\eta = 0.505 \ d = \infty$	3.02		3.83
	$\eta = 0.51 \ d = 10$	1.51		1.52
	$\eta = 0.51 \ d = 100$	1.53		1.55
	$\eta = 0.51 \ d = 1000$	1.72		1.80
	$\eta = 0.51 \ d = \infty$	2.29		3.05
	$\eta = 0.52 \ d = 10$	1.51		1.53
	$\eta = 0.52 \ d = 100$	1.51		1.55
	$\eta = 0.52 \ d = 1000$	1.61		1.75
	$\eta = 0.52 \ d = \infty$	1.84		2.32
	$\eta = 0.55 \ d = 10$	1.51		1.55
	$\eta = 0.55 \ d = 100$	1.47		1.56
	$\eta = 0.55 \ d = 1000$	1.44		1.62
	$\eta = 0.55 \ d = \infty$	1.88		1.81
SqKelly	$\eta = 0.6 \ d = 10$	1.50		1.59
	$\eta = 0.6 \ d = 100$	1.45		1.57
	$\eta = 0.6 \ d = 1000$	1.51		1.52
	$\eta = 0.6 \ d = \infty$	2.42		1.84
		1.98		
a priori Kelly	$\eta = 0.505$	2.77		
	$\eta = 0.51$	1.88		
	$\eta = 0.52$	1.60		
	$\eta = 0.55$	2.14		
	$\eta = 0.6$	3.34		

Table 6. Geometric mean of the ratios of sample sizes to the smallest sample size for each condition described above. The smallest ratio is in bold font. In the simulations, for the hypothesis tests considered, the ONEAudit transformation entails a negligible loss in efficiency compared to ALPHA applied to the “raw” assorter.

6 Conclusions

Ballot-polling risk-limiting audits (BPAs) are algebraically equivalent to card-level comparison risk-limiting audits (CLCAs) using faux cast-vote records (CVRs) that match the overall reported results. Any set of CVRs that reproduces the reported contest tallies (more generally, reproduces reported assorter totals) has the same net overstatement of the margin: they are “overstatement-net-equivalent” (ONE) to the voting system’s tabulation and can be used as if the voting system had exported them.

ONE CVRs let audits use batch-level data far more efficiently than traditional batch-level comparison RLAs (BLCAs) do: create ONE CVRs for each batch, then apply CLCA as if the voting system had provided those CVRs. For real and simulated data, this saves a large amount of work compared to manually tabulating the votes on *every* card in the batches selected for audit, as BLCAs require. If batches are sufficiently homogeneous, the workload approaches that of “pure” CLCA using linked CVRs from the voting system. BLCAs also require locating and retrieving every card in each batch that is selected for audit. That is straightforward when reporting batches are identifiable physical batches, but not when physical batches contain a mixture of ballot cards from different reporting batches, which is common in jurisdictions that use vote centers or do not sort

vote-by-mail ballots before scanning them. In the California presidential election in 2020 and the Georgia election for Secretary of State in 2022, this approach would reduce the workload by a factor of 75 to 380, respectively, compared to the most efficient known method for BLCA.

ONE CVRs also can obviate the need to stratify, to rescan cards, or to use “hybrid” audits when the voting system cannot export a linked CVR for every card: create ONE CVRs for the cards that lack them, then apply CLCA as if the CVRs had been provided by the voting system. The same CLCA software can be used to audit voting systems that can export a linked CVR for every card and also those that cannot. For data from a 2018 pilot audit in Kalamazoo, MI, ONE CLCA gives a measured risk much smaller than that of SUITE (2% versus 3.7%), the hybrid method used in the pilot. Stratification and hybrid audits increase the complexity and opacity of audits, and rescanning substantially increases time and cost. Hence, ONEAudit may be cheaper, faster, simpler, and more transparent than previous methods. Software used to generate the tables and figures is available at <https://www.github.com/pbstark/ONEAudit>.

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References

1. Appel, A.: Sort the mail-in ballot envelopes, or don't? <https://freedom-to-tinker.com/2023/05/01/sort-the-mail-in-ballot-envelopes-or-dont/> (2023), last visited 16 June 2023
2. Appel, A., Stark, P.: Evidence-based elections: Create a meaningful paper trail, then audit. *Georgetown Law Technology Review* **4.2**, 523–541 (2020), <https://georgetownlawtechreview.org/wp-content/uploads/2020/07/4.2-p523-541-Appel-Stark.pdf>
3. Bañuelos, J., Stark, P.: Limiting risk by turning manifest phantoms into evil zombies. Tech. rep., arXiv.org (2012), <http://arxiv.org/abs/1207.3413>, retrieved 17 July 2012
4. Blom, M., Stuckey, P.J., Teague, V.J.: Ballot-polling risk limiting audits for irv elections. In: Krimmer, R., Volkamer, M., Cortier, V., Goré, R., Hapsara, M., Serdült, U., Duenas-Cid, D. (eds.) *Electronic Voting*. pp. 17–34. Springer, Cham (2018)
5. Harrison, A., Fuller, B., Russell, A.: Lazy risk-limiting ballot comparison audits. <https://arxiv.org/abs/2202.02607> (2022). <https://doi.org/10.48550/arXiv.2202.02607>
6. Higgins, M., Rivest, R., Stark, P.: Sharper p-values for stratified post-election audits. *Statistics, Politics, and Policy* **2**(1) (2011). <https://doi.org/10.2202/2151-7509.1031>
7. Lindeman, M., McBurnett, N., Ottoboni, K., Stark, P.: Next steps for the Colorado risk-limiting audit (CORLA) program. <https://arxiv.org/abs/1803.00698> (2018)
8. Lindeman, M., Stark, P.: A gentle introduction to risk-limiting audits. *IEEE Security and Privacy* **10**, 42–49 (2012)

9. Lindeman, M., Stark, P., Yates, V.: BRAVO: Ballot-polling risk-limiting audits to verify outcomes. In: Proceedings of the 2011 Electronic Voting Technology Workshop / Workshop on Trustworthy Elections (EVT/WOTE '11). USENIX (2012)
10. Ottoboni, K., Stark, P., Lindeman, M., McBurnett, N.: Risk-limiting audits by stratified union-intersection tests of elections (SUITE). In: Electronic Voting. E-Vote-ID 2018. Lecture Notes in Computer Science. Springer (2018), https://link.springer.com/chapter/10.1007/978-3-030-00419-4_12, preprint: <https://arxiv.org/abs/1809.04235>
11. Ottoboni, K., Bernhard, M., Halderman, J.A., Rivest, R.L., Stark, P.B.: Bernoulli ballot polling: A manifest improvement for risk-limiting audits. In: Bracciali, A., Clark, J., Pintore, F., Rønne, P., Sala, M. (eds.) Financial Cryptography and Data Security. FC 2019. Lecture Notes in Computer Science, vol 11599. Springer, Cham. (2020). https://doi.org/10.1007/978-3-030-43725-1_16
12. Rhode Island RLA Working Group: Pilot implementation study of risk-limiting audit methods in the state of Rhode Island. <https://www.brennancenter.org/sites/default/files/2019-09/Report-RI-Design-FINAL-WEB4.pdf> (2019)
13. Spertus, J., Stark, P.B.: Sweeter than SUITE: Supermartingale stratified union-intersection tests of elections. In: Electronic Voting, 7th International Joint Conference, E-Vote-ID 2022. Springer Nature, Cham (2022)
14. Sridhar, M., Rivest, R.: k-cut: A simple approximately-uniform method for sampling ballots in post-election audits. In: Bracciali, A., Clark, J., Pintore, F., Rønne, P., Sala, M. (eds.) Financial Cryptography and Data Security. FC 2019. Lecture Notes in Computer Science, vol 11599. Springer, Cham. (2020). https://doi.org/10.1007/978-3-030-43725-1_17
15. Stark, P.: Election audits by sampling with probability proportional to an error bound: dealing with discrepancies. statistics.berkeley.edu/~stark/Preprints/ppebwrwd08.pdf (2008)
16. Stark, P.: A sharper discrepancy measure for post-election audits. *Ann. Appl. Stat.* **2**, 982–985 (2008), <http://arxiv.org/abs/0811.1697>
17. Stark, P.: Super-simple simultaneous single-ballot risk-limiting audits. In: Proceedings of the 2010 Electronic Voting Technology Workshop / Workshop on Trustworthy Elections (EVT/WOTE '10). USENIX (2010), http://www.usenix.org/events/evtwote10/tech/full_papers/Stark.pdf
18. Stark, P.: Sets of half-average nulls generate risk-limiting audits: SHANGRLA. In: Bernhard, M., Bracciali, A., Camp, L., Matsuo, S., Maurushat, A., Rønne, P., Sala, M. (eds.) Financial Cryptography and Data Security, Lecture Notes in Computer Science, 12063. Springer Nature, Cham (2020)
19. Stark, P.: Non(c)esuch ballot-level risk-limiting audits for precinct-count voting systems. In: Katsikas, S.e.a. (ed.) Computer Security. ESORICS 2022 International Workshops. Lecture Notes in Computer Science, 13785. pp. 541–554. Springer, Cham (2023). https://doi.org/10.1007/978-3-031-25460-4_31
20. Stark, P.B.: Conservative statistical post-election audits. *Annals of Applied Statistics* **2**, 550–581 (2008), <http://arxiv.org/abs/0807.4005>
21. Stark, P.B.: Risk-limiting post-election audits: P -values from common probability inequalities. *IEEE Transactions on Information Forensics and Security* **4**, 1005–1014 (2009)
22. Stark, P.B.: ALPHA: Audit that learns from previously hand-audited ballots. *Annals of Applied Statistics* **17**, 641–679 (2023). <https://doi.org/10.1214/22-AOAS1646>

A Detailed simulation results for BPA versus CLCA using ONE CVRs based on contest totals

θ	Method	params	N =10,000, %blank				N =100,000 %blank				N =500,000 %blank			
			10	25	50	75	10	25	50	75	10	25	50	75
0.505	sqKelly		8,852	9,050	9,024	9,247	89,777	90,146	89,742	89,914	456,305	449,438	442,418	449,083
	apKelly	$\eta = 0.505$	9,590	9,804	9,928	9,977	38,995	43,712	54,354	75,746	56,043	66,229	91,978	153,452
	ALPHA	$\eta = 0.505$	8,237	8,635	9,013	9,771	46,485	49,763	59,800	82,477	84,352	95,843	137,445	257,907
		$d = 100$	8,136	8,610	9,033	9,777	43,951	48,756	59,587	82,534	78,259	91,725	134,181	256,568
		$d = 1000$	8,108	8,684	9,078	9,785	42,094	46,855	59,413	82,679	72,004	87,454	131,923	256,439
		$d = \infty$	8,180	8,712	9,194	9,802	37,106	43,424	58,777	83,714	56,227	71,123	118,816	257,356
	ONEAudit	$\eta = 0.505$	8,232	8,637	9,015	9,774	46,408	49,802	59,895	82,625	84,159	95,708	137,878	258,783
		$d = 100$	8,146	8,628	9,042	9,778	43,869	48,777	59,697	82,656	77,946	91,620	134,828	257,724
		$d = 1000$	8,165	8,720	9,096	9,788	42,256	47,124	59,582	82,843	72,313	87,728	132,902	257,921
		$d = \infty$	8,405	8,868	9,279	9,815	43,941	50,644	65,198	85,899	76,545	97,537	162,012	299,847
	apKelly	$\eta = 0.51$	8,550	9,067	9,432	9,936	37,498	40,902	48,425	63,746	80,498	88,848	110,767	153,644
	ALPHA	$\eta = 0.51$	8,244	8,642	9,004	9,771	46,624	49,708	59,802	82,467	84,466	95,854	137,524	257,857
		$d = 100$	8,145	8,599	9,024	9,771	44,062	48,808	59,517	82,466	78,549	91,878	134,385	256,440
		$d = 1000$	8,027	8,613	9,044	9,782	42,099	46,647	59,140	82,503	72,109	87,232	130,818	255,286
		$d = \infty$	7,816	8,453	9,018	9,783	37,017	39,037	50,522	78,914	72,207	70,373	90,357	199,594
	ONEAudit	$\eta = 0.51$	8,230	8,639	9,018	9,775	46,383	49,762	59,993	82,757	84,117	95,717	138,469	259,705
		$d = 100$	8,135	8,622	9,042	9,780	43,944	48,862	59,775	82,753	77,916	91,757	135,334	258,598
		$d = 1000$	8,116	8,701	9,088	9,789	42,090	47,062	59,549	82,893	72,035	87,880	132,796	258,431
		$d = \infty$	8,189	8,732	9,208	9,805	37,260	43,588	59,033	83,958	56,357	71,377	119,701	259,498
	apKelly	$\eta = 0.52$	7,962	8,469	8,708	9,527	62,935	64,271	66,275	71,029	303,860	313,931	311,775	309,128
	ALPHA	$\eta = 0.52$	8,225	8,635	9,001	9,763	46,732	49,794	59,765	82,456	84,869	96,070	137,502	257,670
		$d = 100$	8,151	8,589	9,012	9,768	44,281	48,886	59,471	82,439	79,160	92,479	134,418	256,283
		$d = 1000$	7,918	8,474	8,965	9,774	42,190	46,692	58,267	82,125	74,537	87,468	129,434	253,540
		$d = \infty$	7,771	8,202	8,679	9,732	63,051	53,088	48,048	69,938	297,773	226,001	113,478	150,425
	ONEAudit	$\eta = 0.52$	8,225	8,640	9,024	9,778	46,312	49,832	60,321	82,961	83,836	96,004	139,309	261,953
		$d = 100$	8,145	8,606	9,049	9,783	43,979	48,885	59,968	82,937	78,223	92,171	136,033	260,821
		$d = 1000$	8,047	8,641	9,074	9,790	42,088	47,016	59,510	82,987	72,111	87,778	132,551	259,460
		$d = \infty$	7,839	8,471	9,044	9,791	36,772	39,113	50,998	79,471	70,015	69,134	91,287	203,227
	apKelly	$\eta = 0.55$	8,936	9,118	9,092	9,266	91,420	90,332	90,539	90,201	458,352	456,098	446,626	453,647
	ALPHA	$\eta = 0.55$	8,217	8,638	8,994	9,762	47,181	50,008	59,613	82,380	85,866	96,812	137,675	257,513
		$d = 100$	8,124	8,553	8,971	9,764	45,535	49,303	59,152	82,256	84,063	95,514	134,842	255,529
		$d = 1000$	8,140	8,433	8,791	9,738	51,308	50,587	57,507	81,018	104,307	103,285	130,298	248,422
		$d = \infty$	8,855	8,822	8,493	9,487	89,905	86,699	78,631	62,335	450,623	438,888	381,743	182,325
	ONEAudit	$\eta = 0.55$	8,246	8,638	9,074	9,788	46,245	49,935	61,020	83,616	83,274	96,581	141,269	267,243
		$d = 100$	8,161	8,619	9,074	9,793	44,225	49,076	60,605	83,593	78,916	93,812	139,010	266,118
		$d = 1000$	7,919	8,476	9,003	9,792	42,735	47,184	59,183	83,272	76,051	88,879	133,783	262,572
		$d = \infty$	7,919	8,209	8,624	9,735	70,910	60,933	49,689	68,352	349,194	293,641	136,117	146,015
	apKelly	$\eta = 0.6$	9,289	9,269	9,288	9,430	92,975	92,124	92,959	93,401	467,898	465,514	463,709	469,873
	ALPHA	$\eta = 0.6$	8,289	8,670	8,971	9,750	48,013	50,519	59,460	82,308	89,010	98,375	138,201	256,774
		$d = 100$	8,364	8,605	8,942	9,755	50,576	51,474	59,582	81,981	98,741	103,538	137,676	254,412
		$d = 1000$	8,986	8,848	8,757	9,654	72,492	65,538	61,183	78,971	199,481	167,721	150,812	243,175
		$d = \infty$	9,223	9,163	9,077	9,200	93,077	92,226	89,731	77,554	465,050	458,055	442,870	380,694
	ONEAudit	$\eta = 0.6$	8,256	8,710	9,140	9,814	46,198	50,470	62,126	84,771	84,058	98,076	145,759	276,759
		$d = 100$	8,183	8,637	9,113	9,815	45,145	49,834	62,046	84,620	82,234	97,068	143,571	275,290
		$d = 1000$	8,063	8,423	8,917	9,796	48,570	50,119	59,544	83,731	94,875	101,193	137,250	268,303
		$d = \infty$	8,755	8,691	8,430	9,614	88,367	85,547	71,029	62,327	444,414	429,114	346,251	158,316

Table 7. Mean sample sizes to reject the null that the assorter mean does not exceed $1/2$ when the fraction of valid votes for the winner is 0.505, for various population sample sizes, numbers of blank/invalid cards, based on 1,000 replications. The smallest sample size for each of the 12 conditions is in bold font. Some flavor of ALPHA applied to the ONEAudit transformation of the assorter values had the smallest sample size in 6 of the 12 conditions; some flavor of ALPHA applied to the raw assorter values had the smallest in 4 conditions, and some flavor of *a priori* Kelly applied to the raw assorter values had the smallest in 2 conditions.

θ	Method	params	$N = 10,000$ %blank				$N = 100,000$ %blank				$N = 500,000$ %blank			
			10	25	50	75	10	25	50	75	10	25	50	75
0.51	sqKelly		7,394	7,416	7,468	8,100	71,259	70,984	73,780	71,692	362,456	360,640	363,556	359,819
	apKelly	$\eta = 0.505$	8,295	8,759	9,490	9,946	18,821	22,636	30,713	50,214	21,200	25,247	37,491	71,246
	ALPHA	$\eta = 0.505$ $d = 10$	6,305	6,655	7,597	9,101	19,189	22,492	31,511	56,797	23,919	28,774	46,053	113,995
		$d = 1000$	6,104	6,601	7,600	9,128	17,462	21,409	31,102	56,897	21,477	26,774	44,840	113,564
		$d = \infty$	6,096	6,617	7,687	9,183	16,575	20,806	31,092	57,477	20,182	25,462	44,406	114,501
	ONEAudit	$\eta = 0.505$ $d = 10$	6,329	6,901	8,024	9,296	18,036	23,373	36,453	64,506	22,101	29,744	57,651	153,556
		$d = 100$	6,301	6,658	7,604	9,110	19,127	22,514	31,591	56,982	23,813	28,768	46,223	114,612
		$d = 1000$	6,104	6,618	7,611	9,138	17,496	21,444	31,212	57,127	21,473	26,805	45,047	114,337
		$d = \infty$	6,160	6,673	7,742	9,199	16,855	21,199	31,472	57,727	20,431	26,028	45,217	115,831
			6,694	7,176	8,202	9,336	24,101	29,993	43,326	68,267	35,297	47,303	85,375	191,268
	apKelly	$\eta = 0.51$	6,535	7,032	8,099	9,491	13,455	16,424	21,998	36,589	15,353	18,538	27,698	51,396
	ALPHA	$\eta = 0.51$ $d = 10$	6,311	6,657	7,595	9,101	19,157	22,484	31,518	56,779	23,953	28,780	46,040	113,940
		$d = 100$	6,095	6,568	7,585	9,125	17,545	21,265	31,055	56,806	21,537	26,764	44,821	113,369
		$d = 1000$	5,969	6,517	7,623	9,167	16,212	20,263	30,581	57,146	19,659	24,987	43,438	113,584
		$d = \infty$	5,812	6,405	7,702	9,225	13,587	17,433	28,011	57,190	15,657	19,894	36,657	107,352
	ONEAudit	$\eta = 0.51$ $d = 10$	6,292	6,661	7,611	9,117	19,068	22,503	31,672	57,148	23,768	28,795	46,429	115,157
		$d = 100$	6,107	6,622	7,616	9,143	17,438	21,462	31,282	57,261	21,518	26,801	45,205	114,937
		$d = 1000$	6,111	6,636	7,716	9,198	16,667	20,960	31,289	57,766	20,273	25,678	44,970	115,955
		$d = \infty$	6,349	6,921	8,048	9,308	18,183	23,536	36,810	64,825	22,265	30,008	58,317	155,129
	apKelly	$\eta = 0.52$	5,657	5,948	6,836	8,433	17,817	20,617	24,937	35,984	34,031	39,193	50,230	74,171
	ALPHA	$\eta = 0.52$ $d = 10$	6,315	6,664	7,594	9,098	19,220	22,504	31,524	56,747	24,014	28,837	46,045	113,849
		$d = 100$	6,120	6,555	7,564	9,114	17,616	21,224	30,968	56,693	21,744	26,877	44,688	113,084
		$d = 1000$	5,853	6,330	7,458	9,134	15,812	19,438	29,469	56,485	19,189	24,381	41,799	111,544
		$d = \infty$	5,495	5,815	7,116	9,054	16,431	16,861	21,695	45,940	25,327	22,107	27,601	69,750
	ONEAudit	$\eta = 0.52$ $d = 10$	6,288	6,671	7,628	9,129	19,029	22,509	31,793	57,565	23,725	28,774	46,722	116,513
		$d = 100$	6,098	6,609	7,632	9,156	17,492	21,475	31,424	57,549	21,650	26,912	45,512	116,168
		$d = 1000$	6,009	6,557	7,666	9,196	16,329	20,490	31,046	57,826	19,799	25,256	44,413	116,211
		$d = \infty$	5,840	6,445	7,749	9,249	13,675	17,596	28,400	57,957	15,735	20,029	37,279	109,939
	apKelly	$\eta = 0.55$	7,586	7,636	7,624	8,143	73,964	74,569	77,857	75,379	379,654	379,856	385,180	376,644
	ALPHA	$\eta = 0.55$ $d = 10$	6,332	6,674	7,578	9,096	19,361	22,590	31,486	56,635	24,427	29,020	46,145	113,420
		$d = 100$	6,155	6,523	7,501	9,097	18,335	21,520	30,659	56,336	23,038	27,633	44,458	112,383
		$d = 1000$	6,139	6,228	7,138	9,011	19,584	20,711	27,763	54,375	25,583	27,311	40,208	105,619
		$d = \infty$	7,543	6,890	6,511	8,506	74,146	63,101	33,353	34,003	374,407	322,840	115,810	51,300
	ONEAudit	$\eta = 0.55$ $d = 10$	6,285	6,702	7,700	9,176	18,936	22,696	32,228	58,523	23,654	28,942	47,848	120,450
		$d = 100$	6,122	6,620	7,667	9,194	17,700	21,680	31,893	58,562	21,877	27,271	46,624	119,742
		$d = 1000$	5,848	6,338	7,519	9,194	15,983	19,648	30,124	57,967	19,540	24,758	43,331	117,202
		$d = \infty$	5,557	5,738	7,016	9,052	20,885	18,719	21,218	43,862	49,275	29,202	27,591	64,394
	apKelly	$\eta = 0.6$	9,027	8,968	9,077	9,067	89,418	90,560	90,425	90,184	443,859	450,170	456,126	455,792
	ALPHA	$\eta = 0.6$ $d = 10$	6,395	6,666	7,583	9,090	19,737	22,988	31,536	56,452	25,199	29,536	46,230	113,089
		$d = 100$	6,494	6,626	7,471	9,059	21,103	22,978	30,620	55,890	27,423	30,146	45,321	111,313
		$d = 1000$	7,848	7,233	7,014	8,813	39,196	32,645	30,132	51,644	63,444	49,191	45,960	98,789
		$d = \infty$	8,965	8,703	7,656	7,904	88,829	87,058	79,399	41,150	438,065	430,248	392,792	124,134
	ONEAudit	$\eta = 0.6$ $d = 10$	6,326	6,785	7,790	9,252	18,926	23,047	33,272	60,344	23,681	29,491	49,670	127,072
		$d = 100$	6,200	6,657	7,720	9,249	18,237	22,179	32,733	60,210	22,845	28,311	48,553	126,355
		$d = 1000$	6,002	6,201	7,355	9,193	18,238	20,520	29,486	58,343	23,405	26,534	43,312	119,646
		$d = \infty$	7,049	6,486	6,539	8,723	68,264	52,660	27,505	35,331	341,699	270,043	66,439	50,969

Table 8. Same as table 7, but with a fraction 0.51 of the valid votes for the reported winner. Some flavor of ALPHA applied to the ONEAudit transformation of the assorter values had the smallest sample size for 6 conditions; some flavor of ALPHA applied to the raw assorter values had the smallest for 4 conditions, and some flavor of *a priori* Kelly applied to the raw assorter values had the smallest for 2 conditions.

θ	Method	params	N =10,000, %blank				N =100,000 %blank				N =500,000 %blank			
			10	25	50	75	10	25	50	75	10	25	50	75
0.52	sqKelly		3,416	3,648	4,134	5,502	13,469	14,905	16,084	22,542	38,589	43,864	49,984	64,106
	apKelly	$\eta = 0.505$	5,909	6,588	7,849	9,560	8,974	10,737	15,375	28,590	9,379	11,108	16,791	33,517
	ALPHA	$\eta = 0.505$ $d = 10$	3,454	3,798	4,989	7,544	5,637	6,941	10,800	27,035	6,089	7,304	12,313	36,836
		$d = 100$	3,292	3,700	4,984	7,571	5,257	6,667	10,687	27,128	5,608	6,960	12,179	36,853
		$d = 1000$	3,361	3,880	5,239	7,732	5,333	6,913	11,268	28,144	5,717	7,216	12,865	38,268
	ONEAudit	$\eta = 0.505$ $d = \infty$	4,064	4,663	6,066	8,173	8,547	11,411	19,692	42,424	9,937	13,662	28,041	83,317
		$d = 100$	3,449	3,799	5,001	7,554	5,628	6,947	10,839	27,193	6,099	7,315	12,355	37,080
		$d = 1000$	3,304	3,724	5,005	7,588	5,259	6,700	10,766	27,270	5,637	6,994	12,240	37,131
	ALPHA	$\eta = 0.51$ $d = 10$	3,447	3,954	5,316	7,765	5,527	7,113	11,524	28,473	5,939	7,432	13,169	38,931
		$d = 100$	4,468	5,044	6,319	8,258	12,336	15,708	24,738	46,521	16,996	23,151	43,940	108,570
		$d = \infty$	3,977	4,469	5,716	7,999	5,305	6,516	9,182	17,430	5,582	6,600	9,877	19,842
	ONEAudit	$\eta = 0.51$ $d = 10$	3,451	3,795	4,987	7,543	5,646	6,936	10,799	27,032	6,107	7,298	12,293	36,817
		$d = 100$	3,274	3,684	4,973	7,564	5,219	6,626	10,652	27,067	5,585	6,939	12,112	36,772
		$d = 1000$	3,201	3,746	5,115	7,693	5,014	6,527	10,902	27,729	5,405	6,848	12,423	37,669
	ALPHA	$\eta = 0.52$ $d = 10$	3,447	4,056	5,606	8,006	5,498	7,526	13,818	35,277	5,966	7,970	16,362	54,916
		$d = 100$	3,447	3,802	5,008	7,567	5,627	6,958	10,862	27,315	6,088	7,319	12,393	37,273
		$d = 1000$	3,299	3,723	5,010	7,599	5,261	6,691	10,773	27,381	5,636	6,988	12,279	37,323
	ONEAudit	$\eta = 0.52$ $d = \infty$	3,386	3,900	5,279	7,761	5,372	6,957	11,390	28,430	5,776	7,274	12,988	38,855
		$d = 100$	4,081	4,687	6,093	8,195	8,631	11,513	19,900	42,767	10,022	13,816	28,401	84,337
		$d = 1000$	2,833	3,206	4,158	6,151	3,985	4,835	6,686	12,577	4,050	4,968	7,159	15,237
	ALPHA	$\eta = 0.52$ $d = 10$	3,451	3,798	4,985	7,537	5,652	6,937	10,786	27,011	6,126	7,299	12,277	36,795
		$d = 100$	3,240	3,649	4,929	7,550	5,219	6,553	10,562	26,959	5,546	6,878	11,996	36,562
		$d = 1000$	2,992	3,491	4,903	7,604	4,632	6,024	10,179	26,983	4,842	6,243	11,568	36,439
	ONEAudit	$\eta = 0.52$ $d = \infty$	2,781	3,321	4,805	7,668	4,017	5,166	8,923	25,652	4,148	5,300	9,820	32,686
		$d = 100$	3,440	3,821	5,027	7,590	5,616	6,961	10,952	27,542	6,092	7,343	12,478	37,742
		$d = 1000$	3,292	3,713	5,024	7,621	5,266	6,682	10,824	27,644	5,633	7,010	12,358	37,724
	ALPHA	$\eta = 0.55$ $d = 10$	3,242	3,790	5,184	7,746	5,080	6,660	11,125	28,349	5,466	6,962	12,686	38,721
		$d = 100$	3,479	4,110	5,663	8,053	5,601	7,651	14,122	35,919	6,046	8,120	16,771	56,462
		$d = 1000$	3,592	3,748	4,237	5,423	19,030	20,062	19,874	25,982	75,673	80,329	85,559	94,161
	ONEAudit	$\eta = 0.55$ $d = 10$	3,465	3,804	4,967	7,526	5,713	6,951	10,746	26,930	6,180	7,322	12,252	36,697
		$d = 100$	3,214	3,596	4,831	7,504	5,319	6,495	10,356	26,669	5,538	6,829	11,753	36,083
		$d = 1000$	2,950	3,200	4,370	7,349	4,901	5,628	8,834	24,842	5,168	5,870	9,762	32,989
	ALPHA	$\eta = 0.6$ $d = 10$	3,477	3,217	3,740	6,601	14,747	8,138	6,785	14,998	44,666	12,870	7,540	17,473
		$d = 100$	3,437	3,843	5,110	7,674	5,639	7,067	11,206	28,408	6,094	7,424	12,818	39,168
		$d = 1000$	3,258	3,710	5,066	7,689	5,282	6,695	10,981	28,362	5,616	7,087	12,557	39,012
	ONEAudit	$\eta = 0.6$ $d = \infty$	2,973	3,486	4,951	7,714	4,604	6,030	10,375	28,075	4,836	6,242	11,796	38,400
		$d = 100$	2,701	3,167	4,653	7,643	4,011	4,903	8,214	23,898	4,069	5,041	8,935	29,407
		$d = 1000$	7,501	7,544	7,447	7,605	76,103	75,259	75,515	77,007	386,240	385,732	403,069	385,244
	ALPHA	$\eta = 0.6$ $d = 10$	3,527	3,822	4,926	7,500	5,883	7,052	10,732	26,811	6,347	7,423	12,165	36,439
		$d = 100$	3,445	3,653	4,714	7,435	5,922	6,710	10,160	26,241	6,279	7,150	11,487	35,380
		$d = 1000$	4,591	3,961	4,073	6,886	10,615	8,421	8,536	21,880	11,811	9,324	9,684	28,900
	ONEAudit	$\eta = 0.6$ $d = \infty$	7,466	6,465	4,335	5,508	76,870	64,404	21,902	12,308	356,324	325,161	100,355	15,780
		$d = 100$	3,461	3,916	5,223	7,821	5,710	7,226	11,679	29,901	6,158	7,584	13,382	41,789
		$d = 1000$	3,288	3,732	5,126	7,813	5,376	6,774	11,308	29,686	5,639	7,246	12,978	41,427
	ALPHA	$\eta = 0.6$ $d = 10$	2,900	3,271	4,663	7,667	4,702	5,725	9,589	27,903	4,919	5,948	10,705	38,147
		$d = 100$	3,132	3,041	3,873	6,981	9,126	6,374	6,674	16,535	17,623	8,035	7,137	19,129
		$d = 1000$												

Table 9. Same as table 7, but with a fraction 0.52 of the valid votes for the reported winner. Some flavor of ALPHA applied to the ONEAudit transformation of the assorter values had the smallest sample size for 4 conditions; some flavor of ALPHA applied to the raw assorter values had the smallest for 2 conditions, and some flavor of *a priori* Kelly applied to the raw assorter values had the smallest for 6 conditions, of which 5 used the true population mean.

θ	Method	params	N =10,000, %blank				N =100,000 %blank				N =500,000 %blank			
			10	25	50	75	10	25	50	75	10	25	50	75
0.55	sqKelly		594	748	1,067	1,834	688	845	1,208	2,331	705	787	1,213	2,446
	apKelly	$\eta = 0.505$	2,919	3,426	4,634	7,180	3,435	4,144	6,098	11,800	3,526	4,167	6,242	12,311
	ALPHA	$\eta = 0.505$ $d = 10$	790	1,009	1,625	3,577	946	1,180	1,973	5,799	939	1,113	2,044	6,158
		$d = 100$	790	1,043	1,694	3,683	930	1,187	2,062	6,012	935	1,155	2,117	6,360
		$d = 1000$	1,057	1,342	2,114	4,154	1,234	1,582	2,690	7,135	1,245	1,577	2,773	7,536
	ONEAudit	$\eta = 0.505$ $d = 10$	1,800	2,217	3,315	5,500	3,284	4,501	8,283	20,041	3,767	5,221	10,930	34,385
		$d = 100$	791	1,012	1,633	3,593	945	1,185	1,984	5,842	941	1,114	2,056	6,195
		$d = 1000$	799	1,054	1,710	3,706	942	1,202	2,083	6,067	945	1,170	2,138	6,417
		$d = \infty$	1,108	1,395	2,174	4,205	1,308	1,662	2,792	7,276	1,317	1,656	2,869	7,693
	apKelly	$\eta = 0.51$	2,084	2,489	3,550	5,648	4,965	6,489	10,806	22,868	6,750	9,133	17,700	46,539
	ALPHA	$\eta = 0.51$ $d = 10$	1,659	1,969	2,809	4,689	1,831	2,220	3,256	6,430	1,856	2,203	3,322	6,561
		$d = 100$	790	1,006	1,623	3,574	945	1,179	1,972	5,796	936	1,110	2,038	6,154
		$d = 1000$	778	1,024	1,675	3,668	916	1,173	2,043	5,982	921	1,135	2,092	6,339
	ONEAudit	$\eta = 0.51$ $d = 10$	968	1,254	2,020	4,084	1,122	1,459	2,542	6,955	1,132	1,448	2,610	7,346
		$d = 100$	1,395	1,799	2,906	5,236	1,928	2,710	5,431	15,712	2,023	2,826	6,067	21,572
		$d = 1000$	792	1,011	1,637	3,611	944	1,186	1,992	5,875	941	1,116	2,064	6,236
		$d = \infty$	796	1,052	1,710	3,714	938	1,199	2,083	6,102	941	1,166	2,140	6,452
	ALPHA	$\eta = 0.52$ $d = 10$	1,068	1,352	2,137	4,186	1,248	1,601	2,726	7,224	1,256	1,593	2,801	7,641
		$d = 100$	1,816	2,236	3,339	5,531	3,320	4,549	8,378	20,256	3,806	5,285	11,077	34,892
		$d = \infty$	952	1,163	1,673	2,930	1,045	1,265	1,858	3,656	1,047	1,242	1,869	3,749
	ONEAudit	$\eta = 0.52$ $d = 10$	790	1,004	1,616	3,570	944	1,173	1,968	5,781	932	1,108	2,028	6,143
		$d = 100$	751	997	1,646	3,638	891	1,144	2,001	5,921	896	1,100	2,047	6,280
		$d = 1000$	821	1,096	1,853	3,944	955	1,250	2,274	6,580	960	1,238	2,328	6,974
		$d = \infty$	948	1,283	2,266	4,721	1,117	1,548	3,160	10,491	1,126	1,562	3,304	11,965
	ALPHA	$\eta = 0.55$ $d = 10$	793	1,015	1,649	3,648	945	1,190	2,009	5,954	942	1,120	2,077	6,322
		$d = 100$	786	1,044	1,712	3,737	926	1,191	2,087	6,150	930	1,157	2,144	6,507
		$d = 1000$	983	1,278	2,060	4,153	1,147	1,492	2,602	7,134	1,153	1,477	2,676	7,541
	apKelly	$\eta = 0.55$	1,420	1,830	2,955	5,302	1,965	2,767	5,564	16,093	2,067	2,889	6,242	22,239
	ALPHA	$\eta = 0.55$ $d = 10$	581	737	1,054	1,816	673	844	1,186	2,330	692	780	1,189	2,413
		$d = 100$	788	996	1,606	3,553	939	1,163	1,951	5,742	932	1,099	2,006	6,106
		$d = 1000$	696	926	1,557	3,566	826	1,066	1,887	5,749	836	1,020	1,931	6,099
	ONEAudit	$\eta = 0.55$ $d = 10$	616	831	1,458	3,540	715	945	1,718	5,633	725	891	1,750	5,912
		$d = 100$	594	790	1,381	3,480	681	884	1,581	5,200	691	846	1,589	5,400
		$d = 1000$	795	1,029	1,683	3,739	945	1,204	2,059	6,194	951	1,137	2,127	6,591
		$d = \infty$	764	1,023	1,714	3,803	903	1,176	2,092	6,331	910	1,129	2,148	6,705
	ALPHA	$\eta = 0.6$ $d = 10$	797	1,079	1,867	4,056	925	1,229	2,287	6,870	928	1,215	2,344	7,275
		$d = 100$	863	1,177	2,143	4,664	1,000	1,377	2,825	9,605	999	1,366	2,922	10,626
		$d = \infty$	1,044	1,238	1,557	2,143	2,795	3,568	3,811	5,811	6,072	5,010	7,595	11,746
	ONEAudit	$\eta = 0.6$ $d = 10$	787	995	1,585	3,527	939	1,155	1,930	5,690	940	1,096	1,974	6,050
		$d = 100$	689	872	1,444	3,430	812	999	1,724	5,483	815	949	1,783	5,805
		$d = 1000$	685	775	1,149	2,977	845	908	1,334	4,395	870	840	1,347	4,622
		$d = \infty$	898	831	1,045	2,446	1,537	1,112	1,187	3,097	2,107	1,011	1,189	3,183
	ALPHA	$\eta = 0.6$ $d = 10$	802	1,057	1,753	3,905	951	1,231	2,153	6,596	957	1,170	2,224	7,056
		$d = 100$	730	1,002	1,724	3,922	868	1,157	2,108	6,629	879	1,099	2,162	7,072
		$d = 1000$	646	888	1,607	3,915	750	1,007	1,916	6,511	752	960	1,966	6,889
	apKelly	$\eta = 0.6$	616	835	1,514	3,851	703	931	1,746	6,020	706	896	1,772	6,269

Table 10. Same as table 7, but with a fraction 0.55 of the valid votes for the reported winner. ALPHA applied to the ONEAudit transformation never had the smallest sample size; some flavor of ALPHA applied to the raw assorter values had the smallest (or was tied for smallest) in 3 conditions, and *a priori* Kelly using the true population mean had the smallest (or was tied for smallest) in 10 conditions.

θ	Method	params	$N=10,000$ %blank				$N=100,000$ %blank				$N=500,000$ %blank			
			10	25	50	75	10	25	50	75	10	25	50	75
0.6	sqKelly		199	247	348	668	210	239	376	729	202	240	368	715
	apKelly	$\eta=0.505$	1,564	1,875	2,629	4,688	1,705	2,026	3,038	5,974	1,695	2,049	3,078	6,085
	ALPHA	$d=10$	221	285	478	1,324	234	284	522	1,597	228	282	519	1,560
		$d=100$	264	336	565	1,458	282	342	616	1,765	273	342	609	1,730
		$d=1000$	451	564	897	1,985	497	601	1,022	2,531	492	611	1,022	2,530
		$d=\infty$	924	1,167	1,819	3,459	1,635	2,211	4,199	10,631	1,812	2,578	5,469	17,416
	ONEAudit	$d=10$	222	286	481	1,333	235	285	525	1,610	229	283	523	1,570
		$d=100$	268	341	573	1,472	286	348	625	1,785	277	347	618	1,750
		$d=1000$	478	592	927	2,017	527	634	1,062	2,589	525	646	1,060	2,586
		$d=\infty$	1,086	1,333	1,983	3,593	2,505	3,242	5,553	12,303	3,304	4,559	8,922	23,820
	apKelly	$\eta=0.51$	832	1,000	1,448	2,672	882	1,046	1,573	3,113	880	1,057	1,573	3,113
	ALPHA	$d=10$	220	284	477	1,322	233	283	521	1,596	227	280	518	1,556
		$d=100$	257	329	555	1,449	275	333	608	1,754	265	336	599	1,717
		$d=1000$	409	519	850	1,935	445	549	958	2,451	443	556	960	2,450
		$d=\infty$	697	912	1,539	3,219	933	1,295	2,690	8,163	960	1,372	2,992	10,794
	ONEAudit	$d=10$	223	286	483	1,341	235	286	527	1,622	229	284	524	1,580
		$d=100$	265	339	571	1,477	284	345	624	1,789	275	345	616	1,755
		$d=1000$	456	569	906	2,005	502	607	1,033	2,565	498	618	1,034	2,561
		$d=\infty$	932	1,178	1,835	3,486	1,651	2,235	4,250	10,751	1,829	2,610	5,545	17,659
	apKelly	$\eta=0.52$	450	540	791	1,511	471	548	835	1,667	465	554	834	1,643
	ALPHA	$d=10$	218	281	475	1,317	232	281	518	1,591	226	279	516	1,553
		$d=100$	243	314	539	1,429	260	318	589	1,729	251	321	581	1,695
		$d=1000$	341	443	758	1,839	367	464	848	2,305	361	469	847	2,296
		$d=\infty$	450	615	1,149	2,791	512	702	1,524	5,297	511	722	1,562	5,917
	ONEAudit	$d=10$	222	287	486	1,356	235	287	531	1,641	229	285	528	1,602
		$d=100$	262	335	570	1,485	279	341	621	1,804	271	341	614	1,769
		$d=1000$	417	529	867	1,978	455	560	983	2,516	452	568	980	2,511
		$d=\infty$	711	932	1,570	3,278	951	1,326	2,767	8,375	981	1,404	3,079	11,127
	apKelly	$\eta=0.55$	221	269	384	737	230	263	409	810	223	264	404	787
	ALPHA	$d=10$	213	276	468	1,307	228	277	508	1,575	222	272	509	1,541
		$d=100$	210	280	492	1,371	223	279	540	1,654	215	278	533	1,620
		$d=1000$	225	304	565	1,578	238	307	613	1,920	231	306	612	1,901
		$d=\infty$	233	318	625	1,894	246	326	688	2,487	239	327	686	2,506
	ONEAudit	$d=10$	223	290	499	1,402	236	290	543	1,703	231	288	539	1,669
		$d=100$	247	322	562	1,509	264	326	615	1,839	255	328	608	1,809
		$d=1000$	328	430	760	1,896	352	453	848	2,389	345	457	849	2,381
		$d=\infty$	401	553	1,068	2,741	441	608	1,341	4,817	441	619	1,362	5,233
	apKelly	$\eta=0.6$	163	204	287	542	173	197	309	627	168	199	307	596
	ALPHA	$d=10$	208	269	455	1,288	224	269	496	1,556	218	264	498	1,519
		$d=100$	181	243	427	1,279	191	236	470	1,540	186	235	465	1,503
		$d=1000$	167	222	391	1,232	176	213	426	1,450	171	216	418	1,427
		$d=\infty$	165	219	380	1,200	175	209	408	1,364	169	212	404	1,361
	ONEAudit	$d=10$	225	299	521	1,484	239	297	569	1,821	233	294	567	1,788
		$d=100$	228	306	552	1,559	242	306	606	1,911	233	307	598	1,883
		$d=1000$	244	331	633	1,785	258	340	696	2,214	251	341	689	2,205
		$d=\infty$	251	348	703	2,141	266	360	779	2,909	259	360	779	2,960

Table 11. Same as table 7, but with a fraction 0.6 of the valid votes for the reported winner. *A priori* Kelly applied to the raw assorter values using the correct population mean had the smallest sample size for all 12 conditions.