How our values and means shape optimal transit fare policy

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Abstract

This study examines how optimal fare policies are shaped by the need to balance moral values and financial means. Motivated by the growing interest in fare-free transit (FFT) and the lack of rigorous modeling frameworks to evaluate its impacts, we develop a joint transit design model that incorporates a means-based step-fare structure encompassing full-fare-free (FFF), partial-fare-free (PFF), and standard-discount-fare (SDF) policies. The model accounts for behavioral responses such as the zero-price effect, as well as operational and administrative savings from eliminating fare collection. It is then employed to identify optimal fare policies under both utilitarian and egalitarian objectives across a range of financial scenarios. Applied to the case of Chicago, our analysis yields several key findings. First, optimal policies depend critically on funding levels: FFF is both just and utility-maximizing only when robust financial support is available. Second, contrary to common belief, a utilitarian is more likely than an egalitarian to support FFF, as equity goals often require targeted benefits. Third, increasing financial flexibility does not always lead to more publicly acceptable designs. These findings offer guidance for transit agencies navigating the post-pandemic fiscal landscape and planners pursuing more sustainable, equitable urban mobility.

Keywords: fare-free transit; utilitarian; egalitarian; fare policy; transit design.

1. Introduction

Fare policy plays a central role in shaping the accessibility and financial sustainability of public transportation systems. In the classical transportation economics literature, optimal fare policy is grounded in the principle of marginal cost pricing. The seminal "Mohring effect" (Mohring, 1972) shows that higher demand leads to more frequent service, which in turn reduces passengers' waiting time. As a result, the marginal social cost of an additional rider is below the average cost, implying that achieving social optimum requires a fare lower than the average cost and thus, in most cases, public subsidy. This theoretical result forms the cornerstone of first-best fare policy: the optimal fare equals the marginal external social cost, with any budget gap covered by subsidy.

The classical fare analysis is explicitly utilitarian in nature. It focuses on maximizing aggregate welfare and, in doing so, overlooks how costs and benefits are distributed across different groups. In practice, transit fare policy, like other transportation policies, has ethical as well as economic implications (Lucas et al., 2016). They affect who can afford to use transit, how frequently they use it, and whether they choose transit over other modes. In particular, the burden of fare payments tends to fall more heavily on low-income travelers, who rely on transit not by choice but by necessity. This raises important questions about fairness and calls for integrating equity considerations into fare policy design.

In recent years, these concerns have fueled renewed interest in fare-free transit (FFT) as a strategy to improve access and reduce disparities. Boston's progressive mayor, Michelle Wu, has implemented a limited FFT program in the city. The idea also featured prominently in the platform of Zohran Mamdani, who has launched a closely watched bid for the New York City mayoralty[1]. Advocates argue that eliminating fares lowers financial barriers for disadvantaged

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[1] https://www.npr.org/2025/07/01/nx-s1-5449040/zohran-mamdani-nyc-mayoral-candidate, accessed on 07/25/2025.

groups, encourages mode shift away from private cars, reduces congestion, and makes transit operation more cost-effective (Cats et al., 2017; Bull et al., 2021; Cools et al., 2016; Volinski, 2012; Keblowski, 2020). However, FFT is not without drawbacks. Critics cite the potential for overcrowding, misuse, and safety issues (Hodge et al., 1994), and question its financial sustainability, especially for large agencies where farebox revenue covers a substantial share of operating costs (Kirschen et al., 2022). Without an adequate replacement for lost revenue, FFT risks degrading service quality, which may ultimately harm the very populations it seeks to serve.

This brings us to the second determinant of fare policy: means. Even if one accepts the normative appeal of FFT, its feasibility depends on the fiscal instruments available to support it. In most cities around the world, transit systems must be subsidized through some combination of general taxes, dedicated sales taxes, or transportation-related charges such as congestion tolls or parking fees. Each of these mechanisms carries its own economic and political implications, and their availability varies widely across jurisdictions. In short, optimal fare policy depends not only on what values we prioritize but also on the practical means we have to fund them.

While full FFT remains relatively rare, partial or targeted fare relief policies are widespread. Many agencies offer free or discount fares to specific population groups—such as seniors, students, or low-income riders—while charging standard fares to others (Darling et al., 2021). This approach enables agencies to retain fare revenue from choice riders (e.g., peak-hour commuters) while easing the burden on captive riders who depend more heavily on transit (Keblowski, 2020; Schank and Huang, 2022). If carefully designed, these policies can strike a more sustainable balance between equity and fiscal responsibility (Harmony, 2018). At the same time, they offer more limited gains in ridership and congestion mitigation compared to full FFT, and they forgo the savings associated with eliminating fare collection altogether (Kirschen et al., 2022).

Thus, fare policy is not an all-or-nothing proposition. Rather, it spans a spectrum—from standard fare to full-fare-free systems, and anything in between—and each option embodies different trade-offs between equity, efficiency, and financial viability. *How is optimal transit fare policy shaped by our values and means?*

To address this question, we build on a stylized joint design model originally introduced in Dai et al. (2024). In the model, the quality of transit service affects its usage—through travelers' mode choice—and depends on revenue raised from various sources: transit users (fares), drivers (driver fees), and the general public (dedicated taxes). The present study generalizes the model in two key ways. First, we introduce a means-based step-fare structure that nests full-fare-free (FFF), partial-fare-free (PFF), and standard-discount-fare (SDF) policies within a unified framework. Second, we incorporate several additional features relevant to FFT, including the "zero-price effect" (Shampanier et al., 2007), administrative savings from eliminating fare collection, and potential operational efficiency gains (Volinski, 2012; Keblowski, 2020; Kirschen et al., 2022).

Fare policies are differentiated based on income, which serves as a proxy for rider type. The model is solved under two normative objectives: (i) a utilitarian goal of maximizing average utility across all travelers, and (ii) an egalitarian goal derived from the difference principle of Rawls (1971), which aims to maximize the welfare of the most disadvantaged group. We then evaluate which fare policies emerge as "optimal" under a given normative objective and a set of financial constraints—defined by the ability to raise sales taxes or charge driver fees. We shall refer to a policy that is optimal under the utilitarian goal as *utility-maximizing*, and a policy optimal under the egalitarian goal *just*—since the difference principle is part of Rawls' theory of justice.

Our results, obtained from a case study based in Chicago, offer several interesting insights. First, we find that FFF policy can, under certain conditions, be both utility-maximizing and just. Specifically, when the transit agency has access to ample financial means FFF emerges as optimal under both normative objectives. In such settings, fare elimination leads to significant efficiency gains and delivers real benefits to low-income riders. However, this outcome is contingent on the financial means. When they become more constrained, the optimal policy changes. Taxation proves to be the more powerful lever: its availability has a more decisive impact on the preferred fare structure than driver fees.

Importantly, we find that FFF is not always more egalitarian than other policies. When resources are limited, a step-fare or partial-fare-free (PFF) policy more effectively targets support to disadvantaged riders while preserving enough revenue to sustain acceptable service levels. This finding serves as a cautionary note for equity-oriented advocates:

progressive goals may not always align with fare-free proposals, particularly in the absence of strong public funding mechanisms.

Finally, our analysis also reveals, paradoxically, that lifting all financial constraints can result in more individuals being worse off—relative to the status quo—than retaining relatively tight controls. This outcome holds regardless of the moral principle adopted and carries important implications for the viability of such policies, should they require public approval.

The remainder of the paper is organized as follows: Section 2 reviews related literature; Section 3 presents the model and formal problem formulation; Section 4 illustrates how the impact of full FFT are captured in the model; Section 5 reports and analyzes the numerical results from the Chicago case study; and Section 6 concludes with a summary of findings and directions for future research.

2. Related studies

Our work is closely related to the study of transit fare policy and the practice of fare-free transit. In the following, we review the relevant works in each of these areas.

2.1. Transit fare policy

Transit pricing is a classical problem in transportation economics. Standard theory suggests that the optimal fare should ensure that the total cost borne by a transit passenger—including both fare and time cost—equals the marginal external cost of their ride, which includes the costs imposed on other passengers as well as those incurred by the operator (Turvey and Mohring, 1975). Accordingly, the optimal fare equals the marginal social cost minus the passenger's own time cost (Jara-Diaz et al., 2024). Moreover, it is well established that the optimal service frequency—often jointly determined with fare—is approximately proportional to the square root of demand (i.e., the total number of passengers) (Jansson, 1980; Jara-Díaz and Gschwender, 2003). This "square-root law" highlights the economies of scale inherent in transit operations. Together with the Mohring effect (Mohring, 1972)—where increased demand justifies more frequent service by reducing waiting times for all users—this provides a strong economic rationale for subsidizing fares with public funding to achieve socially optimal service levels.

The classical framework has been augmented by practical considerations such as externalities of travel and mode choice. Parry and Small (2009) analyze the welfare effects of fare adjustments in passenger rail and bus transit. Their analysis derives optimal transit fares by accounting for temporal variation (peak vs. off-peak), various externalities (such as congestion and pollution), and economies of scale. Empirical studies of Washington, DC, Los Angeles, and London demonstrate that transit subsidies are highly efficient: even with a 50% farebox recovery ratio, further fare reductions are well justified. Using a nested logit model to describe travel decisions (including scheduling and mode choice), Basso and Silva (2014) analyze the efficiency of transit subsidies and how it interacts with other urban transport policies, such as congestion pricing and dedicated bus lanes. They find significant substitutability among these policies; in particular, dedicated bus lanes can generate substantial welfare gains "without requiring additional public funding." Building on a similar modeling framework as that of Parry and Small (2009), Börjesson et al. (2017) examine how Stockholm's congestion pricing scheme affects the optimal design of its bus system. They find that congestion pricing reduces the need for transit subsidies, echoing the finding by Basso and Silva (2014). Additionally, lowering service frequency during off-peak hours and deploying larger buses contribute most significantly to welfare gains. Asplund and Pyddoke (2020) develop a stylized transit model featuring a radial spatial structure, two zones (inner and outer), and two time periods (peak and off-peak), to study the welfare effects of optimizing bus fares and service frequencies in the small Swedish city of Uppsala. They find that bus services are oversupplied in the outer zone and that fare adjustments, in general, would have limited welfare impact. More recently, Almagro et al. (2024) extend the model of Parry and Small (2009) to a realistic network setting, calibrated for the City of Chicago using emerging data sources such as ride-hailing trip records and cellphone location data. They find that, to maximize welfare, current fares should be reduced by more than 50%, accompanied by substantial cuts in service frequencies—28% for buses and 10% for trains. Similar adjustments are observed even when transit policies are jointly optimized with congestion pricing.

While transit fare policies in practice often pursue a range of goals, the transportation economics literature—such as the studies discussed above—focuses almost exclusively on system efficiency or aggregate welfare. Some studies report equity implications of fare policies (e.g., Basso and Silva (2014); Almagro et al. (2024)), but their design objectives remain fundamentally utilitarian. In contrast, distributional impacts have received more attention in the transportation planning literature, often explored through empirical case studies (e.g. Nahmias-Biran et al., 2014; Rubensson et al., 2020; Tiznado-Aitken et al., 2021; Brown, 2018). A central concern is the distinction between flat fares and alternative pricing schemes (e.g., distance- or time-based), as flat fares are commonly viewed as regressive—placing a disproportionate burden on low-income travelers (Cervero, 1981; Brown, 2018).

A small body of work has examined fare policy optimization beyond the conventional focus on welfare or efficiency. Borndörfer et al. (2012) formulate an intercity transit network model with O-D-specific fares and explore multiple objectives, including demand, revenue, profit, and welfare maximization. Wang et al. (2021) compare flat and distance-based fare policies under a profit-maximization framework for a single-line service, finding that distance-based fares tend to favor short-distance riders—a result that helps explain their regressive nature. Huang et al. (2021) develop a network-based fare and frequency optimization model that explicitly targets income inequality, using the Gini coefficient as the objective function. Most recently, Dai et al. (2024) propose a comprehensive transit design model that jointly determines fares, frequencies, road pricing, and dedicated tax rates under either a utilitarian (average utility) or egalitarian (utility of the most disadvantaged) objective. Their central aim is to assess whether, and under what conditions, fare-free transit constitutes a just public policy.

2.2. Fare-free transit in practice

Fare-free transit (FFT), in various forms, has been implemented across many parts of the world, with mixed results (Prince and Dellheim, 2019; Keblowski, 2020; Saphores et al., 2020; King and Taylor, 2023). Its success has varied widely depending on social, political, and cultural contexts. This section reviews selected FFT implementations, with particular emphasis on practices in Europe and the U.S.

Compared to other regions, Europe has been more receptive to large-scale FFT initiatives, often framing them as part of broader sustainable development goals (Carr and Hesse, 2020; Keblowski, 2020). As early as the 1970s, Bologna (Italy) launched one of Europe's first major FFT experiments, offering free peak-hour transit as part of its travel demand management program known as "Zone a Traffico Limitato" (ZTL), which aimed to "restore the human dimension of [their] city" (Jäggi et al., 1977, Chapter 3). Similarly, the city of Hasselt (Belgium) implemented full-scale FFT from 1997 to 2012, in an effort to "reinvent the urban transportation system [and develop] a city for people" (Brie, 2019). A more recent and widely studied case is Tallinn (Estonia), which made transit free for residents in 2013 to encourage a mode shift from private cars to public transport and to improve mobility for unemployed and low-income residents (Cats et al., 2017).

Other notable examples include cities in France (Briche et al., 2018; Huré et al., 2025), Poland (Štraub et al., 2023), and Luxembourg, which implemented fare-free transit nationwide (Carr and Hesse, 2020; Gillard et al., 2024). On the whole, Europe's FFT programs have been relatively successful. Tallinn reported a 14% increase in transit ridership following implementation (Cats et al., 2017), while smaller Polish cities such as Lubin and Żory saw ridership doubled or even tripled within a year (Lugowski, 2019). In some cases, such as Hasselt, transit usage remained high even after the FFT program was discontinued—thanks in part to pro-transit land use changes and supportive public policies established during the FFT period (Brie, 2019).

Although the world's first full-scale FFT system was launched in Commerce, California—where City of Commerce Transit remains fare-free to this day[2]—the U.S. has seen relatively few and largely short-lived FFT experiments, mostly in medium-sized cities (Keblowski, 2020). While the goals of these programs often echo those pursued in

[2] https://www.commerceca.gov/city-hall/transportation, accessed on 5/5/2025.

Europe, their outcomes have been shaped by distinct socio-political and spatial contexts that make sustained commitments to FFT more difficult.

One recurring concern in U.S. implementations is the increase in discretionary or "unnecessary" trips, which contribute to crowding and service degradation without clear social or economic benefit (Vuchic, 2005, Chapter 8). U.S. transit agencies have also reported challenges with disruptive or all-day passengers, raising safety and security concerns and undermining overall service quality (Cline Jr et al., 2024; Hodge et al., 1994). A prominent example is Austin, Texas, where a full FFT pilot launched in 1989 was terminated after one year due to a rise in problematic ridership, vandalism, and system abuse (Perone and Volinski, 2003; Ray, 2019).

Beyond operational concerns, structural factors also constrain FFT viability in the U.S. Compared to Europe, American cities tend to be more car-oriented and less transit-accessible, limiting the potential for modal shift even under fare-free policies (Conwell et al., 2023). As a result, full FFT has mostly been adopted in smaller cities with low service densities and modest farebox recovery ratios, where the revenue loss can more easily be offset by alternative funding sources such as local sales taxes (Perone and Volinski, 2003; Volinski, 2012; Keblowski, 2020; Kirschen et al., 2022). In these settings, ridership impacts have been broadly similar to European cases, with substantial surges in demand following FFT implementations (see Kirschen et al., 2022).

More recently, many U.S. transit agencies suspended fare collection during the COVID-19 pandemic, relying on emergency funding to sustain operations (Siddiq et al., 2023). This widespread fare suspension served as a de facto pilot for FFT. Among the few cities to continue the policy post-pandemic is Kansas City, Missouri, which has committed to permanent fare-free service[3]. While the pandemic confounds causal inference, Kansas City experienced smaller ridership declines and a faster rebound compared to peer agencies, suggesting potential retention benefits associated with FFT (Kirschen et al., 2022).

Many local governments and transit agencies have struggled to balance affordability with service quality when implementing full FFT, as these goals are often in tension. Eliminating fares means forgoing a crucial source of revenue, which can threaten service quality unless alternative funding streams are secured (Harmony, 2018). Indeed, financial sustainability remains one of the primary reasons FFT programs are suspended or never implemented (Doxsey and Spear, 1981; Dai et al., 2024). Therefore, many agencies have turned to partial FFT or reduced-fare programs. These initiatives, which constrain fare-free access by time, location, or rider demographics, represent a form of differential pricing intended to target specific policy goals (Keblowski, 2020; Cervero, 1990; Brown, 2018; King and Taylor, 2023). If the aim is to improve access for disadvantaged groups—such as low-income households, older adults, or people with disabilities—then eliminating or reducing fares for these riders is arguably the most direct and cost-effective approach (Harmony, 2018). These groups tend to rely more heavily on public transit and spend a higher share of their income on transportation (Garrett and Taylor, 1999; Darling et al., 2021; Brough et al., 2022).

In the U.S., demographic-based reduced-fare programs are anchored in federal policy. The Federal Transit Law requires federally subsidized transit providers to offer at least a 50% fare discount during off-peak hours for seniors, people with disabilities, and Medicare recipients[4]. Income-based programs, by contrast, are far less widespread: only 17 of the 50 largest U.S. transit agencies offer reduced fares based on income, and just one provides fully fare-free service to very low-income riders (Darling et al., 2021). Eligibility for such programs typically requires proof of income below a set threshold (e.g., a percentage of the federal poverty level) or enrollment in established assistance programs such as SNAP. Despite their limited adoption, income-based programs have demonstrated effectiveness and fiscal viability. For instance, Brough et al. (2022) conducted a controlled experiment in King County, WA, in which low-income travelers in the treatment group received free transit passes. These riders averaged one additional boarding per day relative to the control group, and usage levels dropped back to baseline after the benefit expired. Similarly, Harmony (2018) reviewed three case studies and found that the revenue forgone through reduced-fare programs constituted only a small share of agency operating budgets. A particularly relevant example is the Regional Transportation Authority (RTA) in the Chicago metropolitan area, where fare discounts are funded through a combination of state reimbursements and local sources, including the Motor Fuel Tax (RTA, 2024).

[3] https://ridekc.org/fares/passes, accessed on 5/6/2025.

[4] Per Federal Transit Law, section 5307(c)(1)(D), see https://www.transit.dot.gov/sites/fta.dot.gov/files/2022-01/Chapter-53-as-amended-by-IIJA-redline_0.pdf, accessed on 9/18/2025.

2.3. Gaps in the literature

In summary, while demographic-based differential pricing has been widely implemented by governments around the world, it remains underexplored in the transit design literature. Moreover, there is a lack of modeling frameworks that capture the operational and behavioral impacts of full FFT, including its influence on service provision and mode choice. Our study addresses these gaps by extending the transit design model of Dai et al. (2024) to explicitly incorporate a general step-fare structure and the effects of full FFT, and by using it to evaluate how differing means and social values shape optimal fare policies.

3. Model

3.1. Basic setting

Consider a square city of grid streets with uniform spacing s_r. A bus service network is overlaid on the grid with fixed line spacing S and stop spacing s, as shown in Figure 1. The operator of the bus service chooses a homogeneous headway h, and incurs an operating cost determined by a vector of service variables $\boldsymbol{x}_o \equiv \{h, s, S\}$. In this study, only h is treated as a design variable to simplify analysis[5]. In addition to configuring operations, the operator also need to choose a financial scheme to fund the bus system, defined by a set of parameters $\boldsymbol{x}_f$. We shall discuss finance in detail later.

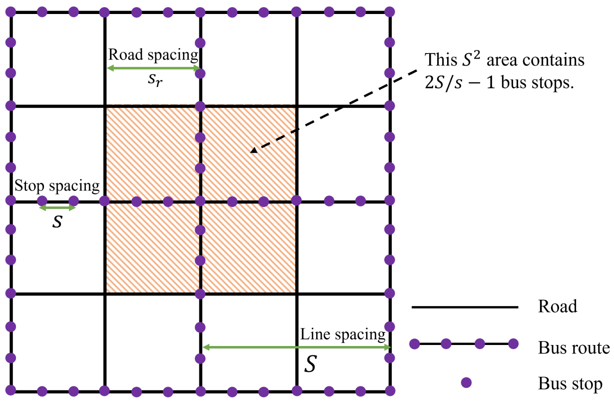


Figure 1: Grid transit network (adopted from Dai et al., 2024).

3.1.1. Travelers and Utility

Travelers, distinguished by their income level, are uniformly distributed in space with a density ρ people per km^2. We model their average daily expenditure (ADE, income less tax and savings) as a random variable $\tilde{e}$ with a probability density function (pdf) $f(\cdot)$ defined on a support $\Xi = [\underline{e}, \overline{e}]$, where $\underline{e}$ and $\overline{e}$ are the smallest and largest ADEs, and $\int_{\underline{e}}^{\overline{e}} f(e)\,\mathrm{d}e = 1$. Each traveler makes on average n trips each day and for each trip they choose a mode $m \in \mathbb{M} = \{b, d\}$, where taking bus is labeled as b and driving as d. Travelers always choose the mode that gives them the highest possible utility, which depends on the benefits derived from (i) the transportation service they receive, or accessibility A_m, and (ii) consuming other goods and services, denoted as E_m. Accordingly, the utility function for a traveler with ADE e choosing mode m is defined as

$$U_m(e) = u(A_m, E_m, e), m \in \mathbb{M}. \tag{1}$$

Following literature (e.g. Parry and Small (2009)), $u(\cdot)$ is assumed to be increasing and quasi-concave in A_m and E_m. We further note that A_m only depends on transit service $\boldsymbol{x}_o$, but E_m depends on both the financial scheme $\boldsymbol{x}_f$ and the traveler's ADE e:

$$A_m = A_m(\boldsymbol{x}_o), m \in \mathbb{M}; \tag{2a}$$
$$E_m = E_m(\boldsymbol{x}_f, e), m \in \mathbb{M}. \tag{2b}$$

[5]Including s and S in the analysis is straightforward but expected to yield no material impact on the main findings, see Section 6.4.2 of Dai et al. (2024) for a discussion.

3.1.2. Accessibility

In a spatially homogeneous setting as considered herein, mobility can be used as a proxy for accessibility. Mobility for mode m, denoted as l_m, is defined as the furthest distance one can reach using the mode within a given time budget t_a. Both private automobiles and buses share the city streets and are subjected to a network level congestion effect modeled by a macroscopic fundamental diagram (MFD) (Geroliminis and Daganzo, 2008), which calculates average speed as a function of the average traffic density in the network. Dai et al. (2024) showed that the average traffic density (k) can be defined as a function of the share of driver among all travelers, γ, and the average speed for mode m, v_m, during the peak hour is related to k via a speed-density function $g_m(\cdot)$:

$$k(\gamma) = \frac{s_r p \rho n t_d}{2w}\gamma, \tag{3}$$

$$v_m = g_m(k), \ \forall m \in \mathbb{M}, \tag{4}$$

where p is the fraction of hourly peak hour traffic in total daily traffic, t_d is the average trip duration, and $g_m(\cdot)$, the speed-density functions of mode m derived from the MFD, is non-increasing in k. The mobility of bus and driving can be derived as (see Dai et al., 2024, for detail):

$$l_d = t_a g_d(k(\gamma)); \tag{5a}$$

$$l_b = \frac{t_a - h - (s+S)/(2v_w)}{1/g_b(k(\gamma)) + t_s/s} + \frac{s+S}{2}, \tag{5b}$$

where v_w is the speed of walking (between a passenger's home/destination and a bus stop) and t_s the time lost per stop. Specifically,

$$t_s = t_{s0} + N_b t_{s1}, \tag{6}$$

$$N_b = \frac{\rho(1-\gamma)hS^2}{2S/s - 1}, \tag{7}$$

where t_{s0} is the time lost to acceleration and deceleration at each stop, t_{s1} is the time lost per passenger boarding, and N_b is the number of passengers per stop per bus. Hence, t_s is also endogenous, affected by both the bus demand $\rho(1-\gamma)$ and the bus supply (as set by the operational variables $\boldsymbol{x}_o$).

The mobility by driving, l_d, is non-increasing in γ due to the congestion effect: the more the bus riders (smaller γ), the less the traffic congestion, hence the greater the mobility by driving. For bus, the relation between l_b and γ is ambiguous. A smaller γ can both enhance l_b by lowering congestion and weaken it by increasing bus demand (hence boarding time). The net effect of γ on the mobility by bus depends on which effect is dominant.

With mode-specific mobility, we define accessibility as

$$A_b = A(l_b(\boldsymbol{x}_o, \gamma)); \tag{8a}$$

$$A_d = A(l_d(\gamma)), \tag{8b}$$

and note that, under the assumption of spatial homogeneity, accessibility always increases with mobility, i.e. $\partial A_m / \partial l_m > 0, \forall m \in \mathbb{M}$.

3.1.3. Expenditure

Each traveler faces an expenditure constraint

$$E_m = e(1 - t - e_0) - nc_m, \tag{9}$$

where t is the tax rate, e_0 is the fraction of the expenditure for necessities (therefore not contributing to utility), and c_m represents the per trip monetary cost of using mode m. Without loss of generality, we assume e_0 be non-increasing in e, indicating that a higher-income traveler would spend a lower share of their income on necessities. The mode-specific transportation cost may vary with income under a means-based step-fare structure, which takes the following form:

$$c_d = c_0 + \tau, \tag{10}$$

$$c_b = \begin{cases} r_1, & \text{if } e \in [\underline{e}, e_r); \\ r_2, & \text{if } e \in [e_r, \overline{e}], \end{cases} , \tag{11}$$

where drivers face a fixed per trip cost c_0 and a driver fee τ (a "tax" levied on drivers), while bus riders pay a discount fare r_1 per trip if their ADE is less than e_r, the discount fare qualification threshold (or discount threshold in short) and a full fare r_2 otherwise.

In summary, the operator can set five financial variables: the tax rate t, the two-tiered fare (r_1, r_2, e_r), and the driver fee τ, collectively represented by $\boldsymbol{x}_f \equiv \{t, r_1, r_2, e_r, \tau\}$. Since r_1 is a discount fare, we have $r_1 \leq r_2$.

3.1.4. Mode choice

We represent mode choice using an indicator function $\mathbb{I}(x \geq y)$, which returns 1 when $x \geq y$ and 0 otherwise. For a given design $(\boldsymbol{x}_o, \boldsymbol{x}_f)$, the mode choice of a traveler with e is given by $\mathbb{I}(U_d(e) \geq U_b(e))$.

Accordingly, the share of drivers is:

$$\gamma = \int_{\underline{e}}^{\overline{e}} f(e)\mathbb{I}(U_d(e) \geq U_b(e))\,\mathrm{d}e. \tag{12}$$

Since the utility of both modes depends on γ through the congestion effect and the boarding delay, the above equation can be viewed as a fixed-point system, whose solution corresponds to a mode choice equilibrium. Formally,

Definition 3.1. (Mode choice equilibrium) Given transit operation design $\boldsymbol{x}_o$, finance scheme $\boldsymbol{x}_f$, and an ADE distribution f, the mode choice equilibrium is achieved when the share of drivers γ satisfies Equation (12).

3.1.5. Bus budget and operating cost

The hourly operating cost, OC, can be estimated based on operating variables and mode share:

$$OC = \frac{4\pi_Q}{hS} + \frac{4\pi_M}{hv_{be}S}, \tag{13a}$$

$$v_{be} = (1/g_b(k(\gamma)) + t_s/s)^{-1}, \tag{13b}$$

where v_{be} is also called the effective operating speed. In Equation (13a), the first term represents the fuel cost, where π_Q is the operating cost per vehicle revenue distance, and the second term captures the labor cost, where π_M is the operating cost per vehicle revenue hour. Note that $4/hS$ is the distance traveled by all buses per unit area per hour, and $4/hv_{be}S$ is the total working hours of all bus drivers per unit area.

The operating budget comes from four revenue streams: fare revenue R, driver fee revenue C, tax revenue T and an exogenous government funding B_0. Due to the step-fare structure, the total fare revenue consists of two parts, namely the discount fare revenue R_1 and the full fare revenue R_2, with $R = R_1 + R_2$. Denoting the total daily operating budget by B, we have

$$B = B_0 + T + R_1 + R_2 + C, \tag{14a}$$

$$T = \rho t \int_{\underline{e}}^{\overline{e}} f(e) e \, \mathrm{d}e, \tag{14b}$$

$$C = \gamma \tau n = \rho \tau n \int_{\underline{e}}^{\overline{e}} f(e) \mathbb{I}(U_d(e) \geq U_b(e)) \, \mathrm{d}e, \tag{14c}$$

$$R_1 = \rho r_1 n \int_{\underline{e}}^{\overline{e}} f(e) \left(1 - \mathbb{I}(U_d(e) \geq U_b(e))\right) \mathbb{I}(e < e_r) \, \mathrm{d}e, \tag{14d}$$

$$R_2 = \rho r_2 n \int_{\underline{e}}^{\overline{e}} f(e) \left(1 - \mathbb{I}(U_d(e) \geq U_b(e))\right) \mathbb{I}(e \geq e_r) \, \mathrm{d}e. \tag{14e}$$

Since the bus service is designed to accommodate peak-hour demand, we assume the budget is allocated proportionally to the level of demand during this period. Recalling that p is employed to denote the ratio of the peak-hour travel demand to the total daily demand, the operator must ensure

$$OC \leq pB. \tag{15}$$

3.2. Design model

3.2.1. Step-fare transit design (SFTD)

The step-fare transit design (SFTD) problem aims to set the operating variables $\boldsymbol{x}_o$ and the financial variables $\boldsymbol{x}_f$ such that a system objective is maximized while satisfying the mode choice equilibrium and the budget constraint.

Since a chief concern of our study is the fairness of transit fare structure, we consider two objectives informed by opposing ideologies: (i) a utilitarian objective defined by the total utility of all travelers (or the utility of an average traveler); and (ii) an egalitarian objective defined by the utility of the least advantaged traveler (i.e. the traveler with the lowest utility among all). The latter is derived from the difference principle in the theory of justice (Rawls, 1971). The total and the lowest utility can be defined, respectively, as

$$U_{\text{total}} = \rho \int_{\underline{e}}^{\overline{e}} f(e) U(e) \, \mathrm{d}e, \quad \text{and} \quad U_{\min} = \min_{e} U(e), \tag{16}$$

where $U(e) = \max_m U_m(e)$—the utility of the chosen mode for ADE level e.

We then define the feasible set of the design variables as

$$\mathbb{X} \equiv \{(\boldsymbol{x}_o, \boldsymbol{x}_f) | h \in [\underline{h}, \overline{h}], s = s_{sq}, S = S_{sq}, t \in [0, \overline{t}], r_1 \in [0, r_2], r_2 \in [0, \overline{r_2}], e_r \in [\underline{e}, \overline{e}], \tau \in [0, \overline{\tau}]\},$$

where $\underline{x}$ and $\overline{x}$ are the variable bounds for the decision variable x. Note that s and S are fixed at the status quo value (noted by subscript sq) which will be defined later. The financial variables all have a lower bound of 0, and the discount fare r_1 has an upper bound r_2. We formulate the SFTD problem as follows:

$$\max_{(\boldsymbol{x}_o, \boldsymbol{x}_f) \in \mathbb{X}} \; U_{\text{total}} \text{ or } U_{\min} \tag{17a}$$

$$\text{subject to: } (12) \text{ and } (15). \tag{17b}$$

3.2.2. Analysis

In this section, we attempt to characterize mode choice equilibrium and its welfare effects, especially regarding the identification of the most disadvantaged traveler. In addition to useful insights, these properties also greatly simplify the design formulations. To this end, we need to introduce the following assumption on mode choice preferences.

Assumption 3.2. At any level of ADE, extra spending power always adds more utility to driving than to transit, i.e. $\frac{\partial U_d}{\partial e} > \frac{\partial U_b}{\partial e}, \forall e \in \Xi$.

In other words, a rise in income always leads to a higher marginal gain for driving than for transit. It follows that a traveler with a sufficiently high income would always prefer driving to transit.

Our first result concerns the mode choice equilibrium for given operational and financial variables. It is an extension from a similar result given in Dai et al. (2024), which assumes a universal fare structure.

Proposition 3.3. *For any* $(\boldsymbol{x}_o, \boldsymbol{x}_f) \in \mathbb{X}$*, there always exists a threshold ADE, denoted as* $\hat{e}$*, such that travelers with* $e > \hat{e}$ *drive and those with* $e < \hat{e}$ *ride bus. The threshold ADE* $\hat{e}$ *and the transportation cost* c_m *paid by different travelers could be determined following one of the four scenarios below:*

- *S1:* $U_d(\underline{e}) \geq U_b(\underline{e})$ *(everyone chooses driving):* $\hat{e} = \underline{e}$*, all travelers pay driver fee* τ*;*
- *S2:* $U_d(\overline{e}) \leq U_b(\overline{e})$ *(everyone chooses bus):* $\hat{e} = \overline{e}$*, travelers with* $e < e_r$ *pay discount fare* r_1*, and travelers with* $e \geq e_r$ *pay full fare* r_2*;*
- *S3: both modes are used and* $r_1 = r_2$ *(uniform fare):* $\hat{e}$ *is the unique solution to* $U_d(e) = U_b(e)$*, travelers with* $e < \hat{e}$ *pay uniform fare* $r_1 = r_2$*, and travelers with* $e \geq \hat{e}$ *pay* τ*;*
- *S4: both modes are used and* $r_1 < r_2$ *(step fare):*
  - *i: no feasible* e *exists such that* $U_d(e) = U_b(e)$*:* $\hat{e} = e_r$*, travelers with* $e < e_r$ *pay* r_1*, and travelers with* $e \geq e_r$ *pay* τ*;*
  - *ii:* $\hat{e}$ *is the unique solution to* $U_d(e) = U_b(e)$ *and* $\hat{e} < e_r$*: travelers with* $e < \hat{e}$ *pay* r_1*, and travelers with* $e \geq \hat{e}$ *pay* τ*;*
  - *iii:* $\hat{e}$ *is the unique solution to* $U_d(e) = U_b(e)$ *and* $\hat{e} > e_r$*: travelers with* $e < e_r$ *pay* r_1*, travelers with* $e_r \leq e < \hat{e}$ *pay* r_2*, and travelers with* $e \geq \hat{e}$ *pay* τ*.*

Proof. See Appendix A.1. □

Note that many of the scenarios above are corner cases. Scenario S1 and S2 are not realistic since only one mode is used. Scenario S3, S4-i and S4-ii are cases where all transit users are charged the same transit fare r_1, so the transportation cost outcome is equivalent to the uniform-fare transit design problem detailed in Dai et al. (2024). Hereafter, we ignore the nuance of the corner cases by assuming scenario S4-iii, where $\underline{e} < e_r < \hat{e} < \overline{e}$, i.e. travelers with $e \in [\underline{e}, e_r)$ ride bus and pay discount fare r_1, travelers with $e \in [e_r, \hat{e})$ ride bus and pay full fare r_2 and travelers with $e \in [\hat{e}, \overline{e}]$ drive and pay driver fee τ. This is formally stated as Assumption 3.4 below.

Assumption 3.4. The transit service and finance designs $(\boldsymbol{x}_o, \boldsymbol{x}_f)$ are such that $\underline{e} < e_r < \hat{e} < \overline{e}$, where $\hat{e}$ is the unique solution to $U_d(e) = U_b(e)$.

Because the traveler with $\hat{e}$ is indifferent to the two modes, we shall refer to $\hat{e}$ as the indifferent ADE hereafter. Note that the share of drivers is related to $\hat{e}$ through

$$\gamma = \int_{\hat{e}}^{\overline{e}} f(e)\,\mathrm{d}e. \tag{18}$$

This implies that we can also replace γ with $\hat{e}$, which has the benefit of being a variable directly affecting utility. Moreover, the mode choice equilibrium condition dictates that

$$U_b(\hat{e}) = U_d(\hat{e}). \tag{19}$$

Proposition 3.3 suggests that we can rewrite the utilitarian objective as

$$U_{\text{total}} = \rho\left(\int_{\underline{e}}^{\hat{e}} U_b(e) f(e)\,\mathrm{d}e + \int_{\hat{e}}^{\overline{e}} U_d(e) f(e)\,\mathrm{d}e\right). \tag{20}$$

Accordingly, the utilitarian SFTD (SFTD-U) model can be specified as (excluding the possibility that only one mode gets used for a given design):

$$\max_{(\boldsymbol{x}_o,\boldsymbol{x}_f)\in\mathbb{X}} \quad U_{\text{total}} = \rho\left(\int_{\underline{e}}^{\hat{e}} U_b(e)f(e)\,\mathrm{d}e + \int_{\hat{e}}^{\bar{e}} U_d(e)f(e)\,\mathrm{d}e\right) \tag{21a}$$

$$\text{subject to:} \quad U_b(\hat{e}) = U_d(\hat{e}), \tag{21b}$$

$$\frac{4\pi_Q}{hS} + \frac{4\pi_M}{hv_{be}S} \leq (T + R_1 + R_2 + C + B_0)p, \tag{21c}$$

where Constraint (21b) imposes mode choice equilibrium and Constraint (21c) ensures the operating cost does not exceed the budget allocated to the peak hour. Moreover, the four endogenous components in the budget are computed by

$$T = \rho t \int_{\underline{e}}^{\bar{e}} f(e)e\,\mathrm{d}e, R_1 = \rho r_1 n \int_{\underline{e}}^{e_r} f(e)\,\mathrm{d}e, R_2 = \rho r_1 n \int_{e_r}^{\hat{e}} f(e)\,\mathrm{d}e, \text{ and } C = \rho\tau n \int_{\hat{e}}^{\bar{e}} f(e)\,\mathrm{d}e. \tag{22}$$

The egalitarian objective involves identifying the most disadvantaged traveler and maximizing their utility. This can be framed as a robust optimization problem that maximizes a worst-case outcome (the utility of the least advantaged traveler in our case). Such a problem is often reformulated as an semi-infinite dimensional problem that turns the inner minimization problem into a set of constraints. Using this idea, we write the egalitarian SFTD (SFTD-E) problem as follows:

$$\max_{(\boldsymbol{x}_o,\boldsymbol{x}_f,z)\in\mathbb{X}\times\mathbb{R}} \quad z \tag{23a}$$

$$\text{subject to:} \quad z \leq U(e), \forall e \in \Xi, \tag{23b}$$

$$\text{(21b) and (21c).} \tag{23c}$$

At first glance, Constraint (23b) seems hopelessly intractable. However, it is usually possible to replace it with a finite number of constraints (known as "cuts"). These cuts may be generated iteratively or ex ante. Our next result clarifies that, in fact, only two cuts would suffice to ensure optimality for the SFTD-E problem.

Proposition 3.5. *For any joint design $(\boldsymbol{x}_o, \boldsymbol{x}_f)$ that satisfy Assumption 3.4, the most disadvantaged traveler has an ADE that is either the lowest ADE $\underline{e}$ or the discount threshold e_r. Moreover, they always ride bus. Thus,* $U_{\min} = \min\{U_b(\underline{e}), U_b(e_r)\}$.

Proof. See Appendix A.2. □

Thus, Problem (24) can be equally formulated as

$$\max_{(\boldsymbol{x}_o,\boldsymbol{x}_f,z)\in\mathbb{X}\times\mathbb{R}} \quad z \tag{24a}$$

$$\text{subject to:} \quad z \leq U_b(\underline{e}), \tag{24b}$$

$$z \leq U_b(e_r), \tag{24c}$$

$$\text{(21b) and (21c),} \tag{24d}$$

4. Impact of full fare-free implementation

When both the discount and full bus fares become zero in Problem (17), i.e. $r_1 = r_2 = 0$, several special effects may arise that warrant careful consideration. Below, we describe how these FFF effects are accounted for in our design framework.

The first is the so-called zero-price (ZP) effect, which refers to a sudden increase in price elasticity when a good becomes completely free. In the literature, ZP effect is often attributed to consumers overestimating the utility of free goods (Shampanier et al., 2007). To account for this phenomenon, we model the effect by increasing bus riders' sensitivity to accessibility A_b when fares are set to zero. Specifically, the perceived utility function under the ZP effect is defined as follows:

$$U'_m(e) = \begin{cases} u(\beta_{ZP} A_m, E_m, e), \text{ if } r_1 = r_2 = 0, m = b; \\ u(A_m, E_m, e), \text{otherwise.} \end{cases}, \tag{25}$$

where $\beta_{ZP} > 1$ is a constant inflator. It is worth emphasizing that, while travelers make mode choice decisions based on their perceived utility U'_m, the bus operator still evaluates the system objective using travelers' actual utility U_m.

Second, boarding may become more efficient when fare collection is eliminated. In bus operations, this allows both the front and rear doors to be used for boarding, thereby reducing the average boarding time per passenger (t_{s1}). We incorporate this effect by scaling t_{s1} with

$$t'_{s1} = \begin{cases} \beta_{BE} t_{s1}, \text{ if } r_1 = r_2 = 0, m = b; \\ t_{s1}, \text{otherwise.} \end{cases} \tag{26}$$

where $\beta_{BE} \in (0, 1)$ is a constant deflator.

Lastly, switching to a fare free operation may help reduce the operating cost because fare enforcement equipment and staff are no longer needed. This could be modeled by scaling down both the distance-based and the time-based operation cost parameters, i.e.,

$$\pi'_Y = \begin{cases} \beta_{CS} \pi_Y, \text{ if } r_1 = r_2 = 0, m = b; \\ \pi_Y, \text{otherwise.} \end{cases}, Y \in \{Q, M\}, \tag{27}$$

where $\beta_{CS} \in (0, 1)$ is a constant deflator.

The three FFF effects, characterized by the three FFF factors, introduce discontinuities into the design model, which significantly complicates its analysis. The good news is that Proposition 3.3 and 3.5, as well as Assumption 3.4, still hold after only minor modifications. Notably, the mode choice equilibrium condition now becomes $U'_b(\hat{e}) = U'_d(\hat{e})$, signifying the fact that users perceive transit utility differently at the zero price point. Due to the discontinuities, it is difficult to find optimal FFF designs as a solution to the SFTD problem. To overcome the challenge, a separate optimization problem is needed, referred to as the full fare-free transit design (FFTD) problem. We first define a basic FFTD problem called the FFTD-NA problem, where none of the FFF features are considered, as the SFTD problem (17) with an additional constraint $r_1 = r_2 = 0$. Then we introduce four variants of of the FFTD problem based on the FFTD-NA problem. We define the FFTD-ZP problem by replacing the mode choice equilibrium constraint by $U'_b(\hat{e}) = U'_d(\hat{e})$, the FFTD-BE problem by replacing t_{s1} with t'_{s1} and the FFTD-CS problem by replacing π_Q with π'_Q and π_M with π'_M. Finally, when all the FFF features are considered all together, we have the FFTD-ALL problem:

$$\max_{(\boldsymbol{x}_o, \boldsymbol{x}_f) \in \mathbb{X}} \quad U_{\text{total}} \text{ or } U_{\min} \tag{28a}$$

$$\text{subject to:} \quad U'_b(\hat{e}) = U'_d(\hat{e}), \tag{28b}$$

$$\frac{4\pi'_Q}{hS} + \frac{4\pi'_M}{hv'_{be}S} \le (T + C + B_0)p, \tag{28c}$$

$$r_1 = r_2 = 0, \tag{28d}$$

where $v'_{be} = \left(1/g_b(k(\gamma)) + \left(t_{s0} + N_b t'_{s1}/s\right)\right)^{-1}$ is the effective operating speed under improved boarding speed per passenger, t'_{s1}.

5. Case study

In this section, we present the results of numerical experiments based on a case study in Chicago. In Section 5.1, we briefly discuss the solution methods for the SFTD and FFTD problems. This is followed by the specification of the utility functions and parameters in Section 5.2. We then compare the SFTD-U and SFTD-E designs in Section 5.3 and the impact of the FFF features in Section 5.4. Lastly, we discuss the moral implications of different fare structures in Section 5.5.

5.1. Solution methods

To determine the optimal design with full FFF features, we have to compare the solution to the SFTD problem and that to the FFTD-ALL problem, and choose the one that provides a better objective function value. To rule out the corner solutions and ensure Assumption 3.4 holds, we impose two additional constraints on indifferent ADE: $\underline{e} \le \hat{e} \le \overline{e}$ and $e_r \le \hat{e}$. For the SFTD problems, infeasibility is reported when no feasible $\hat{e}$ is found to satisfy $U_d(\hat{e}) = U_b(\hat{e})$. Note that it is possible that a uniform-fare design emerge as the optimal solution. Similarly, for the FFTD problem, infeasibility arises when no feasible $\hat{e}$ is found to satisfy $U_d'(\hat{e}) = U_b'(\hat{e})$.

Both SFTD and FFTD problems are treated as optimization problems with respect to $\boldsymbol{x}_o$, $\boldsymbol{x}_f$ and $\hat{e}$. These problems are nonlinear and non-convex, due largely to the mode choice equilibrium condition. While finding global solutions to non-convex problems is generally difficult, our problems are simple enough that we can afford solving each instance with over-the-shelf local search algorithms many times (each with a different starting point). Our experiments indicate that this approach effectively mitigates the risk of being stuck at poor local optimums. All models are coded in MATLAB R2022b and run on a personal computer with Intel(R) Core(TM) i7-10510U CPU @ 1.80GHz and 16 GB RAM. The optimization solver used in the numerical experiment is MATLAB's *fmincon* function, available through the Optimization Toolbox$^{\text{TM}}$.

5.2. Case study setup

The utility functions employed in the case study take the Cobb-Douglas form:

$$U_m(e) = A_m^{\alpha}\left(\frac{E_m}{e}\right)^{1-\alpha} = \left(\frac{l_m}{l_0}\right)^{\alpha}\left(\frac{E_m}{e}\right)^{1-\alpha} = \left(\frac{l_m}{l_0}\right)^{\alpha}\left(1 - t - e_0 - \frac{nc_m}{e}\right)^{1-\alpha}, \tag{29}$$

where l_0 is the mobility of driving under the free-flow travel condition, and $\alpha \in [0,1]$ is the weight of contribution to utility by accessibility. To facilitate the comparison of utilities across modes and ADE levels, we normalize the two components by l_0 and e, respectively. We leave it to the reader to verify that the utility function (29) satisfies Assumption 3.2 as well as the following conditions: For $m \in \mathbb{M}$,

1. (U_m increasing and quasi-concave in A_m) $\frac{\partial U_m}{\partial A_m} > 0$, $\frac{\partial^2 U_m}{\partial A_m^2} \le 0$;
2. (U_m increasing and quasi-concave in E_m) $\frac{\partial U_m}{\partial E_m} > 0$, $\frac{\partial^2 U_m}{\partial E_m^2} \le 0$;
3. (A_m increasing in l_m)$\frac{\partial A_m}{\partial l_m} > 0$.

The parameter values were estimated based on the pre-COVID transit demand data in the city of Chicago and the operation data of the Chicago Transit Authority (CTA). The main data source includes 2019 American community surveys (ACS), Chicago Metropolitan Agency for planning (CMAP) surveys, Consumer expenditure reports as well as National Transit Database (NTD) Operation Statistics. The default values of key parameters in our model are adopted from Dai et al. (2024), summarized in Table 1. The justification for the choice of the default values is provided in Appendix B. Here, we focus on the FFF factors introduced in Section 4, because they are not included in Dai et al. (2024).

Recall that each of the three FFF features addresses a different aspect of transit operation. First, the parameter β_{BE} adjusts downward the boarding time per passenger when fare collection is eliminated. Since CTA has never implemented FFT before, we elect to estimate β_{BE} using empirical evidence collected in other cities. The City of Boston

Table 1: Default parameter values for the design models.

Variable	Description	Value
	Supply	
$\boldsymbol{q}$	key flow coordinates on the MFD	{200, 320, 330, 280} veh/h
$\boldsymbol{k}$	key density coordinates on the MFD	{5, 10, 17, 22} veh/mile-lane
v_w	walking speed	4 km/h
s_r	road spacing	0.4 km
t_{s0}	time lost per stop due to deceleration and acceleration	12 s
t_{s1}	time lost added per boarding passenger	1 s
w	number of lanes	4 lanes
π_Q	maintenance and fuel cost per vehicle revenue km	\$1.5/veh-km
π_M	labor cost per vehicle revenue hour	\$117/veh-hour
	Demand	
t_a	travel time budget	30 min
t_d	average trip duration	21 min
ρ	traveler density	2400 travelers/km^2
$\boldsymbol{e}$	boundaries of ADE distribution	\${31.4, 44.4, 58.1, 78.0, 133.2}
c_0	driving cost per trip	\$5.4
	Decision (status quo)	
h_{sq}	status quo headway	10 min
s_{sq}	status quo stop spacing	0.223 km
S_{sq}	status quo line spacing	0.4 km
t_{sq}	status quo local option sales tax	1.0%
r_{sq}	status quo fare per trip	\$1.25
τ_{sq}	status quo driver fee per trip	\$0
	Calibrated parameters	
B_0	daily government subsidy per square kilometer	\$1320
α	weight factor in utility function	0.35
n	number of trips per person per day	2.65 trips
p	fraction of daily trips in one peak hour	0.085
	FFF factors	
β_{ZP}	Zero-price effect factor on perceived utility	1.12
β_{BE}	Boarding time reduction factor	0.77
β_{CS}	Operation cost reduction factor	0.96

and the Massachusetts Bay Transportation Authority (MBTA) have implemented FFT on three high-ridership routes recently. It was reported that, in the initial pilot period, the boarding time per passenger has decreased by 23%[6]. Given CTA and MBTA are both large metropolitan transit operators (with similar service areas, populations and fares, see NTD, 2021), it is reasonable to expect a similar reduction in boarding time in Chicago should FFT be implemented. Accordingly, we estimate $\beta_{BE} = 0.77$ in our model.

Second, β_{CS} measures the operating cost savings associated with fare collection staff labor and equipment. For bus operation, the magnitude of these savings vary significantly across agencies (ranging from 0.5% to 22%), depending on, among other factors, how much was spent on fare collection efforts before the introduction of FFT (Volinski, 2012). In our study β_{CS} is set to 0.96, because large bus agencies on average save about 4% on fare collection when they implement FFT according to Volinski (2012).

Finally, β_{ZP} gauges the extra benefit of free transit as perceived by travelers. As this effect has a direct impact on mode choice, it can be estimated from the change in the ridership before and after the fare-free implementation. Roughly speaking, we search for a β_{ZP} that induces a relative ridership increase in our model similar to what was observed in reality. In Boston, the fare-free routes see an average ridership increase of about 38%[7]. Following the trends observed

[6] https://www.boston.gov/sites/default/files/file/2022/03/Route28_Report_FINAL.pdf, accessed on 4/6/2025.
[7] ibid.

in Boston, we estimate the increase in ridership that can be attributed to the fare-free implementation is 38%. Using our model and setting β_{BE} and β_{CS} at the values set above, we find when $\beta_{ZP} = 1.12$, a 38% ridership jump is achieved when fare is reduced from the status quo to zero.

5.3. Step-fare transit designs

We test step-fare transit design using three experiments collectively labeled as Experiment 1. In Experiment 1-i, the operator is allowed to collect as much taxes and driver fees as required by the optimization of a given objective. In 1-ii, the tax rate is capped at 1% and the driver fee is capped at $1. The third set mimics the financial situation at the status quo by capping the tax rate at 1% while eliminating the driver fee. For each set, we solve both SFTD-U (utilitarian objective) and SFTD-E (egalitarian objective) problems.

5.3.1. Benefits of step-fare designs

To gauge the benefits of step-fare designs, we solve a uniform-fare version of the design model (where an additional constraint $r_1 = r_2$ is enforced when solving the SFTD) as a benchmark. This means for each experiment set we create four designs, labeled as E_0 (uniform-fare egalitarian), E (step-fare egalitarian), U_0 (uniform-fare utilitarian), and U (step-fare utilitarian).

Table 2 reports all twelve solutions, as well as the conditions at the status quo (SQ). Each solution specifies the headway h, the fares (r_1, r_2), the tax rate (t), the driver fee (τ), the total utility (U_{total}), the utility of the least advantaged traveler $(U_{\min})$, the transit share (for the step-fare designs, the two percentages reported correspond to the shares of travelers charged discount fare r_1 and full fare r_2 respectively) and the Gini coefficient. Note that the Gini coefficient is a measure of distributive impact[8].

Table 2: Uniform-fare vs. Step-fare designs.

Scenario	h	t	r_1	r_2	τ	$U_{\min}$	U_{total}	Transit%		Gini
SQ	0.167	1.0%	1.25		0.00	0.2884	950.1	29%		0.1219
1-i-E_0	0.063	4.3%	0.00		3.49	0.3783	977.9	80%		0.0427
1-i-E	0.061	3.3%	0.00	0.89	4.26	0.3847	978.8	61%	20%	0.0363
1-i-U_0	0.074	3.7%	0.00		1.59	0.3779	1008.1	67%		0.0639
1-i-U	0.070	0.0%	0.00	2.35	3.17	0.3540	1012.5	47%	20%	0.0672
1-ii-E_0	0.125	1.0%	0.25		1.00	0.3470	986.7	51%		0.0913
1-ii-E	0.111	1.0%	0.29	0.71	1.00	0.3540	990.7	30%	21%	0.0881
1-ii-U_0	0.099	1.0%	0.70		1.00	0.3433	991.3	52%		0.0914
1-ii-U	0.099	1.0%	0.00	1.03	1.00	0.3434	992.7	17%	34%	0.0899
1-iii-E_0	0.174	1.0%	1.17		0.00	0.2886	948.9	28%		0.1220
1-iii-E	0.170	1.0%	1.08	1.34	0.00	0.2939	946.9	10%	18%	0.1215
1-iii-U_0	0.155	1.0%	1.42		0.00	0.2867	951.1	29%		0.1222
1-iii-U	0.155	1.0%	1.42	1.42	0.00	0.2867	951.1	29%		0.1222

We first observe that step-fare designs outperform their uniform-fare counterparts in all experiments. This is expected since the uniform-fare version has an additional constraint $r_1 = r_2$, which could not improve the solution. However, the improvements due to the added flexibility in the fare policy are modest even without any financial constraints (Set 1-i), where the objective function is improved by 1.7% for the egalitarian design and 0.4% for the utilitarian design. As the financial constraints tighten up, the improvement diminishes. In Set 1-iii, where the driver fee is forbidden, using a step-fare design makes no difference at all for the utilitarian objective.

When financial constraints are non-existent (Set 1-i), a partial-fare-free (PFF) design (i.e., $r_1 = 0, r_2 > 0$) is both just and utility-maximizing. For an egalitarian, the bus system should be made free for low-income riders and funded by

[8] Gini coefficients falls between 0 (perfect equality) and 1 (inequality). Smaller Gini coefficients indicate more equal distribution.

high taxes (t), high driver fees (τ), and a modest full fare (r_2). Since the bottom 60% of the travelers qualify for free ride, a greater burden of financing the bus system falls onto drivers and higher-income bus riders. This significant redistribution of resources produces a more egalitarian outcome than the status quo and the uniform-fare egalitarian design. For a utilitarian, the bus system is funded by a high full fare (nearly doubling the current fare) and high driver fees, while completely avoiding taxation and keeping the fare free for low-income riders. In this case, nearly half of all travelers are qualified for fare-free transit, implying that the utility gained from waiving the fare for them outweighs the utility lost to higher fare and driver fees paid by the others.

When the financial constraint tightens (Set 1-ii), uniform-fare designs no longer admit an fare-free solution regardless of the objective. Clearly, the limited tax and driver fee revenue cannot sustain a reasonable service level in the absence of fare revenue. With a step-fare structure, the result is somewhat unexpected. Whereas the egalitarian design offers low-income travelers a discount fare slightly less than half of the full fare, the utilitarian design gives them a free ride, electing instead to have the wealthier bus riders bear all the burdens. In other words, a PFF design is not just (in the Rawlsian sense) but utility-maximizing. Of course, the tightened financial conditions mean that the share of travelers who can enjoy free rides plummeted from 47% in 1-i-U to 17% in 1-ii-U. Moreover, for the egalitarian design, the step-fare solution actually collects significantly higher farebox revenues than the uniform-fare solution—in fact, even the discount fare (\$0.29) is higher than the \$0.25 optimal uniform fare. A higher budget leads to a better level of service, which more than compensates the utility lost to a mild fare increase for the most disadvantaged traveler.

Under the financial austerity of the status quo (Set 1-iii), an egalitarian design still favors a step-fare solution, which sets the discount fare at about 80% of the full fare. On the other hand, fare discount no longer moves the needle in a utilitarian design. Indeed, the four solutions in this set, despite the minor variations in fare and headway, all lead to transit shares and Gini indexes quite similar to those achieved at the status quo.

5.3.2. Distributive effects

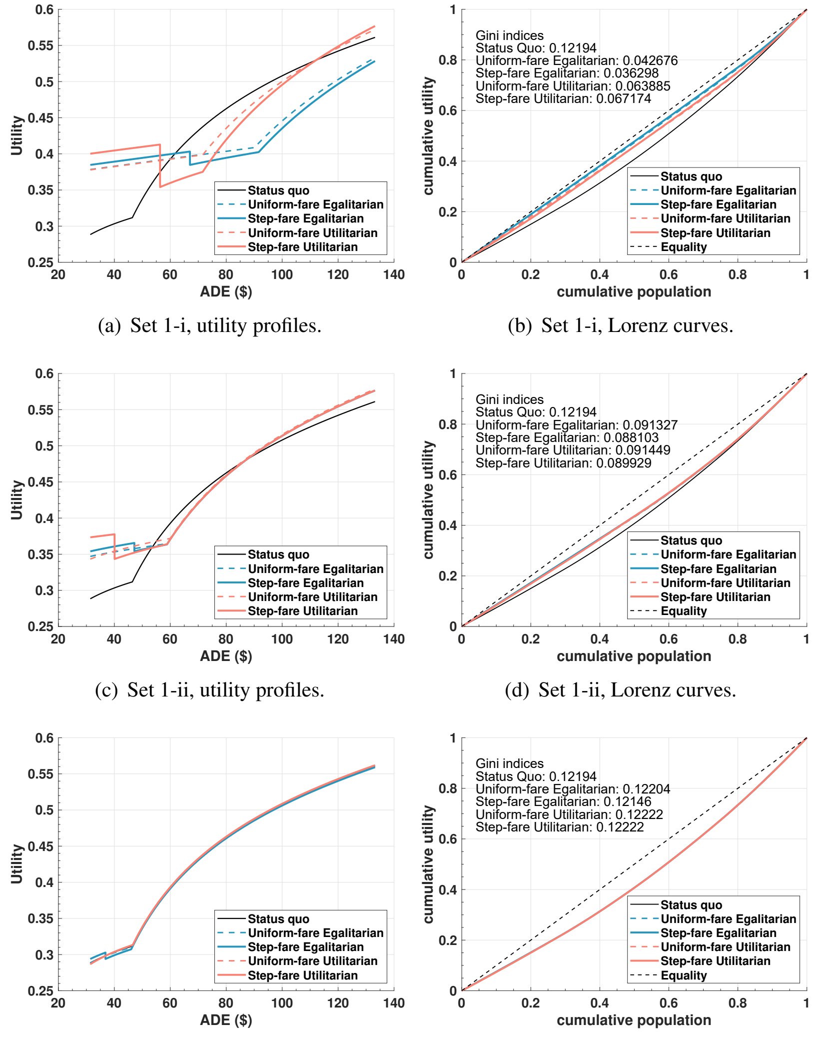
Figure 2 visualizes the distributive effects of optimal step-fare and uniform-fare designs in the three sets of experiments. Shown on the left column were the utility profiles—the utility as a function of ADE—achieved under different designs. The right column reports corresponding Lorenz curves and Gini indices. To plot the Lorenz curve for a given utility profile, we rank the utilities for all ADE levels, and obtain the share of travelers at each utility level. The curve represents the ranked cumulative utilities against the cumulative share of travelers. Following the relation between Gini coefficients and Lorenz curves, we calculate the Gini coefficient of any distribution as twice the area between the 45 degree "perfect equality" line and the corresponding Lorenz curve.

Let us begin with the results of Set 1-i (the first row from Figure 2), where the operator is given the greatest financial freedom. Figure 2(a) indicates that under the step-fare egalitarian designs (blue solid curves), the utility of the traveler with the lowest ADE equals the utility of the traveler at the discount threshold e_r^*. This means both constraints (24b) and (24c) are active at optimality. Clearly, the discontinuity manifested in the utility profiles of step-fare designs results from the fare structure. Comparing to the egalitarian uniform-fare designs (blue dashed lines), the step-fare designs improves the utility of the low income bus riders, while sacrificing the utility of the others. As a result, the Lorenz curve for the egalitarian step-fare design (see the blue solid curve in Figure 2(b)) is the closest to the perfect equality line (with the lowest Gini index of 0.036).

From Figures 2(a) and 2(c) we can see that the utilitarian designs (red solid lines) leverage the step-fare structure to offer the low income bus riders a free transit service. While the policy lifts the utility of low income travelers significantly above the level at the status quo, it comes at a steep cost for those who are stuck with transit but now have to pay more to access the service (observe that the utility drops sharply when the fare jumps at e_r^*). Another group that benefits from the utilitarian design is the drivers on the opposite end of the income spectrum: they too are better off than at the status quo. Taken together, the step-fare utilitarian design is more efficient than the uniform-fare counterpart but less equal in the distribution of benefits when the financial constraints are loose (as reflected by the red Lorenz curves and Gini indices in Figures 2(b) and 2(d)).

With tighter financial constraints, the trends observed above fade away. In Figures 2(e) and 2(f), the differences between the curves associated with different designs are barely noticeable. The implication is clear: without the flexibility to raise revenues from other sources, optimizing the fare structure and operation has limited impact on the distribution of benefits, regardless of the moral principle adopted.

Finally, a comparison of Figures 2(a) and 2(c) reveals a striking and somewhat paradoxical finding: when more financial resources are available, the optimally designed policies end up making more travelers worse off relative to the status quo. This outcome holds regardless of the moral principle adopted, though it is more pronounced under egalitarianism: any traveler with an ADE above \$60 experiences a lower utility under the optimal egalitarian policy than under the status quo. A closer look shows that without financial constraints (Set 1), the egalitarian and utilitarian policies make 45.46% and 43.69% of travelers worse off, respectively. These figures drop to 37.52% and 37.69% under mild financial restrictions (Set 2). In other words, spending more does not necessarily improve conditions for more people. The reason is that greater financial flexibility gives the planner more power to redistribute, but also reduces the incentive to account for those adversely affected by the policy. If we assume each individual votes against policies that leave them worse off, then policies developed under mild financial constraints may face less opposition in a democratic process than those designed without any budgetary limits.



(a) Set 1-i, utility profiles. (b) Set 1-i, Lorenz curves.

(c) Set 1-ii, utility profiles. (d) Set 1-ii, Lorenz curves.

(e) Set 1-iii, utility profiles. (f) Set 1-iii, Lorenz curves.

Figure 2: Distributive effects of the step-fare and uniform-fare designs across three experimental settings. A perfectly equal distribution corresponds to a Lorenz curve that coincides with the 45-degree line. Greater inequality is indicated by a more pronounced "bowing" or convexity of the Lorenz curve away from this line.

5.4. *Full fare free design*

The results from the previous section suggest that a full-fare-free (FFF) policy is rarely justified, regardless of the moral principle applied. However, up to this point, we have not accounted for any potential benefits of the FFF policy, including the zero-price (ZP) effect, the boarding efficiency (BE) effect, and the cost reduction (CR) effect. Could incorporating one or more of these features alter our conclusions? To explore this question, we revisit the setting without financial constraints (scenario i in Experiment 1) and solve four variants of the FFTD problem: (1) ZP—incorporating only the zero-price effect; (2) BE—incorporating only the boarding efficiency effect; (3) CR—incorporating only the cost reduction effect; and (4) ALL—incorporating all three effects. These experiments are labeled as Experiment 2.

5.4.1. Benefits of full-fare-free designs

Table 3 compares the four FFF designs (ZP, BE, CR and ALL) against two benchmarks: an optimal step fare design from Experiment 1-i and a basic FFF design (NA). Note that none of the three FFF features are considered in the benchmark designs. It includes similar metrics as Table 2 except (i) U'_{min} specifies the minimum perceived utility due to the ZP effect; and (ii) The transit fares are omitted because they are fixed at 0 in all FFF designs.

Table 3: Impact of FFF features.

Scenario	h	t	τ	U_{min}	U'_{min}	U_{total}	Transit%	Gini
2-i-E-NA	0.063	4.3%	3.49	0.3783	0.3783	977.9	80%	0.0427
2-i-E-ZP	0.063	4.7%	3.03	0.3757	0.3909	975.7	81%	0.0460
2-i-E-BE	0.065	3.7%	3.58	0.3856	0.3856	993.8	81%	0.0405
2-i-E-CR	0.061	4.1%	3.49	0.3803	0.3803	982.4	81%	0.0423
2-i-E-ALL	0.064	4.0%	3.11	0.3848	0.4004	995.6	82%	0.0434
1-i-E	0.061	3.3%	4.26	0.3847	0.3847	978.8	81%	0.0363
2-i-U-NA	0.074	3.7%	1.59	0.3779	0.3779	1008.1	67%	0.0639
2-i-U-ZP	0.074	4.3%	1.10	0.3743	0.3895	1008.3	67%	0.0692
2-i-U-BE	0.077	3.3%	1.53	0.3843	0.3843	1022.8	67%	0.0624
2-i-U-CR	0.072	3.6%	1.57	0.3797	0.3797	1012.4	67%	0.0635
2-i-U-ALL	0.076	3.8%	1.02	0.3823	0.3978	1026.8	67%	0.0673
1-i-U	0.070	0.0%	3.17	0.3540	0.3540	1012.5	67%	0.0672

The impacts of the BE and CR effects are relatively straightforward, as both unequivocally enhance operational efficiency. The BE effect alone leads to improvements in both U_{min} and U_{total} by 1.9% and 1.5%, respectively, under the egalitarian and utilitarian designs. In contrast, the CR effect has a more modest impact, yielding a 0.5% improvement in the egalitarian design and a 0.4% improvement in the utilitarian design. The ZP effect is more nuanced because it only increases the appeal of transit by manipulating its perceived utility. Whereas the perceived utility for the most disadvantaged bus rider is increased by 3.3% under the egalitarian design, their actual utility fell by 0.7%. This discrepancy seems to highlight the peril of misperception, as the designer is compelled to offer a sub-optimal service to cope with an irrational response to a free good. However, this risk does not seem to generalize to the utilitarian design, for which the ZP effect is in fact a positive influence on the total actual utility, albeit the gain (about 0.02%) is negligible. In other words, travelers' irrationality may sometimes benefit, rather than hinder, system-wide objectives. When all three effects are incorporated, the resulting designs outperform the benchmark by 1.9% and 1.7% under the egalitarian and utilitarian objectives, respectively.

Moreover, the BE and CR effects are both equity-enhancing, since they help close the mobility gap between bus riders and drivers. This is confirmed by the decrease in the Gini indexes for both egalitarian and utilitarian designs. On the other hand, by inducing more travelers to use transit, the ZP effect actually worsens the gap in the actual utility between bus riders and drivers. This has an adversarial impact on equity, as evidenced in a slight increase in the Gini index—measured based on the distribution of actual utility. When considering all FFF effects together, the net impact on equity is generally indeterminate. However, in our numerical study, the adverse influence of the ZP effect outweighed the equity gains from BE and CR: the Gini indices increased by 1.6% and 5.3% under the egalitarian and utilitarian objectives, respectively.

Comparing the FFTD-ALL outcome and the optimal SFTD outcome (2-i-ALL vs. 1-i), we observe that, for both the egalitarian and utilitarian objectives, FFF designs outperform the optimal SFTD ones (which prescribe PFF policies). Under the utilitarian objective, the total actual utility increases by 1.4%. Additionally, the FFF policy improves the utility of the most disadvantaged traveler by a drastic 8.0%, since under the setting 1-i-U, the PFF was discriminatory to those who have to pay for the full fare. For the egalitarian designs, although the objective functions are comparable under the FFF policy and the PFF policy (U_{min} of FFF policy is only 0.03% higher than that of PFF policy), the total utilities and distributive effects are quite different. While the PFF policy is more equitable (as evidenced by a Gini index that is 16% lower), the FFF policy yield a total utility that is 1.7% higher.

5.4.2. Distributive effects

Figure 3 compares the utility profiles of the egalitarian and utilitarian FFF designs with and without the three FFF features. We can see that these features consistently increase utility under the utilitarian design: the red solid line (with the features) remains above the red dashed line (without the features) across all traveler types. A similar pattern holds under the egalitarian design, except for a subset of travelers who experience nearly identical utility under both settings (where the blue solid line and the blue dashed line intersect). A closer inspection reveals that transit users with ADE values near the indifference point (just left of the discontinuity on the blue solid line) experience a slight decrease in utility when FFF features are added. The discontinuity in the solid lines can be attributed to the ZP effect, which causes travelers to overestimate the utility of transit due to its perceived "free" nature. As a result, some travelers who would otherwise drive choose transit instead. Notably, this discontinuity disappears in the perceived utility profiles (dotted lines), as travelers make choices based on perceived rather than actual utilities.

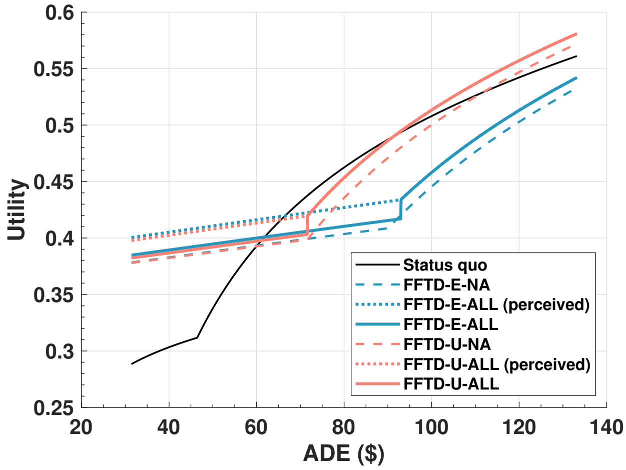


Figure 3: Utility profiles of FFTD solutions, with and without the FFF features.

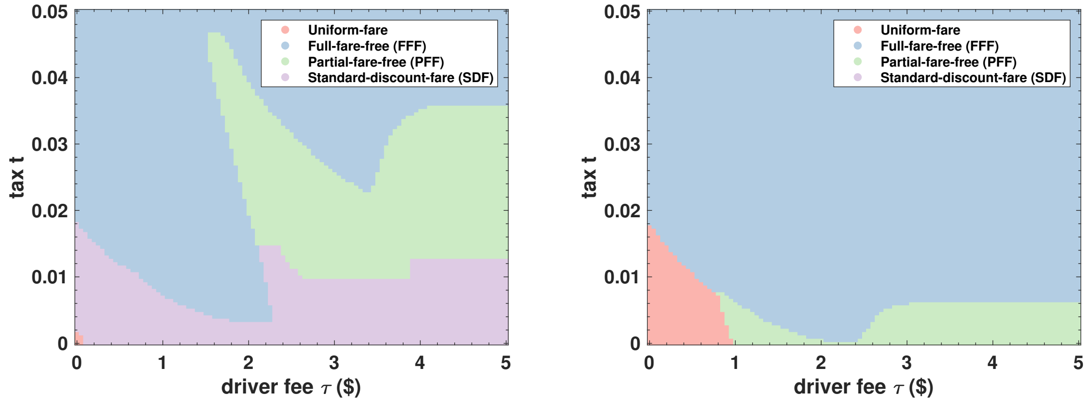
Figure 3 also shows that, while the FFF features disproportionately benefit drivers under the utilitarian design, this imbalance is mitigated in the egalitarian design. In the former, driver fees drop by 36% (which exclusively benefits drivers), accompanied by a 3% increase in taxes—in the end, although transit users benefit from a better service, this modest gain is canceled out by a higher tax burden. In contrast, under the egalitarian design, incorporating the FFF features results in a 7% reduction in the tax rate and an 11% reduction in driver fees. Thus, transit users not only enjoy better service quality but also face a lower tax burden.

5.5. Moral implications of fare structures

In this section we attempt to address the following question: are there correlations between the moral principle adopted in the design and the optimal fare structure, and if so, to what extent do the financial constraints affect them? To this end, we create a large number of experiment sets by varying the financial caps $(\bar{t}, \bar{\tau}) \in [0, 0.05] \times [0, 5]$, i.e., the tax rate cap varies between 0% and 5% and the driver fee cap varies between \$0 and \$5. For each experiment set, we solve both the SFTD problem and the FFTD-ALL problem with the utilitarian and egalitarian objectives, and report the optimal solutions.

5.5.1. Optimal fare structures

This section reports the optimal fare structure under various financial caps $(\bar{t}, \bar{\tau})$. For each experiment set, if the FFTD-ALL outcome is better than the optimal SFTD outcome, the optimal fare structure is full-fare-free (FFF); otherwise, the optimal fare structure could be classified as one of the following:(i) uniform-fare: $r_1^* = r_2^* \neq 0$ or $r_2^* > 0, e_r^* = \underline{e}$ or $r_1^* > 0, e_r^* = \hat{e}$, (ii) partial-fare-free (PFF): $0 = r_1^* < r_2^*, e_r^* \in (\underline{e}, \hat{e})$, or (iii) standard-discount-fare (SDF): $0 < r_1^* < r_2^*, e_r^* \in (\underline{e}, \hat{e})$. Figure 4 delineates the optimal fare-structure regions under utilitarianism and egalitarianism.



(a) Egalitarian design.

(b) Utilitarian design.

Figure 4: Optimal fare structures under different financial restrictions and moral principles.

We begin with the egalitarian design (Figure 4(a)). The findings from Sections 5.3 and 5.4 suggest that maximizing the utility of the most disadvantaged traveler may be achieved through (i) revenue redistribution from higher income riders and drivers (corresponding to either SDF or PFF policy), or (ii) eliminating fare altogether (corresponding to FFF policy). However, all three policies require the authority to tax or charge travelers. Consequently, under extremely tight financial constraints (the left bottom corner, where both tax rate and driver fee caps are close to zero), the just fare policy is in fact a uniform fare for everyone.

As the financial restrictions loosen up, we first notice that SDF dominates the other two policies as long as the tax cap is very low ($t < 0.3\%$). In other words, even if the operator is free to charge drivers at will, the poorest travelers are still better off paying fare to preserve service quality. This is because the revenue generated from driver fees does not necessarily increase with the charge rate. The finding underscores a key limitation of driver fees as a funding mechanism for achieving equitable outcomes.

The boundary separating the FFF policy from the PFF and SDF policies is complicated. In the region with low driver fee caps (the left periphery), a more relaxed financial condition favors FFF over SDF, and the line dividing them slopes downward, indicating that tax and driver fee revenues act as substitutes. However, as long as the driver fee cap is below a certain threshold (roughly \$1.6), PFF is never preferred. Apparently, with limited contributions from drivers, medium-income travelers who must pay fare under PFF are potentially worse off than the poorer traveler who do not have to. Under loose financial conditions (the top right quadrant), FFF and PFF policies deliver similar optimal utility for the most disadvantaged traveler(as we have compared 1-i-E and 2-i-E-ALL in Table 3). As a result, FFF dominates in some sub-areas whereas PFF dominates in others, both marginally.

For the utilitarian, Figure 4(b) reveals a somewhat surprising result: the FFF policy is preferred under all but the most financially constrained conditions. Specifically, it becomes the utility-maximizing policy as long as the tax cap exceeds approximately 1.8%—a level sufficient to support decent transit service without fare collection. Under tighter financial constraints (the red region in the lower-left corner), the uniform-fare policy becomes optimal. However, judging by the size of this red area, the utilitarian is far more likely to favor the uniform-fare policy than the egalitarian. Zooming in on the lower part of the plot, particularly around $t = 0.5\%$, we observe that as the driver fee cap increases from \$0 to \$5, the utility-maximizing policy shifts sequentially from uniform-fare to PFF, then to FFF, and eventually back to PFF. The boundary between FFF and PFF initially slopes downward until the driver fee cap reaches \$2.4, after which it briefly slopes upward before leveling off. This behavior reflects the natural limitations of raising revenue through driver fees—namely, that drivers may shift modes in response to higher charges. Once the driver fee cap exceeds \$2.4, additional utility gains are more effectively achieved by charging fares (as permitted under PFF but not under FFF). After the cap surpasses \$3.17, the total utility cannot further improved without changing the tax cap—hence the flattening of the boundary on the far right.

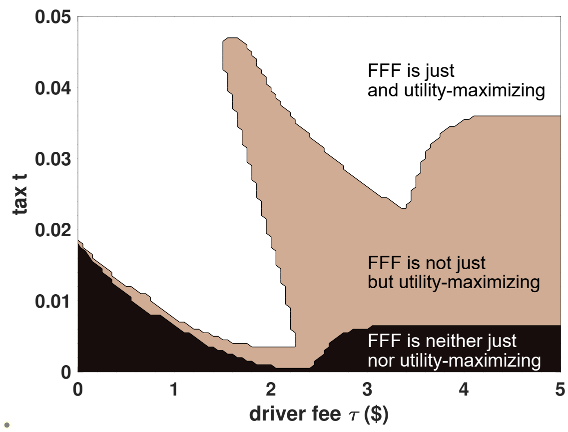
Another interesting result is that, when both tax and driver fee caps are low, the PFF region abruptly transition into a uniform-fare region, without passing through an SDF regime. This absence can be explained as follows: facing tighter budget constraints, the utilitarian prefers to lower the share of travelers receiving free rides by decreasing the discount threshold e_r, rather than increasing the discount fare. Although e_r never reaches its lower bound $\underline{e}$, the transition between the PFF and the uniform-fare policies occurs at a sufficiently low value for e_r. Below that level, providing free rides for the small group of travelers at the expense of high fare for the rest of bus riders no longer outperform charging all bus riders a moderate and uniform fare.

In summary, for the egalitarian, the SDF policy is most suitable when the tax cap is low, while the FFF policy is preferred when the tax cap is high and the driver fee cap is low. In other cases, the FFF and PFF policies perform comparably and are both superior to the SDF policy. On the other hand, the utilitarian prefers a uniform-fare policy when financial constraints are tight. Yet, under a low tax cap and a higher driver fee cap, the PFF is the best option for them. Finally, with a greater leeway for taxation, FFF is always the dominant utilitarian policy. Table 4 offers an overview of the relationships.

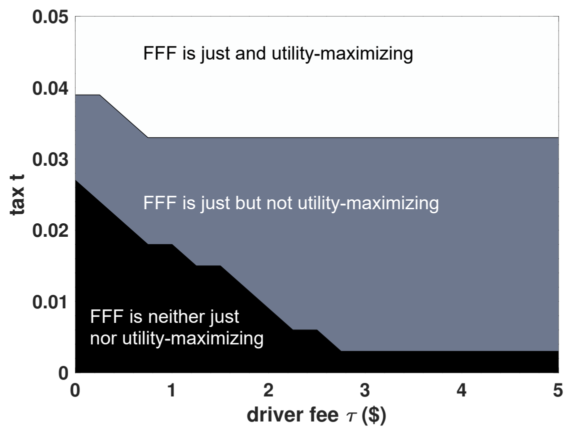
Table 4: Relationship between moral principles, fare structures and financial conditions.

Policy	Egalitarian	Utilitarian
Uniform-fare	very low tax and driver fee caps	low tax and driver fee caps
FFF	high tax cap, or medium tax cap and low driver fee cap	high tax cap, or medium tax cap and high driver fee cap
PFF	medium tax cap and high driver fee cap	low tax cap and high driver fee cap
SDF	low tax cap	NA

5.5.2. *Justification of the full-fare-free policy*



(a) Step-fare structure, with FFF features.



(b) Uniform-fare structure, without FFF features (Dai et al., 2024).

Figure 5: Moral justification of the FFF policy.

Figure 5 summarizes the conditions under which an FFF policy can be justified by the egalitarian, the utilitarian, or both. The left plot is derived by juxtaposing the FFF frontiers from Figure 4, which assume a step-fare structure with all FFF features, whereas the right plot is taken from Dai et al. (2024), which assumes a uniform-fare structure and ignores all FFF features.

Figure 5(a) highlights three subareas in which: (i) FFF is neither just nor utility-maximizing (the black area); (ii) FFF is not just but utility-maximizing (the brown area); or (iii) FFF is both just and utility-maximizing (the white area).

Similarly, Figure 5(b) shows three distinctive subareas, including subareas (i) and (iii), but not (ii). Instead, it contains an area that is just but not utility-maximizing (the gray area).

A common pattern in both plots is that the justification for the FFF policy depends critically on the severity of financial constraints. The less restrictive the constraints, the easier it is to justify FFF—regardless of the moral principle applied. This observation has also been noted in (Dai et al., 2024). However, the differences between the two plots offer some new insights.

First, the black region—where the FFF policy is neither just nor utility-maximizing—shrinks significantly when fare structures are relaxed and FFF features are incorporated. This trend is expected, as both changes increase design flexibility and enhance the general appeal of the FFF policy.

Second, and more unexpectedly, it becomes more difficult for the egalitarian to justify the FFF policy once FFF features are included. This is evident from the smaller "just" region in Figure 5(a) compared to Figure 5(b). At first glance, this seems counterintuitive—after all, making the FFF policy more attractive would appear to benefit the most disadvantaged travelers.

This result can be explained in two ways. First, while the FFF policy eliminates fares for all transit users, SDF and PFF policies allow for more targeted support, which may better align with egalitarian goals. Second, the benefits of FFF features—such as system-wide cost savings—are broadly distributed and not specifically reserved for disadvantaged travelers, thereby diluting their equity-enhancing effect.

6. Conclusion and future directions

We have examined how optimal fare policies are shaped by moral values and financial means. Motivated by the growing interest in fare-free transit (FFT) and the lack of rigorous modeling frameworks to evaluate its impacts, we developed a generalized joint design model building on Dai et al. (2024). Our model introduces a means-based step-fare structure that encapsulates full-fare-free (FFF), partial-fare-free (PFF), and standard-discount-fare (SDF) policies. It also incorporates behavioral responses known to influence FFF ridership and accounts for operational and administrative cost savings associated with eliminating fare collection. These extensions allow us to systematically identify optimal fare policies—under a broad range of financial conditions—according to either a utilitarian objective (maximizing total utility) or an egalitarian one (promoting distributive justice).

The case study based on the City of Chicago reveals several important findings with broader policy implications.

First, optimal fare policies critically depend on financial means, regardless of the moral principle applied. With unlimited access to financial resources, FFF emerges as both utility-maximizing and just. At the other end of the spectrum, a uniform fare for all is preferred. Among the two alternative funding instruments we consider, dedicated taxation proves far more effective than a driver fee. This is expected: while drivers can avoid the fee by switching to transit, no one can evade a broad-based tax. In addition, while differentiated pricing schemes such as SDF and PFF are often favored by an egalitarian, a utilitarian rarely finds them attractive.

Second, a utilitarian is far more likely to endorse FFF than an egalitarian. Under loose financial constraints, FFF dominates all other policies from a utilitarian perspective but performs on par with PFF under egalitarianism. This result may seem counterintuitive—if not outright shocking—to FFT advocates who often associate fare-free policies with egalitarian ideals. Yet a moment's reflection reveals that the logic is sound. Promoting equity typically requires targeted redistribution based on individual attributes, which FFF by design cannot offer. Thus, those aiming to advance distributive justice should consider fare discounts for specific demographics rather than abolishing fares altogether.

Third—and also surprisingly—we find that increasing financial resources does not necessarily lead to more socially acceptable outcomes. In fact, unconstrained funding can empower planners to adopt policies that inadvertently leave a larger share of the population worse off relative to the status quo. This paradox arises because greater redistributional freedom may reduce the incentive to safeguard the welfare of marginally affected groups. As a result, policies developed under moderate financial constraints may attract broader democratic support, even if they fall short of being “optimal” under either egalitarian or utilitarian criteria. This finding underscores the limitations of design models driven by a single normative objective.

For the Chicago case specifically, we find that FFF can be both just and utility-maximizing if the operator has access to a \$1-per-trip driver fee in addition to the existing 1% dedicated sales tax. Without the driver fee, however, neither FFF nor PFF is recommended. Under an egalitarian objective, the optimal policy involves a discount fare set at 80% of the full fare, with eligibility restricted to the bottom 10% of travelers by income. In contrast, a utilitarian would opt for a uniform fare that is higher than the current baseline and applied to all travelers.

Looking ahead, transit agencies in the U.S. are likely to face significant fiscal challenges. Since the COVID-19 pandemic, ridership has yet to return to pre-pandemic levels; in many regions, it remains 20–25% below normal[9]. While federal relief funds[10] have so far prevented major service cuts, those resources are temporary and will soon be exhausted. As agencies seek alternative funding mechanisms to compensate for sustained ridership losses, the modeling framework presented in this paper could serve as a useful foundation. Substantial extensions, however, will be needed to reflect structural shifts such as the growing prevalence of telework as a substitute for commuting.

Several additional avenues of future research are also worth pursuing. First, extending the model to more realistic network settings—incorporating spatial and temporal heterogeneity, richer mode choices (e.g., ride-hailing, commuter rail), and variable service quality—would enhance its practical relevance. Second, future work should explore the interaction between fare policy and other transportation strategies such as congestion pricing and transit-oriented development. Although our model includes a stylized driver fee, it is not designed to manage congestion. A more integrated approach that combines fare policy with initiatives like dedicated bus lanes or coordinated pricing of road and transit use could offer a more comprehensive framework for advancing sustainable urban mobility.

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Appendix A. Proof of main results

Appendix A.1. Proof of Proposition 3.3

If a traveler choose driving, then their ADE must satisfy $\Delta U(e) = U_d(e) - U_b(e) \geq 0$. Based on Assumption 3.2, $\Delta U(e)$ strictly increases with e.

We first deal with two special cases:

1. Single mode choice: When $\Delta U(\underline{e}) \geq 0$, i.e., everyone chooses driving, we have $\hat{e} = \underline{e}$ and everyone pays driver fee τ, which corresponds to scenario S1. Similarly, When $\Delta U(\overline{e}) < 0$, i.e., nobody chooses driving, we have $\hat{e} = \overline{e}$. Travelers with $e < e_r$ pay discount fare r_1 and other travelers pay full fare r_2, which corresponds to scenario S2.
2. Single fare: When $\Delta U(\underline{e}) < 0 < \Delta U(\overline{e})$ and $r_1 = r_2$, i.e., both modes are chosen by some travelers and the reduced fare equals full fare and e_r becomes irrelevant, $U_b(e)$ is a continuous function. Since $U_d(e)$ is also continuous, $\Delta U(e)$ is continuous. The continuity and monotonicity of $\Delta U(e)$ imply that there must exist an e_1 such that $\underline{e} < e_1 < \overline{e}, \Delta U(e_1) = 0, \Delta U(e) < 0\ \forall e \in [\underline{e}, e_1)$ and $\Delta U(e) > 0\ \forall e \in (e_1, \overline{e}]$. So we have $\hat{e} = e_1$. This corresponds to scenario S3 where travelers with $e < \hat{e}$ pays uniform fare $r_1 = r_2$, and others pay driver fee τ.

Per Equation (10), the expenditure function and the utility function for bus riders are discontinuous at e_r when $r_1 < r_2$. As such, $\Delta U(e)$ has a discontinuity at e_r. To facilitate the proof, we introduce two auxiliary functions representing the utility difference between driving and bus for different e:

$$\Delta U_1(e) = U_d(e) - U_{b1}(e), e \in [\underline{e}, e_r]; \tag{A.1a}$$

$$\Delta U_2(e) = U_d(e) - U_{b2}(e), e \in [e_r, \overline{e}]. \tag{A.1b}$$

where U_{b1} and U_{b2} are the utility function for riding bus when the fare is r_1 and r_2 respectively, i.e. $U_b(e) = U_{b1}(e)$ if $e \in [\underline{e}, e_r)$ and $U_b(e) = U_{b2}(e)$ if $e \in [e_r, \overline{e}]$. Because $r_1 < r_2$, we have $\Delta U_1(e_r) < \Delta U_2(e_r)$. Since $\Delta U_1(e)$ and $\Delta U_2(e)$ also strictly increase with e, we have $\Delta U_1(\underline{e}) < \Delta U_1(e_r) < \Delta U_2(e_r) < \Delta U_2(\overline{e})$. Excluding the special case 1, we have $\Delta U_1(\underline{e}) < 0 < \Delta U_2(\overline{e})$. In addition, $\Delta U(e)$ is a piece-wise continuous function:

$$\Delta U(e) = \begin{cases} \Delta U_1(e) = U_d(e) - U_{b1}(e), & \text{if } e \in [\underline{e}, e_r); \\ \Delta U_2(e) = U_d(e) - U_{b2}(e), & \text{if } e \in [e_r, \overline{e}]. \end{cases} \tag{A.2}$$

The location of $\hat{e}$ depends on the location of value 0 within $(\Delta U_1(\underline{e}), \Delta U_2(\overline{e}))$:

1. if $\Delta U_1(\underline{e}) < 0 < \Delta U_1(e_r)$, there exists a unique $e_2 \in (\underline{e}, e_r)$ such that $\Delta U_1(e_2) = 0$, corresponding to S4-ii, i.e., $\hat{e} = e_2$, which is also the unique solution to $U_b(e) = U_d(e)$. Travelers with $e < \hat{e}$ pay discount fare r_1 and others pay driver fee τ;
2. if $\Delta U_1(e_r) \leq 0 < \Delta U_2(e_r)$, only those who are qualified for the reduced fare will ride bus, leading to scenario S4-i where no solution exists for $U_b(e) = U_d(e)$ due to discontinuity of $\Delta U(e)$ at e_r. In this case, we have $\hat{e} = e_r$, travelers with $e < \hat{e}$ pay discount fare r_1 and others pay driver fee τ;
3. if $\Delta U_2(e_r) \leq 0 < \Delta U_2(\overline{e})$, there exists a unique $e_3 \in [e_r, \overline{e})$ such that $\Delta U_2(e_3) = 0$, corresponding to scenario S4-iii where $\hat{e} = e_3$, which is also the unique solution to $U_b(e) = U_d(e)$. Travelers with $e < e_r$ pay discount fare r_1, travelers with $e_r \leq e < \hat{e}$ pay regular fare r_2 and others pay driver fee τ.

Appendix A.2. Proof of Proposition 3.5

We continue to use U_{b1} and U_{b2} as defined in Appendix A.1. For any joint design $\boldsymbol{x} = (\boldsymbol{x}_o, \boldsymbol{x}_f)$ that does not admit a corner solution, we have $\underline{e} < e_r < \hat{e} < \overline{e}$, $U(\underline{e}) = U_{b1}(\underline{e})$ and $U(e_r) = U_{b2}(e_r)$ according to Proposition 3.3. Hence, the travelers with the lowest ADE and the travelers with the threshold ADE e_r always ride bus.

Next, we show that although $U(e)$ is not entirely monotonically increasing on Ξ, U_{b1} and U_{b2} are monotonically increasing on their respective domains by inspecting the partial derivatives:

$$
\begin{aligned}
\frac{\partial U_{b1}}{\partial e} &= \frac{\partial U_{b1}}{\partial E_{b1}} \frac{\partial E_{b1}}{\partial e} \\
&= \frac{\partial U_{b1}}{\partial E_{b1}} \left(1 - t - e_0(e) - e\frac{\partial e_0}{\partial e}\right) > 0, \forall e \in [\underline{e}, e_r], \qquad \text{(A.3)}\\
\frac{\partial U_{b2}}{\partial e} &= \frac{\partial U_{b2}}{\partial E_{b2}} \frac{\partial E_{b2}}{\partial e} \\
&= \frac{\partial U_{b2}}{\partial E_{b2}} \left(1 - t - e_0(e) - e\frac{\partial e_0}{\partial e}\right) > 0, \forall e \in [e_r, \overline{e}], \qquad \text{(A.4)}
\end{aligned}
$$

where E_{b1} and E_{b2} are the expenditure functions for riding bus when the fare is r_1 and r_2 respectively. Both partial derivatives are positive on their domains because $\forall m = b_1, b_2$, $\frac{\partial U_m}{\partial E_m} > 0$, $1-t-e_0(e) > 1-t-e_0(e)-nc_m/e = E_m/e > 0$ and $-e\frac{\partial e_0}{\partial e} > 0$. Therefore, we have

$$U(\underline{e}) = U_{b1}(\underline{e}) \leq U_{b1}(e) = U(e), \forall e \in [\underline{e}, e_r), \qquad \text{(A.5)}$$

$$U(e_r) = U_{b2}(e_r) \leq U_{b2}(e) = U(e), \forall e \in [e_r, \hat{e}), \qquad \text{(A.6)}$$

$$U(e_r) = U_{b2}(e_r) \leq U_{b2}(e) \leq U_d(e) = U(e), \forall e \in [\hat{e}, \overline{e}]. \qquad \text{(A.7)}$$

It follows from the above that the utility for travelers with ADE $\underline{e}$ is the lowest among the utilities for travelers with $e \in [\underline{e}, e_r)$, and the utility for travelers with ADE e_r is the lowest among the utility for travelers with $e \in [e_r, \overline{e}]$, i.e. $\min_{e\in[\underline{e},e_r)} U(e) = U(\underline{e})$ and $\min_{e\in[e_r,\overline{e}]} U(e) = U(e_r)$. We leave it to the reader to verify that the relation between $U(e_r)$ and $U(\underline{e})$ is indeterminate. Thus, $U_{\min} = \min_{e\in\Xi} U(e) = \min\{U(\underline{e}), U(e_r)\} = \min\{U_b(\underline{e}), U_b(e_r)\}$.

Appendix B. Justification of the parameter values in Table 1

Appendix B.1. Demand

Chicago Transit Authority (CTA) covers a service area of 803 square kilometers (km^2) and serves a population of 3,240,768. Thus the demand density in the service area is estimated to be about 4,000 travelers per km^2. The most recent travel survey conducted by the Chicago Metropolitan Agency for Planning (CMAP) (Comeaux, 2021) suggests about 60% of all travelers take transit or drive, with a split of about 30%:70% between them. Thus, for our model, the population density is estimated as $0.6 \times 4000 = 2,400$ travelers/km^2, i.e., $\rho = 2400$ travelers/km^2.

Appendix B.2. Expenditure

A piece-wise uniform distribution is fitted using annual household expenditure quintile data. The average annual household expenditure for the five income quintiles are \$28,672, \$40,472, \$53,045, \$71,173 and \$121,571 respectively[11]. With an average household size of 2.5 (USBLS, 2020), we convert the annual averages to ADE: $\underline{e} = \$31.4, e_{q1} = \$44.4, e_{med} = \$58.1, e_{q3} = \$78.0, \overline{e} = \$133.2$. Let $\boldsymbol{e} \equiv \{\underline{e}, e_{q1}, e_{med}, e_{q3}, \overline{e}\}$ represent the set of ADE quintile averages. Then, these quintile averages are used as boundary values to create four population segments. Within each segment income is assumed to be uniformly distributed. As such, we have the following PDF for the ADE (also see Figure B.6):

$$
f(e) = \begin{cases}
\frac{1/4}{e_{q1}-\underline{e}}, & e \in [\underline{e}, e_{q1}),\\
\frac{1/4}{e_{med}-e_{q1}}, & e \in [e_{q1}, e_{med}),\\
\frac{1/4}{e_{q3}-e_{med}}, & e \in [e_{med}, e_{q3}),\\
\frac{1/4}{\overline{e}-e_{q3}}, & e \in [e_{q3}, \overline{e}],\\
0, & \text{otherwise.}
\end{cases} \qquad \text{(B.1)}
$$

[11] https://data.bts.gov/stories/s/Transportation-Economic-Trends-Transportation-Spen/ida7-k95k/, accessed on 2023/11/01.

The mandatory spending takes up about 50% of ADE for the lowest income quintile, and about 40% of ADE for the highest income quintile (Duly, 2003). For simplicity, the following linear function is used to represent how e_0 varies with e:

$$e_0 = 0.5 - 0.1\frac{e - \underline{e}}{\overline{e} - \underline{e}}. \tag{B.2}$$

Appendix B.3. Cost of driving

The cost of driving per trip is derived from the annual expenditure on driving. In 2019, an average household's expenditure on driving is \$9,972, and a household owns on average 1.9 cars (USBLS, 2020). Therefore, the cost of driving per trip is estimated as $\$c_d = 9972/365/1.9/n = \$14.4/n$, where n, the number of trips per day, is to be calibrated.

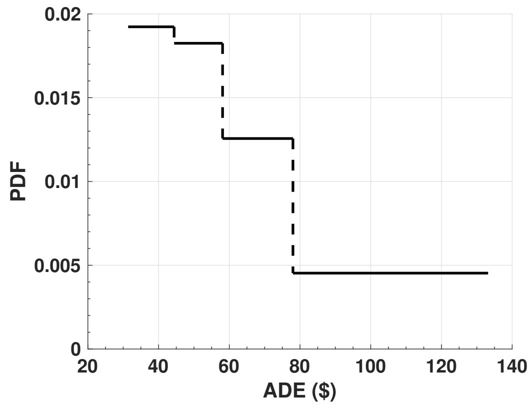


Figure B.6: PDF of the piece-wise uniform ADE (e) distribution.

Appendix B.4. Operation parameters

From literature, the speed of walking (v_w) is taken as 4 km/h, the budget travel time t_a is set to 30 min (Dai et al., 2023), the time lost per stop due to acceleration and deceleration is $t_{s0} = 12$s and the time lost per stop per boarding passenger is $t_{s1} = 1$s/pax (Chen and Nie, 2017).

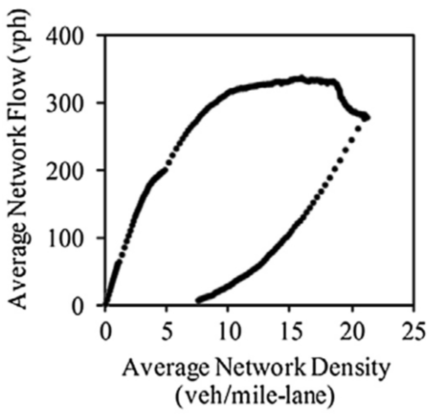
CTA's operating cost is represented both in a per hour measure and a per km measure: \$146.5 per revenue hour or \$7.5 per revenue km (NTD, 2020). To properly account for π_Q and π_M without double counting, we consider the split of the operating cost between labor (π_Q) and fuel (π_M). As labor takes up about 80% of transit operating cost (NTD, 2020), we have $\pi_Q = \$146.5 \times 0.8 = \$117/\text{h}$ and $\pi_M = \$7.5 \times 0.2 = \$1.5/\text{km}$.

The average trip duration in Chicago is $t_d = 21$ min (Comeaux, 2021). Each road in the road system has two lanes in each direction ($w = 4$) and the average block length is a quarter of a mile (or 400 meters) (ChicagoStudies, 2020), which gives $s_0 = 0.4$km.

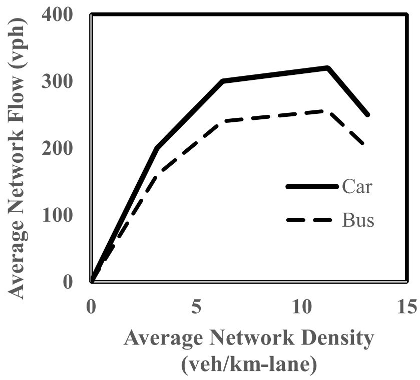
The speed-density function in Chicago is estimated based on existing network fundamental diagrams (NFD) developed by Mahmassani et al. (2013), see Figure 7(a). Since the CTA service area constitutes the core of the Chicago metropolitan area (CMA) region, we assume that the NFD is the same as the CMA NFD. We further approximate the NFD by a piece-wise linear function (B.3) for the ease of implementation. Buses are assumed to run at 80% of the average car speed at the same traffic density. See Figure 7(b) for the linearized NFD for cars and buses—note that the lower part of the original NFD (the post flow-breakdown regime) is left out to avoid unnecessary complications. In 2019, the travel time during peak hour is on average about 31% higher compared to free-flow travel time in Chicago, which means the peak hour travel speed is about 77% of free-flow speed FHWA (2019). We refer to this ratio as congestion index, and ensure that our model produces a similar congestion index under the status quo conditions when we calibrate the parameters. The speed-density functions for cars and buses are, respectively

$$g_d(k) = \begin{cases} \frac{q_1}{k_1}, 0 \le k \le k_1, \\ \frac{q_2-q_1}{k_2-k_1}(1 + \frac{k_1}{k}) + \frac{q_1}{k}, k_1 < k \le k_2, \\ \frac{q_3-q_2}{k_3-k_2}(1 + \frac{k_2}{k}) + \frac{q_2}{k}, k_2 < k \le k_3, \\ \frac{q_4-q_3}{k_4-k_3}(1 + \frac{k_3}{k}) + \frac{q_3}{k}, k_3 < k \le k_4, \\ 0, k > k_4, \end{cases} \quad \text{and } g_b(k) = 0.8 g_c(k). \tag{B.3}$$

The parameters used to specify the above NFD are represented by $\boldsymbol{q} \equiv \{q_1, q_2, q_3, q_4\}$ and $\boldsymbol{k} \equiv \{k_1, k_2, k_3, k_4\}$. Their values are reported in Table 1.



(a) NFD of CMA (Mahmassani et al., 2013).



(b) Linearized NFD of CTA service area.

Figure B.7: Actual and linearized network fundamental diagrams (NFD).

Appendix B.5. Status quo service and policy

The average headway of the current CTA transit system is $h_{sq} = 0.167$ hours (CMAP, 2017) and the average stop spacing is $s_{sq} = 0.223$ km (Pandey et al., 2021). While the single trip ticket prices for CTA "L" trains and buses are \$2.50 and \$2.25 respectively, Almagro et al. (2024) suggests that the actual per trip cost ranges from \$1.09 to \$1.33. For simplicity, we use $r_{sq} = \$1.25$ in our study.

Chicago does not charge drivers a fee for the purpose of funding transit so $\tau_{sq} = \$0$. The Regional Transportation Authority (RTA), the state-level transit authority in Illinois, imposes a 1.0% sales tax dedicated to fund transit projects[12]. Hence, $t_{sq} = 1\%$.

Appendix B.6. Calibration results

In total, four parameters, including the available subsidy (B_0), the exponent in the utility function (α), the number of trips per day (n), as well as the peak hour factor (p), are "calibrated" from the model itself. This means that we search for the values for these parameters such that they reproduce the status quo conditions, including the observed mode split, farebox recovery ratio, congestion index, total budget, and total daily transit ridership. We found that when the subsidy $B_0 = \$1320/\text{day}/\text{km}^2$, the exponent in the utility function $\alpha = 0.35$, the average number of daily trips $n = 2.65$ trips and the peak hour factor $p = 0.085$, the model produces aggregate outcomes that well match observations, as reported in Table B.5. To verify the price elasticity, a 1% increase in the transit fare (from \$1.25 to \$1.2625) reduces the transit market share by about 0.23% (from 28.72% to 28.65%). This gives a fare elasticity of about -0.23, which is within the range established in literature (see e.g., Baum, 1973; Litman, 2004).

Table B.5: Key statistics produced by the calibrated model vs. data.

	Data	Calibrated model
Transit mode share	30% (Comeaux, 2021)	29%
Daily ridership	1.47 million (NTD, 2020)	1.46 million
Farebox	43% (NTD, 2020)	44%
Peak hour budget	\$441/km^2 (NTD, 2020)	\$441/km^2
Congestion index	0.77 (FHWA, 2019)	0.74

[12] https://tax.illinois.gov/localgovernments/masstransit.html, accessed on 2025/05/01.